

# Analysis of the Dynamic Response of Multielement Semiconductor Lasers

ROBERT J. LANG, STUDENT MEMBER, IEEE, AND AMNON YARIV, FELLOW, IEEE

**Abstract**—We present a derivation of the dynamic response of a semiconductor laser consisting of more than one active element. We show that the amplitude and phase of the modulated cavity adiabatically follows the complex resonance of the composite cavity; and using this relation, plus linearized carrier equations, we calculate the parameters characterizing the modulation response of the composite system. In the process, “effective” differential gain constants and linewidth enhancement factors arise which take the place of the corresponding parameters in single-element lasers. In the case of a two-section laser, we show that frequency chirping under modulation is present except under special conditions; we identify those conditions and show how chirping can be avoided.

## I. INTRODUCTION

SINGLE-mode semiconductor lasers (SL's) are desirable for use in fiber optic links because their narrow frequency spread translates into a minimum of pulse spreading in a dispersive fiber. However, most simple Fabry-Perot SL's oscillate in a number of longitudinal modes, particularly under high-frequency modulation. To provide suppression of all but a single mode, more complicated resonator structures have been proposed and demonstrated, including distributed feedback lasers [1], distributed Bragg reflectors [2], axially coupled cavity lasers [3], [4], and laterally coupled cavity lasers [5].

The analysis of such structures is correspondingly more difficult than that of a simple Fabry-Perot cavity. The steady-state lasing frequency can be analyzed by performing a roundtrip analysis of each element of the system including intercavity coupling [5]–[12]. The result of such analyses is typically a nonlinear equation which implicitly defines the lasing frequency  $\omega$ ; the equation depends upon the gain and refractive indexes (and hence, carrier density) of the various cavities, and the requirement that  $\omega$  be real (for steady-state operation) imposes a constraint on the set of carrier densities. The full time evolution under large-signal modulation must include the (also nonlinear) carrier rate equations for the different active elements in the system. In one approach, the electric field in each section of the laser is taken as a dynamical variable, and rate equations for both the electric field and carrier densities are solved for a transient excitation [6], [9], [12]. In the

other approach, a single field equation is used which describes a mode of the composite resonator [7]. The latter treatment has the advantage of including fewer dynamical variables. In either case, the rate equations are nonlinear, due to the nonlinear dependence of characteristic parameters (e.g., lasing frequency, photon lifetime, carrier lifetime) on the dynamical variables. Generally, the only way to solve them is numerically.

Recently, Agrawal [12] presented numerical solutions from a small-signal analysis of a two-active-element laser in which the photon density and carrier densities of each cavity and the relative phase between the two cavities were taken as the dynamical variables. The small-signal modulation response, however, can be solved *analytically* using the modes of the composite resonator. Not only does this give analytic expressions for the small-signal modulation (useful in their own right), but it enables us to see immediately what physical quantities are important and how they affect such things as the relaxation resonance or linewidth enhancement factor. Furthermore, while a smooth, chirpless small-signal response does not guarantee the same large-signal response, a poor small-signal response pretty much rules out the possibility of a reasonable large-signal response. Finally, pathologies in the small-signal response (e.g., singularities, negative differential gain) can indicate the presence of other properties of interest (e.g., bistability).

In this paper, we derive for the first time an analytic solution to the dynamic response for a general multielement semiconductor laser. We show that the dynamic behavior of the optical field can be obtained from an analytic continuation of the steady-state eigenvalue equation to complex frequency, and that the relevant parameters which characterize the modulation can be written in terms of partial derivatives of the steady-state equation. This equation, plus linearized carrier equations, yield complete expressions for the amplitude and frequency fluctuations under current modulation. We apply the formalism to the case of a two-element semiconductor laser and explicitly calculate the modulation response as a function of frequency in terms of the effective gain,  $\alpha$  parameter, and relaxation resonance frequency. We show that frequency modulation (chirping) is generally present in the two-section laser (and, in fact, in any laser of more than two sections); however, it can be eliminated if the operating point is chosen such that the effective  $\alpha$  parameters of the two

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The authors are with the Department of Applied Physics, California Institute of Technology, Pasadena, CA 91125.

cavities are equal [13] or by appropriate splitting of the modulation current [14].

## II. OPTICAL FIELD

The connection between the time-varying medium (the modulated laser cavity) and the time evolution of the laser field is not immediately obvious, so we begin by deriving it. Maxwell's equations for a charge-free time-varying medium lead to a wave equation of the form

$$\nabla^2 E(x, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} [\mu^2(x, \hat{t}) E(x, t)] = 0 \quad (1)$$

where  $E(x, \hat{t})$  is the electric field and  $\mu(x, \hat{t})$  is the index of refraction varying on a time scale  $\hat{t} = \epsilon t$ ,  $\epsilon \ll 1$ , i.e., much more slowly than optical frequencies. The presence of gain and loss can be accommodated by allowing complex  $\mu$  if the loss is approximately constant as a function of frequency. We shall look for solutions of the form

$$E(x, t) = A(x, \hat{t}) e^{j\psi(\hat{t})} \quad (2)$$

where  $A(x, \hat{t})$  is a normalized, also slowly varying field distribution, and  $\psi(\hat{t})$  is complex to accommodate both amplitude and frequency fluctuations. [The  $\hat{t}$  dependence of  $A(x, \hat{t})$  is included because the amplitude at distinct points may vary; the average power fluctuation is absorbed into the imaginary part of  $\psi(\hat{t})$ .] Substitute (2) into (1), and the result, to lowest order in  $\epsilon$ , is

$$\nabla^2 A(x, \hat{t}) + \frac{\mu^2(x, \hat{t})}{c^2} [\dot{\psi}^2 - j\ddot{\psi}] A(x, \hat{t}) = 0 \quad (3)$$

where a dot indicates time differentiation. Now we can separate the equation into time-dependent and time-independent parts:

$$\nabla^2 A(x, \hat{t}) + \frac{\mu^2(x, \hat{t})}{c^2} \omega^2(\hat{t}) A(x, \hat{t}) = 0 \quad (4)$$

$$\dot{\psi}^2 - j\ddot{\psi} \equiv \omega^2(\hat{t}). \quad (5)$$

In (4), time appears only as a parameter. In fact, (4) is precisely the time-independent Helmholtz equation one derives for a time-invariant medium. Consequently, any calculation of the spatial modes [i.e.,  $A(x, \hat{t})$ ] based on this equation is valid to the same order as this equation—in particular, the steady-state equation for a lasing eigenfrequency is valid as an equation for the instantaneous frequency  $\omega(\hat{t})$ . Equation (5) then gives the time evolution of the optical field as a function of the instantaneous eigenfrequency. We can expand (5) in a series in  $\epsilon$  as well, eliminating the nonlinearity and reducing the equation to the simple (and intuitive)

$$\dot{\psi} = \omega(\hat{t}). \quad (6)$$

We are now done with the perturbation expansions in  $\epsilon$ , and so for the remainder of the analysis, we will not make a distinction between  $t$  and  $\hat{t}$ .

Of course, a solution for  $\psi$  requires an expression for the steady-state  $\omega$ , and as mentioned previously, for mul-

tiement SL's,  $\omega$  is generally defined implicitly in a nonlinear equation of the form.

$$F(\omega, n_1, \dots, n_N) = 0 \quad (7)$$

for an  $N$ -active-element laser with carrier densities  $\{n_i\}$ . (Note that the number of "elements" in this analysis refers to the number of active elements. The presence of passive sections of the laser complicates the resonance function  $F$ , but does not increase the number of dynamical variables. Thus, an external-cavity laser is a one-active-element laser; a  $C^3$  laser is a two-active-element laser.) It is not the point of this paper to derive the steady-state equation; that has been done in numerous works for various structures, including two-element axially coupled lasers [7]–[12], multielement axially coupled lasers [6], and two-element laterally coupled lasers [5]. We note, however, that such an equation, although solved for a fixed, real  $\omega$ , is formally equivalent to a solution of (4) for the instantaneous frequency  $\omega(\hat{t})$ ; consequently, we need not return to first principles to find the dynamic response of a multielement device. We may simply analytically continue the dc resonance equation to complex  $\omega$  and use (7) as an instantaneous definition of  $\omega(\hat{t})$  in terms of the independent variables  $\{n_i\}$ .

We shall also require a set of fill factors, defined by

$$\Gamma_i \equiv \frac{p_i \sum_k V_k}{\sum_k p_k V_k} \quad (8)$$

where  $p_k$  is the photon density in the  $k$ th cavity and  $V_k$  is the volume of the  $k$ th cavity; the summation includes both active and passive elements. In general,  $\Gamma_i \neq 1$  (as noted by [6]), and in fact,  $\Gamma_i$  can (and does) vary under modulation. We note that while the  $\Gamma$ 's so defined will depend explicitly upon both  $\omega$  and the  $\{n_i\}$ ,  $\omega$  is already defined in terms of the independent  $\{n_i\}$  by (7).

We now have all of the machinery to carry out the small-signal analysis of the optical field. We linearize (6) and (7) about a steady-state operating point

$$\dot{\psi} \equiv \omega_0 + \Delta\omega(t) - j\dot{\rho}(t)$$

$$n_i \equiv n_{i0} + v_i(t)(\Delta\omega - j\dot{\rho}) \frac{\partial F}{\partial \omega} \Big|_{\omega_0, n_{i0}} + \sum_i v_i \frac{\partial F}{\partial n_i} \Big|_{\omega_0, n_{i0}} = 0. \quad (9)$$

Divide by  $\partial F / \partial \omega$  and take real and imaginary parts to get

$$\dot{\rho} = \sum_i g'_{\text{ieff}} v_i, \quad \Delta\omega = -\sum_i m'_{\text{ieff}} v_i \quad (10)$$

where

$$g'_{\text{ieff}} \equiv \text{Im} \left[ \frac{\partial F / \partial n_i}{\partial F / \partial \omega} \right], \quad m'_{\text{ieff}} \equiv \text{Re} \left[ \frac{\partial F / \partial n_i}{\partial F / \partial \omega} \right].$$

We call  $g'_{\text{ieff}}$  and  $m'_{\text{ieff}}$  the effective differential gain and index constants, respectively. It is clear from the above relations that they have the correct units; it remains to be

shown that they, in fact, play the same role as the corresponding parameters in single-cavity lasers.

III. CARRIER DYNAMICS

Each section of the laser can be treated as an independent carrier pool, described by a volume-averaged rate equation

$$\frac{1}{\tau_i} \equiv \frac{1}{\tau_s} + g'_i \Gamma_i p, \quad \omega_{ieff}^2 \equiv 2g_i \Gamma_i p g'_{ieff}, \quad \alpha_{ieff} \equiv m'_{ieff} / g'_{ieff}$$

$$d_i \equiv \left( j\Omega + \frac{1}{\tau_i} \right), \quad c_{ik} \equiv p g_i \frac{d\Gamma_i}{dn_k}$$

With these definitions, (14) and (15) can be put into matrix form:

$$\begin{pmatrix} j\Omega & 0 & -g'_{ieff} & \cdots & -g'_{Neff} \\ 0 & 1 & \alpha_{ieff} g'_{ieff} & \cdots & \alpha_{Neff} g'_{Neff} \\ \frac{\omega_{ieff}^2}{g'_{ieff}} & 0 & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_{Neff}^2}{g'_{Neff}} & 0 & c_{N1} & \cdots & c_{NN} + d_N \end{pmatrix} \begin{pmatrix} \tilde{\rho} \\ \Delta\tilde{\omega} \\ \tilde{v}_1 \\ \vdots \\ \tilde{v}_N \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \tilde{e}_1 \\ \vdots \\ \tilde{e}_N \end{pmatrix} \quad (17)$$

$$\dot{n}_i = \frac{J_i}{qd} - \frac{n_i}{\tau_s} - g_i(n_i) \Gamma_i(n_1, \dots, n_N) p \quad (11)$$

where  $J_i$  is the  $i$ th pump current density,  $q$  is the charge on a single carrier,  $d$  is the active layer thickness,  $\tau_s$  is the spontaneous lifetime,  $g_i$  is the gain constant,  $\Gamma_i$  is the fill factor mentioned above, and  $p$  is the average power density (so that  $p$  is proportional to  $|\exp j\psi|^2$ ). We linearize this set of equations in the same way we did in (9):

$$J_i \equiv J_{i0} + qd \cdot e_i(t)$$

$$\dot{v}_i = e_i - \left[ \frac{1}{\tau_s} + g'_i \Gamma_i p \right] v_i - 2g_i \Gamma_i p \rho - \sum_k g_i p \frac{d\Gamma_i}{dn_k} v_k \quad (12)$$

where  $g'_i$  is the material (as opposed to the effective) differential gain constant, and the total derivatives of  $\Gamma_i$  are given by

$$\frac{d\Gamma_i}{dn_k} = \frac{\partial\Gamma_i}{\partial n_k} - m'_{keff} \frac{\partial\Gamma_i}{\partial \text{Re}(\omega)} - g_{keff} \frac{\partial\Gamma_i}{\partial \text{Im}(\omega)} \quad (13)$$

The distinction between  $\text{Re}(\omega)$  and  $\text{Im}(\omega)$  must be made here because the fill factor is not an analytic function of  $\omega$ . We have also made use of the relations

$$\text{Re} \frac{\partial\omega}{\partial n_k} = \frac{\partial \text{Re}\omega}{\partial n_k}, \quad \text{Im} \frac{\partial\omega}{\partial n_k} = \frac{\partial \text{Im}\omega}{\partial n_k}$$

We now Fourier transform (10) and (13). The operator  $\partial/\partial t$  becomes a factor  $j\Omega$ , and the equations become

$$j\Omega \tilde{\rho} = \sum_i g'_{ieff} \tilde{v}_i, \quad \Delta\tilde{\omega} = -\sum_i m'_{ieff} \tilde{v}_i \quad (14)$$

$$j\Omega \tilde{v}_i = \tilde{e}_i - [1/\tau_s + g'_i \Gamma_i p] \tilde{v}_i - 2g_i \Gamma_i p \tilde{\rho} - \sum_k g_i p \frac{d\Gamma_i}{dn_k} \tilde{v}_k \quad (15)$$

where a tilde indicates a transformed variable. It is convenient to make some definitions of parameters:

This matrix system can be solved by Cramer's rule to yield

$$\tilde{\rho}(\Omega) = \frac{\begin{vmatrix} 0 & -g'_{ieff} & \cdots & -g'_{Neff} \\ \tilde{e}_1 & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{e}_N & c_{N1} & \cdots & c_{NN} + d_N \end{vmatrix}}{\begin{vmatrix} j\Omega & -g'_{ieff} & \cdots & -g'_{Neff} \\ \frac{\omega_{ieff}^2}{g'_{ieff}} & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_{Neff}^2}{g'_{Neff}} & c_{N1} & \cdots & c_{NN} + d_N \end{vmatrix}} \quad (18a)$$

$$\Delta\tilde{\omega}(\Omega) = \frac{\begin{vmatrix} j\Omega & 0 & -g'_{ieff} & \cdots & -g'_{Neff} \\ 0 & 0 & \alpha_{ieff} g'_{ieff} & \cdots & \alpha_{Neff} g'_{Neff} \\ \frac{\omega_{ieff}^2}{g'_{ieff}} & \tilde{e}_1 & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_{Neff}^2}{g'_{Neff}} & \tilde{e}_N & c_{N1} & \cdots & c_{NN} + d_N \end{vmatrix}}{\begin{vmatrix} j\Omega & -g'_{ieff} & \cdots & -g'_{Neff} \\ \frac{\omega_{ieff}^2}{g'_{ieff}} & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\omega_{Neff}^2}{g'_{Neff}} & c_{N1} & \cdots & c_{NN} + d_N \end{vmatrix}} \quad (18b)$$

$$\bar{v}_i(\Omega) = \frac{\begin{vmatrix} j\Omega & -g'_{1\text{eff}} & \cdots & -g'_{(i-1)\text{eff}} & 0 & \cdots & -g'_{N\text{eff}} \\ \frac{\omega_{1\text{eff}}^2}{g'_{1\text{eff}}} & c_{11} + d_1 & \cdots & c_{1(i-1)} & \bar{e}_1 & \cdots & c_{1N} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \frac{\omega_{N\text{eff}}^2}{g'_{N\text{eff}}} & c_{N1} & \cdots & c_{N(i-1)} & \bar{e}_N & \cdots & c_{NN} + d_N \end{vmatrix}}{\begin{vmatrix} j\Omega & -g'_{1\text{eff}} & \cdots & -g'_{N\text{eff}} \\ \frac{\omega_{1\text{eff}}^2}{g'_{1\text{eff}}} & c_{11} + d_1 & \cdots & c_{1N} \\ \vdots & \vdots & & \vdots \\ \frac{\omega_{N\text{eff}}^2}{g'_{N\text{eff}}} & c_{N1} & \cdots & c_{NN} + d_N \end{vmatrix}} \quad (18c)$$

In many cases, the optical power density in an individual element is desired (e.g., the power emitted from one of the exit facets); in this case, the photon density in the  $i$ th cavity  $p_i$  is given by (8) to be

$$p_i = \Gamma_i p \quad (19)$$

which yields the modulation response

$$\bar{\rho}_i(\Omega) = \Gamma_i \bar{\rho}(\Omega) + \sum_k \frac{d\Gamma_i}{dn_k} \bar{v}_k(\Omega) \quad (20)$$

where the total derivatives  $d\Gamma_i/dn_k$  are defined by (13).

#### IV. APPLICATIONS

We shall first check the formalism by applying it to the well-known case of a single-cavity, simple Fabry-Perot resonator. In this case,  $c_{11}$  can be absorbed into the term  $1/\tau_1$  (in practice, it can be neglected entirely) and (18) becomes

$$\bar{\rho}(\Omega) = \bar{e}_1(\Omega) \frac{\frac{g'_{1\text{eff}}}{j\Omega + 1/\tau_1}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1}}, \quad \Delta\bar{\omega}(\Omega) = \bar{e}_1(\Omega) \cdot j\Omega \alpha_{1\text{eff}} \frac{\frac{g'_{1\text{eff}}}{j\Omega + 1/\tau_1}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1}} \quad (21)$$

Equation (21) is exactly the frequency response one derives from the more familiar theory [15], exhibiting a relaxation resonance at frequency  $\omega_{1\text{eff}}$  and residual phase modulation that goes to zero at zero frequency. Furthermore, we can check the values of  $g'_{1\text{eff}}$ ,  $\omega_{1\text{eff}}$ , and  $\alpha_{1\text{eff}}$  by direct calculation from the simple eigenvalue equation for the lasing frequency:

$$F(\omega, n_1) \equiv \frac{1}{R^2} \exp\left[(\gamma(n_1) - \gamma_0)L - \frac{2j\omega\mu(n_1)L}{c}\right] - 1 = 0 \quad (22)$$

where  $\gamma(n_1)$  is the power gain per unit length,  $\gamma_0$  is the loss,  $L$  is the length of the laser,  $\mu(n_1)$  is the index of refraction, and  $R$  is the mirror reflectivity. Applying (10)

to (22), we find

$$g'_{1\text{eff}} = \frac{\gamma'_c}{2\mu}, \quad m'_{1\text{eff}} = \frac{\omega\mu'}{\mu}, \quad \alpha_{1\text{eff}} = \frac{2\omega\mu'}{\gamma'_c} \quad (23)$$

which we recognize as being precisely the material differential gain constant, differential index, and linewidth enhancement factor. Similarly,  $\omega_{1\text{eff}}$  is the relaxation resonance familiar from McCumber's theory [16].

Thus reassured, we now turn to uncharted territory and analyze the response of a two-element laser. We shall not calculate here the effective modulation quantities; the algebra is straightforward given  $F(\omega, n_1, n_2)$ , but the exact form of  $F$  depends on the particular configuration chosen. As above, the diagonal elements  $c_{ii}$  can be absorbed into the term  $1/\tau_i$  of (16), i.e.,

$$\frac{1}{\tau_i} \equiv \frac{1}{\tau_s} + g'_i \Gamma_i p + g_i p \frac{d\Gamma_i}{dn_i}$$

Then, evaluating (18) for a two-active element laser yields

$$\bar{\rho}(\Omega) = \bar{e}_1(\Omega) \left\{ \frac{\frac{g'_{1\text{eff}} - g'_{2\text{eff}} \frac{c_{21}}{j\Omega + 1/\tau_1}}{j\Omega + 1/\tau_2}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} + \bar{e}_2(\Omega) \left\{ \frac{\frac{g'_{2\text{eff}} - g'_{1\text{eff}} \frac{c_{12}}{j\Omega + 1/\tau_2}}{j\Omega + 1/\tau_1}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} \quad (24)$$

$$\Delta\tilde{\omega}(\Omega) = \tilde{e}_1(\Omega) \left\{ \frac{\frac{g'_{1\text{eff}}}{j\Omega + 1/\tau_1} \left[ j\Omega \left( \alpha_{1\text{eff}} - \alpha_{2\text{eff}} \frac{g'_{2\text{eff}}}{g'_{1\text{eff}}} \frac{c_{21}}{j\Omega + 1/\tau_2} \right) + \frac{(\alpha_{1\text{eff}} - \alpha_{2\text{eff}})\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2} \right]}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} + \tilde{e}_2(\Omega) \left\{ \frac{\frac{g'_{2\text{eff}}}{j\Omega + 1/\tau_2} \left[ j\Omega \left( \alpha_{2\text{eff}} - \alpha_{1\text{eff}} \frac{g'_{1\text{eff}}}{g'_{2\text{eff}}} \frac{c_{12}}{j\Omega + 1/\tau_1} \right) + \frac{(\alpha_{2\text{eff}} - \alpha_{1\text{eff}})\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} \right]}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\}. \quad (25)$$

The expressions for  $\tilde{v}_i(\Omega)$  are similarly obtainable (and equally formidable); however, the relations simplify somewhat if  $c_{ij}\tau_i \ll 1$  for all off-diagonal elements  $c_{ij}$  (which has been suggested, for example, for the laterally coupled cavity laser [5]). In this case, there is no difference between  $\tilde{\rho}(\Omega)$  and  $\tilde{\rho}_i(\Omega)$ , and (24) and (25) reduce to

ulation disappears. Alternatively, it may be nulled out by splitting the current modulation such that

$$\frac{\tilde{e}_1}{g_1\Gamma_1} = \frac{\tilde{e}_2}{g_2\Gamma_2}$$

$$\tilde{\rho}(\Omega) = \tilde{e}_1(\Omega) \left\{ \frac{\frac{g'_{1\text{eff}}}{j\Omega + 1/\tau_1}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} + \tilde{e}_2(\Omega) \left\{ \frac{\frac{g'_{2\text{eff}}}{j\Omega + 1/\tau_2}}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} \quad (26)$$

$$\Delta\omega(\Omega) = \tilde{e}_1(\Omega) \left\{ \frac{\frac{g'_{1\text{eff}}}{j\Omega + 1/\tau_1} \left[ j\Omega \alpha_{1\text{eff}} + \frac{(\alpha_{1\text{eff}} - \alpha_{2\text{eff}})\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2} \right]}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\} + \tilde{e}_2(\Omega) \left\{ \frac{\frac{g'_{2\text{eff}}}{j\Omega + 1/\tau_2} \left[ j\Omega \alpha_{2\text{eff}} + \frac{(\alpha_{2\text{eff}} - \alpha_{1\text{eff}})\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} \right]}{j\Omega + \frac{\omega_{1\text{eff}}^2}{j\Omega + 1/\tau_1} + \frac{\omega_{2\text{eff}}^2}{j\Omega + 1/\tau_2}} \right\}. \quad (27)$$

In Figs. 1 and 2, we have plotted  $\tilde{\rho}(\Omega)$  and  $\Delta\tilde{\omega}(\Omega)$  as a function of frequency for a particular set of effective modulation constants with the  $c_{ij}$  taken to be zero. The first feature to note is the presence of two resonance terms in the denominators. They combine to give a single resonance at the Pythagorean sum of the two resonance frequencies (as can be seen from Figs. 1 and 2). The resonant frequencies are proportional to  $g'_{i\text{eff}}$ ; consequently, any increase in  $g'_{i\text{eff}}$  over the material  $g'_i$  will result in an enhanced modulation bandwidth. Such behavior has been demonstrated in passive-active coupled cavities [13].

Another feature of interest is the fact that the frequency modulation does not go to zero as  $\Omega \rightarrow 0$ . In fact, while the first term in the square brackets in (27) is simply the residual FM found in any single-cavity semiconductor laser which disappears at zero frequency, the second term causes the frequency response to level off at low frequencies (in fact, it can be seen from (27) that if the difference in the effective  $\alpha$  parameters is of order 1, the frequency response is flat up to the relaxation resonance). This results in an undesirable FM response under modulation. It should be noted that for multielement lasers, the effective linewidth enhancement factor is generally not equal to the material enhancement factor (the corresponding quantity for the single-element laser)—it depends strongly on the particular operating point selected. So, if an operating point is chosen such that the effective  $\alpha$  parameters for the two lasers are exactly equal, the low-frequency FM mod-

as was done in [4]. (It is also equally possible to null out the AM response to obtain a purely FM laser, although the regime of linear operation (and hence validity of the small-signal response) is rather small [17].) It is clear from (27) that for structures with more than two elements, there will generally be some frequency modulation. While in some structures the elements of the  $\{c_{ij}\}$  matrix are negligible (e.g., the aforementioned laterally coupled laser), in many they are not (e.g., a  $C^3$  laser); for such geometries, (20), (24), and (25) must be used and the structure of the response is somewhat more complicated than that displayed in Figs. 1 and 2.

In summary, we have presented for the first time analytic expressions for the dynamic response of multielement semiconductor lasers. In the process, we derive effective differential gains, indexes, and  $\alpha$  parameters which play the role of the corresponding material quantities in single-element semiconductor lasers. We have verified the general formalism in the single-element case and applied it to a special case of a two-element semiconductor laser. We have shown that frequency chirping is generally present, even at zero frequency in two-element lasers, yet it can be avoided either by modulating at an operating point where the effective  $\alpha$  parameters for the two cavities are equal or, alternatively, by driving the currents to both cavities in a fixed relationship.

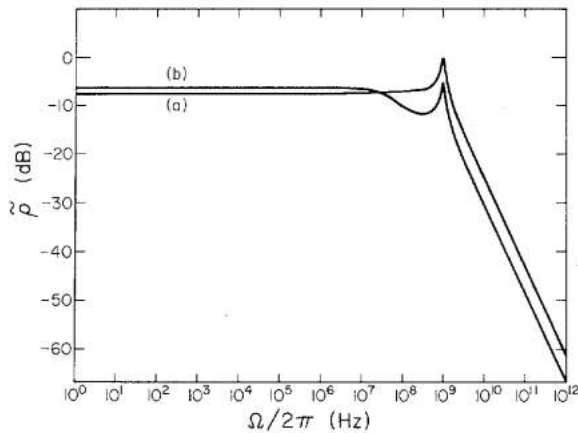


Fig. 1. Amplitude modulation response for parameters  $\omega_{1\text{eff}} = 6 \cdot 10^9$  rad/s,  $\omega_{2\text{eff}} = 1 \cdot 10^9$  rad/s,  $\tau_1 = 10^{-9}$  s,  $\tau_2 = 5 \cdot 10^{-9}$  s,  $g'_{1\text{eff}} = 1.2 \cdot 10^{-5}$  cm<sup>3</sup>/s,  $g'_{2\text{eff}} = 3.0 \cdot 10^{-6}$  cm<sup>3</sup>/s,  $\alpha_{1\text{eff}} = 5.01$ ,  $\alpha_{2\text{eff}} = 5.00$ . (a) Amplitude modulation response (arbitrary zero) for  $\tilde{\epsilon}_2 = 0$ . (b) Amplitude modulation response (same zero) for  $\tilde{\epsilon}_1 = 0$ .

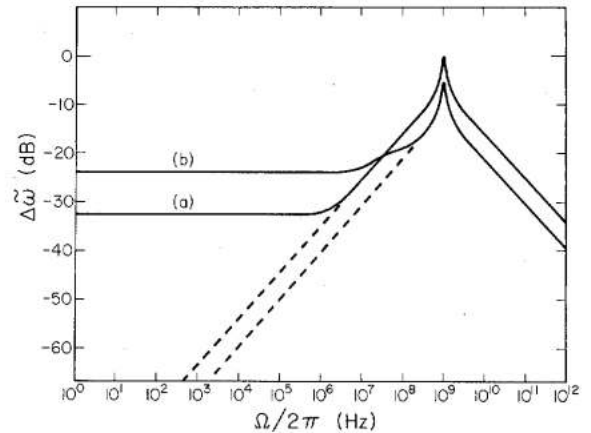


Fig. 2. Frequency modulation response for same parameters as in Fig. 1. (a)  $\tilde{\epsilon}_2 = 0$ , (b)  $\tilde{\epsilon}_1 = 0$ . Dashed lines indicate the residual phase modulation present in single-cavity lasers.

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**Robert J. Lang** (S'83), for a photograph and biography, see p. 451 of the May 1985 issue of this JOURNAL.

**Amnon Yariv** (S'56–M'59–F'70), for a photograph and biography, see p. 138 of the February 1985 issue of this JOURNAL.