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Analysis of Simply Supported Rectangular Kirchhoff Plates by the Finite Fourier Sine Transform Method

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Abstract**—*In this work, the boundary value problem of simply supported rectangular Kirchhoff plates subjected to applied transverse loads is solved by the method of finite Fourier sine transform. The finite Fourier sine transform method was adopted as the analytical research tool due to the Dirchlet boundary conditions of the plate problem. Application of the finite Fourier sine transform to the fourth order governing partial differential equation of the Kirchhoff plate problem and the associated boundary conditions simplified the problem to an algebraic problem in the transform domain. The solution is obtained in the plate domain by inversion. The problem was solved for general distributed load p(x, y), point load applied at an arbitrary point on the plate, uniformly distributed patch load over the plate region $x_0 \le x \le x_1$, $y_0 \le y \le y_1$, and uniformly distributed load over the entire plate. The finite Fourier sine transform solutions obtained in each case were found to be identical solutions obtained with the Navier's double trigonometrical series method as presented in Timoshenko and Woinowsky-Krieger. The finite Fourier sine transform method was found to yield exact solutions to the classical thin plate flexure problem for simply supported edges.*

Keywords**— *Finite Fourier sine transform method, Kirchhoff plate, Dirichlet boundary conditions, distributed transverse load, patch load, point load, Navier's double trigonometric series method.***

I. INTRODUCTION

Plates are three dimensional structural members with extensive applications in civil, mechanical, aeronautical, naval and geotechnical engineering used to carry external loads by the development of bending resistance about the two axes of the plate [1, 2, 3].

The term plate theory denotes an approximate theory used to determine the stress fields and deformation field in elastic bodies one dimension of which (the plate thickness, *h*) is small compared with the other dimensions (the width and length of a rectangular plate supported at the edges or the diameter of a circular plate) [4]. The approximations consist of the introduction of certain simplifying assumptions into the governing kinematic, stress strain and equilibrium equations of the mathematical theory of elasticity [3, 4, 5]. These simplifications yield results which do not differ significantly from those obtained from the exact equations for the range of definition of the problem. The simplifications used in various plate theories derive from the definition of a plate as a three dimensional structure with one small dimension; and also from the consequences of Bernoulli-Navier's hypothesis for beams when extended to plates.

In the classical Kirchhoff –Love's plate theory, the influence of transverse shear strains is assumed to be negligible, and a simultaneous consideration of kinematics, stress-strain law and the differential equation of equilibrium for an infinitesimal plate element results in a fourth order partial differential equation as the governing equation of equilibrium [7]. Consequently, the number of boundary conditions appurtenant to the support conditions appears to be in disagreement with the order of the governing partial differential equation [3, 8]. This limits the validity of the expressions for the shearing forces to the open region of the plate middle surface and introduces Kirchhoff's shearing forces for the boundary of the plate. Three actual boundary conditions at each edge of the plate have to be replaced by two approximate conditions transformed in the Kirchhoff sense [2, 4].

Despite the shortcomings of the classical Kirchhoff-Love plate theory, it is well documented that for the majority of engineering applications, the theory gives sufficiently accurate results. The limitations and imperfections of the classical Kirchhoff-Love plate theory have led to the development of other plate theories. Some of these are Reissner plate theory [9, 10]; Mindlin plate theory [11], Henky refined plate theory [12, 13], Shimpi refined plate theory [14], Higher Order Plate Deformation theory [2], Third Order plate theory [15], Leung's Plate theory and Osadebe plate model [16]. Modified plate theories have also been used in plate bending analysis [17].

The plate problem in general is a boundary value problem which is a system of differential equations to be satisfied in the plate domain and the associated boundary conditions to be satisfied at the plate boundaries [18]. The plate problem

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has been solved successfully in the technical literature using two basic approaches – classical methods – Navier's double trigonometric series and Levy's single trigonometric series methods, and Numerical or Approximate Methods – Finite Difference Method [17], Finite Element Method, Boundary Integral Method, Variational Methods [20, 21] (Ritz Variational method [22], Galerkin Variational Method), Integral transform methods (Laplace transforms, Fourier transforms and Hankel transforms) [23] and Perturbation methods. In this work, the finite Fourier sine transform method is applied to solve the boundary value problem of simply supported rectangular Kirchhoff plates under given transverse loads.

II. METHODOLOGY

The finite transforms follow from the theory of Fourier series [24, 25]

The finite sine transform S_n of a function of x,$f(x)$ is defined as

$$S_n = S(f(x)) = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx\,; \qquad (1)$$

$m = 1, 2, 3 \ldots,\quad n = 1, 2, 3 \ldots$

where $0 \le x \le l$, $S_n = S(f(x))$ is the finite Fourier sine transform of $f(x)$

with its inverse sine transform as

$$S^{-1}(S_n) = f(x) = \sum_{n=1}^{\infty} S_n \sin\frac{n\pi x}{l} \qquad (2)$$

where S^{-1} is the inverse finite sine transform

The finite sine transform is commonly used with Dirichlet boundary conditions, that specify the value of $f(x)$ at the domain boundaries;$x = 0$, and $x = l$.

The Finite cosine transform C_n of a function of x,$f(x)$, normally used with Neuman boundary conditions that specify the value of $\frac{\partial f}{\partial x}$ at the domain boundaries,$x = 0$, and $x = l$ is defined as

$$C_n = C(f(x)) = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx\,, \qquad (3)$$

$n = 1, 2, 3 \ldots$

The inverse cosine transform, denoted by C^{-1}, is defined by:

$$C^{-1}(C_n) = f(x) = \frac{C_0}{2} + \sum_{n=1}^{\infty} C_n \cos\frac{n\pi x}{l} \qquad (4)$$

These transforms reduce a partial differential equation PDE to an ordinary differential equation ODE.

If $f(x, y)$ is a function of two independent variables x and y, defined in a given region $0 \le x \le a$, $0 \le y \le b$, its double finite Fourier sine transform $F_s(f(x, y))$ is defined by [26, 27]

$$F_s(f(x,y)) = F(m,n) = \int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy \qquad \ldots(5)$$

The inverse double finite Fourier sine transform is given by the double series:

$$F_s^{-1}(f(m,n)) = f(x,y) = \frac{4}{ab}\sum_m^{\infty}\sum_n^{\infty} F(m,n)\sin\left(\frac{m\pi x}{a}\right)\sin\frac{n\pi y}{b},$$

$m, n = 1, 2, 3 \qquad \ldots(6)$

where $F_s^{-1}(f(m,n))$ is the inverse double finite Fourier sine transform of $(f(m,n))$

III. APPLICATION OF THE FINITE FOURIER SINE TRANSFORM METHOD

The governing partial differential equation to be solved is given by Kirchhoff plate equation:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y)}{D} \qquad (7)$$

where $p(x, y)$ is the transverse distributed load acting on the plate, D is the flexural rigidity of the plate material, $D = \frac{Eh^3}{12(1-\mu^2)}$, E = Young's modulus of elasticity, h = plate thickness, μ is the Poisson's ratio, and $w(x, y)$ is the transverse deflection of plate, x and y are the space variables in the plane of the plate, and$0 \le x \le a$, $0 \le y \le b$.

The boundary conditions for simple supports at the plate edges: $x = 0, a$; $y = 0, b$ are $w = 0$ on $x = 0, a$,$y = 0, b$, $w_{xx} = 0$ on $x = 0$, a, $w_{yy} = 0$ on $y = 0$, b

where $w_{xx} = \frac{\partial^2 w}{\partial x^2}$, and $w_{yy} = \frac{\partial^2 w}{\partial y^2}$

Transverse Distributed load $p(x, y)$

For distributed transverse load $p(x, y)$, taking the finite Fourier Sine transforms of both sides of Equation (7), we have Equation (8)

$$\int_0^b\int_0^a \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right)\sin\alpha_m x \sin\beta_n y\, dxdy$$

$$= \frac{1}{D}\int_0^b\int_0^a p(x,y)\sin\alpha_m x\sin\beta_n y\, dxdy \qquad (8)$$

where $\alpha_m = \frac{m\pi}{a}$, $\quad \beta_n = \frac{n\pi}{b}$

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Using the linearity property of the finite Fourier Sine transform, and noting the finite Fourier Sine transform of $\frac{\partial^4 w}{\partial x^4}$ given by Equation (9)

$$\int_0^b\int_0^a \frac{\partial^4 w}{\partial x^4}\sin\alpha_m x\sin\beta_n y\,dxdy$$

$$= -\alpha_m\int_0^b\left[(-1)^m w_{xx}|_{x=a} - w_{xx}|_{x=0}\right]\sin\beta_n y\,dy$$

$$+\alpha_m^4\int_0^a\int_0^b w(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy$$

$$= \alpha_m^4 w(m,n) \quad \ldots(9)$$

since $w_{xx}|_{x=a} = w_{xx}|_{x=0} = 0$

where

$$w(m,n) = \int_a^b\int_0^a w(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy \quad (10)$$

$w(m, n)$ = finite Fourier Sine transform of the transverse deflection function $w(x, y)$, we have Equation (11):

$$\int_0^b\int_0^a\left(\alpha_m^4 + 2\alpha_m^2\beta_n^2 + \beta_n^4\right)w(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy$$

$$= \frac{1}{D}\int_a^b\int_0^a p(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy \quad (11)$$

$$\left(\alpha_m^2 + \beta_n^2\right)^2\int_0^b\int_0^a w(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy$$

$$= \frac{1}{D}\int_0^b\int_0^a p(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy \quad (12)$$

$$\left(\alpha_m^2 + \beta_n^2\right)^2 w_{mn} = \frac{p_{mn}}{D} \quad (13)$$

where p_{mn} is the finite Fourier Sine transform of the transverse distributed load

Hence,

$$w_{mn} = \frac{p_{mn}}{D(\alpha_m^2 + \beta_n^2)^2} \quad (14)$$

By inversion,

$$w(x,y) = \frac{4}{Dab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\alpha_m x\sin\beta_n y \quad (15)$$

$$w(x,y) = \frac{4}{abD}\sum_m^{\infty}\sum_n^{\infty}\frac{p_{mn}\sin\alpha_m x\sin\beta_n y}{(\alpha_m^2 + \beta_n^2)^2} \quad (16)$$

$$= \frac{4}{abD}\sum_m^{\infty}\sum_n^{\infty}\frac{p_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2} \quad (17)$$

$$= \frac{4}{\pi^4 abD}\sum_m^{\infty}\sum_n^{\infty}\frac{p_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \quad (18)$$

This satisfies the boundary conditions

Case of concentrated load $p(x,y) = P_0$ at $x = \xi,\ y = \eta$ inside the plate domain

The load $p(x, y)$ is represented using Dirac delta functions as

$$p(x,y) = P_0\delta(x-\xi)\delta(y-\eta) \quad (19)$$

where P_0 is a constant and $\delta(x-\xi)\delta(y-\eta)$ are the Dirac delta functions, ξ and η are the coordinates of application of point load on the plate.

then, $0 \le \xi \le a;\ 0 \le \eta \le b$

$$\int_0^b\int_0^a \nabla^4 w(x,y)\sin\alpha_m x\sin\beta_n y\,dxdy$$

$$= \frac{1}{D}\int_0^b\int_0^a P_0\delta(x-\xi)\delta(y-\eta)\sin\alpha_m x\sin\beta_n y\,dxdy \quad (20)$$

$$\left(\alpha_m^4 + 2\alpha_m^2\beta_n^2 + \beta_n^4\right)w_{mn} = \frac{P_0}{D}\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b} \quad (21)$$

$$w_{mn} = \frac{\frac{P_0}{D}\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2} \quad (22)$$

$$= \frac{P_0}{D\pi^4}\frac{\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \quad (23)$$

By inversion,

$$w(x,y) = \frac{4}{ab}\sum_m^{\infty}\sum_n^{\infty} w_{mn}\sin\alpha_m x\sin\beta_n y, \quad (24)$$

$m,n = 1,2,3,4 \ldots$

$$= \frac{4}{ab}\sum_m^{\infty}\sum_n^{\infty}\frac{P_0}{D\pi^4}\frac{\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}$$

$\ldots(25)$

$$= \frac{4P_0}{Dab\pi^4}\sum_m^\infty\sum_n^\infty \frac{\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(26)$$

$$w(x,y) = P_0 K(x,y,\xi,n) \quad (27)$$

K = Kernel or Green function.This, satisfies the boundary conditions.

The bending moments are found from the bending moment displacement relations.

For point load P at the centre of the plate, $x=\xi=a/2$, $y=\eta=b/2$ the deflection, and loading moment value become

$$w\left(x=\frac{a}{2},\ y=\frac{b}{2}\right) = \frac{4P_0}{D\pi^4 ab}\sum_m^\infty\sum_n^\infty \frac{\sin^2\frac{m\pi}{2}\sin^2\frac{n\pi}{2}}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(28)$$

$m = 1, 3, 5 \ldots \qquad n = 1, 3, 5 \ldots$

$$M_{xx} = \frac{4P\alpha}{\pi^2}\sum_m^\infty\sum_n^\infty \frac{(-1)^{m+n-2}(m^2\alpha^2+0.3n^2)}{(m^2\alpha^2+n^2)^2} \quad (29)$$

$$M_{yy} = \frac{4P\alpha}{\pi^2}\sum_m^\infty\sum_n^\infty \frac{(-1)^{m+n-2}(n^2+0.3m^2\alpha^2)}{(m^2\alpha^2+n^2)^2} \quad (30)$$

Uniformly distributed patch load over the plate region $x_0 \le x \le x_1$ $y_0 \le y \le y_1$

The plate deflection due to a uniformly distributed patch load over the region $x_0 \le x \le x_1$, $y_0 \le y \le y_1$ is found by integration of the point load solution; as

$$w(x,y) = \frac{4}{Dab\pi^4}\sum_m^\infty\sum_n^\infty\int_{y_0}^{y_1}\int_{x_0}^{x_1} \frac{p(\xi,\eta)\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}\sin\frac{m\pi x}{a}\sin\frac{m\pi y}{b}\,d\xi d\eta}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(31)$$

$$= \frac{16p_0}{\pi^6 D}\sum_m^\infty\sum_n^\infty \frac{S_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad (32)$$

$$S_{mn} = \sin\frac{m\pi\xi}{a}\sin\frac{m\pi u}{2a}\sin\frac{n\pi\eta}{b}\sin\frac{n\pi v}{2b} \quad (33)$$

where, $u=(x_0-x_1)$, $v=(y_1-y_0)$

From the bending moment displacement equations,

$$M_{xx} = \frac{16p_0}{\pi^4}\sum_m^\infty\sum_n^\infty \frac{S_{mn}\left(\frac{m^2}{a^2}+\mu\frac{n^2}{b^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(34)$$

$$M_{yy} = \frac{16p_0}{\pi^4}\sum_m^\infty\sum_n^\infty \frac{S_{mn}\left(\frac{n^2}{b^2}+\mu\frac{m^2}{a^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(35)$$

Uniformly Distributed load p_0 over the entire plate

The plate deflection for uniformly distributed load p_0 on the entire plate domain is found by integrating the point load solution over the entire plate are as:

$$w(x,y) = \frac{4}{Dab\pi^4}\sum_m^\infty\sum_n^\infty\int_0^b\int_0^a \frac{p_0\sin\frac{m\pi\xi}{a}\sin\frac{n\pi\eta}{b}\sin\frac{m\pi x}{a}\sin\frac{m\pi y}{b}\,d\xi d\eta}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad (36)$$

$$= \frac{16p_0}{D\pi^6}\sum_m^\infty\sum_n^\infty \frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad (37)$$

$m = 1, 3, 5 \ldots \qquad n = 1, 3, 5 \ldots$

From the bending moment-displacement equations, the bending moment distributions become

$$M_{xx} = \frac{16p_0}{D\pi^4}\sum_m^\infty\sum_n^\infty \frac{\left(\frac{m^2}{a^2}+\mu\frac{n^2}{b^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(38)$$

$$M_{yy} = \frac{16p_0}{D\pi^4}\sum_m^\infty\sum_n^\infty \frac{\left(\frac{n^2}{b^2}+\mu\frac{m^2}{a^2}\right)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2} \quad \ldots(39)$$

$m, n = 1, 3, 5, 7 \ldots$

IV. RESULTS AND DISCUSSIONS

The Finite Sine transform method has been applied to the boundary value problem of simply supported rectangular Kirchhoff plates under general distributed load $p(x, y)$, point

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load P at (ξ, η) uniformly distributed patch load over a given area of the plate and uniformly distributed load over the entire plate. The deflection functions computed are shown in each case as Equations(18), (26), (32) and (37). The deflection functions were found to be identical with the Navier's double trigonometric series solution for the deflection in each case. The double series of infinite terms obtained for the deflection $w(x, y)$ for point loads is a rapidly convergent series and the deflection at any point can be obtained with good accuracy by considering only the first few terms. For point load applied at the plate centre, the deflection was observed to be symmetrical about the plate axes of symmetry, and maximum deflection was found to occur at the plate centre. The series solutions obtained for both $w(x, y)$, M_x and M_y (for point load) were found to diverge at the point of application of the point load.

Variation of maximum deflection at the plate centre, with the plate aspect ratio are tabulated inTable 3

For the case of uniformly distributed load on the entire plate, the displacement was found to be identical to the corresponding Navier's double trigonometric series solution. The deflection was found to be a double series of infinite terms, symmetrical about the two axes of symmetry of the plate. Bending moment functions found from the moment displacement relations were similarly found to be doubly symmetrical about the plate axes. Maximum values of deflection and bending moments for various b/a values were found to occur at the plate centre and are tabulated as shown in Table 1. The convergence characteristics of the series of w, M_{xx}, M_{yy} are shown in Table 2, which shows the deflections converge faster than the bending moments.

Table.1: Deflection and Bending Moment coefficients for Simply Supported Rectangular Kirchhoff Plates under uniformly distributed loads

b/a	$w_{\max} = F\dfrac{pa^4}{D}$ Timoshenko and Woinowsky-Krieger	Present study (deflection coefficient)	$M_{xx\,\max}$ Timoshenko and Woinowsky-Krieger	$M_{xx\,\max}$ Present study (moment coefficient)	$M_{yy\,\max}$ Timoshenko and Woinowsky-Krieger	$M_{yy\,\max}$ Present study
1.0	4.06×10^{-3}	4.062×10^{-3}	0.0479	0.047886	0.0479	0.047886
1.1	4.85×10^{-3}	4.85×10^{-3}	0.0554	0.0554	0.0493	0.0493
1.2	5.64×10^{-3}	5.64×10^{-3}	0.0627	0.0627	0.0501	0.0501
1.3	6.83×10^{-3}	6.83×10^{-3}	0.0694	0.0694	0.0503	0.0503
1.4	7.05×10^{-3}	7.05×10^{-3}	0.0755	0.0755	0.0502	0.0502
1.5	7.72×10^{-3}	7.724×10^{-3}	0.0812	0.08116	0.0498	0.049843
1.6	8.30×10^{-3}	8.30×10^{-3}	0.0862	0.0862	0.0492	0.0492
1.7	8.83×10^{-3}	8.83×10^{-3}	0.0908	0.0908	0.0486	0.0486
1.8	9.31×10^{-3}	9.31×10^{-3}	0.0948	0.0948	0.0479	0.0479
1.9	9.74×10^{-3}	9.74×10^{-3}	0.0985	0.0985	0.0471	0.0471
2	10.13×10^{-3}	10.12866×10^{-3}	0.1017	0.101683	0.0464	0.046350
3	12.23×10^{-3}	12.2328×10^{-3}	0.1189	0.118861	0.0406	0.0406266
4	12.82×10^{-3}	12.81865×10^{-3}	0.1235	0.12346	0.0384	0.038415
5	12.97×10^{-3}	12.97×10^{-3}	0.1246	0.124625	0.0375	0.03745
∞	13.02×10^{-3}	13.0208×10^{-3}	0.1250	0.1250	0.0375	0.0375

Table.2: Convergence study for Deflection and Bending moments at the center of simply supported square Kirchhoff plates under uniform load

No of terms	$w_{\max}$ $\dfrac{pa^4}{D}\times 10^{-2}$	$M_{xx\,\max}$ $pa^2\times 10^{-2}$	$M_{yy\,\max}$ $pa^2\times 10^{-2}$
1	0.416	5.34	5.34
2	0.405	4.69	4.69
3	0.406	4.86	4.94
4	0.406	4.81	4.90
Exact	0.406	4.79	4.79

Table.3: Simply supported rectangular Kirchhoff plates under point load at the center

b/a	w_{max} Timoshenko and Woinowsky-Krieger	w_{max} Present study
1.0	0.01160	0.01160
1.2	0.01353	0.01353
1.4	0.01464	0.01464
1.6	0.01570	0.01570
1.8	0.01620	0.01620
2	0.01651	0.01651

V. CONCLUSIONS

The double finite Fourier sine transform method has been used to derive analytic flexural solutions of simply supported rectangular Kirchhoff plates under general distributed load, point load at ξ, η, uniform patch load and uniform load on the entire plate. The analysis is performed without any assumption of the displacement trial (shape) function which illustrates the advantage of the method. The method is an efficient and accurate analytical tool for Kirchhoff plate bending analysis and can be extended to other boundary value problems of plates such as buckling and vibration.

REFERENCES

[1] Reddy, J.N. (1999): Theory and Analysis of Elastic Plates. Taylor and Francis London.

[2] Chandrashekhara K. (2011) Theory of Plates. University Press (India) Limited Hyderabad.

[3] Szilard, (2004): Theories and Applications of Plate Analysis Classical, Numerical and Engineering Methods. John Wiley and sons Inc.

[4] Panc, V. (1975): Theories of Elastic Plates. Noordhoff International Publishing,Leyden, the Netherlands.

[5] Ventsel and Krauthammer (2001) Thin Plates and Shells Theory, Analysis and Applications. Marcel Dekker Inc New York, USA.

[6] Kapadiya H.N. and Patel A.D. (2015): Review of Bending Solutions of Thin Plates. International Journal of Scientific Research and Development (IJSRD) Vol. 3 Issue 03, 2015, ISSN 2321-0613, pp 1709-1712.

[7] Osadebe N.N., Ike C.C., Onah H., Nwoji C.U. and Okafor F.O. (2016). Application of Galerkin-Vlasov Method to the Flexural Analysis of Simply Supported Rectangular Kirchhoff Plates under Uniform Loads. Nigerian Journal of Technology NIJOTECH Vol. 35, No 4, October, 2016 pp. 732-738.

[8] Timoshenko S. and Woinowsky-Krieger S (1999) Theory of Plates and Shells. Mc Graw Hill Book Co. Singapore.

[9] Reissner, E. (1945): The Effect of Transverse Shear Deformation on the Bending of Elastic Plates. Journal of Applied Mechanics, 12.

[10] Reissner, E. (1947): On Bending of Elastic plates. Quart. Appl. Math; 5.

[11] Mindlin, R.D (1951): Influence of Rotary Inertia and Shear on Flexural Motions of Isotropic Elastic Plates. J. Appl. Mech Vol 18 No 1, pp.31-38.

[12] Hencky, H. (1913): Der Spannungszustand in rechteckigen Platten R. Oldenbourg, Mùńchen u Berlin.

[13] Hencky, H. (1947): Über die Berücksichtignng der schubuerzerrungen in ebenen Platten Ing Arch 16.

[14] Shimpi, R.P. (2002): Refined Plate Theory and its Variants. AIAA Journal vol 40, No 1 January, 2002.

[15] Reddy, J.N. (1984): A Simply Higher Order Theory for Laminated Composite Plates. ASME Journal of Applied Mechanics, Vol 51, pp. 745-752.

[16] Osadebe, N.N. (1997): Differential Equations for Small Deflection Analysis of Thin Elastic Plates Possessing Extensible Middle Surface. University of Nigeria Virtual Library.

[17] Suetake Y (2006) Plate Bending Analysis by using a Modified Plate Theory. CMES Vol.11 No.3, pp. 103-110, Tech Science Press.

[18] Sebastian V.K. (1983): An Elastic Solution for Simply Supported Rectangular Plates. Nigerian Journal of Technology (NIJOTECH) Vol. 7, No.1 September, 1983, pp. 11-16.

[19] Ezeh J.C. Ibearugbulam O.M. and Onyechere C.I. (2013): Pure Bending Analysis of thin Rectangular Flat Plates using Ordinary Finite Difference Method. International Journal of Emerging Technology and Advanced Engineering (IJETAE), Volume 3, Issue 3, March 2013, www.ijetae.com, ISSN: 2250-2459.

[20] Aginam C.H., Chidolue C.A. and Ezeagu C.A. (2012): Application of Direct Variational Method in the Analysis of Isotropic thin Rectangular Plates ARPN Journal of Engineering and Applied Sciences

Vol. 7, No 9, September 2012. Asian Research Publishing Network (ARPN) ISSN 1819-6608.

[21] Ibearugbulem O.M., Ettu L.O. and Ezeh J.C. (2013): Direct Integration and Work Principle as New Approach in Bending Analysis of Isotropic Rectangular Plates. The International Journal of Engineering and Science (IJES) Volume 2, Issue 3, 2013 pp 28-36, ISSN 2319-1813.

[22] Balasubramanian Ashwin (2011): Plate Analysis with Different Geometries and Arbitrary Boundary Conditions. MSc Thesis: Mechanical Engineering Faculty of Graduate School, the University of Texas at Arlington, December, 2011.

[23] Zhoug Yang, Zhao Xue-Feng, and Liu Hang (2014): Vibration of plate on foundation with four edges free by finite cosine integral transform method. Latin American Journal of Solids and Structures, Vol 11, No 5, Rio de Janiero, October, 2014.

[24] Lokenath D. and Bhatta D. (2007): Integral Transforms and their Applications, Second Edition. Chapman and Hall/ CRC Taylor and Francis Group LLC New York.

[25] Brown D. (2002): Integral Transforms and their Applications, Third Edition. Texts in Applied Mathematics 41, Springer.

[26] Antimorov M.Ya, Kalyshkin A. and Vaillanconot Remi (1993): Applied Integral Transforms Centre des Researches Mathematiques, Universite de Montreal Volume 2 CRM Monograph Series. American Mathematical Society Providence, Rhode Island USA.

[27] Andrews L.C. and Shivamoggi B.K. (1999): Integral Transforms for Engineers. University of Central Florida, SPIE Optical Engineering Press, A publication of SPIE, The International Society for Optical Engineering, Bellingham, Washington USA.