

# ANALYSIS OF THE DOUBLY-CHARMED TETRAQUARK MOLECULAR STATES WITH THE QCD SUM RULES

Qi Xin, Zhi-Gang Wang<sup>1</sup>

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

## Abstract

In the present work, we investigate the scalar, axialvector and tensor doubly-charmed tetraquark molecular states without strange, with strange and with doubly-strange via the QCD sum rules, and try to make assignment of the  $T_{cc}^+$  from the LHCb collaboration in the scenario of molecular states. The predictions favor assigning the  $T_{cc}^+$  to be the lighter  $DD^*$  molecular state with the spin-parity  $J^P = 1^+$  and isospin  $I = 0$ , while the heavier  $DD^*$  molecular state with the spin-parity  $J^P = 1^+$  and isospin  $I = 1$  still escapes experimental detections, the observation of the heavier  $DD^*$  molecular state would shed light on the nature of the  $T_{cc}^+$ . All the predicted doubly-charmed tetraquark molecular states can be confronted to the experimental data in the future.

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Key words: Tetraquark molecular state, QCD sum rules

## 1 Introduction

In recent years, a series of charmonium-like (and bottomonium-like) states were observed by the international high energy experiments [1], those states lie nearby the thresholds of two charmed (or bottom) mesons and have hidden-charm (or hidden-bottom), and are potential excellent candidates for the tetraquark states or tetraquark molecular states. Up to now, those exotic  $X$ ,  $Y$ ,  $Z$  states have posed a big challenge and a big opportunity for the hadron spectroscopy, and provide us with an active research field as an extension of the conventional quark model.

In 2007, the Belle collaboration observed the  $Z_c(4430)$  in the  $\pi^\pm\psi'$  mass spectrum in the  $B \rightarrow K\pi^\pm\psi'$  decays [2]. In 2014, the LHCb collaboration provided the first independent confirmation of the existence of the  $Z_c^-(4430)$  and established its spin-parity  $J^P = 1^+$  by analyzing the  $B^0 \rightarrow \psi'\pi^-K^+$  decays [3]. In 2013, the BESIII collaboration (also the Belle collaboration) observed the  $Z_c(3900)$  in the  $\pi^\pm J/\psi$  mass spectrum in the process  $e^+e^- \rightarrow \pi^+\pi^-J/\psi$  [4] ([5]). The  $Z_c(4430)$  and  $Z_c(3900)$  have non-zero electric charge and have the valence quarks  $c\bar{c}u\bar{d}$  or  $c\bar{c}d\bar{u}$ , and they are very good candidates for the hidden-charm tetraquark (molecular) states.

In 2015, the LHCb collaboration observed the pentaquark candidates  $P_c(4380)$  and  $P_c(4450)$  in the  $J/\psi p$  mass spectrum in the  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays [6]. In 2019, the LHCb collaboration observed the pentaquark candidate  $P_c(4312)$  and confirmed the pentaquark structure  $P_c(4450)$ , and proved that it consists of two narrow overlapping peaks  $P_c(4440)$  and  $P_c(4457)$  [7]. The  $P_c(4312)$ ,  $P_c(4440)$  and  $P_c(4457)$  have positive electric charge and have the valence quarks  $c\bar{c}uud$ , and they are very good candidates for the hidden-charm pentaquark (molecular) states.

In 2017, the LHCb collaboration observed the doubly-charmed baryon  $\Xi_{cc}^{++}$  in the  $\Lambda_c^+ K^- \pi^+ \pi^+$  mass spectrum [8]. The observation of the  $\Xi_{cc}^{++}$  provides us with the crucial experimental input on the strong correlation between the two charm quarks, which is of great importance on the spectroscopy of the doubly-charmed baryon states, tetraquark states and pentaquark states to gain a deeper insight into the mechanism of low energy QCD, and stimulates many interesting works, for example, the doubly-heavy pentaquark states [9].

In the same road, very recently, the LHCb collaboration reported an important observation of the doubly-charmed tetraquark candidate  $T_{cc}^+$  in the  $D^0 D^0 \pi^+$  mass spectrum at the European Physical Society conference on "high energy physics 2021" [10]. Subsequently, the LHCb collaboration formally announced the observation of the exotic state  $T_{cc}^+$  just below the  $D^0 D^{*+}$  threshold

<sup>1</sup>E-mail: zgwang@aliyun.com.

using a data set corresponding to an integrated luminosity of  $9\text{fb}^{-1}$  acquired with the LHCb detector in proton-proton collisions at center-of-mass energies of 7, 8 and 13 TeV [11, 12]. The Breit-Wigner mass and width are  $\delta M_{BW} = -273 \pm 61 \pm 5_{-14}^{+11}$  KeV below the  $D^0 D^{*+}$  threshold and  $\Gamma_{BW} = 410 \pm 165 \pm 43_{-38}^{+18}$  KeV [10, 11, 12]. The exotic state  $T_{cc}^+$  is consistent with the ground state isoscalar tetraquark state with a valence quark content of  $cc\bar{u}\bar{d}$  and spin-parity  $J^P = 1^+$ , and exploring the  $DD$  mass spectrum disfavors interpreting the  $T_{cc}^+$  as the isovector state. The observation of the  $T_{cc}^+$  is a great breakthrough beyond the  $\Xi_{cc}^{++}$  for the hadron physics, and it is the firstly observed doubly-charmed tetraquark candidate with the typical quark configuration  $cc\bar{u}\bar{d}$ , it is a very good candidate for the tetraquark (molecular) state having doubly-charm.

Before and after the observation of the doubly-charmed tetraquark candidate  $T_{cc}^+$ , there have been several works on the doubly-charmed tetraquark (molecular) states using the QCD sum rules [13, 14, 15, 16, 17, 18, 19], the nonrelativistic quark model [20, 21, 22, 23, 24, 25] ([26]), (the coupled-channel analysis [27, 28, 29, 30, 31, 32, 33]), the heavy quark symmetry [34, 35] ([36]), the lattice QCD [37], (the effective Lagrangian approach [38]), etc. The predicted masses vary from about 250 MeV below to 250 MeV above the mass of the measured value of the  $M_{T_{cc}^+}$ . The closeness to the  $DD^*$  threshold makes one think immediately about the possibility of assigning it to be a  $DD^*$  molecular state.

In the past years, the QCD sum rules have become a powerful theoretical approach in exploring the masses and decay widths of the  $X$ ,  $Y$  and  $Z$  states to diagnose their natures, irrespective of the hidden-charm (or hidden-bottom) tetraquark states or tetraquark molecular states. In Refs.[13, 14, 15, 16, 17, 18, 19], the diquark-antidiquark type doubly-heavy tetraquark states are investigated by the QCD sum rules. In Refs.[16, 17], we investigate the scalar, axialvector, vector, tensor doubly-charmed tetraquark states with QCD sum rules systematically by carrying out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way and taking the energy scale formula to determine the suitable energy scales of the QCD spectral densities. The predicted masses for the axialvector-diquark-scalar-antidiquark type and axialvector-diquark-axialvector-antidiquark type axialvector doubly-charmed tetraquark states  $cc\bar{u}\bar{d}$  are  $3.90 \pm 0.09$  GeV, which is in excellent agreement with the experimental value from the LHCb collaboration [10, 11, 12].

If we perform Fierz rearrangements for the axialvector-diquark-scalar-antidiquark type four-quark axialvector currents  $J_\mu(x)$  both in the color and Dirac spinor spaces [16], we can obtain special superpositions of the color-singlet-color-singlet type currents,

$$\begin{aligned}
J_\mu(x) &= \varepsilon^{ijk} \varepsilon^{imn} Q_j^T(x) C \gamma_\mu Q_k(x) \bar{u}_m(x) \gamma_5 C \bar{d}_n^T(x), \\
&= \frac{i}{2} [\bar{u}i\gamma_5 Q \bar{d} \gamma_\mu Q - \bar{d}i\gamma_5 Q \bar{u} \gamma_\mu Q] + \frac{1}{2} [\bar{u}Q \bar{d} \gamma_\mu \gamma_5 Q - \bar{d}Q \bar{u} \gamma_\mu \gamma_5 Q] \\
&\quad - \frac{i}{2} [\bar{u}\sigma_{\mu\nu} \gamma_5 Q \bar{d} \gamma^\nu Q - \bar{d}\sigma_{\mu\nu} \gamma_5 Q \bar{u} \gamma^\nu Q] + \frac{i}{2} [\bar{u}\sigma_{\mu\nu} Q \bar{d} \gamma^\nu \gamma_5 Q - \bar{d}\sigma_{\mu\nu} Q \bar{u} \gamma^\nu \gamma_5 Q], \\
&= \frac{i}{2} J_\mu^1(x) + \frac{1}{2} J_\mu^2(x) - \frac{i}{2} J_\mu^3(x) + \frac{i}{2} J_\mu^4(x), \tag{1}
\end{aligned}$$

where  $Q = c, b$ , the  $i, j, k, m, n$  are color indexes, the  $C$  is the charge conjugation matrix. The currents  $J_\mu^1(x)$ ,  $J_\mu^2(x)$ ,  $J_\mu^3(x)$  and  $J_\mu^4(x)$  couple potentially to the color-singlet-color-singlet type tetraquark states or two-meson scattering states. The decays to the two-meson states can take place through the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism if they are allowed in the phase-space.

In fact, there exist spatial separations between the diquark and antidiquark pair, the currents  $J_\mu(x)$  should be modified to  $J_\mu(x, \epsilon)$ ,

$$J_\mu(x, \epsilon) = \varepsilon^{ijk} \varepsilon^{imn} Q_j^T(x) C \gamma_\mu Q_k(x) \bar{u}_m(x + \epsilon) \gamma_5 C \bar{d}_n^T(x + \epsilon), \tag{2}$$

where the four-vector  $\epsilon^\alpha = (0, \vec{\epsilon})$ . The repulsive barrier or spatial distance between the diquark and antidiquark pair frustrates the Fierz rearrangements or recombinations [39, 40, 41, 42, 43], though

in practical calculations we usually take the local limit  $\epsilon \rightarrow 0$ , we should not take it for granted that the Fierz rearrangements are feasible [43], so we cannot obtain the conclusion that the  $T_{cc}^+$  has the  $D^*D - DD^*$  Fock component according to the properties of the currents  $\bar{q}i\gamma_5 c \otimes \bar{q}\gamma_\mu c$  in Eq.(1). At most, we can only obtain the conclusion tentatively that the  $T_{cc}^+$  has a diquark-antidiquark type tetraquark Fock component with the spin-parity  $J^P = 1^+$  and isospin  $I = 0$  [16]. It is interesting to explore whether or not there exists a color-singlet-color-singlet type tetraquark Fock component for the  $T_{cc}^+$  state, or in the other words, whether or not there exists a color-singlet-color-singlet type tetraquark state which has a mass about 3875 MeV based on the QCD sum rules.

In the present work, we extend our previous works [16, 17] to explore the color-singlet-color-singlet type scalar, axialvector and tensor doubly-charmed tetraquark states via the QCD sum rules by accomplishing the operator product expansion up to the vacuum condensates of dimension 10 and taking account of the  $SU(3)$  breaking effects in a consistent way, and make possible assignment of the  $T_{cc}^+$  in the scenario of the hadronic molecules (or color-singlet-color-singlet type tetraquark states) based on the QCD sum rules. Furthermore, we obtain reliable predictions for the doubly-charmed scalar, axialvector and tensor tetraquark molecular states without strange, with strange and with doubly-strange, which can be confronted to the experimental data in the future.

The article is arranged as follows: we acquire the QCD sum rules for the masses and pole residues of the doubly-charmed tetraquark molecular states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

## 2 QCD sum rules for the doubly-charmed tetraquark molecular states

Firstly, let us write down the two-point correlation functions  $\Pi(p)$ ,  $\Pi_{\mu\nu}(p)$  and  $\Pi_{\mu\nu\alpha\beta}(p)$  in the QCD sum rules,

$$\begin{aligned}\Pi(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J(x) J^\dagger(0) \} | 0 \rangle, \\ \Pi_{\mu\nu}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle, \\ \Pi_{\mu\nu\alpha\beta}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ J_{\mu\nu}(x) J_{\alpha\beta}^\dagger(0) \} | 0 \rangle,\end{aligned}\quad (3)$$

where the currents

$$J(x) = J_{D^*D^*}(x), J_{D^*D_s^*}(x), J_{D_s^*D_s^*}(x), \quad (4)$$

$$\begin{aligned}J_\mu(x) &= J_{DD^*,L,\mu}(x), J_{DD^*,H,\mu}(x), J_{DD_s^*,L,\mu}(x), J_{DD_s^*,H,\mu}(x), J_{D_sD_s^*,\mu}(x), \\ &J_{D_1D_0^*,L,\mu}(x), J_{D_1D_0^*,H,\mu}(x), J_{D_{s1}D_0^*,L,\mu}(x), J_{D_{s1}D_0^*,H,\mu}(x), J_{D_{s1}D_{s0}^*,\mu}(x),\end{aligned}\quad (5)$$

$$\begin{aligned}J_{\mu\nu}(x) &= J_{D^*D^*,L,\mu\nu}(x), J_{D^*D^*,H,\mu\nu}(x), J_{D^*D_s^*,L,\mu\nu}(x), J_{D^*D_s^*,H,\mu\nu}(x), J_{D_s^*D_s^*,L,\mu\nu}(x), \\ &J_{D_s^*D_s^*,H,\mu\nu}(x),\end{aligned}\quad (6)$$

$$\begin{aligned}J_{D^*D^*}(x) &= \bar{u}(x)\gamma_\mu c(x) \bar{d}(x)\gamma^\mu c(x), \\ J_{D^*D_s^*}(x) &= \bar{q}(x)\gamma_\mu c(x) \bar{s}(x)\gamma^\mu c(x), \\ J_{D_s^*D_s^*}(x) &= \bar{s}(x)\gamma_\mu c(x) \bar{s}(x)\gamma^\mu c(x),\end{aligned}\quad (7)$$

$$\begin{aligned}
J_{DD^*,L,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) i\gamma_5 c(x) \bar{d}(x) \gamma_\mu c(x) - \bar{u}(x) \gamma_\mu c(x) \bar{d}(x) i\gamma_5 c(x) \right], \\
J_{DD^*,H,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) i\gamma_5 c(x) \bar{d}(x) \gamma_\mu c(x) + \bar{u}(x) \gamma_\mu c(x) \bar{d}(x) i\gamma_5 c(x) \right], \\
J_{DD_s^*,L,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) i\gamma_5 c(x) \bar{s}(x) \gamma_\mu c(x) - \bar{q}(x) \gamma_\mu c(x) \bar{s}(x) i\gamma_5 c(x) \right], \\
J_{DD_s^*,H,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) i\gamma_5 c(x) \bar{s}(x) \gamma_\mu c(x) + \bar{q}(x) \gamma_\mu c(x) \bar{s}(x) i\gamma_5 c(x) \right], \\
J_{D_s D_s^*,\mu}(x) &= \bar{s}(x) i\gamma_5 c(x) \bar{s}(x) \gamma_\mu c(x), \tag{8}
\end{aligned}$$

$$\begin{aligned}
J_{D_1 D_0^*,L,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) c(x) \bar{d}(x) \gamma_\mu \gamma_5 c(x) + \bar{u}(x) \gamma_\mu \gamma_5 c(x) \bar{d}(x) c(x) \right], \\
J_{D_1 D_0^*,H,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) c(x) \bar{d}(x) \gamma_\mu \gamma_5 c(x) - \bar{u}(x) \gamma_\mu \gamma_5 c(x) \bar{d}(x) c(x) \right], \\
J_{D_{s_1} D_0^*,L,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) c(x) \bar{s}(x) \gamma_\mu \gamma_5 c(x) + \bar{q}(x) \gamma_\mu \gamma_5 c(x) \bar{s}(x) c(x) \right], \\
J_{D_{s_1} D_0^*,H,\mu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) c(x) \bar{s}(x) \gamma_\mu \gamma_5 c(x) - \bar{q}(x) \gamma_\mu \gamma_5 c(x) \bar{s}(x) c(x) \right], \\
J_{D_{s_1} D_{s_0}^*,\mu}(x) &= \bar{s}(x) c(x) \bar{s}(x) \gamma_\mu \gamma_5 c(x), \tag{9}
\end{aligned}$$

$$\begin{aligned}
J_{D^* D^*,L,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) \gamma_\mu c(x) \bar{d}(x) \gamma_\nu c(x) - \bar{u}(x) \gamma_\nu c(x) \bar{d}(x) \gamma_\mu c(x) \right], \\
J_{D^* D^*,H,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{u}(x) \gamma_\mu c(x) \bar{d}(x) \gamma_\nu c(x) + \bar{u}(x) \gamma_\nu c(x) \bar{d}(x) \gamma_\mu c(x) \right], \\
J_{D^* D_s^*,L,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) \gamma_\mu c(x) \bar{s}(x) \gamma_\nu c(x) - \bar{q}(x) \gamma_\nu c(x) \bar{s}(x) \gamma_\mu c(x) \right], \\
J_{D^* D_s^*,H,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{q}(x) \gamma_\mu c(x) \bar{s}(x) \gamma_\nu c(x) + \bar{q}(x) \gamma_\nu c(x) \bar{s}(x) \gamma_\mu c(x) \right], \\
J_{D_s^* D_s^*,L,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{s}(x) \gamma_\mu c(x) \bar{s}(x) \gamma_\nu c(x) - \bar{s}(x) \gamma_\nu c(x) \bar{s}(x) \gamma_\mu c(x) \right], \\
J_{D_s^* D_s^*,H,\mu\nu}(x) &= \frac{1}{\sqrt{2}} \left[ \bar{s}(x) \gamma_\mu c(x) \bar{s}(x) \gamma_\nu c(x) + \bar{s}(x) \gamma_\nu c(x) \bar{s}(x) \gamma_\mu c(x) \right], \tag{10}
\end{aligned}$$

and  $q = u, d$ . We construct the color-singlet-color-singlet type local four-quark currents  $J(x)$  and  $J_\mu(x)$  to interpolate the scalar and axialvector tetraquark molecular states, respectively, and add the subscripts  $L$  and  $H$  to distinguish the lighter and heavier states in the same doublet due to the mixing effects, as direct calculations indicate that there exists such a tendency. In fact, the tensor currents  $J_{L,\mu\nu}(x)$  and  $J_{H,\mu\nu}(x)$  couple potentially to the axialvector and tensor tetraquark molecular states, respectively, we also add the subscripts  $L$  and  $H$  to distinguish the lighter and heavier states in the same doublet. The currents  $J_{DD^*,L,\mu}(x)$  ( $J_{D_1 D_0^*,H,\mu}(x)$ ,  $J_{D^* D^*,L,\mu\nu}(x)$ ) and  $J_{DD^*,H,\mu}(x)$  ( $J_{D_1 D_0^*,L,\mu}(x)$ ,  $J_{D^* D^*,H,\mu\nu}(x)$ ) have the isospin  $(I, I_3) = (0, 0)$  and  $(1, 0)$ , respectively, though they have the same constituents, we can distinguish them therefore the molecular states according to the isospins; while the currents  $J_{DD_s^*,L,\mu}(x)$  ( $J_{D_{s_1} D_0^*,H,\mu}(x)$ ,  $J_{D^* D_s^*,L,\mu\nu}(x)$ ) and  $J_{DD_s^*,H,\mu}(x)$  ( $J_{D_{s_1} D_0^*,L,\mu}(x)$ ,  $J_{D^* D_s^*,H,\mu\nu}(x)$ ) have the isospin  $I = \frac{1}{2}$ . Under parity transform

$\widehat{P}$ , the current operators have the properties,

$$\begin{aligned}
\widehat{P}J(x)\widehat{P}^{-1} &= +J(\tilde{x}), \\
\widehat{P}J_\mu(x)\widehat{P}^{-1} &= -J^\mu(\tilde{x}), \\
\widehat{P}J_{L,\mu\nu}(x)\widehat{P}^{-1} &= -J_{L,\mu\nu}(\tilde{x}), \\
\widehat{P}J_{H,\mu\nu}(x)\widehat{P}^{-1} &= +J_{H,\mu\nu}(\tilde{x}),
\end{aligned} \tag{11}$$

where the coordinates  $x^\mu = (t, \vec{x})$  and  $\tilde{x}^\mu = (t, -\vec{x})$ . We can rewrite Eq.(11) in more explicit form,

$$\begin{aligned}
\widehat{P}J_i(x)\widehat{P}^{-1} &= +J_i(\tilde{x}), \\
\widehat{P}J_{L,0i}(x)\widehat{P}^{-1} &= +J_{L,0i}(\tilde{x}), \\
\widehat{P}J_{H,ij}(x)\widehat{P}^{-1} &= +J_{H,ij}(\tilde{x}),
\end{aligned} \tag{12}$$

$$\begin{aligned}
\widehat{P}J_0(x)\widehat{P}^{-1} &= -J_0(\tilde{x}), \\
\widehat{P}J_{L,ij}(x)\widehat{P}^{-1} &= -J_{L,ij}(\tilde{x}), \\
\widehat{P}J_{H,0i}(x)\widehat{P}^{-1} &= -J_{H,0i}(\tilde{x}),
\end{aligned} \tag{13}$$

where the coordinate indexes  $i, j = 1, 2, 3$ , and we can see clearly that there are both positive and negative components, and they couple potentially to the axialvector/tensor and pseudoscalar/vector tetraquark molecular states, respectively. At the hadron side, we separate the contributions of the axialvector/tensor tetraquark molecular states explicitly by choosing the suitable tensor structures.

Now, let us insert a complete set of intermediate hadronic states with the same quantum numbers as the currents  $J(x)$ ,  $J_\mu(x)$  and  $J_{\mu\nu}(x)$  into the correlation functions  $\Pi(p)$ ,  $\Pi_{\mu\nu}(p)$  and  $\Pi_{\mu\nu\alpha\beta}(p)$  respectively to obtain the hadronic representation [44, 45, 46], and isolate the ground state doubly-charmed scalar/axialvector/tensor tetraquark molecule contributions,

$$\begin{aligned}
\Pi(p) &= \frac{\lambda_T^2}{M_T^2 - p^2} + \dots \\
&= \Pi_T(p^2) + \dots, \\
\Pi_{\mu\nu}(p) &= \frac{\lambda_T^2}{M_T^2 - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots \\
&= \Pi_T(p^2) \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots, \\
\Pi_{L,\mu\nu\alpha\beta}(p) &= \frac{\lambda_T^2}{M_T^2 (M_T^2 - p^2)} \left( p^2 g_{\mu\alpha} g_{\nu\beta} - p^2 g_{\mu\beta} g_{\nu\alpha} - g_{\mu\alpha} p_\nu p_\beta - g_{\nu\beta} p_\mu p_\alpha + g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right) \\
&\quad + \frac{\lambda_{T-}^2}{M_{T-}^2 (M_{T-}^2 - p^2)} \left( -g_{\mu\alpha} p_\nu p_\beta - g_{\nu\beta} p_\mu p_\alpha + g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right) + \dots \\
&= \widetilde{\Pi}_T(p^2) \left( p^2 g_{\mu\alpha} g_{\nu\beta} - p^2 g_{\mu\beta} g_{\nu\alpha} - g_{\mu\alpha} p_\nu p_\beta - g_{\nu\beta} p_\mu p_\alpha + g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right) \\
&\quad + \widetilde{\Pi}_{T-}(p^2) \left( -g_{\mu\alpha} p_\nu p_\beta - g_{\nu\beta} p_\mu p_\alpha + g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right), \\
\Pi_{H,\mu\nu\alpha\beta}(p) &= \frac{\lambda_T^2}{M_T^2 - p^2} \left( \frac{\widetilde{g}_{\mu\alpha} \widetilde{g}_{\nu\beta} + \widetilde{g}_{\mu\beta} \widetilde{g}_{\nu\alpha}}{2} - \frac{\widetilde{g}_{\mu\nu} \widetilde{g}_{\alpha\beta}}{3} \right) + \dots, \\
&= \Pi_T(p^2) \left( \frac{\widetilde{g}_{\mu\alpha} \widetilde{g}_{\nu\beta} + \widetilde{g}_{\mu\beta} \widetilde{g}_{\nu\alpha}}{2} - \frac{\widetilde{g}_{\mu\nu} \widetilde{g}_{\alpha\beta}}{3} \right) + \dots,
\end{aligned} \tag{14}$$

where the pole residues  $\lambda_T$  and  $\lambda_{T^-}$  are defined by

$$\begin{aligned}
\langle 0|J(0)|T_{cc}(p)\rangle &= \lambda_T, \\
\langle 0|J_\mu(0)|T_{cc}(p)\rangle &= \lambda_T \varepsilon_\mu, \\
\langle 0|J_{L,\mu\nu}(0)|T_{cc}(p)\rangle &= \frac{\lambda_T}{M_T} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^\alpha p^\beta, \\
\langle 0|J_{L,\mu\nu}(0)|T_{cc}^-(p)\rangle &= \frac{\lambda_{T^-}}{M_{T^-}} (\varepsilon_\mu p_\nu - \varepsilon_\nu p_\mu), \\
\langle 0|J_{H,\mu\nu}(0)|T_{cc}(p)\rangle &= \lambda_T \varepsilon_{\mu\nu},
\end{aligned} \tag{15}$$

the  $\varepsilon_\mu/\varepsilon_{\mu\nu}$  are the polarization vectors of the doubly-charmed axialvector/tensor tetraquark molecular states.

In the present work, we accomplish the operator product expansion up to the vacuum condensates of dimension 10 and take account of the vacuum condensates  $\langle \bar{q}q \rangle$ ,  $\langle \frac{\alpha_s GG}{\pi} \rangle$ ,  $\langle \bar{q}g_s \sigma Gq \rangle$ ,  $\langle \bar{q}q \rangle^2$ ,  $\langle \bar{q}q \rangle \langle \frac{\alpha_s GG}{\pi} \rangle$ ,  $\langle \bar{q}q \rangle \langle \bar{q}g_s \sigma Gq \rangle$ ,  $\langle \bar{q}g_s \sigma Gq \rangle^2$  and  $\langle \bar{q}q \rangle^2 \langle \frac{\alpha_s GG}{\pi} \rangle$  with the assumption of vacuum saturation in a consistent way [47], where  $q = u, d$  or  $s$ . It is straightforward but tedious to accomplish the operator product expansion, for the technical details, one can consult Refs.[48, 49, 50, 51]. Moreover, we neglect the small masses of the  $u$  and  $d$  quarks, and take account of the terms proportional to  $m_s$  considering the light flavor  $SU(3)$  mass-breaking effects.

Then we match the hadron side with the QCD side of the correlation functions  $\Pi_T(p^2)$  and  $p^2 \tilde{\Pi}_T(p^2)$  below the continuum thresholds  $s_0$  and accomplish Borel transform in regard to the variable  $P^2 = -p^2$  to obtain the QCD sum rules:

$$\lambda_T^2 \exp\left(-\frac{M_T^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right), \tag{16}$$

the QCD spectral densities  $\rho_{QCD}(s)$  are available via contacting us via E-mail. We differentiate Eq.(16) in regard to  $\tau = \frac{1}{T^2}$ , and obtain the QCD sum rules for the masses of the scalar/axialvector/tensor doubly-charmed tetraquark molecular states  $T_{cc}$  without strange, with strange and with doubly-strange,

$$M_T^2 = -\frac{\int_{4m_c^2}^{s_0} ds \frac{d}{d\tau} \rho_{QCD}(s) \exp(-\tau s)}{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp(-\tau s)} \Big|_{\tau=\frac{1}{T^2}}. \tag{17}$$

We acquire the masses and pole residues by solving the coupled equations (16)-(17).

### 3 Numerical results and discussions

We take the conventional values of the vacuum condensates  $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$ ,  $\langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ ,  $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$ ,  $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$ ,  $\langle \frac{\alpha_s GG}{\pi} \rangle = 0.012 \pm 0.004 \text{ GeV}^4$  at the energy scale  $\mu = 1 \text{ GeV}$  [44, 45, 46, 52], and take the  $\overline{MS}$  masses  $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$  and  $m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV}$  from the Particle Data Group [1]. In addition, we take account of the energy-scale dependence of the quark condensates,

mixed quark condensates and  $\overline{MS}$  masses [53],

$$\begin{aligned}
\langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\
\langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33-2n_f}}, \\
\langle \bar{q}g_s\sigma Gq \rangle(\mu) &= \langle \bar{q}g_s\sigma Gq \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\
\langle \bar{s}g_s\sigma Gs \rangle(\mu) &= \langle \bar{s}g_s\sigma Gs \rangle(1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33-2n_f}}, \\
m_c(\mu) &= m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33-2n_f}}, \\
m_s(\mu) &= m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33-2n_f}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2 t} + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \tag{18}
\end{aligned}$$

where  $t = \log \frac{\mu^2}{\Lambda^2}$ ,  $b_0 = \frac{33-2n_f}{12\pi}$ ,  $b_1 = \frac{153-19n_f}{24\pi^2}$ ,  $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$ ,  $\Lambda = 213$  MeV, 296 MeV and 339 MeV for the quark flavors  $n_f = 5, 4$  and  $3$ , respectively [1]. There are  $u, d, s$  and  $c$  quarks, we choose the quark flavor numbers  $n_f = 4$ , and evolve the QCD spectral densities  $\rho_{QCD}(s)$  to the suitable energy scales  $\mu$  to extract the molecule masses.

In the hidden-heavy  $Q\bar{Q}q\bar{q}'$  systems and doubly-heavy  $QQ\bar{q}\bar{q}'$  systems, we can introduce the effective heavy quark masses  $\mathbb{M}_Q$  and divide the tetraquark (molecular) states into both the heavy degrees of freedoms and light degrees of freedoms. We neglect the small  $u$  and  $d$  quark masses, then obtain the heavy degrees of freedoms  $2\mathbb{M}_Q$  and light degrees of freedoms  $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2\mathbb{M}_Q)^2}$  [16, 17, 49, 50, 51, 54, 55, 56]. We can also introduce the effective  $s$ -quark mass  $\mathbb{M}_s$  according to the light flavor  $SU(3)$  breaking effects, then the light degrees of freedoms  $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2\mathbb{M}_c)^2 - k\mathbb{M}_s}$  with  $k = 0, 1, 2$  [57, 58]. In the present work, we choose the effective  $c/s$ -quark masses  $\mathbb{M}_c = 1.82$  GeV and  $\mathbb{M}_s = 0.2$  GeV, and use the modified energy scale formula  $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2\mathbb{M}_c)^2 - k\mathbb{M}_s}$  to enhance the pole contributions and improve the convergent behavior of the operator product expansion.

The energy gaps between the ground states and first radial excited states are about  $0.5 \sim 0.6$  GeV for the traditional heavy mesons and quarkonia [1], we tentatively take the constraint  $\sqrt{s_0} = M_T + 0.4 \sim 0.6$  GeV, as there are two color-singlet clusters, each cluster has the same quantum numbers as that of the corresponding heavy meson, and search for the continuum threshold parameters  $s_0$  and Borel parameters  $T^2$  to satisfy pole dominance at the hadron side and convergence of the operator product expansion at the QCD side via trial and error.

Eventually, we obtain the Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities, pole contributions, and the contributions of the highest dimensional vacuum condensates, which are presented clearly in Table 1. In the Table, we use the notations  $D, D^*, \dots$  to represent the color-singlet constituents in the molecular states with the same quantum numbers as the mesons  $D, D^*, \dots$ . From the Table, we can see clearly that the pole contributions or ground state contributions, which are defined by

$$\text{pole} = \frac{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{\infty} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right)}, \tag{19}$$

are larger than (40–60)% at the hadron side, the central values are larger than 50%, the elementary criterion of the pole dominance is satisfied very good. Moreover, the contributions of the vacuum condensates of dimension 10 are  $|D(10)| < 1\%$  or  $\ll 1\%$  at the QCD side, where the contributions from the QCD spectral densities  $\rho_{QCD,n}(s)$  having the vacuum condensates of dimension  $n$  are defined by

$$D(n) = \frac{\int_{4m_c^2}^{s_0} ds \rho_{QCD,n}(s) \exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0} ds \rho_{QCD}(s) \exp\left(-\frac{s}{T^2}\right)}, \quad (20)$$

the convergent behaviors of the operator product expansion are very good, the other elementary criterion is also satisfied very good.

Now we take account of all the uncertainties of the input parameters, and obtain the masses and pole residues of the scalar/axialvector/tensor doubly-charmed tetraquark molecular states without strange, with strange and with doubly-strange, which are presented explicitly in Table 2. From the Tables 1–2, we can see clearly that the modified energy scale formula  $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2M_c)^2} - kM_s$  is well satisfied.

In Fig.1, as an example, we plot the masses of the axialvector doubly-charmed tetraquark molecular states  $(D^*D - DD^*)_L$  and  $(D^*D + DD^*)_H$  with variations of the Borel parameters at much larger ranges than the Borel widows. From the figure, we can see clearly that there appear very flat platforms in the Borel windows, the regions between the two short perpendicular lines. The uncertainties originate from the Borel parameters are rather small, and we expect to make reliable predictions. In the present work, we choose the uniform constraints  $\sqrt{s_0} = M_T + 0.50 \sim 0.55 \pm 0.10$  GeV and  $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2M_c)^2} - kM_s$ , uniform pole contributions (40–60)%, uniform convergent behaviors  $|D(10)| < 1\%$  or  $\ll 1\%$ , and very flat Borel platforms, and we expect to make robust predictions.

There exist both a lighter state and a heavier state for the  $cc\bar{u}\bar{d}$  and  $cc\bar{q}\bar{s}$  tetraquark molecular states, the lighter state  $(D^*D - DD^*)_L$  with the isospin  $(I, I_3) = (0, 0)$  has a mass  $3.88 \pm 0.11$  GeV, which is in excellent agreement with the mass of the doubly-charmed tetraquark candidate  $T_{cc}^+$  from the LHCb collaboration [10, 11, 12], and supports assigning the  $T_{cc}^+$  to be the  $(D^*D - DD^*)_L$  molecular state, as the  $T_{cc}^+$  has the isospin  $I = 0$ . In other words, the exotic state  $T_{cc}^+$  maybe have a  $(D^*D - DD^*)_L$  Fock component. The heavier state  $(D^*D + DD^*)_H$  with the isospin  $(I, I_3) = (1, 0)$  has a mass  $3.90 \pm 0.11$  GeV, the central value lies slightly above the  $DD^*$  threshold, the strong decays to the final states  $DD\pi$  are kinematically allowed but with small phase-space. While in the scenario of the tetraquark states, the doubly-charmed tetraquark states with the isospin  $(I, I_3) = (1, 0)$  and  $(0, 0)$  have degenerated masses  $3.90 \pm 0.09$  GeV [16, 17].

If we choose the same input parameters, the  $DD^*$  molecular state with the isospin  $I = 1$  has slightly larger mass than the corresponding molecule with the isospin  $I = 0$ , it is indeed that the isoscalar  $DD^*$  molecular state is lighter. To reduce systematic uncertainties and make more reliable predictions, we choose the same pole contributions to acquire all the molecule masses, irrespective of the isospins  $I = 1, \frac{1}{2}$  and 0, and tentatively assign the LHCb's  $T_{cc}^+$  as the molecular state with the isospin  $I = 0$ , because exploring the  $DD$  mass spectrum disfavors interpreting the  $T_{cc}^+$  as the isovector state [11, 12].

From Table 2, we can see clearly that there exists a small mass gap between the centroids of the isoscalar  $(D^*D - DD^*)_L$  and isovector  $(D^*D + DD^*)_H$  states, about 0.02 GeV, while the mass gap between the centroids of the two states  $(D_s^*D - D_sD^*)_L$  and  $(D_s^*D + D_sD^*)_H$  with the isospin  $I = \frac{1}{2}$  is about 0.01 GeV. The small but finite mass gaps maybe originate from the spin-spin and tensor interactions between the constituent quarks, as the isospin breaking effect of the  $u$  and  $d$  quarks is about  $\delta m = 3.4$  MeV at the energy scale  $\mu = 1$  GeV [1], which cannot account for the mass gaps 0.02 GeV and 0.01 GeV. For the molecular states  $(D_0^*D_1 + D_1D_0^*)_L$ ,  $(D_0^*D_1 - D_1D_0^*)_H$ ,  $(D_0^*D_{s1} + D_{s0}^*D_1)_L$  and  $(D_0^*D_{s1} - D_{s0}^*D_1)_H$ , there exists a P-wave in the color-singlet constituents, the P-wave is embodied implicitly in the underlined  $\underline{\gamma_5}$  in the scalar currents



$\bar{q}i\gamma_5\gamma_5c$ ,  $\bar{s}i\gamma_5\gamma_5c$  and axialvector currents  $\bar{q}\gamma_\mu\gamma_5c$ ,  $\bar{s}\gamma_\mu\gamma_5c$ , as multiplying  $\gamma_5$  to the pseudoscalar currents  $\bar{q}i\gamma_5c$ ,  $\bar{s}i\gamma_5c$  and vector currents  $\bar{q}\gamma_\mu c$ ,  $\bar{s}\gamma_\mu c$  changes their parity. We should introduce the spin-orbit interactions to account for the large mass gaps between the lighter and heavier states ( $L, H$ ), i.e.  $((D_0^*D_1 + D_1D_0^*)_L, (D_0^*D_1 - D_1D_0^*)_H), ((D_0^*D_{s1} + D_{s0}^*D_1)_L, (D_0^*D_{s1} - D_{s0}^*D_1)_H)$ .

From Tables 1-2, we can see that the continuum threshold parameters  $\sqrt{s_0} = M_T + 0.50 \sim 0.55 \pm 0.10$  GeV, one maybe worry about the contaminations from the two-meson or three-meson scattering states, which contribute self-energies. In fact, the renormalized self-energies contribute a finite imaginary part to modify the dispersion relation, we can take account of the finite width effects by the simple replacement of the hadronic spectral densities,

$$\lambda_T^2 \delta(s - M_T^2) \rightarrow \lambda_T^2 \frac{1}{\pi} \frac{M_T \Gamma_T(s)}{(s - M_T^2)^2 + M_T^2 \Gamma_T^2(s)}, \quad (21)$$

where the  $\Gamma_T(s)$  are the energy dependent widths. All in all, we can take account of the two-meson or three-meson scattering states reasonably by adding a finite width to the tetraquark (molecular) states. Direct calculations indicate that the finite widths cannot affect the masses  $M_T$  significantly, and can be safely absorbed into the pole residues saving the physical widths are not large enough. For detailed discussions about this subject, one can consult Ref.[59]. Furthermore, we choose the local four-quark currents, which couple potentially to the molecular states rather than to the scattering states, although the couplings to the scattering states are unavoidable considering the same quantum numbers. The traditional mesons are spatial extended objects and have average spatial sizes  $\sqrt{\langle r^2 \rangle} \neq 0$ . In the local limit  $r \rightarrow 0$ , the  $D$  and  $D^*$  mesons lose themselves and merge into color-singlet-color-singlet type tetraquark states. We expect that the  $D$ ,  $D^*$  and  $T_{cc}$  mesons have average spatial sizes of the same order, the couplings to the continuum states are small, as the overlappings of the wave-functions are rather small. In the QCD sum rules, we use the nomenclature "molecular states" in the sense of color-singlet-color-singlet type structures, and the color-singlet clusters have the same quantum numbers as the traditional mesons, in fact, they are also compact tetraquark states according to the local currents, just like the color-antitriplet-color-triplet (diquark-antidiquark) type tetraquark states. If they are the usually called "molecular states" indeed, they should have the spatial extension about (or larger than) 1 fm, which is much larger than the size of the traditional mesons, and it is not robust to interpolate them with the local four-quark currents.

The decay properties of the doubly-charmed tetraquark molecular states are rather simple as the decays can take place easily through the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism if kinematically allowed, we can search for those scalar/axialvector/tensor doubly-charmed tetraquark molecular states without strange, with strange and with doubly-strange presented in Table 2 in the  $DD\pi$ ,  $DD\gamma$ ,  $DD\pi\pi$ ,  $DD_s\gamma$ ,  $DD_s\pi\pi$ ,  $DD_s\pi\pi\pi$ ,  $D_sD_s\gamma$ ,  $D_sD_s\pi\pi\pi$  invariant mass distributions at the BESIII, LHCb, Belle II, CEPC, FCC, ILC in the future. For example, the production cross sections of the tetraquark state  $cc\bar{u}\bar{d}$  with the spin-parity  $J^P = 1^+$  were explored before the observation of the  $T_{cc}^+$  by the LHCb collaboration [60].

Now we explore the three-body strong decays  $T_{cc}^+ \rightarrow D^0 D^{*+} \rightarrow D^0 D^0 \pi^+$ ,  $D^0 D^+ \pi^0$ , and  $T_{cc}^+ \rightarrow D^+ D^{*0} \rightarrow D^+ D^0 \pi^0$  to acquire additional support of assigning the  $T_{cc}^+$  to be the  $D^*D - DD^*$  molecular state. We write down the three-point correlation function  $\Pi_{\alpha\mu}(p, q)$ ,

$$\Pi_{\alpha\mu}(p, q) = i^2 \int d^4x d^4y e^{ip \cdot x} e^{iq \cdot y} \langle 0 | T \left\{ J_5^D(x) J_\alpha^{D^*}(y) J_\mu^{DD^*,L}(0) \right\} | 0 \rangle, \quad (22)$$

where the currents,

$$\begin{aligned} J_5^D(x) &= \bar{c}(x) i\gamma_5 u(x), \\ J_\alpha^{D^*}(y) &= \bar{c}(y) \gamma_\alpha d(y), \end{aligned} \quad (23)$$

interpolate the  $D$  and  $D^*$  mesons, respectively. At the hadron side, we isolate the ground state

$T_{cc}$	Isospin	$T^2(\text{GeV}^2)$	$\sqrt{s_0}(\text{GeV})$	$\mu(\text{GeV})$	pole	$ D(10) $
$D^*D^*$	1	2.8 – 3.2	$4.55 \pm 0.10$	1.7	(41 – 61)%	< 1%
$D_s^*D^*$	$\frac{1}{2}$	2.9 – 3.3	$4.65 \pm 0.10$	1.7	(42 – 62)%	$\ll$ 1%
$D_s^*D_s^*$	0	3.2 – 3.5	$4.80 \pm 0.10$	1.8	(42 – 61)%	$\ll$ 1%
$D^*D - DD^*$	0	2.9 – 3.3	$4.45 \pm 0.10$	1.4	(42 – 62)%	$\ll$ 1%
$D^*D + DD^*$	1	2.6 – 3.0	$4.40 \pm 0.10$	1.4	(42 – 63)%	$\ll$ 1%
$D_s^*D - D_sD^*$	$\frac{1}{2}$	3.0 – 3.4	$4.50 \pm 0.10$	1.5	(40 – 62)%	< 1%
$D_s^*D + D_sD^*$	$\frac{1}{2}$	2.9 – 3.3	$4.50 \pm 0.10$	1.5	(40 – 60)%	$\ll$ 1%
$D_s^*D_s$	0	3.0 – 3.4	$4.60 \pm 0.10$	1.5	(41 – 63)%	$\ll$ 1%
$D_0^*D_1 - D_1D_0^*$	0	5.6 – 7.0	$6.35 \pm 0.10$	4.6	(41 – 60)%	$\ll$ 1%
$D_0^*D_1 + D_1D_0^*$	1	4.7 – 6.1	$5.90 \pm 0.10$	4.0	(42 – 61)%	$\ll$ 1%
$D_0^*D_{s1} - D_{s0}^*D_1$	$\frac{1}{2}$	5.8 – 7.2	$6.50 \pm 0.10$	4.6	(43 – 60)%	< 1%
$D_0^*D_{s1} + D_{s0}^*D_1$	$\frac{1}{2}$	4.7 – 6.1	$6.05 \pm 0.10$	4.0	(42 – 62)%	< 1%
$D_{s1}D_{s0}^*$	0	4.9 – 6.3	$6.20 \pm 0.10$	4.0	(43 – 61)%	< 1%
$D^*D^* - D^*D^*$	0	3.2 – 3.6	$4.55 \pm 0.10$	1.7	(42 – 61)%	< 1%
$D^*D^* + D^*D^*$	1	3.0 – 3.4	$4.55 \pm 0.10$	1.7	(41 – 60)%	< 1%
$D_s^*D^* - D_s^*D^*$	$\frac{1}{2}$	3.3 – 3.7	$4.65 \pm 0.10$	1.7	(40 – 59)%	$\ll$ 1%
$D_s^*D^* + D_s^*D^*$	$\frac{1}{2}$	3.1 – 3.5	$4.65 \pm 0.10$	1.7	(42 – 61)%	< 1%
$D_s^*D_s^* - D_s^*D_s^*$	0	3.6 – 4.0	$4.80 \pm 0.10$	1.8	(40 – 60)%	$\ll$ 1%
$D_s^*D_s^* + D_s^*D_s^*$	0	3.4 – 3.9	$4.80 \pm 0.10$	1.8	(41 – 61)%	< 1%

Table 1: The Borel parameters, continuum threshold parameters, energy scales of the QCD spectral densities, pole contributions, and the contributions of the vacuum condensates of dimension 10 for the ground state doubly-charmed axialvector tetraquark molecular states.

$T_{cc}$	Isospin	$M_T(\text{GeV})$	$\lambda_T(\text{GeV}^5)$
$D^*D^*$	1	$4.04 \pm 0.11$	$(4.99 \pm 0.66) \times 10^{-2}$
$D_s^*D^*$	$\frac{1}{2}$	$4.12 \pm 0.10$	$(5.74 \pm 0.78) \times 10^{-2}$
$D_s^*D_s^*$	0	$4.22 \pm 0.10$	$(7.46 \pm 0.89) \times 10^{-2}$
$D^*D - DD^*$	0	$3.88 \pm 0.11$	$(1.92 \pm 0.29) \times 10^{-2}$
$D^*D + DD^*$	1	$3.90 \pm 0.11$	$(1.50 \pm 0.22) \times 10^{-2}$
$D_s^*D - D_sD^*$	$\frac{1}{2}$	$3.97 \pm 0.10$	$(2.40 \pm 0.41) \times 10^{-2}$
$D_s^*D + D_sD^*$	$\frac{1}{2}$	$3.98 \pm 0.11$	$(2.06 \pm 0.30) \times 10^{-2}$
$D_s^*D_s$	0	$4.10 \pm 0.12$	$(2.31 \pm 0.45) \times 10^{-2}$
$D_0^*D_1 - D_1D_0^*$	0	$5.79 \pm 0.15$	$(2.13 \pm 0.19) \times 10^{-1}$
$D_0^*D_1 + D_1D_0^*$	1	$5.37 \pm 0.13$	$(1.30 \pm 0.11) \times 10^{-1}$
$D_0^*D_{s1} - D_{s0}^*D_1$	$\frac{1}{2}$	$5.93 \pm 0.27$	$(2.80 \pm 0.33) \times 10^{-1}$
$D_0^*D_{s1} + D_{s0}^*D_1$	$\frac{1}{2}$	$5.54 \pm 0.20$	$(1.51 \pm 0.16) \times 10^{-1}$
$D_{s1}D_{s0}^*$	0	$5.67 \pm 0.27$	$(1.77 \pm 0.27) \times 10^{-1}$
$D^*D^* - D^*D^*$	0	$4.00 \pm 0.11$	$(2.47 \pm 0.32) \times 10^{-2}$
$D^*D^* + D^*D^*$	1	$4.02 \pm 0.11$	$(2.83 \pm 0.30) \times 10^{-2}$
$D_s^*D^* - D_s^*D^*$	$\frac{1}{2}$	$4.08 \pm 0.10$	$(2.81 \pm 0.40) \times 10^{-2}$
$D_s^*D^* + D_s^*D^*$	$\frac{1}{2}$	$4.10 \pm 0.11$	$(3.19 \pm 0.44) \times 10^{-2}$
$D_s^*D_s^* - D_s^*D_s^*$	0	$4.19 \pm 0.09$	$(3.49 \pm 0.49) \times 10^{-2}$
$D_s^*D_s^* + D_s^*D_s^*$	0	$4.20 \pm 0.10$	$(4.00 \pm 0.53) \times 10^{-2}$

Table 2: The masses and pole residues of the ground state doubly-charmed tetraquark molecular states.

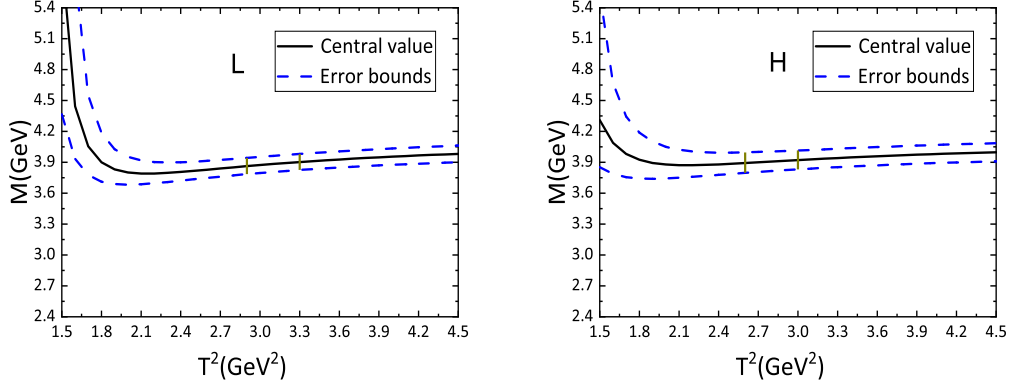


Figure 1: The masses with variations of the Borel parameters for the axialvector tetraquark molecular states, where the  $L$  and  $H$  denote the lighter and heavier  $DD^*$  states, respectively.

contributions,

$$\begin{aligned} \Pi_{\alpha\mu}(p, q) = & \left\{ \frac{f_D M_D^2 f_{D^*} M_{D^*} \lambda_T G}{\sqrt{2} m_c} \frac{1}{(M_T^2 - p'^2)(M_D^2 - p^2)(M_{D^*}^2 - q^2)} \right. \\ & \left. + \frac{C_{DD^*}}{(M_D^2 - p^2)(M_{D^*}^2 - q^2)} + \dots \right\} g_{\alpha\mu} + \dots \end{aligned} \quad (24)$$

$$\begin{aligned} \langle 0 | J_5^D(0) | D(p) \rangle &= \frac{f_D M_D^2}{m_c}, \\ \langle 0 | J_\alpha^{D^*}(0) | D^*(q) \rangle &= f_{D^*} M_{D^*} \xi_\alpha, \\ \langle D(p) D^*(q) | T_{cc}(p') \rangle &= i \xi^*(q) \cdot \varepsilon(p') \frac{G}{\sqrt{2}}, \end{aligned} \quad (25)$$

the  $\xi_\alpha$  is polarization vector of the  $D^*$  meson, the  $G$  is the hadronic coupling constant in the lagrangian,

$$\mathcal{L} = \frac{G}{\sqrt{2}} T_{cc,\alpha}^{+\dagger} (D^0 D^{*+, \alpha} - D^+ D^{*0, \alpha}) + h.c.. \quad (26)$$

For the definition of the unknown constant  $C_{DD^*}$ , one can consult Refs.[61, 62]. We choose the tensor structure  $g_{\alpha\mu}$  to study the hadronic coupling constant  $G$  and accomplish the operator product expansion up to the vacuum condensates of dimension 10 at the QCD side. Then we take the rigorous quark-hadron duality below the continuum thresholds, set  $p'^2 = 4p^2$ , and perform the double Borel transforms with respect to the variables  $P^2 = -p^2$  and  $Q^2 = -q^2$ , respectively to obtain the QCD sum rules [61, 62],

$$\begin{aligned} & \frac{f_D M_D^2 f_{D^*} M_{D^*} \lambda_T G}{4\sqrt{2} m_c} \frac{1}{\widetilde{M}_T^2 - M_D^2} \left[ \exp\left(-\frac{M_D^2}{T_1^2}\right) - \exp\left(-\frac{\widetilde{M}_T^2}{T_1^2}\right) \right] \exp\left(-\frac{M_{D^*}^2}{T_2^2}\right) \\ & + C_{DD^*} \exp\left(-\frac{M_D^2}{T_1^2} - \frac{M_{D^*}^2}{T_2^2}\right) = \int_{m_c^2}^{s_D^0} ds \int_{m_c^2}^{u_{D^*}^0} du \rho_{QCD}(s, u) \exp\left(-\frac{s}{T_1^2} - \frac{u}{T_2^2}\right), \end{aligned} \quad (27)$$

where  $\widetilde{M}_T^2 = \frac{M_T^2}{4}$ , the  $s_D^0$  and  $u_{D^*}^0$  are the continuum threshold parameters, the  $T_1^2$  and  $T_2^2$  are the Borel parameters, and the  $\rho_{QCD}(s, u)$  is the QCD spectral density, which is neglected for simplicity.

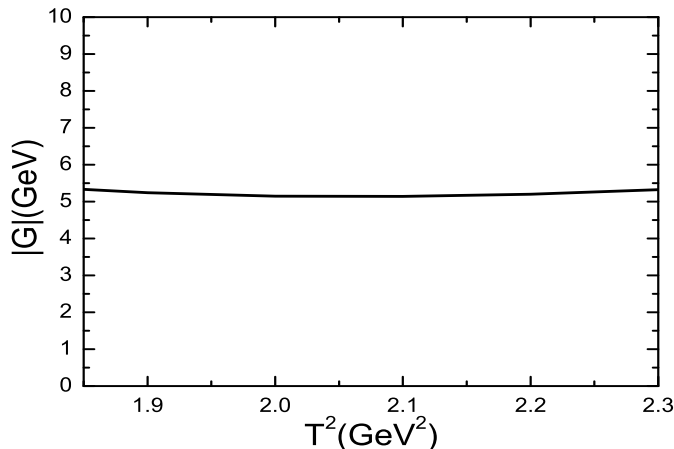


Figure 2: The hadronic coupling constant  $G$  with variations of the Borel parameter  $T^2$ .

Again, we set  $T_1^2 = T_2^2 = T^2$  for simplicity [62], and choose the hadronic parameters  $M_D = 1.87$  GeV,  $f_D = 208$  MeV,  $s_D^0 = (2.4 \text{ GeV})^2$ ,  $M_{D^*} = 2.01$  GeV,  $f_{D^*} = 263$  MeV,  $u_{D^*}^0 = (2.5 \text{ GeV})^2$  from the QCD sum rules [63]. The unknown parameter is fitted to be  $\sqrt{2}C_{DD^*} = -0.0215 \text{ GeV}^8$  to obtain platform in the Borel window  $T^2 = (1.9 - 2.3) \text{ GeV}^2$ , see Fig.2. Finally, we acquire the central value,

$$|G| = 5.2 \text{ GeV}, \quad (28)$$

and  $\frac{|G|}{\sqrt{2}} = 3.7 \text{ GeV}$ , which is in very good agreement with the values  $g_{T_{cc}D^*+D^0} = 3658.30 \text{ MeV}$  and  $g_{T_{cc}D^*0D^+} = -3921.04 \text{ MeV}$  from the unitary method with the coupled channel effects [32], which lead to remarkable agreement with the experimental data on the decays  $T_{cc}^+ \rightarrow D^0 D^0 \pi^+$  [11, 12]. Now we can obtain the conclusion tentatively that the present predictions support assigning the  $T_{cc}^+$  as the  $D^*D - DD^*$  molecular state, the central value  $|G| = 5.2 \text{ GeV}$  serves as a crude estimation, more detailed studies (including error analysis) of the decays of all the molecular states shown in Table 2 will be our next work.

Generally speaking, a physical tetraquark (molecular) state maybe have several Fock components, we can choose any current with the same quark structure as one of the Fock components to interpolate this tetraquark (molecular) state due to the non-vanishing current-hadron coupling constant. In both the scenarios tetraquark states and molecular states, we can reproduce the mass of the  $T_{cc}^+$ , and therefore obtain the conclusion tentatively that there maybe exist a color-singlet-color-singlet type molecular state and a color-antitriplet-color-triplet type tetraquark state, which have almost degenerated masses, or exist one axialvector tetraquark state with both the color-singlet-color-singlet type and color-antitriplet-color-triplet type Fock components. The present work does not exclude assigning the  $T_{cc}^+$  as the diquark-antidiquark type tetraquark state. More experimental and theoretical works are still needed before reaching definite conclusion.

## 4 Conclusion

In the present work, we investigate the scalar/axialvector/tensor doubly-charmed tetraquark molecular states without strange, with strange and with doubly-strange via the QCD sum rules by performing the operator product expansion up to the vacuum condensates of dimension 10 and taking account of all the light flavor  $SU(3)$  breaking effects in a consistent way. We use the modified energy

scale formula to acquire the suitable energy scales of the QCD spectral densities, and obtain the masses of the ground state scalar/axialvector/tensor doubly-charmed tetraquark molecular states. The present calculations favor assigning the doubly-charmed tetraquark candidate  $T_{cc}^+$  to be the lighter  $D^*D - DD^*$  tetraquark molecular state with the spin-parity  $J^P = 1^+$  and isospin  $I = 0$ , while the heavier  $D^*D + DD^*$  molecular state with the spin-parity  $J^P = 1^+$  and  $I = 1$  still escapes experimental detections, the observation of the heavier  $D^*D + DD^*$  molecular state would shed light on the nature of the  $T_{cc}^+$ , because in the scenario of the tetraquark states, the doubly-charmed tetraquark states with the isospin  $(I, I_3) = (1, 0)$  and  $(0, 0)$  have degenerated masses. Moreover, we make predictions for other scalar/axialvector/tensor doubly-charmed tetraquark molecular states, which can be searched for in the  $DD\pi$ ,  $DD\gamma$ ,  $DD\pi\pi$ ,  $DD_s\gamma$ ,  $DD_s\pi\pi$ ,  $DD_s\pi\pi\pi$ ,  $D_sD_s\gamma$ ,  $D_sD_s\pi\pi\pi$  invariant mass distributions at the BESIII, LHCb, Belle II, CEPC, FCC, ILC in the future.

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