Analysis of Retrial Queueing System M/G/1 with Impatient Customers, Collisions and Unreliable Server Using Simulation

János Sztrik[1], Ádám Tóth[1](✉), Elena Yu. Danilyuk[2], and Svetlana P. Moiseeva[2]

[1] University of Debrecen, 26 Kassai Road, Debrecen, Hungary
{sztrik.janos,toth.adam}@inf.unideb.hu
[2] National Research Tomsk State University, Tomsk, Russian Federation

Abstract. In this paper, we consider a retrial queueing system of $M/G/1$ type with an unreliable server, collisions, and impatient customers. The novelty of our work is to carry out a sensitivity analysis applying different distributions of service time of customers on significant performance measures for example on the probability of abandonment, the mean waiting time of an arbitrary, successfully served, impatient customer, etc. A customer is able to depart from the system in the orbit if it does not get its appropriate service after a definite random waiting time so these will be the so-called impatient customers. In the case of server failure, requests are allowed to enter the system but these will be forwarded immediately towards the orbit. The service, retrial, impatience, operation, and repair times are supposed to be independent of each other. Several graphical illustrations demonstrate the comparisons of the investigated distributions and the interesting phenomena which are obtained by our self-developed simulation program. The achieved results are compared to the results of the [2] to check how the system characteristics changes if we use other distributions of service time and to present the advantages of performing simulations in certain scenarios.

Keywords: Retrial queue · Impatient customers · Collisions · Unreliable server · Simulation · Sensitivity analysis

1 Introduction

With the growing number of users, devices, and networks it is crucial developing and applying new methods and ideas for designing communication systems even

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in the case of the existing systems. Across industries, more and more companies expand their services using a higher number of devices and cloud networking resulting in big data transmission. Consequently, creating mathematical and simulation models of modern telecommunication systems are necessary because these investigations can lessen the hardship of modifying or creating systems. In real life, in many cases, customers encountering the service units in busy state may make a decision to attempt to be served after some random time remaining in the system. Instead of residing in a queue these customers are located in a virtual waiting room called orbit and can be modeled with retrial queues. Queuing systems with retrial queues are widely used tools modelling emerging problems in major telecommunication systems, such as telephone switching systems or call centres. Many papers dealt with these types of systems which can be viewed in the following works like in [4,9,19].

Models with customers impatience in queues like the process of reneging and balking have been studied by various authors in the past. Most recent results about systems having the impatience property can be found for example in [7,8,16].

In certain scenarios during the transmission of a message, another message may appear in the channel which makes both impossible to decode causing a conflict. This can happen due to the limited number of communication channels and sometimes the launched uncoordinated attempts leading to the loss of the transmission and consequently the necessity for retransmission. In such cases, these requests go into orbit and after a random waiting time other attempts will be initiated in order to reach the service facility again. Investigating and building up efficient procedures for preventing conflicts and corresponding message delays are needed. Of course, there are papers that have studied retrial queues with collisions see for example [11–15].

Seeking in the available literature it is assumed that the components of the system are accessible all the time. In practice, this is quite unrealistic and scientists can not ignore examining the reliability of retrial queueing systems because server breakdowns and repairs have a great influence on the system characteristics and the performance measures. In real-life systems typical problems arise like a power outage, human errors, or in wireless communication packets can suffer transmission failure, interruptions throughout their transfer and unfortunately it can happen at any time. These systems with an unreliable server were analyzed in several papers, for example in [3,6,10,18,20].

In the paper of [2] a retrial queueing system of $M/M/1$ type with Poisson flow of arrivals, impatient customers, collisions, and unreliable service device is presented. In that, an asymptotic analysis method is used to define the stationary distribution of the number of customers in the orbit. We investigate the same model as in [2], but the results are gathered by our simulation program package. With this approach, it is possible to calculate performance measures that can not be determined or almost impossible to give exact formulas using numerical or asymptotic analysis. Various software packages exist which are capable to describe and perform an evaluation of complex systems if all the random variables are exponentially distributed but undoubtedly the usage of simulation has a tremendous advantage: besides exponential, any other distribution can be integrated into the code. The novelty of our work is the inclusion of other distributions of service time in the previously developed models to carry out a sensitivity analysis to see whether the observed curiosities are valid for this model or how this modification alters the performance measures. To do so we use stochastic simulation because using this method it is feasible to calculate the desired measures while obtaining analytical results, which in this case, are a difficult task if at all possible. With the help of this program, we present graphical results revealing interesting phenomena.

2 System Model

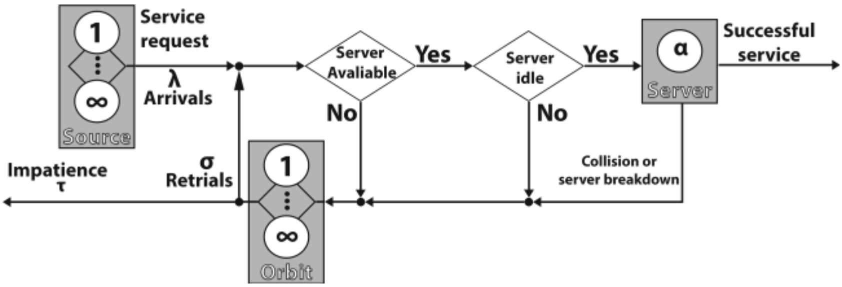

Fig. 1. The considered model.

We consider a queueing system of $M/G/1$ with collisions, impatience of the customers, and an unreliable server which is shown in Fig. 1. The system arrival process is characterized by the Poisson process with a rate of λ. The arriving customer occupies instantly the service unit in idle state and the distribution of its service is according to exponentially, gamma, Pareto, lognormal, hypo-exponentially, and hyper-exponentially distributed random variable with the same mean value and variance but with different parameters. Otherwise, it is forwarded toward the orbit. The retrial time of the requests is assumed to be exponentially distributed with a rate of σ. In the case of a busy server an arriving customer brings about a collision and both requests enter the orbit. It is supposed that the server is unreliable so it breaks down from time to time according to an exponential distribution with parameter γ_0 when the server is idle and with parameter γ_1 when it is busy. In that period generation of new requests continues but each of them is sent to orbit. After a breakdown, it is immediately sent for repair and the recovery process is also an exponential random variable with the rate γ_2. Every customer possesses an "impatience" property meaning that a customer may depart from the system earlier after waiting a random time in the orbit. The distribution of the impatient time follows an exponential distribution with parameter τ. In this unreliable model after interruption or breakdown, it is supposed that requests immediately are placed in orbit. Every service is independent of the other service including the interrupted ones, too.

3 Simulation Results

To obtain the results of our simulation program a statistic package is used that was developed by Andrea Francini in 1994 [5]. With the help of this tool, it is possible to make a quantitative estimation of the mean and variance values of the desired variables using the method of batch means. There are n observations in every batch and the useful run is divided into a predetermined number of batches. In order for the estimation to work correctly, the batches are necessary to be long enough and approximately independent. It is one of the most popular confidence interval techniques for a steady-state mean of a process. The following works contain more detailed information about this method in [1]. The simulations are performed with a confidence level of 99.9%. The relative half-width of the confidence interval required to stop the simulation run is 0.00001.

3.1 First Scenario

The realization of the sensitivity analysis includes four different distributions of service time to compare the performance measures with each other. In every case, the parameters are selected in a way that the mean and variance would be equal. To accomplish that we applied a fitting process that is required to be done and [17] contains detailed information about the whole process describing every used distribution. Two scenarios are developed to investigate the effect of the various distributions. Table 2 shows the chosen parameters of the distribution of service time while Table 1 the values of other parameters. In the first one, the squared coefficient of variation is greater than one and the following distributions are used: hyper-exponential, gamma, Pareto, and lognormal. Results in connection with the second scenario (when the squared coefficient of variation is less than one) were also examined but because of the page limitation, these will be intended to be published in the extended version of the paper.

Table 1. Numerical values of model parameters

σ	γ_0	γ_1	γ_2	τ
0.01; 0.001	0.1	0.2	1	0.02; 0.002

In Figs. 2 and 3 the comparison of steady-state distribution of the number of customers in the orbit can be seen when the distribution of service time of the incoming customers is different. It demonstrates the probability $(P(i))$ of

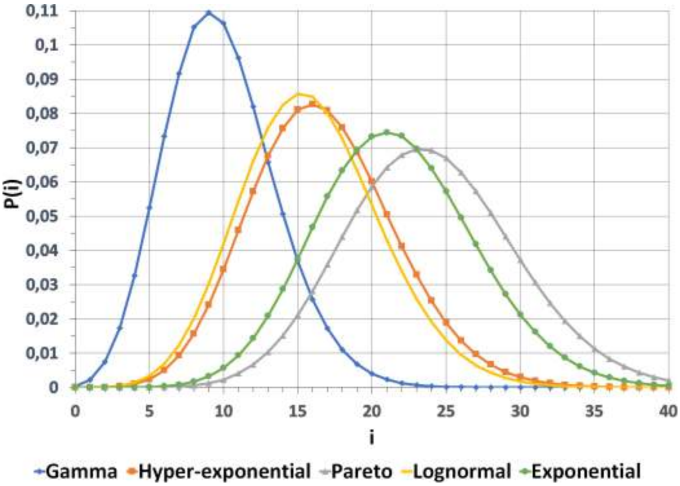

Fig. 2. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.01$, $\tau = 0.02$, $\lambda = 0.7$.

how many customers (i) residing in the orbit. Taking a closer look at the curves in more detail they coincide with normal distribution regardless of the used parameter setting. The figures also show the case of exponential distribution with the same mean as the other applied distributions. The mean number of customers in the orbit significantly differs from each other, at gamma distribution

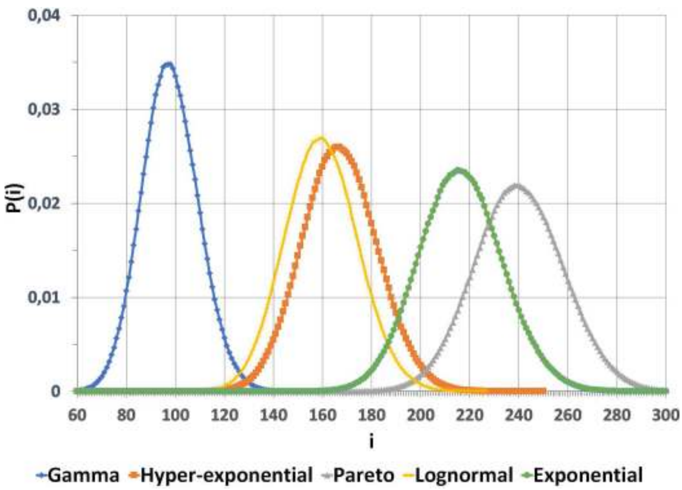

Fig. 3. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.001$, $\tau = 0.002$, $\lambda = 0.7$.

Table 2. Parameters of service time of incoming customers

Distribution	Gamma	Hyper-exponential	Pareto	Lognormal
Parameters	$\alpha = 0.0816$	$p = 0.4607$	$\alpha = 2.040$	$m = -1.292$
	$\beta = 0.0816$	$\lambda_1 = 0.9214$	$k = 0.5098$	$\sigma = 1.6075$
		$\lambda_2 = 1.0786$		
Mean	1			
Variance	12.25			
Squared coefficient of variation	12.25			

customers spend the fewest at Pareto distribution the highest time for waiting which is quite interesting.

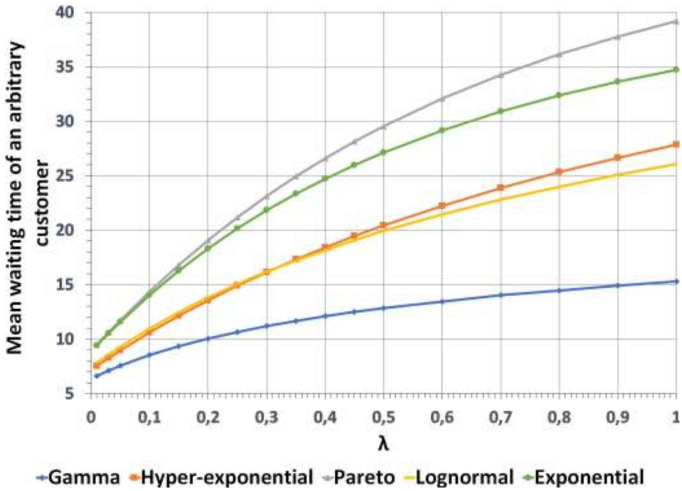

Fig. 4. Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.01$, $\tau = 0.02$.

The mean waiting time of an arbitrary customer is presented in the function of the arrival intensity of incoming customers in Figs. 4 and 5. Even though the mean and the variance are identical huge gaps develop among the applied distributions. With the increment of the arrival intensity, the mean waiting time of an arbitrary customer increases as well. The same tendency is observable when we use other values of retrial and impatience time. The usage of gamma distribution results in lower mean waiting time compared to the others, especially versus gamma and Pareto distributions.

Figure 6 and 7 demonstrate the development of the probability of abandonment of a customer besides increasing arrival intensity. This measure shows the probability that an arbitrary customer leaves the system throughout the orbit

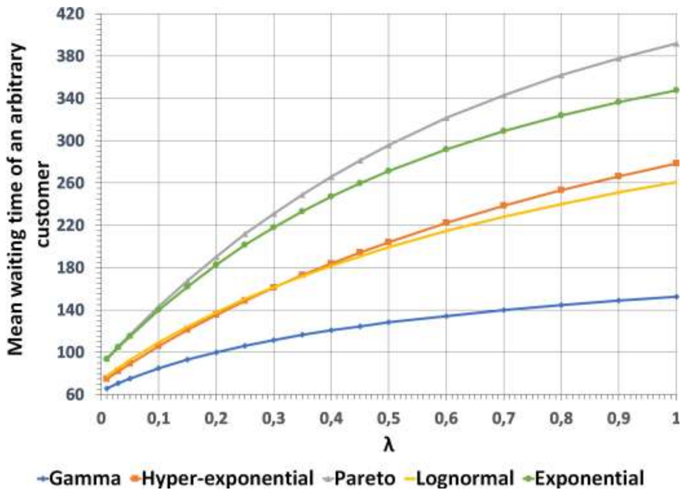

Fig. 5. Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.001$, $\tau = 0.002$.

which means the request does not get its appropriate service requirement (impatient customers). As λ increases the value of this performance measure raises as well which is true for every used distribution but the difference is quite high among them. At gamma distribution, the tendency of leaving the system earlier is much less than the others especially compared to Pareto and exponential distributions. Taking a closer look at the Fig. 6 and 7 the obtained values of

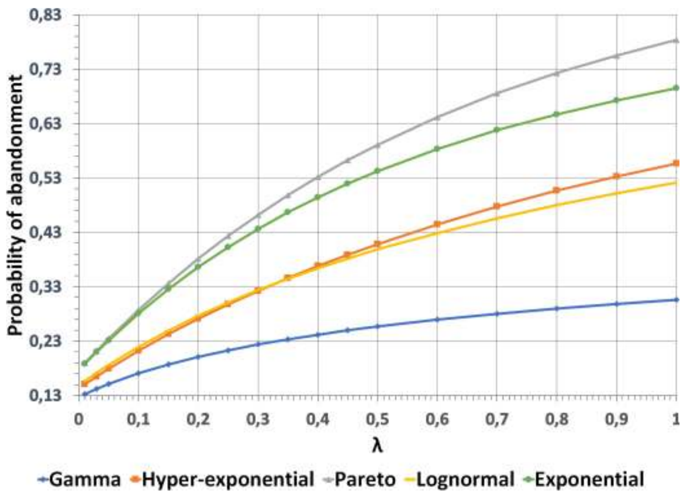

Fig. 6. Comparison of probability of abandonment, $\sigma = 0.01$, $\tau = 0.02$.

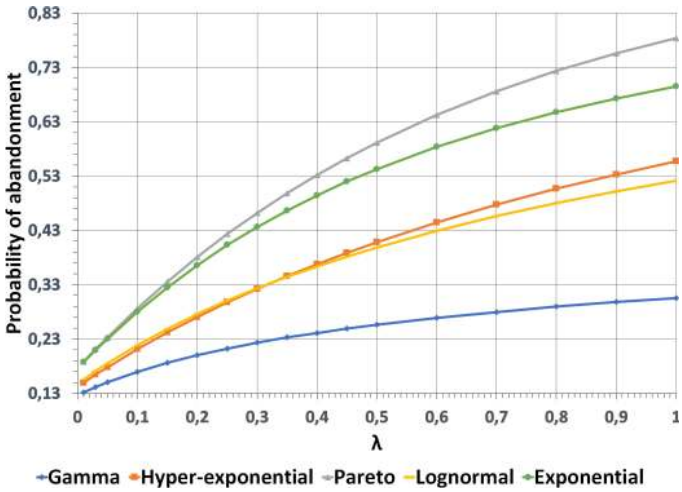

Fig. 7. Comparison of probability of abandonment, $\sigma = 0.001$, $\tau = 0.002$.

this measure are basically identical because the relationship remains the same between σ and τ.

3.2 Second Scenario

After observing the results and the tendencies of the previous section we modified the parameters of service time of incoming customers to see how this new parameter setting affects the performance measures. In this scenario, the squared coefficient of variation is less than one meaning that instead of hyper-exponential we used hypo-exponential distribution. We go over the same figures as in the first scenario but with the new applied parameters of service time which can be viewed in Table 3. All the other parameters remained unchanged (see Table 1).

Table 3. Parameters of service time of incoming customers

Distribution	Gamma	Hypo-exponential	Pareto	Lognormal
Parameters	$\alpha = 1.6$	$\mu_1 = 4$	$\alpha = 2.6125$	$m = -0.2428$
	$\beta = 1.6$	$\mu_2 = 1.3333$	$k = 0.6172$	$\sigma = 0.6968$
Mean	1			
Variance	0.625			
Squared coefficient of variation	0.625			

Figures 8 and 9 display the steady-state distribution of the number of customers in the orbit using various distributions of service time. The obtained curves are much closer to each other with this parameter setting even though

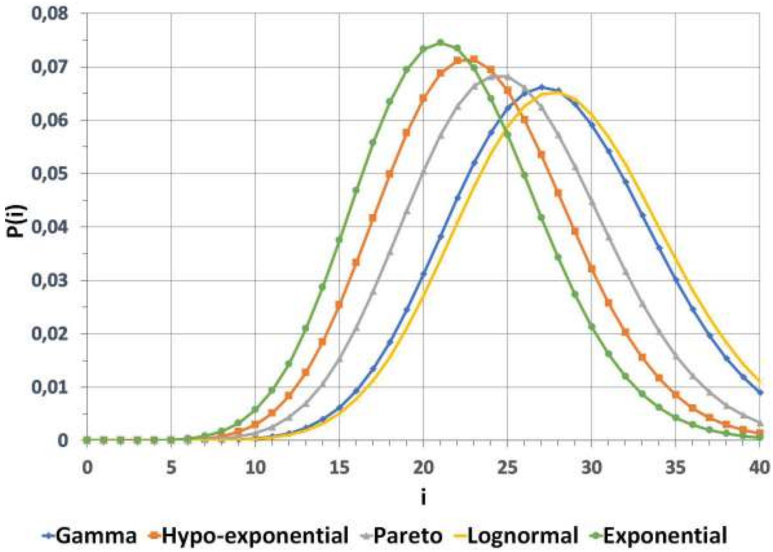

Fig. 8. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.01$, $\tau = 0.02$, $\lambda = 0.7$.

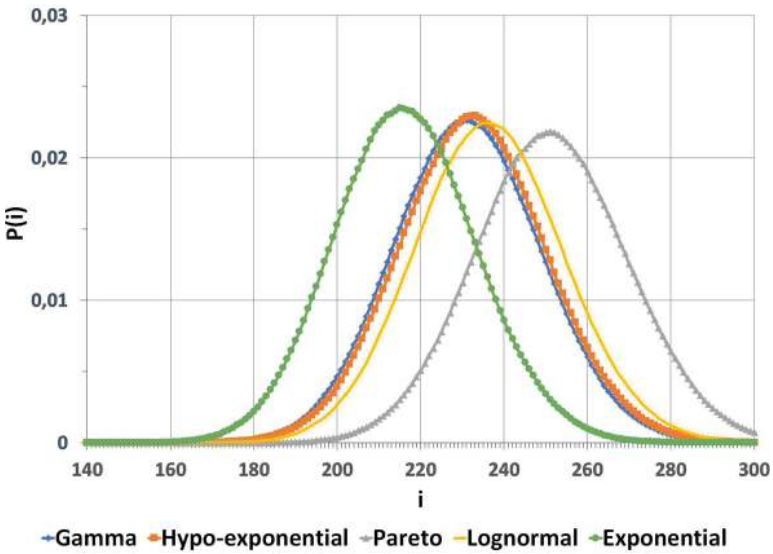

Fig. 9. Distribution of the number of customers in the orbit using various distributions, $\sigma = 0.001$, $\tau = 0.002$, $\lambda = 0.7$.

the difference is still quite significant in Figure 9 and the shape of the curves resemble the normal distribution. The value of the mean number of customers is higher in the case of gamma and lognormal distribution compared to Figures 2 and 3.

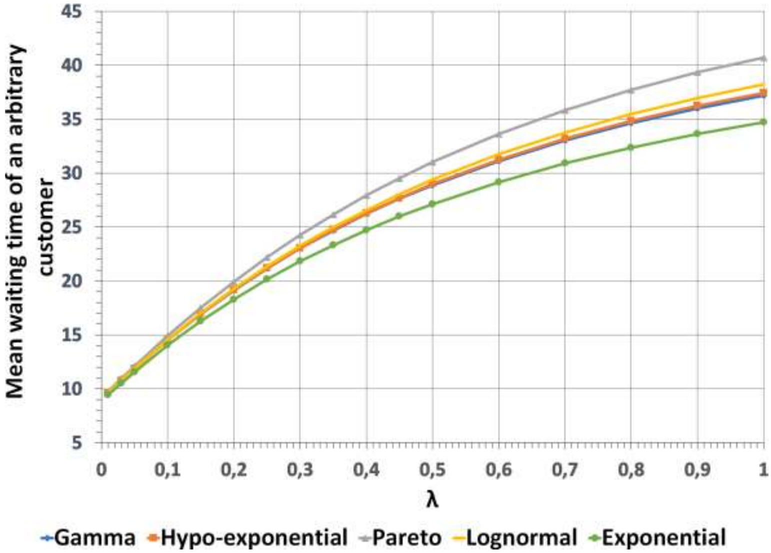

Fig. 10. Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.01$, $\tau = 0.02$.

The next two figures (Figs. 10 and 11) are related to the mean waiting of an arbitrary customer. Evaluating the results it can be stated that very slight differences occur although in the case of Pareto distribution the values are a little bit higher. Otherwise, they almost overlap each other and the same tendency can be observed in both figures. The mean waiting time increases with the increment of arrival intensity. Obviously, the achieved results indicate that the characteristics of the system are different using these parameters of service time among the applied distributions collated in this scenario with the former one.

Finally, to have a total comparison between the investigated scenarios Fig. 12 exhibits the probability of abandonment in the function of arrival intensity. After examining the two previous figures it is no wonder how this measure develops. The realization of the attained values shows how close the applied distributions with each other and the probability that an arbitrary customer leaves the system from the orbit increases besides higher arrival intensity. The represented values are almost totally identical with the results of Fig. 12 when $\sigma = 0.001$ and $\tau = 0.002$ as in the previous section.

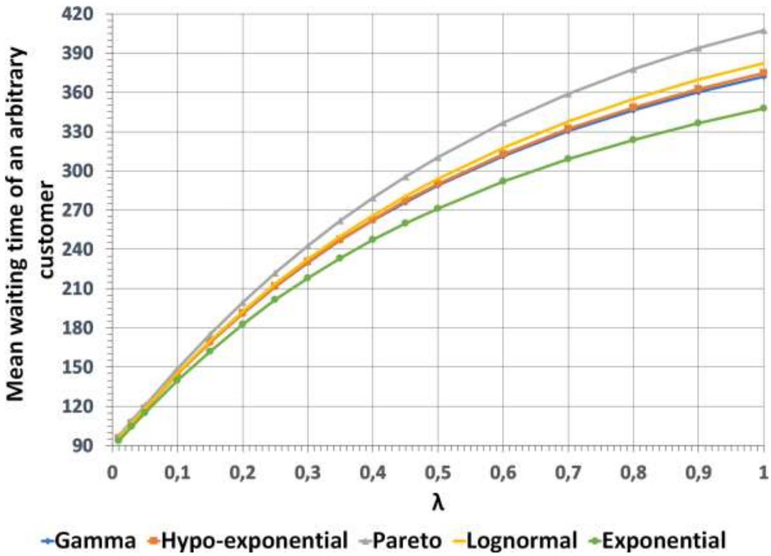

Fig. 11. Mean waiting time of an arbitrary customer vs. arrival intensity using various distributions, $\sigma = 0.001$, $\tau = 0.002$.

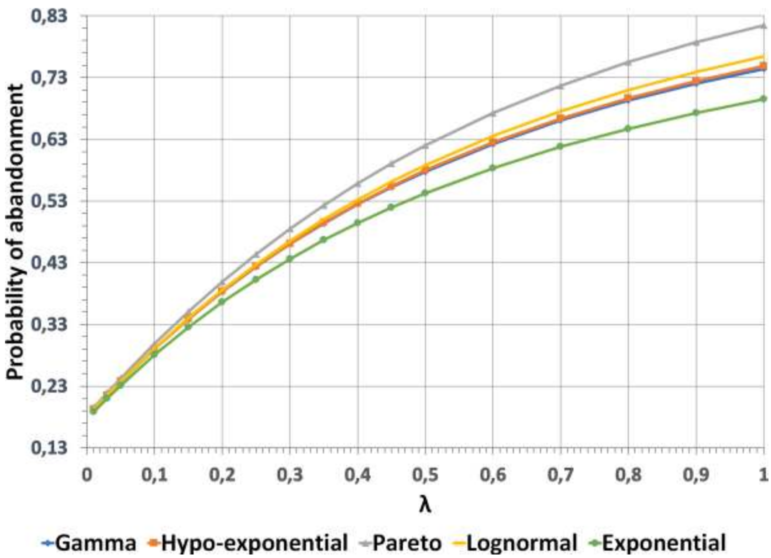

Fig. 12. Comparison of probability of abandonment, $\sigma = 0.01$, $\tau = 0.02$.

4 Conclusion

We studied the development of performance measures like the mean number of customers in the orbit or the mean waiting time of an arbitrary customer in a retrial queueing system of type $M/G/1$ with a non-reliable server and impatient customers in the orbit. Simulation has been carried out, the obtained results demonstrate that the number of customers in the orbit corresponds to the normal distribution in the case of every applied distribution. It is also displayed how the different distributions affect the performance measures despite the equality of mean value and variance when the squared coefficient of variation is more than one. In the case of the other scenario when the squared coefficient of variation is less than one, results clearly illustrated the moderate effect on the performance measures compared to the first scenario. In the future, we would extend this sensitivity analysis including more distributions or expanding the system with other features like two-way communication or other operation modes during a server failure.

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