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# Analysis of the Buzz Formation Models: Models of New Marketing

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## Abstract

The theme of this chapter was to review differential equation models of diffusion phenomena that have been developed in various fields of research and to conduct a comparative review of the models through the application of actual data. Among the many models created for diffusion phenomena, the models examined in this chapter are large number models that explain macro changes. These models were developed in various research fields. However, comparing these models reveals that although many models are used only for individual fields of study, mathematically similar models are often observed. It is not necessarily apparent the type of model that would be most effective for any specific diffusion phenomena case. Applications of specific models can be observed by reviewing individual research examples. However, the effectiveness of each model is not clarified in the absence of a comparison across various models. The purpose of this chapter was to apply various models to different types of data, and thereby verify the suitability of each model to specific phenomena.

**Keywords:** the modified exponential curve type, the sigmoid curve type, process models, Gompertz curve models, modified exponential curve models, model that combines two factors

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## 1. Introduction

Along with the rapid spread of social media, the competitive environment of the marketplace will change, inevitably altering marketing effectiveness as well. This chapter examines diffusion phenomena based on marketing research [1–3].

There are two types of differential equation models to express diffusion phenomena and used widely as demand forecast models for durable consumer goods in marketing research: (a) the

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sigmoid curve type and (b) the modified exponential curve type. The former is suitable for human interaction types of diffusion phenomena and the latter is suitable for external information source diffusion phenomena. This analysis examines the effectiveness of these models on the marketing field.

## 2. Process model

Based on the development of Internet, marketing (especially consumer) is changing. In these situations, this analysis examines the effectiveness of above models on the marketing field. One of the major characteristics of the models discussed in this chapter is that they are process models in marketing. Many of the marketing models used in the field of social science are structure models. For example, multidimensional scaling, factor analysis, and password cracking analysis models are all structure models that statically describe structures on a unitemporal basis. Process models on the marketing, in contrast, describe dynamic changes and include time ( $t$ ) among their independent variables. There are several types of process models on the marketing. Models on the marketing describing diffusion phenomena typically apply differential equations as the basic model building block.

$$dP/dt = f(P, t) \quad (1)$$

Here,  $P$  is the diffusion rate ( $0 \leq P \leq 1$ ),  $t$  represents time, and  $f$  is a specific function.  $dP/dt$  is the differential coefficient for  $P$  with respect to  $t$ . This differential equation model on the marketing is an effective method when describing dynamic situations.  $dP/dt$  represents how much  $P$  changes per unit change in  $t$ , and its value changes depending on the values of  $P$  and  $t$  as shown in Formula (1). Differential equation of (1) may not always be solved. However, it can often be easily solved when  $f(P, t)$  is factorized as follows:

$$f(P, t) = f_p(P) \cdot f_t(t) \quad (2)$$

This means (1) can be rewritten as

$$dP/dt = f_p(P) \cdot f_t(t) \quad (3)$$

$f_p$  is a function that does not contain  $t$  and  $f_t$  is a function that does not contain  $P$ .

Eq. (3) is a separation of variables. Thus, the solution can be given as

$$\int \frac{dP}{f_p(P)} = \int f_t(t) dt + C \quad (4)$$

$C$  is an arbitrary constant determined by the initial state and other factors.

Compared to structure models, differential equation models have characteristics as follows:

- a. Nonlinearity: solutions for (4) are mostly nonlinear models with complex functions compared to many linear structure models.
- b. In most cases,  $t$  is the only independent variable.

These characteristics show the fundamental differences between differential equation models and structure models. Structure models, like regression analysis, can use as many independent variables as necessary. Moreover, this model focuses on how to select independent variables. Conversely, differential equation models only have  $t$  as an independent variable and focus on how to assume the function for  $f$  in Formula (1). Differential equation models provide not only high adaptability to data but also support of existing theory.

Among many streams of diffusion research, marketing research pedigree and epidemiology are leading fields in which differential equation models have made significant progress. Epidemiology in particular has accumulated many differential equation models since the beginning of twentieth century and it has influenced the pedigree of other models. Among these pedigrees, differential equation models have been developed for such practical applications as product demand forecasts and infectious disease spread predictions.

Despite the application of these models in sociology diffusion research, a leading sociologist of his day and sporadic achievements in the field of social psychology is an example, and these types of models are yet to become mainstream.

However, this chapter emphasizes that the use of differential equation models is not simply limited to prediction applications, but is closely related to the internal mechanism of understanding diffusion phenomena. These models are used to mathematically express diffusion phenomena mechanisms, not just as an equation applicable to the data. Many generalizations regarding diffusion phenomena. Differential equation models can be an effective means of empirically verifying these suppositions.

### 3. Examination of four models

The next examines the theoretical reasonableness of some models in marketing. This chapter examines the following types of diffusion phenomena models.

1. Gompertz curve models.
2. Sigmoid type models.
3. Modified exponential curve models.
4. Model that combines two factors.

In addition to the above types, normal probability curve models, Weibull curve models, and Pólya-Eggenberger curve models have been used in marketing research to describe product diffusion. There are also many examples of applications in such social development models as

population increase and economy growth in the marketing field. These are examples of the application of diffusion curves, however, they can be difficult to use as process models. Thus, these models are not included below.

### 3.1. Gompertz curve models

Gompertz curve models are characteristically used to explain diffusion phenomena mechanisms through two contradicting factors: (a) a factor that reinforces diffusion processes along with an increasing rate of diffusion, and (b) a factor that interrupts diffusion processes as time passes. For example, it is frequently observed in some kinds of trend phenomena that (a) expansion of a trend will reinforce the pressure of trend adoption while (b) attraction to a trend, such as novelty, decreases over time passes.

In a Gompertz curve model, let factor (a) be  $k$  and  $P$  and factor (a) be  $b^t$  ( $k$  and  $b$  are constants and  $0 < b < 1$ ). Then the following differential equation can be obtained by connecting both sides in the form of product.

$$dP/dt = k \cdot b^t \cdot P \quad (5)$$

Solving this gives

$$P(t) = 1 / \left\{ 1 + \frac{q_0}{p_0} (t+1)^{-k} \right\} \quad (6)$$

(note that  $C = P_0 / \exp(n)$ ,  $n = k / \log b$ , and  $a = \exp(n)$ ).

Because  $0 < b < 1$ , this converges to  $P(t) = C$  when  $t \rightarrow \infty$ .

Example of a Gompertz curve model apply to diffusion phenomena.

However, research examples show that Gompertz curve models may be suitable as an index of the absolute quantity of some types of phenomena, such as product demand and sales, rather than considering  $P$  to be a diffusion rate ( $0 \leq P \leq 1$ ).

The convergence value of  $P(t)$  in Eq. (6) is  $P(t) = C$ , when  $t \rightarrow \infty$ . However, in general,  $C \neq 1$  and the solution for Eq. (5) is

$$P(t) = P_0 \exp(kt) \quad (7)$$

$$P(t) \rightarrow \infty \text{ when } t \rightarrow \infty.$$

Considering the characteristics of Gompertz curve models, it would be better to think of  $P$  as an index of an absolute quantity ( $0 \leq P$ ).

### 3.2. Sigmoid type models

It has been observed in merchandise diffusion phenomena that having an increasing number of people who have already adopted an event will increase the pressure on people who have

not yet adopted the event. Examples proving this concept include demonstration effects in marketing, as group pressure in psychology and as imitation in sociology.

The following differential equation models a simple sigmoid curve.

$$dP/dt = kP(1 - P) \tag{8}$$

$P$  represents the diffusion rate ( $0 \leq P \leq 1$ ) and  $k$  is a constant ( $k > 0$ ). This model shows that a new adoptee is created in proportion to the number of people who have already adopted ( $P$ ) from among nonadoptees ( $1 - P$ ). This model can also represent the creation of new adoptees from their random interactions with people who have already adopted. Here,  $k$  is a parameter indicating interaction efficiency.

Solving Formula (8) gives

$$P(t) = 1 / \left( 1 + \frac{q_0}{p_0} e^{-kt} \right) \tag{9}$$

In this equation,  $p_0$  and  $q_0$  are the proportions of adoptees and nonadoptees at  $t = 0$ . ( $p_0 + q_0 = 1$ ).

Although there are several examples of applying this famous sigmoid curves, it has limits as a model of diffusion phenomena. More specifically, parameter  $k$  representing interaction efficiency in Formula (2.1.1) should be a variable that decreases with time rather than a constant. Thus, replacing  $k$  with the function  $g(t)$  that decreases over time in Formula (8) gives

$$dP/dt = g(t) \cdot P(1 - P) \tag{10}$$

Suggested solutions for  $g(t)$  include (i) a harmonic function, (ii) an exponential function, among others.

A model by Dodd (i) assumes a harmonic sequence of

$$g(t) = \frac{k}{t + 1} \tag{11}$$

This gives ( $k$  is a constant)

$$dP/dt = \frac{k}{t + 1} P(1 - P) \tag{12}$$

Solving this gives

$$P(t) = 1 / \left\{ 1 + \frac{q_0}{p_0} (t + 1)^{-k} \right\} \tag{13}$$

Ref. [4]<sup>6)</sup>

A model by Hernes (ii) is the expression below,

$$g(t) = A \cdot b^t \quad (14)$$

assumes an exponential function ( $A$  and  $b$  are constants and  $(0 < b < 1)$ ), thus

$$dP/dt = A \cdot b^t P(1 - P) \quad (15)$$

Solving this gives

$$P(t) = 1/(1 + k \cdot a^{bt}) \quad (16)$$

(note that  $k = p_0/(a(1 - p_0))$ ,  $\log a = A/\log b$ ).

It is hard to say if these modified suggestions were examined thoroughly as there were not many practical application examples when compared to simple sigmoid curve.

### 3.3. Modified exponential curve models

Modified exponential curve models are used to represent external information sources that independently influence the society as a fundamental mechanism while sigmoid type models are used to represent interactions among the members of a specific social group. Information sources that influence the society require the probability of interactions among members to be fixed over time. These information sources include mass media, bulletin boards, and salespersons. Suppose that new adoptees can be created through contact with external information source, nonadoptees would also contact these available information sources with a fixed probability. This would lead to the differential equation

$$dP/dt = k(1 - P) \quad (17)$$

This equation indicates a new adoptee is created with a fixed probability  $k$  among nonadoptees  $(1 - P)$ .

By solving Eq. (17),

$$P(t) = 1 - q_0 e^{-kt} \quad (18)$$

Given that initial condition is  $q_0 = 1$ ,

$$P(t) = 1 - e^{-kt} \quad (19)$$

This is a modified exponential curve. A modified exponential curve model can be contrasted to a sigmoid type model across several points. For example, the number of new adoptees ( $dP/dt$ ) per unit of time is maximized when  $P = 0.5$  for a simple sigmoid curve, while the number maximizes at  $P = 0$  for a modified exponential curve. A simple sigmoid curve displays a typical S-shape while a modified exponential curve does not display an S-shape but rather a sharp curve at its beginning.

There are fewer examples of applications of modified exponential curve models compared to sigmoid type models. Note that both used revised modified exponential curve models. This may be because results suggested by modified exponential curve models will not be satisfactory except in unique cases [5–23].

### 3.4. Model that combines two factors

As described previously, sigmoid type models and modified exponential curve models are used to contrast (i) interactions among members within a group, and (ii) contact with external information sources, respectively. This chapter suggested the following model that combines two factors.

$$dP/dt = k_1P(1 - P) + k_2(1 - P) \quad (20)$$

The parameter  $k_1$  is called the coefficient of imitation, corresponding to the factor of the sigmoid curve model. The parameter  $k_2$  is called the coefficient of innovation, corresponding to the factor of the modified exponential curve model. There are two types of new adoptees in a diffusion phenomenon.

New adoptees can be divided into two groups: (a) new adoptees that have been influenced by existing adoptees, and (b) new adoptees who made their own adoption decision independently and with no influence from the existing adoptees. According to Moore (1991), the former are equivalent to followers while the latter are equivalent to opinion leaders in diffusion theory.  $k_1$  and  $k_2$  correspond to (a) and (b).

Solutions for Eq. (20) are slightly complicated as shown:

$$P(t) = \frac{1 - \exp\{-(k_1 + k_2)t\}}{k_1/k_2 \cdot \exp\{(-k_1 + k_2)t\} + 1} \quad (21)$$

However, solutions are subjected to the initial condition of  $P(0) = 0$ . When using this model, diffusion phenomenon types should be considered based on the value of  $k_1/k_2$ . The larger the value of  $k_1/k_2$ , the closer the shape of curve (21) resembles a sigmoid curve. The closer  $k_1/k_2$  comes to 0 and the closer (21) comes to a modified exponential curve. As shown in Section 1, as  $k_1/k_2$  gets larger the closer the curved line comes to resemble the S shape of a sigmoid curve. Note in Section 1, the values of  $k_1$  and  $k_2$  are set to  $P = 0.5$  when  $t = 10$ .

This model has been used widely in marketing research as forecasters of product demand. These models can be described as flexible in that they do not have the strict theoretical rules seen in sigmoid models and modified exponential curve models. Moreover, these models can be applied in many situations beside product diffusion research. However, this model displays several challenges when applied to actual data. Because of the complexity of Formula (21) compared to other models, it can be extremely difficult to estimate parameters by modifying models to reach a linear least-squares method. A short-cut convention method is as a discrete parameter estimation method. However, this will not give the most suitable parameter estimate. Such an approximate estimation method can be problematic when compared with the suitability of other models.

Also in these models, there are two unrestrained parameters even with the initial condition of  $P(0) = 0$ .

In the absence of more thorough examinations, some flexibility is necessary to compare this model to other models.

## 4. Conclusion

The theme of this chapter was to review differential equation models of diffusion phenomena that have been developed in various fields of research and to conduct a comparative review of the models through the application of actual data.

Among the many models created for diffusion phenomena, the models examined in this chapter are large number models that explain macro changes. These models were developed in various research fields. However, comparing these models reveals that although many models are used only for individual fields of study, mathematically similar models are often observed. It is not necessarily apparent the type of model that would be most effective for any specific diffusion phenomena case. Applications of specific models can be observed by reviewing individual research examples. However, the effectiveness of each model is not clarified in the absence of a comparison across various models.

The purpose of this chapter was to apply various models to different types of data, and thereby verify the suitability of each model to specific phenomena.

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