Analysis of Self-Organized Criticality in the Olami-Feder-Christensen model and in real earthquakes

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In recent years there has been an intense debate on earthquake predictability [1] and a great effort in studying earthquake triggering and interaction [2–5]. Along these lines the possible application of the self-organized criticality (SOC) paradigm [6–14] has been discussed.
In general, the term self-organized criticality refers to the intrinsic tendency of a large class of spatially extended dynamical systems to spontaneously organize into a dynamical critical state. The idea of the seismogenic crust as a self-organized complex system was introduced over the years as a possible explanation for the widespread occurrence of space-time long-range correlations in earthquakes dynamics, similar to those observed in critical phase transitions. Actually, Earthquakes trigger dynamic and static stress changes. The first acts at short time and spatial scales, involving the brittle upper crust, while the second involves relaxation processes in the asthenosphere and acts at long time and spatial scales [15–21].
By means of a new analysis, based on the Olami-Feder-Christensen (OFC) model, we show that it is possible to reproduce statistical features of earthquakes catalogs within a SOC scenario taking into account long-range interactions. Since its introduction by Olami, Feder and Christensen in 1992 [12], the OFC model has played a key role in modelling earthquakes phenomenology. In its original version the OFC model consists of a two-dimensional square lattice of $N = L^2$ sites, each one connected to its four nearest neighbors and carrying a seismogenic force represented by a real variable Fi, which initially takes a random value in the interval (0 , Fth). In order to mimic a uniform tectonic loading all the forces are increased simultaneously and uniformly, until one of them reaches the threshold value Fth and becomes unstable. The driving is then stopped and an avalanche (earthquake) starts. The number of topplings during an avalanche defines its size S, while the dissipation level of the dynamics is controlled by the parameter α. The model is conservative if α=0.25, while it is dissipative for α=0.25.
Despite its simplicity, the OFC model exhibits a rich behavior resembling real seismicity, such as the presence of aftershocks and foreshocks [14]. However the presence of criticality in the non-conservative version of this model is controversial and it is still debated [22,23], also in relation with the influence of topology. We consider the dissipative OFC model on a *small world* topology [24] and our first goal is to show that, at variance with OFC models on other topologies which are critical only in the conservative case, it clearly reaches a critical state characterized

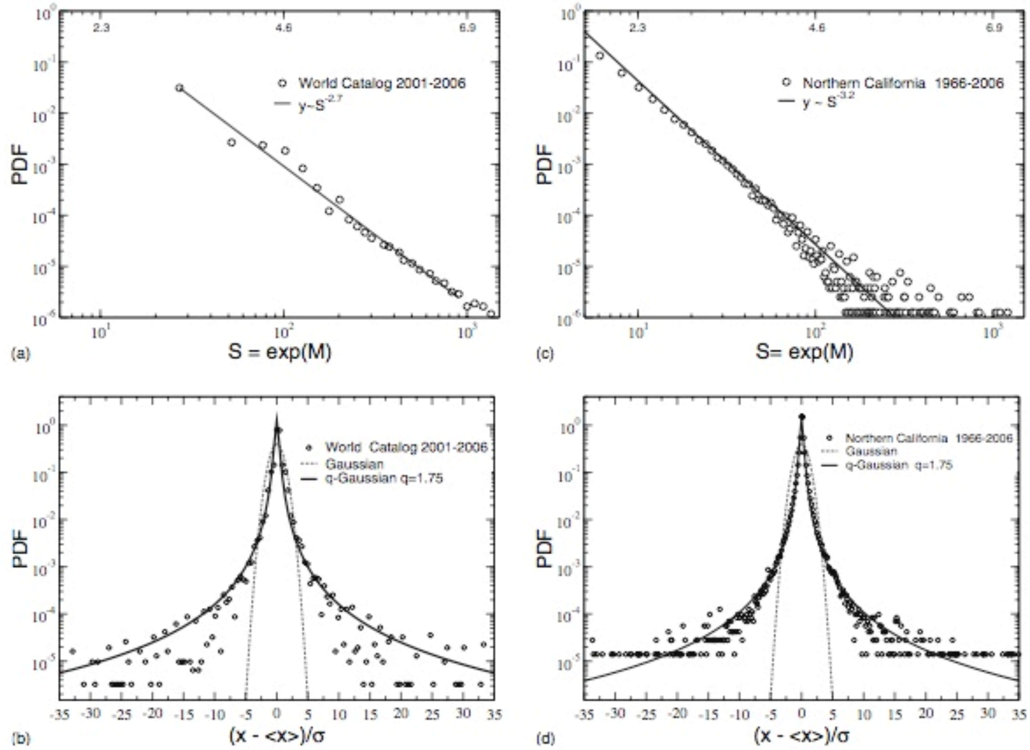

FIG. 2. Power-law distribution of $S=\exp(M)$, M being the magnitude for the world catalog (a) and for the northern California catalog (c). The correspondent power-law fits are also reported. We show the corresponding values of the magnitude M in the upper part of the figures. PDFs of the energy differences $x(t)=S(t+1)-S(t)$ are shown in (b) for the worldwide seismic catalog and in (d) for the northern California catalog. In both the figures the data have been fitted with a q-Gaussian (full line) with $q\sim 1.75\pm 0.15$. A standard Gaussian is plotted as a dotted line in all the figures for comparison. See the text for further details.

by both power law behavior of earthquakes size distribution and finite size scaling of cut-offs.

Furthermore [25], we show that when criticality appears, the probability density functions (PDFs) for the avalanche size differences at different times have fat tails with a q-Gaussian shape. This behavior does not depend on the time interval adopted and is found also when considering energy differences between real earthquakes. In fact, we repeated the previous analysis for the worldwide seismic catalog available online and for a more complete seismic data set, i.e., the Northern California catalog, and we observed a similar scenario (see figure above). Finally, we demonstrate that it is possible to explain such a behavior analytically simply assuming the absence of correlations among the sizes (released energies) of the avalanches (earthquakes).

These results on one hand give further support to the hypothesis that seismicity can be explained within a dissipative self-organized criticality framework when long-range interactions are considered. On the other hand, although temporal and spatial correlations among avalanches (earthquakes) do surely exist and a certain degree of statistical predictability is likely possible, they indicate that it is not possible to predict the magnitude of seismic events.

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