# ANALYSES OF THE SEPARATED BOUNDARY <br> <br> LAYER FLOW ON THE SURFACE AND <br> <br> LAYER FLOW ON THE SURFACE AND <br> $\because$ IN THE WAKE ..OF BLUNT TRAILING <br> EDGE AIRFOILS 

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| $c$ or C | Airfoil chord |
| :---: | :---: |
| $C_{\text {D }}$ | Drag coefficient |
| ${ }^{C} D_{p}$ | Drag coefficient due to non-zero pressure coefficient in the wake |
| $C_{D_{q}}$ | Drag coefficient due to momentum deficit in the wake |
| ${ }^{C^{\text {dtotal }}}$ | Total drag coefficient; $C_{D_{\text {total }}}=C_{D_{p}}+C_{D_{q}}$ |
| ${ }^{\text {c }}$ | Pressure coefficient; $\left(P-P_{\infty}\right) / \frac{1}{2} \rho U_{\infty}{ }^{2}$ |
| $C_{L}$ | Lift coefficient; Lift/ $\frac{1}{2} \mathrm{PU}_{\infty}{ }^{2}$ |
| $f\left(n_{1}\right), f\left(n_{2}\right), f\left(n_{3}\right)$. | Functions for defining the velocity |
| $f\left(n_{4}\right), f\left(n_{5}\right)$ | Profile similarity in the separated flow |
| H | Form factor; $\delta * / \theta$ |
| $\mathrm{K}_{\mathrm{c}}$ | Calibration constant for the disc probe |
| $\mathrm{K}_{1}, \mathrm{~K}_{2}$ | Value of similarity parameter at the upper or lower edge of the wake boundary layer |
| $P_{5}$ | Local static pressure in the boundary layer or in the wake |
| $\mathrm{P}_{\mathrm{T}}$ | Local total pressure in the boundary layer or in the wake |
| $P_{\infty}$ | Free stream static pressure |
| $\mathrm{q}_{\infty}$ | Free stream dynamic head, $\frac{1}{2} \mathrm{U}_{\infty}{ }^{2}$. |
| $\mathrm{R}_{\mathrm{N}}, \mathrm{R}_{\mathrm{e}}$ | Reynolds number |
| t | Thickness of the blunt trailing edge |
| u, U | $\times$ component velocity |
| Uw, Um | $x$ component velocity on the locus of minimum velocity |
| Ve | Hot wire cooling velocity |

## $\infty$

e
u
$\ell, L$
T.E.
o, w
y component, velocity
Turbulent fluctuations in $y$ component velocity
Distance measured on the chord of the airfoil from the leading edge

Distance measured in the airfoil wake from the trailing edge

Angle of attack in degrees
Parameters for the similarity of velocity profiles
Momentum thickness
Boundary layer thickness
Displacement thickness
Density
Dynamic viscosity
Kinamatic viscosity

Free stream values
Edge of the boundary layer
Upper edge of the boundary layer wake
Lower edge of the boundary layer wake
Trailing edge values
Values at the wall

## SUMMARY

This report presents the results of theoretical and experimental investigations of viscous flow phenomena associated with sharp and blunt trailing edge airfoils. Experimental measurements were obtained for a 17 percent thick, high performance $G A(W)-1$ airfoil. This airfoil was chosen because it exhibits typical turbulent separation characteristics, in that the point of separation moves gradually upstream on the airfoil upper surface as the angle of attack is increased and an extensive region of flow separation is present on the upper surface at the $C_{L_{\text {max }}}$ condition. The airfoil is equipped with detachable trailing edges for the purpose of studying effects of trailing edge thickness.

Experimental measurements consist of velocity and static pressure profiles which were obtained by the use of forward and reverse total pressure probes and disc type static pressure probes. These measurements were obtained over the surface and in the wake of sharp and blunt trailing edge airfoils. Measurements of the upper surface boundary layer were obtained in both the attached and separated flow regions. In addition, static pressure data were acquired, and skin friction on the airfoil upper surface was measured with a specially constructed device. Comparison of the present viscous flow data with the data of Wichita State University on the surface of a GA(W)-1 airfoil indicated reasonable agreement in the attached flow region. In the separated flow region, considerable differences exist between these two sets of measurements.

Analytical studies were performed for the separated turbulent boundary layer flow on the airfoil surface and in the wake of blunt and sharp trailing edge airfoils. A multi-layered physical flow model for the flow in the separated boundary layer region on the airfoil surface has been developed, and similarity parameters have been derived for the separated flow velocity profiles. A mathematical model for the flow in the wake of the blunt base airfoils has also been developed. This model, which represents an initial value problem, consists of a set of integral equations for solution of the characteristic flow quantities in the wake behind the blunt trailing edge up to the point of confluence. These equations are coupled, ordinary nonlinear and nonhomogeneous differential equations, and can be solved by available single-step or multi-step computerized numerical methods. The detailed wake flow study indicated that the velocity profiles for the flow in the wake were not strictly similar, but that the shape of the similarity curve depended upon such factors as the ratio of minimum to edge velocity and the wake boundary layer thickness. Second order effects (due to the deviation of wake velocity profile similarity) on such parameters as integral area function, growth rate function, and wake boundary layer stretch parameter, were addressed by the derivation of auxiliary equations. Thus, the overall accuracy of the present theoretical method for computing the wake flow and profile drag of blunt and sharp trailing edge airfoils was improved.

## 1. INTRODUCTION

In view of the current emphasis on energy conservation, it is particularly significant to consider the development of more accurate and reliable aerodynamic methods for the prediction of profile drag and separated flow characteristics for advanced airfoil sections. A theoretical method for predicting the drag of airfoils having attached flow and sharp trailing edges (ref. 1) has already been developed and uses an approach in which the wake flow characteristics are developed analytically and integrated to calculate profile drag. The method has proved quite effective for this application, and warrants extension to cover the cases of blunt-base airfoils and airfoils having regions of separated flow. Thus, the objectives of this contract effort are to develop the blunt-base airfoil drag prediction method and to initiate studies of separated boundary layer flows on airfoil upper surfaces.

The choice of an airfoil section with non-zero trailing edge thickness can be made for one of several reasons - either structural or aerodynamic. In the latter case, the use of a blunt trailing edge has been suggested by Whitcomb (ref. 2) and Pearcey (ref. 3) as a possible means of delaying the onset of shock-induced separation and drag rise at high subsonic speeds. The advantages which are obtained by relaxing the condition of zero thickness at the trailing edge are schematically illustrated in Figure 1-1. Referring to the nomenclature of this figure, we have:

$$
\int_{\theta=0}^{x / c=1.0} \tan \theta d(x / c)=\left(\frac{y_{u}}{c}\right)_{\max }-\left(\frac{y_{u}}{c}\right)_{T . E .}
$$

which says that finite values of trailing edge thickness, $\left(y_{u} / C\right)$ T. $\mathrm{E}_{\text {. }}$, allows a greater airfoil thickness ratio for the same surface slope distribution. Alternatively, a more favorable slope distribution can be obtained for the same airfoil thickness ratio with the use of a thick trailing edge. The distribution of airfoil upper surface slope at transonic speeds is very important (ref. 4) from the standpoint of obtaining "shockless flow conditions" and minimizing or avoiding shock-induced separation effects. Finite values of trailing edge thickness can also be favorable in minimizing the flow unsteadiness due to shock movement and buffetting. The underlying reasons for these favorable blunt-base characteristics are schematically illustrated in figure $1-2$. As shown in this figure, when the boundary layer on the upper surface of an airfoil near the trailing edge starts becoming moderately thick due to shock-induced separation, the blunt trailing edge acts as a shield in order to retain the lower surface pressure distribution, and hence the location of the forward stagnation point on the airfoil surface remains unchanged. The invariant location of the forward stagnation point in transonic flight results in a reduction of flow unsteadiness and buffetting caused by the rapidly fluctuating shock location on the upper surface of the wing section.

The gain in aerodynamic performance which is achieved by the use of a rather thick airfoil trailing edge is shown in figure 1-3(a) and 1-3(b). Experimental data are presented in figure 1-3(a) which show superior lift performance for the blunt-base airfoil when approaching separation. In this case, the thickness ratio is higher for the blunt trailing edge airfoil than for the corresponding sharp trailing edge airfoil. The improvement in the separation boundary for a given airfoil thickness ratio is shown in figure 1-3(b). In practice, the base drag penalty would probably prohibit the use of very thick trailing edge sections, such as shown here. However, there is a definite value of trailing edge thickness - for any given airfoil at a specified lift coefficient and Mach number which will give the best aerodynamic performance in terms of ML/D and reduced buffetting. The profile drag prediction methods developed herein will facilitate the conduct of parametric (computational) studies as required to define this optimal trailing edge thickness for any given application.

Due to the action of the viscosity of the fluid on the surface of the airfoil, self-forming vortex singularities are continuously generated. In general, these singularities move with the flow; however, in the case of blunt trailing edge airfoils, depending upon the trailing edge thickness and the characteristics of the boundary layer on the surface of the airfoil in the vicinity of the trailing edge, a standing vortex [fig. 1-4(a)] is formed behind the trailing edge. Such a standing vortex singularity of ten produces the desired steady potential flow, depending upon the strength of the singularity. Although not the subject of this report, there are several practical methods for stabilization of such standing vortices. Some of these are illustrated in figure 1-4, where the character of the flow in the wake of the airfoil is also shown.

In the case of sharp or blunt trailing edge airfoils which exhibit trailing edge stall, the typical shape of the $C_{L}-\alpha$ curve is shown in figure l-5(a). The nonlinearity in lift curve slope begins at $\alpha=\alpha_{1}$ and the condition of the viscous flow at $\alpha=\alpha_{1}$ is shown in figure 1-5(b). This figure shows that the turbulent boundary layer separation on the upper surface of the airfoil first appears at $\alpha=\alpha_{1}$ near the trailing edge. As the angle of attack is increased beyond $\alpha_{1}$, the point of separation moves upstream and figure 1-5(c) shows the existence of the finite region of flow separation at $\alpha=\alpha_{2}$. The viscous flow conditions at $\alpha=\alpha_{3}$, which correspond to airfoil $C_{L_{\text {max }}}$, are shown in figure $1-5(\mathrm{~d})$. In this case, the length of the separated flow region has increased considerably. The increase in the angle of attack, $\Delta \alpha=\alpha_{3}-\alpha_{1}$, is observed experimentally to have a range of $8^{\circ}$ to $12^{\circ}$ for single component airfoils with trailing edge stall. Theoretical methods and computer program subroutines, which were developed in references 5 and 6 , can predict aerodynamic characteristics such as pressure distribution, pitching moment and profile drag of sharp trailing edge single-component airfoil quite accurately when there is no flow separation. However, reliable theoretical methods for the prediction of airfoil aerodynamic performance in the presence of separated boundary layer flow are not presently available.

The preceding discussion points out the desirability of developing theoretical methods for (a) computing the wake flow characteristics and profile drag for blunt-base airfoils and (b) computing the characteristics
of the separated turbulent boundary layer on airfoil surfaces. Specifically, the method should be generalized so as to be valid at both high and low angles of attack. Therefore, the objectives to be accomplished in the present studies are:
(1) To obtain measurements of velocity and static pressure profiles in the wake of a blunt trailing edge $G A(W)-1$ single component airfoil for the purpose of developing a physical flow model for the blunt-base airfoil wake. Also, to make comparisons of the characteristic features of the wake flows for sharp and blunt trailing edge airfoils, at distances "far" from the trailing edge, for similarity of velocity and static pressure profiles.
(2) To derive equations to provide a generalized mathematical model for computations of the wake flow behind arbitrary shaped blunt-base airfoils.
(3) To obtain boundary layer data, static pressure profiles, and wall shear on the upper surface of the blunt-base airfoil in the region of approaching turbulent boundary layer separation and the separated turbulent boundary layer region. Develop a physical flow model and investigate similarity parameters for characteristic layers of the separated boundary layer velocity profiles.


Figure 1-1. Advantages of the Use of Finite Trailing Edge Thickness Airfoil Section


Figure 1-2. Surface and Wake Pressures in the Vicinity of Thick and Sharp Trailing Edges


Figure 1-3(a). The Effect of Removing the Rear Position of a Given Airfoil


Figure 1-3(b). The Effect of Moving the Maximum-Thickness Position to the Trailing Edge
(a) Formation of Standing Vortex Behind Blunt Base Airfoils


Figure 1-4. Illustration of Some Fundamental Phenomena Behind Blunt Base Airfoils

(a). Typical $C_{L}-\alpha$ curve for T.E. Stall Airfoil
!nitial Turbulent Separation Point

(c). Viscous flow Conditions at $\alpha=\alpha_{2}$

Edge of B.L.

(b). Viscous Flow Conditions at $\alpha=\alpha_{1}$

Figure 1-5. Schematic Representation of the Viscous Flow Phenomena over the Airfoil Surface Exhibiting Trailing Edge Type Stall

## II. THEORETICAL STUDY

Theoretical work performed during the present study consists of (i). development of a few similarity parameters for the separated flow on the surface of airfoils, (ii) development of similarity parameters or a one parameter family of velocity and static pressure profiles in the wake of blunt-base single component airfoils, and (iii) the development of physical and mathematical models for the flow in the wake of blunt-base single component airfoils. Items (i) and (ii) are described in Sections IV. 1 and IV. 2 of RESULTS AND DISCUSSION, respectively. The development of physical and mathematical models for the blunt-base wake flow is given in the following paragraphs.

## Il.1 Physical Flow Model for Blunt-Base Wake Flow

Figure II-1(a) shows the flow model for the wake behind the blunt-base airfoils. This flow model was derived from the experimental data behind the present sharp and blunt trailing edge airfoils. As can be seen from figure II-1(a), the wake flow is divided into Regions I and IV. Results of analysis of experimental data in Region IV indicated that the wake flows for both blunt and sharp trailing edge airfoils are "dynamically similar" in this region. This implies that velocity profile similarity function and the variations of static pressure and shear stress when expressed as the function of dimensionless wake flow parameters are identical in Region IV for blunt and sharp trailing edge airfoils. Region IV consists of two layers, namely upper wake layer 3-4 and lower wake layer 1-2 [Figure 1I-1(a)]; the locus of minimum velocity is at the juncture of these layers. For positive lift coefficients, the velocity on the locus of the outer layer or the upper layer 3-4 is higher than the velocity on the locus of the lower wake layer $1-2$; at a distance "very far" from the airfoil trailing edge these velocities become equal to each other.

The details of the flow in Region $l$ behind the blunt trailing edge airfoils are shown schematically in Figure ll-i(b). This figure shows that the flow in Region 1 is further subdivided into Zones $L_{1}, L_{2}$, and $L_{3}$. These zones are formed due to the mixing of the airfoil upper and lower surface boundary layers behind the blunt trailing edge. This mixing phenomena causes the formation of "qualitatively similar" flow to that formed by the two counter flowing plane parallel jets. As shown in Figure ll-1(b), the flow in Region 1 can be divided into three layers, namely upper wake layer $C_{2} Z_{2} B_{2}$, reverse velocity core $C_{2} C_{4}$, and lower wake layer $C_{4} Z_{4} B_{5}$. The internal boundaries of the upper and lower wake boundary layers are shown as loci $U C_{1} C_{2} C$ and $L C_{3} C_{4} C$, respectively.

At a certain point C , called the point of confluence, the lines of constant mass or the dividing stream lines $U M_{1} M_{2} \mathrm{C}$ and $\mathrm{LM}_{3} \mathrm{M}_{4} \mathrm{C}$ meet each other. The point of confluence is equivalent to the trailing edge point for the sharp
trailing edge airfoils. The line of constant mass is defined such that [fig. li-1(b)],

$$
\begin{align*}
& \int_{y_{z_{2}}}^{y_{m_{2}}} u d y=-\int_{y_{c_{2}}}^{y_{z_{2}}} u d y-u_{C R}\left(\frac{y_{c_{2}}+y_{c_{4}}}{2}\right) \\
& \int_{y_{z_{4}}}^{y_{m_{4}}} u d y=\int_{y_{z_{4}}}^{y_{c_{4}}} u d y+u_{C R}\left(\frac{y_{c_{2}}+y_{c_{4}}}{2}\right)
\end{align*}
$$

where

$$
\left.\left.\begin{array}{rl}
y_{m_{2}}, y_{m_{4}}= & \begin{array}{rl} 
& \text { ordinates of the upper and lower dividing streamlines, } \\
& \text { respectively }
\end{array} \\
y_{z_{2}}, y_{z_{4}}= & \text { ordinates of the upper and lower zero velocity lines, } \\
& \text { respectively }
\end{array}\right\} \begin{array}{rl}
y_{c_{2}}, y_{c_{4}}= & \text { ordinates of the upper and lower edges of the reverse } \\
& \text { velocity core, respectively }
\end{array}\right\}
$$

From the definition of the constant mass lines, equations (II-1), it can be seen that the streamlines, which exist between the upper or lower constant mass lines and the edges of the reverse velocity core, are closed streamlines as shown in figure ll-1 (b).

A characteristic feature of the flow in Region 1 is that the flow velocity in the reverse velocity core is not constant. The velocity in the core is zero at the location of the airfoil blunt trailing edge, and decreases with respect to chordwise distance aft of the trailing edge and reaches the minimum negative value at the end of Zone $L_{2}$. The core velocity in the Zone $L_{2}$ continually increases from the minimum negative value and becomes equal to zero at the end of Zone $L_{2}$ at the point of confluence. The typical variation of the experimentally measured static pressure coefficient on the characteristic loci of the wake flow behind the blunt trailing edge is shown schematically in figure $11-2$. A few interesting observations can be made from these figures. The pressure coefficients in Zone $L_{1}$ on the characteristic loci are approximately constant on the upper edge of the wake, wake center, and the lower edge of the wake; the $C_{p}$ on the center of the wake is closer to the upper edge of the wake and its value remains in between the values of $C_{p}$ on the loci of upper and lower edges of the wake. In Zone $L_{2}$ the $C_{p}{ }^{\prime} s$ increase quite rapidly; up to the point of confluence, on the three characteristic loci. This phenomena is similar to the pressure distribution on the airfoil surface in the vicinity of the sharp trailing edge. The static pressure then
continuously decreases in Region IV on these loci, however, the value of $C_{p}$ on the locus of minimum velocity has higher value than at either at the same chordwise location. This trend in variation of $C_{p}$ in Region IV behind a blunt trailing edge airfoil is similar to the one observed for sharp trailing edge airfoils.

### 11.2 Description of Mathematical Model for Region I

The time-averaged governing equations of motion in the wake of the blunt trailing edge airfoils for Region 1 can be written as follows:

$$
\begin{align*}
\rho u \frac{\partial u}{\partial x}+\rho v \frac{\partial u}{\partial y}=- & \frac{\partial P}{\partial x}+\frac{\partial \tau x y}{\partial y}-X \text { Momentum Equation }  \tag{11-2}\\
\rho u \frac{\partial v}{\partial x}+\rho v \frac{\partial v}{\partial y}=- & \frac{\partial P}{\partial y}+\frac{\partial \tau y x}{\partial x}-Y \text { Momentum Equation }  \tag{11-3}\\
& \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0-\text { Continuity } \tag{11-4}
\end{align*}
$$

In the following derivatives, the $y$-momentum equation is not used; however, the variation of the pressure normal to the wake boundary layer is taken into account by the use of Leibnitz's rule. The Euler equation, applied to upper and lower edges of the wake can be written as

$$
\frac{1}{\rho} \frac{d P_{e_{u}}}{d x}=-U_{e_{u}} \frac{d U_{e u}}{d x}
$$

and

$$
\begin{equation*}
\frac{1}{\rho} \frac{d P_{e_{\ell}}}{d x}=-U_{e_{\ell}} \frac{d U_{e}}{d x} \tag{11-5}
\end{equation*}
$$

The effect of local curvature on the flow development in Region $I$ is assumed negligible and the flow density is assumed approximately constant in equations ( $11-2$ ) through ( $11-5$ ). The shear stress terms appearing in equation ( $11-2$ ) is the algebraic sum of laminar and turbulent contributions, i.e.

$$
\begin{equation*}
\tau_{x y}={ }^{\tau} y x=\mu \frac{\partial u}{\partial y}-\overline{\rho u^{\prime} v^{\prime}} \tag{11-6}
\end{equation*}
$$

$\begin{aligned} \text { where } \overline{u^{\prime} v^{\prime}}= & \text { time average product of fluctuating velocity components in } \\ & x \text { and } y \text { directions. }\end{aligned}$
The typical velocity profile in Region 1 is shown in figure $11-3$. Experimental measurements indicate that across the reverse velocity core layer $y_{5}-y_{4}$ the magnitude of velocity $U_{m}$ and the value of static pressure coefficients, $C_{p}$, can be considered approximately constant at a given $x$ location. The present experimental data in Region 1 also suggest that by
the appropriate choice of dimensionless parameters and fluctuations the velocity profiles for the lower layer $y_{4}-y_{1}$ and the upper wake layer $y_{8}-y_{5}$ can be made "similar." The further discussion on this matter is given in Section IV.2.2, of RESULTS AND DISCUSSION. The similarity parameter and variable for the upper wake boundary layer $y_{8}-y_{5}$ are defined as

$$
\begin{equation*}
\xi u=\frac{y-y_{5}}{y_{5} c-y_{5}} ; \quad P(\xi u)=\frac{U_{e_{u}}-u}{U_{e_{u}}-U} \tag{11-7}
\end{equation*}
$$

and for the lower wake layer $\mathrm{y}_{4}-\mathrm{y}_{1}$,

$$
\begin{equation*}
\xi \ell=\frac{y_{4}-y}{y_{4}-y_{l_{c}}} ; \quad P(\xi \ell)=\frac{U_{e_{\ell}}-u}{U_{e_{\ell}}-U\left(y_{4}\right)} \tag{11-8}
\end{equation*}
$$

where

$$
\begin{gathered}
y_{l_{c}}=\text { distance } y \text { in the lower wake layer } y_{4}-y_{1} \text { when } u=0.5\left(U_{e_{\ell}}+U_{\left(y_{4}\right)}\right) \\
y_{5_{c}}=\text { distance } y \text { in the upper wake layer } y_{8}-y_{5} \text { where } u=0.5\left(U_{e_{u}}+U_{\left(y_{5}\right)}\right) \\
U_{e_{u}}=\text { velocity on the locus of the upper edge of boundary layer } \\
U_{e \ell(x)}=\text { velocity on the locus of the lower edge of the boundary layer } \\
U_{\left(y_{4}\right)}, U\left(y_{5}\right) \text {, and } U_{m}=\text { velocity in the reverse velocity core, and } \\
K_{l_{u}}=\frac{y_{8}-y_{5}}{y_{5}-y_{5}} ; \quad K_{l}=\frac{y_{4}-y_{1}}{y_{4}-y_{l_{c}}} .
\end{gathered}
$$

As the velocity profiles for layer $y_{8}-y_{5}$ and $y_{4}-y_{1}$ are similar, the integral equations which are ordinary differential equations, can be derived for these layers. The momentum integral equation for the layer $y_{8}-y_{5}$, for example, can be derived by integrating individual terms of equation (11-2) from $y=y_{5}$ to $y=y_{8}$ and by making use of the similarity or the velocity profile for this layer as given by equation (l|-7). During the mathematical manipulations, use is made of equations (II-4) and (II-5), Leibnitz's rule and the following applicable boundary conditions:
at

$$
y=y_{8}: \quad u=U_{e_{u}}, P\left(\xi_{u}\right)=0, \quad \xi u=K_{1 u} ; \tau=\tau\left(y_{8}\right)=0, \frac{1}{\rho} \frac{d P_{e u}}{d x}=-U_{e_{u}} \frac{d U_{e u}}{d x}
$$

and

$$
C_{p s}=C_{p_{e u}}
$$

at

$$
y=y_{5}: \quad u=U_{\left(y_{5}\right)}=U_{m}, P(\xi u)=1.0 ; \xi(, u)=0.0, c_{p}=c_{P\left(y_{5}\right)}, V_{(y)}=v\left(y_{5}\right)
$$

and

$$
\tau=\tau\left(y_{5}\right) .
$$

The momentum integral equation for the layer $y_{8}-y_{5}$ can be derived as described above by the use of these boundary conditions. This can be written as

Momentum Integral Equation for Layer $y_{8}-y_{5}$
$-\frac{d}{d x}\left[\left(y_{5 c}-y_{5}\right)\left(u_{e u}\right)\left(u_{e u}-U_{\left(y_{5}\right)}\right)\right] \int_{0}^{K_{1}} P_{(\xi u)} d \xi$
$+\frac{d}{d x}\left(y_{5}-y_{5}\right)\left(U_{e_{u}}-u_{\left(y_{5}\right)}\right)^{2} \int_{0}^{K_{1_{u}}} p^{2}(\xi u) d \xi$
$-\left(\frac{d y_{5}}{d x}\right)\left(u_{\left(y_{5}\right)}\right)\left(U_{e_{u}}-u_{\left(y_{5}\right)}\right)-\left(\frac{d U_{e u}}{d x}\right)\left(y_{5 c}-y_{5}\right)\left(u_{e_{u}}-U_{\left(y_{5}\right)}\right) \int_{0}^{K_{1} u} P(\xi u) d \xi$
$+\int_{y_{5}}^{y_{8}} u_{e_{u}} \frac{d U_{e u}}{d x} d y+v_{\left(y_{5}\right)}\left\{U_{e_{u}}-u_{\left(y_{5}\right)}\right\}$
$+\frac{U_{\infty}{ }^{2}}{2} \frac{d}{d x} \int_{y_{5}}^{y_{8}} C_{p} d z-\frac{U_{\infty}{ }^{2}}{2}\left(C_{p\left(y_{8}\right)}\right) \frac{d y_{8}}{d x}+\frac{U_{\infty}{ }^{2}}{2} C_{P\left(y_{5}\right)} \frac{d y_{5}}{d x}=-\frac{\tau\left(y_{5}\right)}{\rho}$

Similarly the Momentum Integral Equation for Layer $y_{4}-y_{1}$ is:

$$
\begin{aligned}
& -\frac{d}{d x}\left[\left(y_{4}-y_{1_{c}}\right)\left(U_{e_{\ell}}\right)\left(U_{e_{\ell}}-U_{\left(y_{4}\right)}\right)\right] \int_{0}^{K_{1}} P(\xi \ell) d_{\xi \ell} \\
& +\frac{d}{d x}\left[\left(y_{4}-y_{1_{c}}\right)\left(U_{e_{\ell}}-U_{\left(y_{4}\right)}\right)^{2}\right] \int_{0}^{K_{1}} P^{2}\left(\xi_{\ell}\right) d \xi_{\ell}+\left(\frac{d y_{4}}{d x}\right)\left(U_{\left(y_{4}\right)}\right)\left(U_{e_{\ell}}-U_{\left(y_{4}\right)}\right)
\end{aligned}
$$

$$
\begin{align*}
& -\left(\frac{d U_{e_{\ell}}}{d x}\right)\left(y_{4}-y_{1_{c}}\right)\left(U_{e_{\ell}}-U\left(y_{4}\right)\right) \int_{0}^{K_{1 \ell}} P\left(\xi_{\ell}\right) d \xi_{\ell} \\
& +\int_{y_{1}}^{y_{4}} U_{e_{\ell}} \frac{d U_{e_{\ell}}}{d x} d y-v_{\left(y_{4}\right)}\left\{U_{e_{\ell}}-U_{\left(y_{4}\right)}\right\} \\
& +\frac{U_{\infty}^{2}}{2} \frac{d}{d x}\left[\int_{y_{1}}^{y_{4}} C_{p} d y\right]-\left(\frac{U_{\infty} 2}{2}\right)\left(\frac{d y_{4}}{d x}\right)\left(C_{p\left(y_{4}\right)}\right)+\frac{U_{\infty}^{2}}{2} C_{p}\left(y_{1}\right) \frac{d y_{1}}{d x}=\frac{{ }^{\tau}\left(y_{4}\right)}{\rho} \tag{11-10}
\end{align*}
$$

Across the reverse flow velocity core layer $y_{5}-y_{4}$, the magnitude of the velocity and the static pressure coefficient $C_{p}$ are assumed constant; this fact is verified from the present experimental measurements. Under this assumption the Momentum Integral Equation for the Layer $y_{5}-y_{4}$ can be derived as

$$
\begin{equation*}
u_{m} \frac{d U_{m}}{d x}\left(y_{5}-y_{4}\right)+\frac{U_{\infty}{ }^{2}}{2}\left(y_{5}-y_{4}\right) \frac{d C_{p_{m}}}{d x}=\frac{{ }^{\tau}\left(y_{5}\right)}{\rho}-\frac{{ }^{\tau}\left(y_{4}\right)}{\rho} \tag{11-11}
\end{equation*}
$$

Equations (11-9) through (11-11) contain shear stress terms, e.g. ${ }^{\tau}\left(y_{5}\right)$, ${ }^{\tau}\left(y_{4}\right)$, etc. on the characteristic loci in Region 1. Shear measurements in ${ }^{2}$, the airfoil wake were not obtained during the present test; hence, auxiliary equations for shear stress in terms of dependent variables cannot be derived. One way to circumvent this problem is to eliminate these terms by adding equations ( $11-9$ ), ( $11-10$ ), and ( $11-11$ ). The resulting Momentum Integral Equation for the Entire Layer $y_{8}-y_{1}$ in Region 1 is then given by

$$
\begin{aligned}
& -\frac{d}{d x}\left(u_{e_{\ell}}\right)\left(u_{e_{\ell}}-u_{m}\right)\left(y_{4}-y_{l_{c}}\right) \int_{0}^{K_{1 \ell}} P\left(\xi_{\chi}\right) \\
& +\frac{d}{d x}\left(u_{e_{\ell}}-u_{m}\right)^{2}\left(y_{4}-y_{1_{c}}\right) \int_{0}^{K_{1 \ell}} P^{2}\left(\xi_{\ell}\right) d_{\xi_{l}} \\
& +\left(u_{m}\right)\left(u_{e_{\ell}}-u_{m}\right)\left(\frac{d y_{4}}{d x}\right)-\left(\frac{d u_{\ell \ell}}{d x}\right)\left(u_{e_{\ell}}-u_{m}\right)\left(y_{4}-y_{1 c}\right) \int_{0}^{K_{1 \ell}} P\left(\xi_{\ell}\right) d \xi_{\ell}
\end{aligned}
$$

$$
\begin{align*}
& +\left(\frac{u_{\infty}^{2}}{2}\right)\left(\frac{d C_{\mathrm{pe} \mathrm{\ell}}}{d x}\right)\left(y_{4}-y_{1}\right)-\frac{d}{d x}\left[\left(u_{e_{u}}\right)\left(u_{e_{u}}-u_{m}\right)\left(y_{5 c}-y_{5}\right)\right] \int_{0}^{K_{1 \ell}} P\left(\xi_{u}\right) d \xi_{u} \\
& +\frac{d}{d x}\left[\left(u_{e_{u}}-u_{m}\right)^{2}\left(y_{5 c}-y_{5}\right)\right] \int_{0}^{k_{1 u}} P^{2}\left(\xi_{u}\right) d \xi_{u} \\
& -\left(u_{m}\right)\left(U_{e_{u}}-u_{m}\right)\left(\frac{d y_{5}}{d x}\right)-\left(\frac{d U_{e u}}{d x}\right)\left(U_{e_{u}}-u_{m}\right)\left(y_{5_{c}}-y_{5}\right)_{\int_{0}}^{k_{l_{u}}} P_{\left(\xi_{u}\right) d \xi_{u}} \\
& +\left(\frac{u_{\infty}{ }^{2}}{2}\right)\left(-\frac{d C_{\rho_{e u}}}{d x}\right)\left(y_{8}-y_{5}\right)+\left(v_{\left(y_{5}\right)}\right)\left(u_{e_{u}}-u_{m}\right)-\left(v_{\left(y_{4}\right)}\right)\left(u_{e_{\ell}}-u_{m}\right)+\left(u_{m} \cdot \frac{d u_{m}}{d x}\right)\left(y_{5}-y_{4}\right) \\
& +\frac{U_{\infty}{ }^{2}}{2} \frac{d}{d x}\left\{\int_{y_{1}}^{y_{4}} C_{P} d y\right\}+\frac{U_{\infty} 2}{2}\left(y_{5}-y_{4}\right)\left(\frac{d C_{P_{m}}}{d x}\right) \\
& +\frac{U_{\infty}^{2}}{2} \frac{d}{d x} \int_{y_{5}}^{y_{8}} c_{p} d y+\left(\frac{U_{\infty}^{2}}{2}\right)\left(\frac{d y_{1}}{d x}\right) c_{p\left(y_{1}\right)}-\left(\frac{U_{\infty}^{2}}{2}\right)\left(\frac{d y_{8}}{d x}\right) c_{p\left(y_{8}\right)}=0 . \tag{11-12}
\end{align*}
$$

If the assumption is made that the rate of growth of the layers $y_{8}-y_{5}$ or $y_{4}-y_{1} \Gamma$ (Fig. $\left.1 \Pi-3\right)$ is controlled by the transverse perturbation velocity; and as experimental data indicates, the existence of approximate velocity profile similarity, then the growth rate for these layers can be expressed by the following equations:

$$
\begin{equation*}
\frac{d}{d x}\left(y_{5_{c}}-y_{5}\right)=c_{l_{u}} \frac{u_{e_{u}}-u_{m}}{u_{e_{u}}+u_{m}} \tag{11-13}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d}{d x}\left(y_{4}-y_{1_{c}}\right)=c_{1_{\ell}} \frac{u_{e_{\ell}}-u_{m}}{u_{e_{\ell}}+u_{m}} \tag{11-14}
\end{equation*}
$$

If the velocity profiles in the airfoil wake were strictly similar, then $C_{l_{u}}$ $\mathrm{C}_{1 \ell}$ would be constants. However, experimental data indicate that the similarity of the velocity profiles is somewhat dependent upon such factors as ratio $U_{m} / U_{e}$, and the ratio of thickness at the upper and lower wake boundary layers. These second-order effects on the values of $C_{1 u}$ and $C_{1 l}$, which are discussed in Section IV.2.4, are the functions of the dimensionless groups
formed of dependent variables such as wake boundary layer thickness and velocities on characteristic loci for the flow in the wake.

Equation (11-12) contains the terms like $\mathrm{dy}_{4} / \mathrm{dx}, \mathrm{dy} \mathrm{y}_{5} / \mathrm{dx}, \mathrm{dy} \mathrm{l}_{1} / \mathrm{dx}, \mathrm{dy} \mathrm{y}_{8} / \mathrm{dx}$, $V\left(y_{4}\right)$ and $V\left(y_{5}\right)$. Auxiliary equations have to be obtained for the above terms as a function of main dependent variables consisting of core velocity $U_{m}$, thickness of the upper wake boundary layer $y_{8}-y_{5}$, thickness of the reverse velocity core $y_{5}-y_{4}$ and the thickness of the lower wake layer $y_{4}-y_{1}$. This is necessary for the purpose of maintaining the same number of main equations as the number of the above-mentioned main dependent variables. The above task can be accomplished by making use of momentum theorem in the flow direction and perpendicular to the flow direction, by use of the continuity equation and by the use of appropriate velocity profile similarity conditions; the control volumes chosen for this purpose are $U B_{1} B_{2} C_{2} U$ and $L B_{4} B_{5} C_{4} L$ shown in figure ll-1(b). Thus, the following auxiliary equations can be derived:

$$
\begin{align*}
\begin{aligned}
\frac{d y_{4}}{d x}=-\left[\{ \frac { d } { d x } ( y _ { 4 } - y _ { 1 c } ) \} \left\{-s_{1}\right.\right. & +s_{2}\left(\frac{u_{m}}{U_{e_{l}}}\right) \\
& \left.+s_{2} \cdot\left(y_{4}-y_{1 c}\right)\left\{\frac{d}{d x}\left(\frac{u_{m}}{U_{e_{l}}}\right)\right\}\right]
\end{aligned} \\
\begin{aligned}
\frac{d y_{1}}{d x}=-\left[\left\{\frac{d}{d x}\left(y_{4}-y_{1 c}\right)\right\} \cdot\right. & \left\{s_{3}+s_{2}\left(\frac{u_{m}}{U_{e_{l}}}\right)\right. \\
& \left.\left.+s_{2} \cdot\left(y_{4}-y_{1 c}\right) \frac{d}{d x}\left(\frac{u_{m}}{U_{e_{l}}}\right)\right\}\right] \\
\frac{d y_{5}}{d x}=\left[\{ \frac { d } { d x } ( y _ { 5 c } - y _ { 5 } ) \} \left\{-s_{1}\right.\right. & +s_{2}\left(\frac{u_{m}}{U_{e_{u}}}\right) \\
& \left.+s_{2}\left(y_{5 c}-y_{5}\right)\left\{\frac{d}{d x}\left(\frac{u_{m}}{U_{e_{u}}}\right)\right\}\right] \\
\frac{d y_{8}}{d x}= & \left.\left\{\frac{d}{d x}\left(y_{5 c}-y_{5}\right)\right\}\left\{s_{3}+s_{2}\left(\frac{u_{m}}{U_{e_{u}}}\right)\right\}+s_{2}\left(y_{5 c}-y_{5}\right)\left\{\frac{d}{d x}\left(\frac{u_{m}}{U_{e_{u}}}\right)\right\}\right]
\end{aligned} \tag{11-15}
\end{align*}
$$

The expressions for the $y$-component velocity at the lower and upper boundaries of the reverse velocity core, in terms of quantities composed of the main dependent variables and velocities at the outer edges of the wake boundary layer velocity profiles in Region 1, can be derived as:

$$
\begin{align*}
v_{\left(y_{5}\right)}=- & \left\{\frac{s_{3}+s_{2}\left(\frac{u_{m}}{U_{e_{u}}}\right)}{s_{2}+s_{4}\left(\frac{u_{m}}{U_{e_{u}}}\right)}\right\} \cdot\left\{\kappa_{1 u} \frac{d}{d x}\left(y_{5 c}-y_{5}\right)\right\} \cdot\left(u_{e_{u}}-u_{m}\right) \cdot \\
& \left\{\left(\frac{1}{k_{l_{u}}}\right) \frac{\left(s_{2}\right)\left(s_{3}-s_{5}\right)+\left(\frac{u_{m}}{u_{e_{u}}}\right)\left\{s_{2}^{2}+s_{4}\left(s_{3}-s_{5}\right)\right\}+\left(\frac{u_{m}}{U_{e_{u}}}\right)^{2} s_{2} \cdot s_{4}}{\left(s_{5}-s_{2}\right)+2 . s_{2}\left(\frac{u_{m}}{u_{e_{u}}}\right)+s_{4} \cdot\left(\frac{u_{m}}{U_{e_{u}}}\right)^{2}}\right. \tag{11-19}
\end{align*}
$$

and

$$
\begin{align*}
v\left(y_{4}\right)=- & \frac{s_{3}+s_{2}\left(\frac{U_{m}}{U_{e_{\ell}}}\right)}{\left\{\frac{s_{2}+s_{4}\left(\frac{U_{m}}{U_{e_{\ell}}}\right)}{}\right) \cdot\left\{K_{1_{\ell}} \frac{d}{d x}\left(y_{4}-y_{1_{c}}\right)\right\} \cdot\left(u_{e_{\ell}}-u_{m}\right) \cdot} \\
& \left\{\left(\frac{1}{K_{1}}\right) \frac{\left(s_{2}\right)\left(s_{3}-s_{5}\right)+\left(\frac{u_{m}}{U_{e_{\ell}}}\right)\left\{s_{2}^{2}+s_{4}\left(s_{3}-s_{5}\right)\right\}+\left(\frac{u_{m}}{U_{e_{\ell}}}\right)^{2} s_{2}: s_{4}}{\left(s_{5}-s_{2}\right)+2 s_{2}\left(\frac{U_{m}}{U_{e_{\ell}}}\right)+s_{4} \cdot\left(\frac{U_{m}}{U_{e_{\ell}}}\right)^{2}}\right. \tag{11-20}
\end{align*}
$$

where

$$
\begin{aligned}
& S_{1}=2 \int_{0}^{K_{1}} P_{(\xi)} d \xi-\int_{0}^{K_{1}} P_{(\xi)}^{2} d \xi \\
& S_{2}=\int_{0}^{K_{1}} P{ }_{(\xi)} d \xi-\int_{0}^{K_{1}} P_{(\xi)}^{2} d \xi
\end{aligned}
$$

$$
S_{3}=\int_{0}^{K_{1}}\left(1-P_{(\xi)}\right)^{2} d \xi
$$

$$
\left.S_{4}=\int_{0}^{K_{1}} P_{(\xi)}^{2}\right) d \xi, \quad S_{5}=\int_{0}^{K_{1}}\left(1-P_{(\xi)}\right) d \xi
$$

and

$$
k_{1 u}=\frac{y_{8}-y_{5}}{y_{5 c}-y_{5}} ; \quad k_{1_{\ell}}=\frac{y_{4}-y_{1}}{y_{4}-y_{1_{c}}}
$$

The values of the integrals

$$
\int_{0}^{K_{1}} P_{(\xi)} d \xi \quad \text { and } \quad \int_{0}^{K_{1}} P_{P}^{2}(\xi) d \xi
$$

which are needed for the evaluation of $S_{1}$ through $S_{5}$, as a function of $K_{1}$ are shown in figure IV-39 of Section IV-2.4. The curves shown in this figure have been obtained from the experimental data. The equation for the variation of the width of the reverse velocity core can be obtained from equations (11-15) and (11-17) as

$$
\begin{align*}
\frac{d}{d x}\left(y_{5}-y_{4}\right)= & {\left[-s_{1}\left\{\frac{d}{d x}\left(y_{5 c}-y_{5}\right)+\frac{d}{d x}\left(y_{4}-y_{1 c}\right)\right\}\right.} \\
& +s_{2}\left\{\left(\frac{u_{m}}{U_{e_{u}}}\right) \cdot \frac{d}{d x}\left(y_{5 c}-y_{5}\right)+\left(\frac{u_{m}}{U_{e_{\ell}}}\right) \cdot \frac{d}{d x}\left(y_{4}-y_{1 c}\right)\right\} \\
& \left.+s_{2}\left\{\left(y_{5 c}-y_{5}\right) \frac{d}{d x}\left(\frac{u_{m}}{U_{e_{u}}}\right)+\left(y_{4}-y_{1 c}\right) \frac{d}{d x}\left(\frac{u_{m}}{U_{e_{l}}}\right)\right\}\right] \tag{11-21}
\end{align*}
$$

Referring to figures $11-1(b)$ and $11-2$, it can be seen that the five main or primary variables have to be computed for the purpose of complete specification of the flow in Region 1 behind the blunt trailing edge airfoils. They are:
(i) Magnitude of the velocity $U_{m}$ in the reverse velocity core layer,
(ii) Thickness of the reverse velocity core $\left(y_{5}-y_{4}\right)(x)$,
(iii) Thickness of the lower wake boundary layer $\left(y_{4}-y_{1}\right)(x)$,
(iv) Thickness of the upper wake boundary layer $\left(y_{8}-y_{5}\right)(x)$, and
(v) Orientation in space of the locus of the lower edge, $y_{l}(x)$.

The five main or primary equations, for the computations of the above five dependent variables, are equations ( $11-12$ ) through ( $11-19$ ). These equations contain terms such as

and the $C_{p}$ distributions at the edges of various layers. In order to be able to solve these main equations, however, the values of the above quantities either have to be known a priori and/or more auxiliary equations are required which express the above quantities as the functional relationships in terms of main dependent variables. As the viscous flow, in the wake of blunt base airfoils is turbulent, theoretical expressions for the above quantities are not available as in the case of laminar boundary layers. Recourse is then made to experimental measurements, and dimensional analyses to obtain empirical expresșions for the above parameters by the use of experimental data for the particular flow, which is being investigated. This matter is discussed in Section IV. 2 of RESULTS AND DISCUSSION.

The above five primary equations are ordinary, nonlinear, nonhomogeneous, and coupled differential equations. These equations can be further reduced into the form,

$$
\begin{equation*}
\bar{\gamma}^{\prime}=F(x, \bar{Y}) ; \quad \bar{Y}_{(a)}=Y \bar{a} \tag{11-22}
\end{equation*}
$$

where $\quad \bar{Y}^{\prime}=$ vector notation for five dependent variables to be computed,
$x=$ independent variable - distance along the airfoil wake
$\bar{Y}(a)=i n i t i a l$ conditions at the trailing edge which are specified,
and symbol $1=d / d x$.
The mathematical problem represented by equation (11-22) is called an initial value problem. Various single-step or multi-step numerical methods, such as the single-step Euler method and multi-step predictor corrector or the Runge-Kutta method and a few others, are available for obtaining solutions to the above set of differential equations. The choice of a method depends upon the particular problem and is governed by the desired accuracy, time of computation, core size available in a particular computer and other dictating factors.


Figure 11-1(a).
Physical Model for viscous Flow in the Wa...
Trailing Edge Single-Comer
Trailing Edge Single-Component Airfoil the Wake of
(Continued)


Figure ll-1(b). Details of Flow in Region 1


Figure $|1-2|$ Typical Pressure Distribution on the Characteristic Loci
in the Wake Behind Blunt Trailing Edge Airfoils


Figure 11-3. Typical Velocity Profile in Region 1

## III. EXPERIMENTAL WORK

The experimental program was conducted for the blunt and sharp trailing edge $G A(W)-1$ airfoils in the research wind tunnel facility at the LockheedGeorgia Company. Measurements in the wake flow boundary layer were obtained for both the blunt and sharp trailing edge airfoils. These measurements were obtained for the purpose of studying the relative development of the viscous flow behind the blunt and sharp trailing edge airfoil wakes, for the development of the physical flow model, and for formulating relationships between fundamental flow parameters. The above information is needed in the present analytical method for the solution of the wake flow behind arbitrary blunt and sharp trailing edge single-component airfoils. Experimental data were also obtained in the separated boundary layer on the surface of the airfoil for the purpose of future development of the mathematical model for separated boundary layer development on airfoils in the presence of extensive flow separation. In this section, a brief description is given of the airfoil model, the experimental facility, the probes for measurement of total and static pressure, and the types of tests. Reference 1 gives the detailed description of tunnel instrumentation, side-wall blowing requirements in the tunnel working section and the data reduction and acquisition system.

## III. 1 Description of Airfoil Models

The wing model was milled from aluminum stock so as to provide a GA(W)-1 airfoil section contour with the trailing edge thickness approximately 2 percent of the airfoil chord. A sharp trailing edge extension was fabricated which could be attached to the present basic blunt trailing edge $G A(W)-1$ airfoil model. The photographs of the present blunt trailing edge model and the sharp trailing edge extensions are shown in figure lll-1. Section geometries for the present sharp and blunt trailing edge airfoils are shown plotted in figure III-2. Geometry of the GA(W)-1 airfoil of reference 7, with 0.7 percent trailing edge thickness is also plotted in figure lll-2 for the purpose of illustrating perspective comparison of three different trailing edge geometrical shapes of the GA(W)-1 airfoil section. The chord of the present blunt trailing edge airfoil model is equal to 26.11 cm ( 10.28 inches) and the present sharp trailing edge airfoil has a 27.94 cm ( 11 inches) long chord. The airfoil model has a span of 76.20 cm ( 30.0 inches) to facilitate a floor-to-ceiling mount in the wind tunnel test section.

For the purpose of measurements of airfoil surface pressure distribution, a total of 40 static pressure orifices are provided on the blunt trailing edge model, 22 on the upper surface and 18 on the lower surface. For the sharp trailing edge airfoil configuration, an additional two orifices are provided near the trailing edge on the upper surface. Tables III.1 and III.2 show the coordinates of the present blunt and sharp trailing edge $G A(W)-1$ airfoils, respectively; the coordinates shown are nondimensionalized with respect to their own chord. Tables III.3 and III. 4 show the chordwise locations for the static pressure orifices for the present blunt and sharp trailing edge airfoils.

## Ill.2 Wind Tunnel Facility

The experimental work was conducted in the MTF (Model Test Facility) wind tunnel which is briefly described. The test section is rectangular with a width of 109.2 cm ( 43 inches), height of 76.2 cm ( 30.0 inches) and the length of 121.9 cm ( 48 inches). The wind tunnel is a closed-circuit tunnel, powered by a 400 horsepower synchronous speed induction motor. The velocity range of the wind tunnel facility with an empty test section is 0 to 91.44 meters per second ( $300 \mathrm{ft} / \mathrm{sec}$ ) with a maximum dynamic pressure of 4984 newtons per square-meter ( 104.1 psf ). The dynamic pressure variations in the test section can be maintained within approximately 0.1 percent during a typical boundary layer survey which takes about ten minutes in this facility. At a dynamic pressure of 2394 newtons per square meter, the nominal Reynolds number in the test section of $4.035 \times 10^{6}$ per meter of chord length.

The test section is equipped with the side-wall boundary layer control slots through which higher energy, with the desired blowing coefficient $C_{\mu}$, is blown to prevent the boundary layer from getting thick and separating on the upper and lower side walls. The side-wall boundary layer thickens and separation takes place due to the adverse pressure gradients created upstream of the model leading edge as the model angles of attack are increased up to $C_{L_{\text {max }}}$ conditions. The boundary layer control slots are located approximately one chord length upstream of the model leading edge on the upper and lower walls of the test section, and these are 77.2 cm ( 30.4 inches) wide. The heights of the blowing slots can be varied from 0.025 to 0.25 centimeters. The method which was used to determine the appropriate blowing requirements for a given airfoil in the tunnel at a specified angle of attack is described in detail in reference 1 which also shows the schematics of the present boundary layer control system used in the tunnel.

The freestream total and static pressures are measured in the settling chamber ahead of the test section and at the entrance to the test section, respectively, by the use of the pressure orifice rings. Pressure leads are installed so that transducers can sense total pressure and the difference between test section total and static pressures. The stagnation temperature of the freestream flow is measured by means of a calibrated thermistor located in the settling chamber. The detailed description of the wind tunnel and the additional instrumentation is given in references 1 and 5.
111.3 Special Instrumentation - Pressure Probes, Wall Shear Device, and Hot-Wire Anemometer

[^0]supports, and the probe drive mechanism, is shown in figure III-4, The photographs of the special type of wall shear measuring devices and the airfoil model showing the locations for inlaying these devices are included in figure lll-5. The details of additional instrumentation used for the acquisition and reduction of experimental measurements is discussed in reference 1.
111.3.1 Total and static pressure probes. - Measurements of total and static pressures were made by making use of forward and reverse total pressure probes and disc-type static pressure probes. The total pressure probes were made from 0.127 cm ( 0.050 inch ) outside diameter tubing which was flattened to $.0635 \mathrm{~cm}(.025 \mathrm{inch})$ at the ends. The detailed drawings of the forward and reverse total pressure probe inserts are shown in figure lili-6. For the measurements of static pressure in viscous flow, two interchangeable, disc-type static pressure probe inserts were used, the semi-circular disc was used for the measurements within the boundary layer on the surface of the airfoil and the circular disc was used for the measurements of static pressure profiles in the wake boundary layer. Both the discs were 0.157 cm (. 062 inch) thick and the diameter of the discs was equal to $0.317 \mathrm{~cm}(0.125 \mathrm{inch})$. The detailed drawings of the circular and semi-circular static pressure probe inserts are shown in figure llI-7. As shown in this figure the semi-circular disc, used for the measurement of static pressure profiles for the boundary layer on the airfoil surface, is flattened on one edge so that the pressures could be measured close to the wall. All probe pressure instrumentation was calibrated to $\pm 2.5 \mathrm{psi}$ range transducers.

For the purpose of determining both the magnitude and the direction of the viscous flow on the surface as well as in the wake of the two-dimensional airfoil model, forward and reverse total pressure probe inserts were used as shown schematically in figure lll-3. If the pressure indicated by the forward total pressure probe insert, whose tip faces the oncoming flow, has a higher value than the local static pressure and the pressure indicated by the reverse pressure probe tip at the same y-location, then the flow direction is forward or positive at that location in the flow. However, if the pressure indicated by the reverse total pressure probe insert is higher than the pressure indicated by the forward pressure probe tip, then the direction of the flow velocity is negative or reverse at that location in the flow; when this was the case, it was observed that the static pressure was lower than the reverse total pressure. For the low-speed flow encountered in the present tests, the magnitude of the flow velocity can be obtained by the use of a modified Bernoulli equation applied locally in the viscous flow. Thus,

$$
\mathrm{P}_{\mathrm{T}_{\mathrm{R}} \text { or }}=\mathrm{P}_{\mathrm{S}_{\mathrm{C}}}+\frac{1}{2} \rho(\mathrm{p}) \mathrm{V}_{\mathrm{R}^{\text {or }}}^{2}
$$

where
$P_{S_{C}}=$ corrected static pressure at a point in the flow
$P_{T_{F}}=$ total pressure indicated by the forward total pressure probe tip, ${ }^{P_{T_{F}}}$ being higher than ${ }_{P_{S}}$ and $P_{T_{R}}$

$$
\begin{aligned}
V_{F} & =\text { magnitude of forward velocity, } P_{T_{F}} \text { being higher than } P_{S_{C}} \text { and }{ }^{P_{T_{R}}} \\
P_{T_{R}} & =\text { total pressure indicated by reverse total pressure probe tip, } \\
& P_{T_{R}}>P_{T_{F}} \\
V_{R} & =\text { magnitude of the negative or reverse velocity, } P_{T_{R}}>P_{T_{F}} \\
{ }^{\rho}(p) & =P_{S_{C}} / R T ; R=\text { universal gas constant } \\
T & =T_{O}=\text { freestream stagnation temperature. }
\end{aligned}
$$

111.3.2 Static pressure probes. - The static pressure disc probes were calibrated in the Lockheed-Georgia wall-jet facility shown schematically in figure lll-8. The calibration was performed by traversing the disc probes across the thin attached boundary layer at freestream dynamic pressures of 322 and 548 newtons $/ \mathrm{m}^{2}$ ( 6.72 and 11.44 psf ). A static pressure orifice was located on the wall at the same axial position as the disc probes. The wall static pressure was used as the true value of local static pressure and was assumed to remain constant across the boundary layer.

Disc probes are normally calibrated by determining the calibration constant from the expression: $K_{c}=\left(P_{S_{c}}-P\right) / q$, where $P_{S_{c}}$ is the true static pressure, $P^{\prime}$ is the indicated static pressure, and $q$ is the true value of the local dynamic pressure. However, since the true value of the local dynamic pressure within the airfoil wake or surface boundary layer is usually unknown, this calibration was performed by determining the calibration constant from the expression,

$$
K_{c}=\frac{P_{S_{c}}-P^{\prime}}{q^{\prime}}
$$

where

$$
\begin{aligned}
\mathrm{q}^{\prime} & =\mathrm{P}_{\mathrm{T}}-\mathrm{P}^{\prime}=\text { apparent dynamic pressure } \\
\mathrm{p}^{\prime} & =\text { indicated static pressure } \\
\mathrm{P}_{\mathrm{S}_{\mathrm{C}}} & =\text { true static pressure. }
\end{aligned}
$$

The plots of calibration curves obtained by traversing both the surface boundary layer and wake disc probes are shown in figures 111-9(a) and III-9(b). These figures show that the value of $\mathrm{K}_{\mathrm{c}}$ is independent of the probe location relative to the wall and the value of the apparent local dynamic pressure within the boundary layer. The values of the calibration constants thus found are 0.314 for the airfoil wake disc probe and 0.335 for the airfoil surface boundary layer probe. The results of additional verification surveys are shown in figure III-10. These data show the indicated and corrected values of a pressure coefficient for both probes. The local wall static pressure coefficients are shown for comparison. The true value of the static pressure coefficients are calculated by the use of the following expression,

$$
C_{P_{S_{T}}}=C_{P_{S_{m}}}+K_{c} \frac{q^{\prime}}{q_{\infty}}
$$

where

$$
\begin{aligned}
C_{P_{S_{T}}} & =\text { true value of static pressure coefficient in the viscous flow } \\
& C_{P_{S_{m}}}=\text { measured value of static pressure coefficient by the disc probe } \\
q^{\prime} & =\text { measured or apparent local dynamic pressure } \\
& =P_{T}-P^{\prime} \\
P^{\prime} & =\text { measured or apparent static pressure by the disc probe } \\
P_{T} & =\text { measured local total pressure } \\
q_{\infty} & =\text { freestream dynamic pressure } \\
K_{C}= & \text { calibration constant for the disc probe whose value depends upon } \\
& \text { the geometrical dimension of the probe only. }
\end{aligned}
$$

Figure $111-11$ shows the sensitivity of the typical disc probe to the variation in the pitch and yaw angle. These data indicate considerable tolerance to the variations in pitch angle. The yaw angle curve, however, shows a tolerance "bucket" which is relatively narrow ( $\pm 3^{\circ}$ for the disc probe data shown). This indicates that the probe can be used with confidence in twodimensional flows but is unreliable in three-dimensional flows or the flow where the streamline direction in the yaw plane is unknown.
111.3.3 Wall Shear Measuring Device. - Photographs of the wall shear measuring device are shown in figure $111-5$. These devices are inserted into the model for measuring the skin friction at several chordwise locations. Figure III-12 shows the schematics of the present wall shear device and the dimensions of the tube. The total pressure tube measures pressure in the boundary layer flow very close to the wall, the opening of the mouth of the total pressure tube is at the same chordwise location (but different spanwise location) as the static pressure orifice.

The present wall shear measuring device operates on the local dynamic similarity principle, Ludwieg and Tillman have presented evidence which suggests that

$$
\begin{equation*}
\frac{u}{u^{*}}=f\left(\frac{u^{*} y}{v}\right) \tag{111-3}
\end{equation*}
$$

where $u^{*}=$ friction velocity $\sqrt{\tau_{0} / \nu}$
$v=$ kinematic viscosity of the fluid
$u=$ velocity in boundary layer at a distance $y$ from surface

The validity of equation ( $111-3$ ) is true in a limited region near the surface and is independent of the pressure gradient or the surface roughness. If $P_{T}$ is the total pressure measured by the surface pitot tube, shown in figure ||1-12, and $P_{S}$ is the static pressure measured at the same chordwise location by the static pressure orifice, then ( $P_{T}-P_{S}$ ) is the dependent variable which is the function of variables such as density $\rho$, wall shear stress $\tau_{0}$, kinematic viscosity $v$, and the outside diameter $d$ at the opening of the surface pressure tube. Thus, we can write

$$
\begin{equation*}
\frac{\left(P_{T}-P_{S}\right) d^{2}}{\rho v^{2}}=F \cdot\left(\frac{T_{0} d^{2}}{\rho v^{2}}\right) \tag{111-4}
\end{equation*}
$$

The displacement of the effective center and the scale effects on the reading of the tube are absorbed into the above equation. The form of function $F$ in equation ( $111-4$ ) can be determined by performing the calibration of the surface total pressure tube in a circular pipe. In this case the wall shear stress $\tau_{0}$ can be calculated from the pressure drop as given by the following equation:

$$
\begin{equation*}
\tau_{0}=\left(P_{1}-P_{2}\right) \frac{D}{4 L} \tag{111-5}
\end{equation*}
$$

where

$$
\begin{aligned}
\tau_{0} & =\text { wall shear stress coefficient } \\
\left(P_{1}-P_{2}\right) & =\text { pressure drop over the length } L \\
D & =\text { inside diameter of the circular pipe. }
\end{aligned}
$$

Preston (ref. 9) obtained the form of the function $F$ in equation (111-4) by calibrating the geometrically similar circular surface total pressure tubes in the circular pipes. For the present surface total pressure tube, the ratio of the internal to the external diameter was kept at the value of 0.6 . This was done for the purpose of obtaining the geometrical similarity between the present surface tube and those used by Preston to develop the form of the function $F$ obtained experimentally by calibrating in circular pipes. Under these circumstances, the use of Preston's calibration curve for wall shear determination on the surface of the airfoil can be truly justified with the use of additional corrections. The skin friction data, obtained by the use of the present surface tube, for the $G A(W)-1$ airfoil are discussed in Section IV.1.3, where the limitations on the use of this type tube are also discussed.
111.3.4 Hot-wire anemometer probe and analog system. - The $X$ hot-wire anemometer system was used to obtain mean velocity measurements and, more importantly, for the measurement of local turbulent shear $\bar{u}^{\top} v^{\top}$ in the wake boundary layer and boundary layer on the upper surface of a GA(W)-1 airfoil. The $X$ hot-wire consists of two thin electrically heated wires suspended between two pairs of needle points, such that both wires are normal to each other. Operation of the hot-wire anemometer for the measurement of viscous flow velocity fields is governed by the laws of convective heat transfer. The
application of these laws to the velocity field determination by hot-wire anemometry is discussed in reference 1. The use of analog equipment which was used in the present study for the purpose of segregating the mean and fluctuating velocity component is briefly outlined in the following paragraphs.

If the calibrated hot-wire output has been linearized, then the strength of the signal from hot wire \#1 (fig. III-13) which is proportional to the cooling velocity can be written as

$$
v_{e(t)}^{l}=|u(t)+v(t)| \cos \alpha
$$

and from hot wire \#2

$$
v_{e(t)}^{2}=|u(t)-v(t)| \cos \beta
$$

where

$$
\begin{aligned}
u(t) & =X \text { component of instantaneous velocity } \\
& =\bar{U}+u^{\prime}(t) \\
\bar{U} & =\text { time-averaged } X \text {-component velocity } \\
V(t) & =Y \text { component of instantaneous velocity } \\
& =\bar{V}+v^{\prime}(t) \\
\bar{V} & =\text { time-averaged } Y \text {-component velocity } \\
v^{\prime}(t) & =\text { fluctuating } Y \text {-component velocity. }
\end{aligned}
$$

When the flow direction and the $X$ hot-wire probe are aligned and the turbulence intensity in the flow is low, then according to Champagne and Sleicher (ref. 15), equations (III-6) and (III-7) can be further simplified as follows:

$$
\begin{align*}
& u_{(t)}=\frac{v_{e}^{1}(t)+v_{e}^{2}(t)}{2} \\
& v(t)=\frac{v_{e}^{1}(t)-v_{e}^{2}(t)}{2}
\end{align*}
$$

An analog system was designed to analyze two linearized hot-wire signals

(1) ū mean velocity in the direction of flow,
(2) $v^{\prime}$ fluctuating component of velocity in the direction normal to mean flow, and
(3) $\bar{u}^{\top} v^{\prime}$ time-averaged product of fluctuating components to give local shear stress in the boundary layer.

Figure 11I-14(a) shows the block diagram for obtaining the above input quantities. The detailed electronic circuit of the system is shown in figure 1II-14(b). The main objective of using an $X$ hot-wire system for the present studies was to obtain the turbulent shear stress profiles in the wake flow boundary layer. This objective was not fulfilled during the present experimentation as the above output quantities were found to have several irregularities and discrepancies. The reasons for this behavior has not been discovered and, as a result, the measurements by the hot-wire anemometer are not reported.

### 111.4 Measurements of Airfoil Profile Drag

It is of interest to determine the contribution to the measured value of profile drag due to the finite value of the static pressure in the wake of the airfoil. This can be accomplished by performing certain manipulations to the available experimental method such as due to Betz or Jones. The expression due to Betz (ref. 16) for the computations of profile drag of airfoils from the measurements of pressures in the wake behind the airfoil is given by:

$$
\begin{align*}
C_{D_{\text {total }}} & =\int_{-y_{l}}^{y_{u}}\left\{\frac{P_{T_{\infty}}-P_{T}(y)}{q_{\infty}}\right\} d(y / c)+\int_{-y_{l}}^{y_{u}}\left[\left\{\left(\frac{P_{T_{\infty}}-P_{S}(y)}{q_{\infty}}\right)^{\frac{1}{2}}-\left(\frac{P_{T}(y)-P_{S}(y)}{q_{\infty}}\right)^{\frac{1}{2}}\right\}\right. \\
& \left.\cdot\left[\left(\frac{P_{T_{\infty}}-P_{S}(y)}{Q_{\infty}}\right)^{\frac{1}{2}}+\left(\frac{P_{T}(y)-P_{S}(y)^{\prime}}{q_{\infty}}\right)^{\frac{1}{2}}-2\right\}\right] d(y / c) \tag{111-9}
\end{align*}
$$

where

$$
\begin{aligned}
C_{D_{\text {total }}} & =\text { airfoil profile drag coefficient } \\
\mathrm{P}_{\mathrm{T}} & =\text { total pressure } \\
\mathrm{P}_{\mathrm{S}} & =\text { static pressure } \\
\mathrm{Y}_{\mathrm{U}}, \mathrm{y}_{\ell} & =\text { upper and lower edges of the wake where total pressure } \\
& \text { has reached freestream value } \\
\mathrm{q} & =\text { dynamic pressure. }
\end{aligned}
$$

Subscripts:

$$
\begin{aligned}
\infty & =\text { freestream value } \\
\mathrm{e} & =\text { edge of boundary layer } \\
\ell & =\text { lower edge } \\
u & =\text { upper edge } \\
(y) & =\text { at any point } y \text { in the wake boundary layer }
\end{aligned}
$$

Now

$$
\begin{equation*}
P_{T_{e}}=P_{T_{\infty}}=P_{\infty}+\frac{1}{2} \rho_{\infty} U_{\infty} 2 \tag{111-10}
\end{equation*}
$$

and

$$
\begin{gather*}
\mathrm{P}_{\mathrm{T}(y)}=\mathrm{P}_{\mathrm{S}}(y)+\frac{1}{2} \rho \mathrm{u}^{2} \\
\frac{\mathrm{P}_{\mathrm{T}_{e}}-{ }^{P_{T}}(y)}{\mathrm{q}_{\infty}}=-\left\{\frac{\mathrm{P}_{\mathrm{S}}(y)-\mathrm{P}_{\infty}}{\mathrm{q}_{\infty}}\right\}+\frac{1}{\mathrm{q}_{\infty}}\left\{\frac{1}{2} \quad \rho_{\infty} u_{\infty}^{2}-\frac{1}{2} \rho u^{2}\right\}
\end{gather*}
$$

Assume flow is incompressible $\rho=\rho_{\infty} \approx \cdot \rho_{e}=$ constant

$$
\begin{gather*}
C_{P(y)}=\frac{P_{S}(y)-P_{\infty}}{q_{\infty}} \\
\frac{P_{T_{\infty}}-P_{T}(y)}{q_{\infty}}=1-C_{P}(y)-\left(\frac{u}{U_{\infty}}\right)^{2} \\
\frac{P_{T_{\infty}}-P_{S}(y)}{q_{\infty}}=\frac{P_{\infty}+q_{\infty}-P_{S}(y)}{q_{\infty}}=1-C_{P}(y)  \tag{111-13}\\
\frac{P_{T}(y)-P_{S}(y)}{q_{\infty}}=\frac{P_{S}(y)+\frac{1}{2} \rho u^{2}-P_{S}(y)}{q_{\infty}}=\left(\frac{u}{U_{\infty}}\right)^{2} \tag{111-14}
\end{gather*}
$$

By substituting equations (11|-12), (III-13), and (III-14), equation (1II-9) can be further simplified to

$$
\begin{aligned}
C_{D_{\text {total }}}= & \int_{-y_{l}}^{y_{u}} \frac{P_{T_{\infty}}-P_{T}(y)}{q_{\infty}} d(y / c)+\int_{-y_{l}}^{y_{u}}\left[\left\{\frac{P_{T_{\infty}}-P_{S}(y)}{q_{\infty}}\right\}-\right. \\
& \left.\frac{P_{T}(y)-P_{S}(y)}{Q_{\infty}}-2\left\{\frac{\left(P_{T_{\infty}}-P_{S}(y)\right)^{\frac{1}{2}}}{q_{\infty}}-\frac{\left(P_{T}(y){ }^{-P_{S}(y)}\right)^{\frac{1}{2}}}{q_{\infty}}\right\}\right] d(y / c) \\
= & \int_{=y_{l}}^{y_{u}}\left(1-C_{P}(y)-\frac{u^{2}}{U_{\infty}^{2}}\right) d y / c+\int_{-y_{l}}^{y_{u}}\left(1-C_{P}(y)-\left(\frac{u}{U_{\infty}}\right)^{2}\right. \\
& \left.-2\left(1-C_{P(y)}\right)^{\frac{1}{2}}+2\left(\frac{u}{\frac{u}{U}}\right)\right)_{d(y / c)}
\end{aligned}
$$

or

$$
c_{D_{\text {total }}}=\int_{-y_{\ell}}^{y_{u}} 2 \frac{u}{U_{\infty}}\left(1-\frac{u}{U_{\infty}}\right) d(y / c)
$$

$$
\begin{equation*}
+2 \int_{-y_{\ell}}^{y_{u}}\left(1-c_{P(y)}\right) d(y / c)-2 \int_{-y_{l}}^{y_{u}}\left(1-c_{P(y)}\right)^{\frac{1}{2}} d(y / c) \tag{111-15}
\end{equation*}
$$

It can be seen from the above equation (111-15) that the measured airfoil profile drag coefficient is composed of two terms, namely (i) wake momentum deficit and (ii) the drag due to the finite value of static pressure coefficient in the wake. Thus,

$$
\begin{align*}
& C_{D_{\text {total }}}= C_{D_{\text {Momentum }}}+C_{D_{\text {Non-zero static }}}  \tag{111-16}\\
& \text { Deficit } \\
& \text { in Wake }
\end{align*}
$$

where

$$
C_{D_{q}}=C_{\substack{D_{\text {Momentum }}^{\text {Deficit }} \\ \text { inWake }}}=2 \int_{-y_{\ell}}^{y_{u}} \frac{u}{U_{\infty}}\left(1-\frac{u}{U_{\infty}}\right) d(y / c) \quad \text { and }
$$

$$
C_{D_{P}}=C_{\substack{D_{\text {Non-zero static }}^{\text {pressure }}}}=2 \int_{-y_{\ell}}^{y_{u}}\left\{\left(1-C_{P(y)}-\left(1-C_{P(y)}\right)^{\frac{1}{2}}\right\} d(y / c)\right.
$$

Equation (111-16) is very convenient in the experimental determination of airfoil profile drag by performing the measurements for the velocity profile and the static pressure profile. The wake velocity profile can be obtained by the use of such instrumentation as a hot-wire anemometer, laser velocimeter, or by the measurements of total and static pressure profiles across the wake boundary layer. The accurate value of the static pressure coefficient in the wake is extremely hard to measure and the error and the contribution to the measured airfoil drag coefficient due to non-zero static pressure coefficient can be determined by the use of equation (111-16) from the velocity and static pressure profile measurements at various chordwise locations in the airfoil wake. By computing both the components of the total drag at various chordwise locations in the wake, it becomes a simpler task to determine the "true" measured drag coefficient of an airfoil. This is further discussed in the Section IV.2.3 of the RESULTS AND DISCUSSION.

### 111.5 Data Reduction and Analyses

The data processing system utilized for this study contract was set up for three specific purposes, namely (i) static pressure distribution on the surface of the airfoil model, (ii) measurements of boundary layer and wall shear on the surface of the airfoil, and (iii) wake flow boundary layer quantities such as total pressure profiles, velocity profiles and the static pressure profiles downstream of the model trailing edge. These measurements were made using a Data Acquisition Unit (D.A.U.) which was controlled by a real-time digital computer which activated scanivalive units for obtaining the wall static pressure distribution and traversed a pressure and hot-wire anemometer probe for the velocity and static pressure profiles. The schematics of the system used for acquiring and reducing the test data is shown in figure lli-15. The detailed description of this sytem, including data reduction equations and the data acquisition unit, has been reported in references 1 and 5.

Two versions of the data analysis programs were developed to calculate the needed quantities. One version, named Wall Boundary Layer Data Analysis Program, was for the study of the viscous flow development on the surface of the airfoil. The second version, named Wake Boundary Layer Data Analysis Program, was for the analyses of the flow in the wake of sharp and blunt trailing edge airfoils. The wall boundary layer data analyses program computes and prints out the following quantities:

- Velocity profile, U(y)

$$
\left.\begin{array}{l}
U_{(y)}=\left\{\sqrt{\left(P_{T}(y)-P_{S}(y)\right.}\right) /\left(0.5 \times P_{(y)}\right)
\end{array}\right\}
$$

- Various integral thickness such as displacement thickness $\delta \%$, Momentum thickness $\theta$, ratios $H=\delta \% / \theta$ and $\tilde{H}=\delta \% * / \theta$
o Various "similarity" parameters and functions for separated flow boundary layer velocity profiles. The objective was to determine suitable parameters which would give velocity profile similarity for different layers of separated flow boundary layer velocity profile. Investigated similarity parameters and functions are discussed in Section IV.1.5.

The wake boundary layer data analyses program computes the prints out the following quantities:

- Wake boundary layer velocity profile $u(y) / U_{\infty}$ and $u(y) / U_{e m}$ where $\quad U_{e m}=0.5\left(U_{e_{u}}+U_{e_{\ell}}\right)$ $U_{e_{u}}=$ velocity at the upper edge of wake $U_{e_{\ell}}=$ velocity at the lower edge of the wake
o Integral of the static pressure profiles across the various layers of the wake flow behind the sharp and blunt trailing edge airfoils
- Various integral thicknesses and their ratios
- Several velocity profile similarity parameters across layers of the blunt and sharp trailing edge airfoils wake velocity profiles. These are discussed in Section IV.2.4.
o Drag coefficient due to (i) non-zero static pressure coefficient, ( $\mathrm{i} i$ ) momentum deficit in the wake, and (iii) sum of (i) and (ii).

TABLE 111.1
COORDINATES OF THE PRESENT BLUNT
TRAILING EDGE GA(W)-1 AIRFOIL

## AIRFOIL CHORD $=26.11 \mathrm{CM}$

| Station Percentage |
| :---: |
| Chord |
| $X / C$ |

0.0
0.002745
0.01095
0.02453
0.04336
0.06722
0.0959
0.12907
0.1664
0.2075
0.2520
0.2994
0.3492
0.4009
0.4378
0.4863
0.5350
0.5837
0.6323
0.6809
0.7296
0.7782
0.82685
0.8755
0.9242
0.9484
0.9728
1.0

Airfoil Section Upper Ordinate Percentage Chord Y/C
0.0
0.01601
0.03099
0.04391
0.05606
0.06686
0.07655
0.08518
0.09285
0.09951
0.10503
0.1094
0.1125
0.1142
0.1151
0.1146
0.1127
0.1102
0.1049
0.09756
0.08976
0.07903
0.0683
0.05464
0.04293
0.03512
0.02927
0.02195

Airfoil Section Lower Ordinate Percentage Chord Y/C
0.0
$-0.01163$
-0.02088
-0.02833
-0.03568
-0.04237
-0.04867
$-0.05404$
-0.05886
-0.06276
-0.06579
-0.0677
-0.0685
-0.06827
-0.0683
-0.06732
-0.06439
-0.06048
-0.05464
-0.04781
-0.03804
-0.02927
-0.01951
-0.01073
-0.00488
-0.00098
0.0
0.00195

TABLE 111.2

## COORDINATES OF THE PRESENT SHARP TRAILING EDGE GA(W)-1 AIRFOIL

AIRFOIL CHORD $=27.94 \mathrm{CM}$

0.0
0.002565
0.01023
0.02293
0.04052
0.06283
0.08962
0.1206
0.1555
0.1939
0.2355
0.2798
0.3263
0.3746
0.4091
0.4545
0.5
0.5455
0.5909
0.6364
0.6818
0.7273
0.7727
0.8182
0.8636
0.8864
0.9091
0.9318
0.9545
0.9772
1.0

Airfoil Section Upper Ordinate Percentage Chord Y/C
0.0
0.01492
0.02888
0.04092
0.05224
0.0623
0.07133
0.07937
0.08652
0.09273
0.09787
0.1019
0.1048
0.1064
0.1073
0.1068
0.105
0.1027
0.09773
0.09091
0.08364
0.07364
0.06364
0.05091
0.04
0.03273
0.02727
0.02045
0.01364
0.00682
0.0

Airfoil Section Lower Ordinate Percentage Chord Y/C
0.0
$-0.01 \overleftrightarrow{83}$
$-0.01946$
-0.02640
-0.03325
-0.03948
-0.04535
-0.05036
-0.05485
-0.05848
-0.06130
-0.06310
-0.06383
-0.06362
-0.06364
-0.06273
-0.06
-0.05636
-0.05091
-0.04455
-0.03545
-0.02727
-0.01818
-0.010
-0.00455
-0.00091
0.0
0.00182
0.00227
0.00091
0.0

TABLE 111.3
STATIC PRESSURE ORIFICE LOCATIONS ON present blunt trailing edge galw -1 AIrfoil

AIRFOIL CHORD $=26.11 \mathrm{CM}$

| Orifice Number | Upper Surface Location X/C | Orifice Number | Lower Surface Location X/C |
| :---: | :---: | :---: | :---: |
| 1 | 0.0067 | 21 | 0.963 |
| 2 | 0.0133 | 22 | 0.985 |
| 3 | 0.0264 | 25 | 0.0532 |
| 4 | 0.0532 | 26 | 0.1065 |
| 5 | 0.1065 | 27 | 0.1601 |
| 6 | 0.1601 | 28 | 0.2134 |
| 7 | 0.2134 | 29 | 0.2671 |
| 8 | 0.2671 | 30 | 0.321 , |
| 9 | 0.321 | 31 | 0.374 |
| 10 | 0.374 | 32 | 0.428 |
| 11 | 0.428 | 33 | 0.482 |
| 12 | 0.482 | 34 | 0.535 |
| 13 | 0.482 | 35 | 0.588 |
| 14 | 0.588 | 36 | 0.642 |
| 15 | 0.642 | 37 | 0.696 |
| 16 | 0.696 | 38 | 0.749 |
| 17 | 0.749 | 39 | 0.8025 |
| 18 | 0.8025 | 40 | 0.856 |
| 19 | 0.856 | 41 | 0.963 |
| 20 | 0.909 | 42 | 0.985 |

TABLE 111.4
STATIC PRESSURE ORIFICE LOCATIONS ON PRESENT SHARP TRAILING EDGE GA(W)-1 AIRFOIL

AIRFOIL CHORD $=27.94 \mathrm{CM}$

| Orifice Number | Upper Surface Location X/C | Orifice Number | Lower Surface Location X/C |
| :---: | :---: | :---: | :---: |
| 1 | 0.0063 | 22 | 0.092 |
| 2 | 0.0125 | 23 | 0.95 |
| 3 | 0.025 | 24 | 0.99 |
| 4 | 0.0496 | 25 | 0.05 |
| 5 | 0.10 | 26 | 0.10 |
| 6 | 0.15 | 27 | 0.15 |
| 7 | 0.20 | 28 | 0.20 |
| 8 | 0.25 | 29 | 0.25 |
| 9 | 0.30 | 30 | 0.30 |
| 10 | 0.35 | 31 | 0.35 |
| 11 | 0.40 | 32 | 0.40 |
| 12 | 0.45 | 33 | 0.45 |
| 13 | 0.55 | 34 | 0.50 |
| 14 | 0.55 | 35 | 0.55 |
| 15 | 0.60 | 36 | 0.60 |
| 16 | 0.65 | 37 | 0.65 |
| 17 | 0.70 | 38 | 0.70 |
| 18 | 0.75 | 39 | 0.75 |
| 19 | 0.80 | 40 | 0.80 |
| $20^{*}$ | 0.85 | 41 | 0.90 |
| 21 | 0.90 | 42 | 0.95 |



Figure III-1. Photographs of the Present Blunt Trailing Edge GA(W)-1 Airfoil Model and the Sharp Trailing Edge Extension
T.E. GEOMETRY OF

BLUNT T.E. GA(W)-1
AIRFOIL OF REF. 7
PRESENT BLUNT T.E. (2\%) GRESENT BLUNT T G ) 1 AIRFOIL

Figure III-2. Geometry of GA(W)-1 Airfoils with Three Different Trailing Edge Thicknesses


Figure ili-3. Schematics of the Probe Assembly


Figure III-4. Photograph of the Probe Assembly in the Test Section of the Wind Tunnel


Figure $111-5$. Photograph of the Shear Measuring Device


Reverse Total Pressure Probe


Forward Total Pressure Probe

$\rightarrow-.0098 \quad$| $\rightarrow .005$ | Flatten End |
| :--- | :--- |
| of Tube to |  |
| dim. shown |  |

Figure III-6. Detailed Drawing of the Forward and Reverse Total Pressure Probe Inserts


Figure 111-7. Detailed Drawings of the Disc-type Static Pressure. Probes


Figure III-8. Instrumentation Calibration Facility


Figure 1ll-9(a). Calibration of Disc Type Static Pressure Probes as a Function of Apparent Dynamic Pressure


Figure 111 -9(b). Calibration of Disc Type Static Pressure Probes as a Function of Distance Within Boundary Layer



Figure 1II-10. Comparison of Corrected and Uncorrected Pressure Coefficient in the Thin Attached Boundary Layer Disc Type Static Pressure Probe


Figure $|l|-11$. Sensitivity of the Disc Probes to the Variation in Yaw and the Pitch
O.D. - Outer Diameter of the Total Pressure Tube $=0.051 \mathrm{Cm}$.
I.D. - Inner Diameter of the Total Pressure Tube $=.0306 \mathrm{Cm}$.

(a) Wall Shear Tube Positioned for Skin Friction Measurements

(b) Cross-Sectional View of the Total Pressure Tube at the Opening

Figure 1II-12. Schematics of Wall Shear Measuring Device

(1)
$v_{e}(t)=|u(t)+v(t)| \cos \alpha$
$v_{e}(t)=|u(t)-v(t)| \cos \beta$
decomposition of the hot-wire signals

Figure III-13. Schematics of $X$ Hot-Wire Anemometer and Decomposition of Signals

Figure 111-14(a). Block Diagram of Analog System


A $\rightarrow$ SUMMER MODULE
B $\rightarrow$ INTEGRATOR MODULE
c $\rightarrow$ LOW PASS FILTER
D $\rightarrow$ MULTIPLIER MODULE

Figure III-14(b). Detailed Circuitry of Each Operational Module
BLOCK A
$(1) \quad(2)$
$E(t)+{ }^{(2)}(t)$
$(1) \quad(2)$
$(1) \quad(2)$
$E(t)+\underset{E}{(t)}$
$(1) \quad(2)$
$E(t) \leftrightarrow E(t)$

or

BLOCK B


Figure 111-14. Block Diagram and Electronic Circuit of the Analog System


Figure III-15. The Data Acquisition and Reduction System

## IV. RESULTS AND DISCUSSION

Measurements were obtained on the surface of the $G A(W)-1$ airfoil and in its wake at various angles of attack. The $G A(W)-1$ airfoil was chosen for the tests because it represents advanced supercritical technology and more importantly, it possesses the progressive trailing edge stall characteristics needed for the turbulent separated flow boundary layer on airfoils. The measurements on the surface of the airfoil consisted of surface static pressures, wall shear, and boundary layer velocity profiles in the regions of the attached and separated boundary layer flow. The above measurements were obtained for sharp and blunt trailing edge $G A(W)-1$ airfoils. The measurements were taken with the objective of understanding the behavior of the separated flow boundary layer such that the data would facilitate the formulation of a physical flow model of the separated flow boundary layer. A generalized mathematical model, for the computation of the separated flow quantities for arbitrary airfoil configuration by integral methods, can then be developed for this physical model by investigation of fundamental flow parameters and parametric relationships and by making use of experimentally measured quantities. The above measurements were obtained for sharp and II blunt trailing edge $G A(W)-1$ airfoil sections.

The measurements in the wake consisted of total pressures, static pressures, and velocity profiles. Both forward and reverse total pressures were measured for the purpose of determining both magnitude and direction of the velocity behind the blunt-base airfoil in the neighborhood of the trailing edge. The above measurements were obtained for the blunt and sharp trailing edge airfoils at several angles of attack. Velocity and static pressure profiles were obtained at several chordwise locations in the wake for the purpose of developing a physical flow model for the wake flow behind the blunt trailing edge airfoils. Parameters for the similarity of the velocity and static pressure profiles were investigated and the relationships among various physical parameters are derived in this section. Establishment of these relationships between various physical parameters is of vital importance for the prediction of drag of arbitrary airfoil sections by the generalized theoretical method such as described in this report.

> IV. 1 Correlative, Comparative, and Investigative Analysis for the Flow. Phenomena on Airfoil Surfaces

The experimental data for the pressure distribution and boundary layer development on the surface of the airfoil are compared for a sharp trailing edge $G A(W)-1$ airfoil and for $G A(W)-1$ airfoils with trailing edge thicknesses of 0.7 percent and 2 percent. Comparison between theoretical computations, experimental data for surface pressure, and boundary layer data are presented and discussed in this section. Physical flow models for the separated flow and similarity parameters for separated turbulent boundary layer velocity profiles are investigated with the help of experimentally measured quantities and dimensional analysis.
IV.1.1 Comparison of experimental pressure distribution for GA(W)-1 airfoils with three trailing edge thicknesses. - Figures IV-1 (a) through (d) show the comparisons of experimental pressure distributions on the present sharp and $2 \%$ blunt trailing edge airfoils. The above figures show some interesting phenomena associated with the flow in the circulation zone (Region 1). The surface pressure distributions at an angle of $\alpha=0^{\circ}$ for sharp and blunt trailing edges differ considerably from one another as shown in figure IV-1(a). The difference in pressure distributions and lift coefficients between sharp and blunt trailing edge airfoils decreases as the angle of attack is increased. At an angle of attack $\alpha=14.4^{\circ}$, the minimal difference in both the pressure distribution as well as the location of the point of separation ( $\Delta x / c \approx 0.05$ ) exist between the present sharp and blunt trailing edge airfoils. The above phenomena can be explained qualitatively with the aid of the schematic illustrations shown in figure IV-2. For low angles of attack, the boundary layer displacement thickness is small compared to the trailing edge thickness thickness and the point of confluence for both the blunt and sharp trailing edge airfoils lie in the neighborhood of their geometrical trailing edges as shown in figures $\operatorname{IV}-2(a)$ and (b). This results in significant difference in the equivalent fluid airfoil shape near the back end of the airfoil. In other words, the thickness and the camber distributions of the equivalent fluid airfoils, for airfoils with sharp and blunt trailing edges, are significantly different at low angles of attack. It is known from both experimental observations and theoretical computations that the pressure distribution on the airfoil surface can be altered substantially, at a constant angle of attack, by the variation of airfoil geometry near the back end. At a high angle of attack, when the boundary layer displacement thickness near the trailing edge is substantially higher than the trailing edge thickness, as shown in figures IV-2(c) and IV-2(d), only minor differences in the equivalent fluid airfoil shape exist between sharp and blunt trailing edge airfoils. The above phenomena explain why only minor differences exist in pressure distribution between sharp and blunt trailing edge airfoils at $\alpha=14.4^{\circ}$ [fig. $\operatorname{IV}-1(\mathrm{~d})$ ] whereas significant differences in the pressure distribution exists at $\alpha=0^{\circ}$ as is shown in figure $\operatorname{IV-1}(\mathrm{a})$.

Figure. IV-3(a) and (b) show the comparison of measured pressure distributions for blunt trailing edge $G A(W)-1$ airfoils, as reported in references 7 and 8, with the present data. The present blunt trailing edge airfoil has the trailing edge thickness $t / c=.02$ whereas the airfoils in references 7 and 8 have trailing edge thickness, $t / c=.007$. The differences in pressure distribution, shown in figures $\operatorname{IV}-3(a)$ and 3 (b), can be explained in the light of the hypothesis of the previous paragraph due to the difference in trailing edge thickness. Figures IV-4(a) and $4(\mathrm{~b})$ show the comparison of the measured pressure distribution for the present sharp trailing edge GA(W)-1 airfoil with the data of References 7 and 8 . The explanation for the slightly higher values of pressure coefficient on the present sharp trailing edge airfoil as compared with the other two data cannot be given very precisely. However, it needs to be pointed out that the measured aerodynamic characteristics of airfoils exhibiting trailing edge stall are quite sensitive to the wind tunnel test section dimensions compared to model dimensions and, more importantly, to the effectiveness of the sidewall boundary layer control. The differences in the measured data, shown in figures IV-4(a) and 4(b), can be
very well attributed to the effect of the sidewall boundary layer and relative dimensions of the test section with the airfoil model.
IV.1.2 Correlations between theoretical predictions and experimental data for surface pressure distribution and boundary layer qualities. Inviscid and viscous pressure distributions were computed by the use of theoretical methods developed in reference 5 and further modified by H. Morgan in reference 6. The above pressure distribution computations were made for the present sharp and blunt trailing edge GA(W)-1 airfoils at various angles of attack from $\alpha=0^{\circ}$ to $\alpha=14.4^{\circ}$, and comparison was made with the present experimental data. Boundary layer computations on the upper surface of the present sharp and blunt trailing edge airfoils were performed using boundary conditions of experimental pressure distributions, theoretical converged pressure distribution, and inviscid pressure distribution. The computations of the boundary layer on the airfoil surface, when the inviscid pressure distribution is used, are done with a newly developed boundary layer code. The purpose of this new boundary layer code is to realistically (compared to experimental data) predict boundary layer development, with the boundary conditions of potential pressure distribution, on the airfoil surface up to the point of separation. This boundary layer code is being further refined for even better predictions.

Figure IV-5(a) shows plots of theoretical inviscid, viscous (or converged), and experimental pressure distributions for a sharp T.E. airfoil at an angle of attack $\alpha=0.0^{\circ}$. This figure shows that the loss in lift due to viscous effects is insignificant and computed pressure distributions agree well with experimental data except at the trailing edge on the upper surface. The slight discrepancy in the pressure distribution, as compared to experimental data, is probably due to imperfections in the model contour due to manufacturing tolerances. Figure IV-5(b) shows the plot of computed boundary layer parameters on the upper surface of the sharp trailing edge airfoil at $\alpha=0^{\circ}$ using the theoretical converged pressure distribution as the boundary condition, whereas in figure IV-5(c) boundary layer parameters are computed using the experimental pressure distribution. Measured boundary layer quantities, obtained from velocity profile data near the trailing edge on the upper surface of the airfoil, are also shown in figures $\operatorname{IV-5(b)}$ and 5(c). Thus, computed boundary layer parameters agree better with experimental boundary layer data near the trailing edge when the experimental pressure distribution is used as can be seen from comparisons shown in figures IV-5(b) and 5(c). Figure IV-6 shows the computed theoretical inviscid and viscous pressure distribution and comparison with experimental data for the present blunt-base GA(W)-1 airfoil at an angle of attack $\alpha=0^{\circ}$. Inviscid pressure distribution shows a higher aft end loading which is reduced when the effect of viscosity is introduced. The agreement between theoretical computations and the experimental pressure distribution is quite good for the blunt-base GA(W)-1 airfoil at a low angle of attack as shown in figure IV-6. Surface boundary layer measurements were not obtained at $\alpha=0^{\circ}$ for the blunt base airfoil and hence boundary layer computations were not performed for this condition. Boundary layer computations are usually shown plotted versus $S / C$ where $S$ is the distance along the airfoil surface and $S=0$ corresponds to the origin of the boundary layer development, whereas the pressure distribution
is usually shown as $C_{p}$ versus $X / C$. Thus, in order to study the boundary layer phenomena with respect to the shape of the pressure distribution on the upper surface of the airfoil, the plots of $S / C$ versus $X / C$ for angles of attack $\alpha=0^{\circ}, 6^{\circ}, 10.3^{\circ}, 14.4^{\circ}, 16^{\circ}$, and $18.4^{\circ}$ are shown in figure $\operatorname{IV}-7$.

Figures IV-8(a) and IV-8(b) show, for the sharp trailing edge airfoil at an angle of attack $\alpha=6^{\circ}$, the comparison between computations and experimental data for the pressure distribution and boundary layer quantities. The computed viscous pressure distribution compares very well with experimental data and hence the predicted boundary layer quantities, using the computed pressure distribution, agree quite well with the experimentally measured boundary layer values at the airfoil trailing edge. Figure $\operatorname{lV}-9(a)$ shows the plots of three types of pressure distribution, namely inviscid, viscous or converged solution, and experimental data for the present blunt-base GA(W)-1 airfoil at an angle of attack $\alpha=6^{\circ}$. This figure shows that for the blunt-base airfoils at moderate to high angles of attack, even in the presence of fully attached flow conditions on the airfoil surface, the computed pressure distribution does not agree quite as well as the experimental data in contrast to the case with sharp trailing edge $G A(W)-1$ airfoil. Another phenomena, which can be observed from figure $\operatorname{IV}-\mathrm{g}(\mathrm{a})$, is that the difference between inviscid and viscous or converged solution results is very slight. This points out that the present method and procedure used in references 5 and 6 for the purpose of accounting the loss in lift due to boundary layer effects for blunt base airfoils needs theoretical refinements, for higher values of trailing edge thickness (such as 12.00 percent used presently). It is believed that the most promising theoretically well-founded procedure for the purpose of computing the viscous pressure distribution for blunt-base airfoils would be to first calculate the viscous flow in the wake of the blunt-base airfoil up to the point of confluence. The point of confluence is equivalent to the virtual trailing edge point and the shape of the equivalent or fluid airfoil can then be computed from the knowledge of the viscous flow behind the blunt base of a given airfoil. The above procedure would also be applicable for the situation when flow separation is present on the surface of the blunt-base airfoil. The results of computations for boundary layer parameters and comparisons with experimentally measured quantities are shown in figures IV-9(b), 9(c), and 9(d). The converged or computed viscous solution is used as the boundary condition for computations shown in figure IV-9(b), and the experimental pressure distribution is used for figure IV-9(c). As can be expected, the correlation between results of the boundary layer computation is better when the experimental pressure distribution is used as the boundary condition rather than the computed converged viscous pressure distribution. Figure IV-(d) shows the results of boundary layer computations, by the use of the new modified boundary layer theory, when the inviscid pressure distribution is used for boundary layer calculations. The new modified boundary layer theory is developed such that the true boundary layer development, i.e. corresponding to experimentally determined values, can be computed from the upstream stagnation point up to the point of separation on the airfoil surface by the use of the inviscid pressure distribution. The above method is used as a constituent for the semi-interim methods, which are being developed at Lockheed-Georgia Company, for the prediction of stall characteristic for airfoils exhibiting trailing edge stall. As can be seen from figure IV-9(d), the boundary layer
prediction compares reasonably well by the use of the new modified method in conjunction with the inviscid pressure distribution.

Results of computations of inviscid and viscous pressure distributions for a sharp trailing edge $\mathrm{GA}(\mathrm{W})-1$ airfoil at an angle of attack $\alpha=10.3^{\circ}$ are shown in figure IV-10, which also shows the experimentally measured pressure distribution. Boundary layer separation occurs on the upper surface between $\mathrm{X} / \mathrm{C}$ of 0.85 to 0.9 as seen from the levelling of $C_{p}$ on the upper surface. This results in the computed viscous pressure distributions being different from the experimental data even though flow separation is limited to only 10 to 15 percent chordwise near the trailing edge on the upper surface. The reason for the above discrepancy is that the boundary layer methods in reference 5 are limited to attached flow conditions only. Boundary layer data on the upper surface of the sharp trailing edge airfoil at $\alpha=10.3^{\circ}$ were not obtained and hence computations for the boundary layer were not performed for this condition. Figure IV-11(a) shows the plots of computed pressure distributions and comparison with experimental data for the present blunt-base airfoil at an angle of attack of $10.3^{\circ}$. In this case, the computed viscous pressure distribution differs only slightly from the inviscid pressure distribution and the comparison between the converged solution and experimental data suffers because the effect of viscosity is not properly accounted at the present time for the blunt trailing edge airfoil which additionally exhibited flow separation. Figure IV-11(b) shows the boundary layer computational results with the use of the experimental pressure distribution at $\alpha=10.3^{\circ}$ for the blunt-base airfoil. This figure shows that calculated boundary layer quantities agree fairly well with experimental data up to the point of separation when the experimental pressure distribution is used to calculate viscous flow quantities on airfoil surfaces by boundary layer computational methods such as that of reference 5. Downstream of the point of separation, the results of computed boundary layer parameters do not agree with the experimental data even when the experimental pressure distribution is used as input to the boundary layer program. The reason for the above phenomena is the fact that the physical flow model for the separated viscous flow is very different from that of the attached boundary layer flow. Hence, the theoretical equations for the attached boundary layer flow cannot be used to predict the realistic development of the separated flow with the use of boundary conditions of either the experimental or the computed experimental pressure distribution. Boundary layer computations with converged or computed viscous pressure distributions are not presented, because as shown in figure IV-11(a), the computed pressure distribution does not agree quite well with the experimental pressures and the separated flow conditions exist on the upper surface of the blunt-base airfoil at $\alpha=10.3^{\circ}$. Results are shown in figure IV-11(c) for the boundary layer computed with potential flow pressures by the use of the modified boundary layer method. These results show that the separation point location as well as the momentum and displacement thickness distribution can be predicted quite reasonably up to the turbulent separation point in this manner. Until such time that methods of predicting separated flow on an airfoil surface become available, such a method can be useful in an empirical manner to assess the aerodynamic characteristics and development of high performance supercritical airfoil sections.

Figures IV-12(a) through $12(\mathrm{~d})$ show the pressure distribution and integral boundary layer quantities for the present $G A(W)-1$ airfoils at an angle of attack $\alpha=14.4^{\circ}$; the results of computations are compared with experimental measurements in these figures. The point of separation on the upper surface of the blunt base airfoil, as seen from the experimental pressure distribution of figure IV-12(b), is in the neighborhood of $X / C=0.75$ whereas the velocity profile data indicate that the point of separation lies in the range $0.7 \leq X / C$ $\leq 0.81$ for $\alpha=14.4^{\circ}$. The turbulent flow separation reported in reference 7 for a blunt trailing edge $G A(W)-1$ airfoil, as obtained from tuft and pressure data, is at $X / C \approx 0.65$. The pressure distribution for the present sharp trailing edge airfoil [fig. IV-12(a)] indicate separation at $X / C$ between 0.65 and 0.7. These measurements thus suggest that the chordwise location of the separation point at a constant angle of attack is affected by the value of trailing edge thickness, the trend being that the separation point moves downstream with increasing values of trailing edge thickness (up to approximately 2 percent trailing edge thickness).

It is interesting to note, from measured boundary layer quantities shown in figures $\operatorname{IV}-12(\mathrm{c})$ and $12(\mathrm{~d})$, that ivalues of momentum thickness in the separated boundary layer region becomes negative downstream of $\mathrm{S} / \mathrm{C} \approx 1.0$ or $X / C \approx .875$. In addition, the rate of growth of the boundary layer displacement thickness, with respect to the distance along the surface of the airfoil, increases abruptly in the separated flow region as compared to the attached boundary layer region. Figure IV-13(a) shows the pressure distribution comparisons at $\alpha=16^{\circ}$, which corresponds to $C_{L_{\text {max }}}$ conditions at a Reynolds number of approximately 1 million for the blunt trailing edge $G A(W)-1$ airfoil. Figures $\operatorname{IV}-13(b)$ and $\operatorname{IV}-13(c)$ show the results of boundary layer computations at $\alpha=16^{\circ}$ for the blunt trailing edge airfoil using experimental and inviscid pressure distributions, respectively. The boundary layer integral thicknesses obtained from the measured velocity profile data are also shown in these figures.

From the computational results shown in figures $\operatorname{IV}-5$ through 13, it is possible to make some specific conclusions regarding the present state of the art in computing airfoil aerodynamic characteristics. The computed viscous pressure distribution agrees very well with experimental data for the sharp trailing edge airfoil exhibiting trailing edge stall up to an angle of attack which corresponds to incipient separation in the neighborhood of the trailing edge. The above statement is true also for the blunt-base airfoils with trailing edge thicknesses less than approximately 1 percent of the airfoil chord. The computed viscous pressure distribution for the $G A(W)-1$ airfoil, shown in figures $\operatorname{IV}-14(a)$ and $14(\mathrm{~b})$ for a trailing edge thickness of approximately 0.7 percent (which was used in reference 8 ) compared with the experimental data of reference 8 shows that excellent correlation is obtained up to $\alpha \approx 8^{\circ}$ which corresponds to the beginning of separation near the trailing edge on the airfoil upper surface. However, when the trailing edge thickness is increased to 2 percent, as with the present blunt trailing edge $G A(W)-1$ airfoil, the correlation between the computed viscous pressure distribution and experimental data suffers at angles lower than the beginning of incipient trailing edge separation. The boundary layer effects in computing the shape of the equivalent fluid airfoil, for airfoils with trailing edge thickness
greater than 1 percent, are not adequate and as a result very little difference in computed potential and viscous pressure distributions exist for such airfoils for angles of attack smaller as well as higher than for which incipient trailing separation appears on airfoil surface. When flow separation is present on either the blunt or sharp trailing edge airfoils, the use of the boundary layer methods of reference 5 with the use of the experimental pressure distribution gives reasonable predictions up to the chordwise location which is upstream of the separation point. However, after the point of separation and in the separated flow region, the calculated integral thicknesses are under-predicted as compared to the present experimental data. The modified boundary layer method (which is developed so far) for the purpose of computing boundary layer development on the airfoil surface with the potential flow pressure distribution, gives reliable results in the attached flow region and is able to predict the location of the point of separation satisfactorily when a separated flow region is present on the airfoil surface. This method has not been checked out totally and thus its full validity needs to be established by comparing with boundary layer experimental data on several airfoil configurations.

Figures $\operatorname{IV}-15(\mathrm{a})$ and $15(\mathrm{~b})$ show the comparison of the present boundary layer measurements on the blunt trailing edge $G A(W)-1$ airfoil with the experimental data of reference 7. The comparison shown in figure IV-15(a) is for an angle of attack $\alpha=10.3$ whereas results shown in figure IV-15(b) are for $\alpha=14.4^{\circ}$. The comparison for $\alpha=10.3^{\circ}$ indicates that the agreement between two sets of data is good up to $S / C \approx 0.88(X / C \approx 0.81)$ which is close to the location of the separation point. Downstream of the separation point, the rate of growth of displacement thickness is higher for the present data than that of reference 7. Data shown in figure IV-15(b) shows qualitatively the similar differences between two sets of data up to the point of separation $S / C \approx 0.8$ (or $X / C_{s e p} \approx 0.7$ ). However, after the point of separation the momentum thickness becomes negative for the present data in the separated flow region whereas the data of reference 7 shows positive values of momentum thickness throughout the separated flow region. The reason momentum thickness is negative in the separated flow region of the present data is due to the fact that the magnitude of the negative velocity as well as thickness of the reverse flow layer is much higher for the present measurements in the separated flow region than for the data of reference 7 .
IV.1.3 Wall shear correlation on upper surface of present sharp trailing edge $\overline{\mathrm{GA}}(\mathrm{W})-1$ airfoil. - The description of the present wall shear measuring device was given in Section III, and the schematic representation of this device was shown in figure ll|-12. This device operates on the principle of local dynamic similarity and if the mouth of the tube lies well within the boundary layer, then the "law of the wall" can be used to obtain and use a calibration curve which is independent of pressure gradient or surface roughness. The calibration curves, for wall shear determination by the use of the presently used device, has been obtained by Preston (ref. 9), Bradshaw (ref. 10), and Patel (ref. 11) and are shown in figures IV-16 and IV-17. Figure IV-17 shows that all the three calibration curves probably fall within the experimental scatter. The curve for the effect of the displacement of the
effective center of the surface pitot tube is included in figure IV-16 as a function of the same independent variable.

It is of interest to investigate the value of wall shear that can be calculated by the use of a razor blade calibration curve in conjunction with the measurements on the airfoil surface by the use of surface pitot tube. The flow phenomena for the determination of wall shear stress in such a case is illustrated schematically in figure $\operatorname{lV}-18$. The razor blade (strictly used for measurements in a laminar sub-layer) corresponds to the surface pitot tube flow situation when the surface pitot has a zero wall thickness. In such a case the dimension $y_{c}$, shown in figure $I V-18$ can be approximately used in place of $h$, as is shown in this figure, $y_{c}$ is equal to the height of the effective center from the wall for the surface pitot tube. if the measurements of total pressure are within the laminar sub-layer, then the following equation can be used to determine the wall shear:

$$
\begin{equation*}
\frac{u}{u^{*}}=\frac{u * y}{v} \tag{IV-1}
\end{equation*}
$$

where

$$
\begin{aligned}
& u *=\text { friction velocity }=\sqrt{\tau_{0} / \rho} \\
& \tau_{0}=\text { wall shear stress } \\
& y=\text { distance from the wall } \\
& v=\text { kinematic viscosity. }
\end{aligned}
$$

However, in order to take into account edge effects, the calibration as proposed by East (ref. 12) is given by the following expression:

$$
\begin{equation*}
y^{*}=-0.23+0.618 x^{*}+0.0165 x^{* 2} ; 1.2 \leq y^{*} \leq 3.8 \tag{IV-2}
\end{equation*}
$$

where

$$
\begin{aligned}
y * & =\log _{10}\left(\frac{\tau \Delta h^{2}}{\rho v^{2}}\right) \\
h & =y_{c} \text { in figure } \mid v-18 \\
x * & =\log _{10}\left(\Delta \mathrm{Ph}^{2} / \rho v^{2}\right) \\
\Delta p & =\text { pressure measured by surface pitot minus local static pressure. }
\end{aligned}
$$

Figures $\operatorname{IV}-19(a)$ through $\operatorname{IV}-19(c)$ show the plots of measured values of the wall shear stress coefficients on the upper surface of the present sharp T.E. GA $(W)-1$ airfoil at angles of attack of $\alpha=6^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$, respectively. The computations of $C_{f}$, from the measurements by surface pitot tube and by the use of the calibrations of Preston and East, was accomplished as described in the previous paragraphs. Theoretical computations.for the wall shear, performed by the use of the methods of reference 5 are also shown in figures IV-19(a) through 19(c) for comparison with experimentally determined
data. The above figures show that trends of computed wall shear compare quite well with experimental data when the boundary layer flow is attached. When the flow is approaching separation, the results of theoretical computations for the wall shear have higher values than the experimental data as is clearly seen in figure $\operatorname{lV}-19(c)$. Thus, vanishing of computed values of skin friction cannot be used effectively for determining the location of the point of separation on the airfoil surface. The values of wall shear by the use of East's calibration seem to indicate the higher value of the $C_{f}$. An exception to this occurs when the boundary layer is becoming quite thick or is approaching separation. The reason for this observed phenomena can be attributed to the fact that strictly speaking, the razor blade calibration is valid only in the laminar sub-layer whereas the measurements by the presently used surface pitot tube may be outside the laminar sub-layer at forward chordwise locations and at low angles of attack.
IV.1.4 Analysis of boundary layer flow on the airfoil surface and data presentation. - The detailed measurements of velocity and static pressure profiles were obtained on the upper surface of the present blunt-base GA(W)-1 airfoil at angles of attack $\alpha=6^{\circ}, 10.3^{\circ}, 14.4^{\circ}, 16^{\circ}$, and $18^{\circ}$. These measurements were obtained for the purpose of studying the mechanism of the flow approaching separation and the separated flow region on the typical trailing edge stall airfoils. The present measurements were obtained with a specific purpose of aiding in the development of the method of predicting the turbulent separated boundary layer characteristics on the surface of an arbitrary shaped airfoil in analogy with the confluent boundary layer method of reference 14. The development of such a multi-layer separated flow prediction method can be broken down into three stages. The first stage deals with the prediction of the location of the incipient separation by potential pressure distributions. The second stage concerns itself with the development of a physical flow model including characteristic fundamental physical parameters and parametric relationships in a separated flow. The third stage uses the information developed in the second stage to develop a generalized mathematical model. The validity and accuracy of the resulting theoretical method for the prediction of the separated flow solution on the airfoil surface depends strongly on the basic physical modeling which, in turn, relies heavily on understanding the experimentally measured phenomena in the separated flow. The measurements and the analysis of the viscous flow presented in this section is directed toward the development of a theoretical method of predicting the development of separated flow characteristics on high performance supercritical airfoil sections.

The comparison of velocity profiles, between the present data and the data of reference 7 , on the upper surface of a blunt trailing edge $G A(W)-1$ airfoil are shown in figures $\operatorname{lV}-20(a)$ through $20(c)$. This comparison is done in the attached flow region, in the region where the turbulent boundary layer is approaching separation, and in the region of separated boundary layer flow. Very little work was done until the present time in the area of developing measurement techniques in separated flows over an airfoil surface. Consequently, the state of the art in separated flow measurement is in its infancy. Hence, comparison of separated flow boundary layer data and the data for the boundary layer approaching separation, shown in figures IV-20(b) and 20(c),
are presented for the purpose of illustrating the differences in the measured data due to the use of different types of instrumentation. Figure IV-20(a) illustrates that the agreement between the present data and those of reference 7 is quite good when the boundary layer flow is attached to the surface of the airfoil. However, as the turbulent boundary layer approaches separation, the differences in the two measurements become apparent as is seen in figure IV-20(b). In the region where separated boundary layer flow exists, the differences between the present data and those of reference 7 become quite noticeable as indicated in figure $\operatorname{IV}-10(c)$. Data of reference 7 show that the magnitude of the negative velocity as well as the thickness of the reverse flow is substantially different from the present measurements. Present data shows that the magnitude of the negative velocity in the separated flow region varies from $u / u_{e}=0$ to $u / u_{e}=-.4$ and the thickness of the negative velocity layer may become as high as 40 percent of the total boundary layer thickness as the airfoil trailing edge is approached. However, the data of reference 7 shows only very shallow regions of reverse flow, i.e. both the magnitude and thickness of the negative velocity layer are substantially less than the present data. The measurements in reference 7 for the separated flow region were carried out by the use of a flat tube probe or cylindrical tube 5-hole probe. As reported in the above reference, these probes encountered violent fluctuations in height of less than about .05 chord above the airfoil surface in the separated flow region. The present measurements in the separated flow region were obtained with the use of forward and reverse total pressure probes and disc-type static pressure probe and no such violent fluctuations in the measured values of total and static pressures were encountered during the test.

The systematic variation of the measured static pressure profiles in the boundary layer, for the flow approaching separation and in the separated flow region of the present blunt-base GA(W)-1 airfoil are shown in figure IV-21 at various chordwise locations on the upper surface. The measured velocity profiles at the corresponding $x$-locations are also shown for the purpose of studying the static pressures variations in the separated flow region. Measured variations of static pressure across the attached turbulent boundary layer indicates an interesting phenomena. This is contrary to the assumption made in the turbulent boundary layer theory which says that $\partial P / \partial y=0$. The reasons for the variation of static pressures across the boundary layer can be attributed to curvature effects and more importantly to the presence of a thick, although attached, boundary layer on the surface of the advanced thick supercritical airfoils. The shape of the static pressure profile across the separated boundary layer flow is also worth noticing. Present measurements suggest that the static pressure is approximately constant up to a $y$-location above the airfoil surface where the flow has negative velocity and then the pressure increases in the boundary layer up to $i$ ts edge. Such information is extremely valuable in the development of physical and mathematical models for computational methods for predicting separated flow development on arbitrary shaped airfoils. The comparison of the static pressure profiles between the present data and those of reference 7 are shown in figure IV-22. This comparison is shown for three flow regions, namely attached, approaching separation, and separated flow boundary layers. The static pressure profile data, which were deduced from the measurements of pressure readings by the
five-hole cylindrical probe in reference 7 , shows constant static pressure in the boundary layer including the separated flow region whereas, as discussed previously, definite trends in the static pressure profile variations are observed for the present data. The static pressure variation measurements in the boundary layer on the airfoil surface for the present studies were performed by the use of a semicircular disc probe which is schematically shown in Figure lil-7. The measured static pressure profiles at different chordwise locations for angles of attack $\alpha=6^{\circ}, 10.3^{\circ}, 14.4^{\circ}$, and $16^{\circ}$ are shown in figures $\operatorname{IV}-23(a), \operatorname{IV}-23(b), \operatorname{IV}-23(c)$, and $\operatorname{IV}-23(d)$, respectively. These measurements are for the present blunt trailing edge $G A(W)-1$ airfoil.
IV.1.5 Physical flow model and velocity profile similarity for separated boundary layer flow. - Plots of measured velocity profiles for an angle of attack $\alpha=14.4^{\circ}$ at various chordwise locations on the upper surface of the present blunt trailing edge airfoil are shown in figure IV-24. This figure clearly illustrates how the turbulent boundary layer flow develops from fully attached flow to approaching separation and then finally remains as fully separated flow over the rest of the airfoil surface up to the trailing edge. The physical flow model for the separated flow boundary layer can be constructed with the aid of the velocity profile plots shown in figure IV-24. This flow model, which typically exists on airfoils exhibiting trailing edge stall, is shown in figure $\operatorname{IV}-25$. This flow model qualitatively represents the typical development of a separated flow boundary layer on an airfoil surface. The viscous flow in the separated boundary layer region is shown divided into four layers. The reason for dividing the flow into various layers is for the purpose of investigating and establishing the conditions for similarity or a one parameter family of velocity profiles across the various layers. If it is possible to find suitable dimensionless parameters, which would render the "similar velocity profiles" for the various layers, then it would be possible to derive a set of ordinary differential equations for the various layers from the governing partial differential equations. The number of ordinary differential equations required in a set depends upon the number of dependent variables required for completely specifying the shape of the velocity profiles for the separated viscous flow shown in figure IV-25. Various groups of dimensionless parameters and functions were considered for investigating velocity profile similarity for the layer $1-3$ shown in figure $\operatorname{IV}-25$. These parameters are shown in figures $\operatorname{IV}-26(a)$ through $\operatorname{IV}-26(e)$. The definitions of the similarity parameters and similarity functions are schematically illustrated in these figures. Plots shown in figures IV-26 illustrate the effectiveness of various parameters in achieving velocity profile similarity for layer 1-3; the similarity parameter and functions chosen in figures $\operatorname{IV}-26(\mathrm{~d})$ and $\operatorname{IV}-26(e)$ are seen to be most effective as evidenced by less scatter in the experimental data points. The least-square fit for the relationship between similarity functions and similarity variables for the curves shown in figure $\mathrm{IV}-26$ are given by

$$
\begin{aligned}
f\left(n_{1}\right)= & \left(1.0-0.4824 n_{1}-0.0643 n_{1}^{2}\right. \\
& \left.-0.0667_{1}^{3}-0.0087 n_{1}^{4}\right)
\end{aligned}
$$

$$
\begin{align*}
& f_{\left(n_{2}\right)}=\left(0.5+0.725 n_{2}-0.669 n_{2}^{2}+0.389 n_{2}^{3}-0.0833 n_{2}^{4}\right) \\
& f_{\left(n_{3}\right)}=\left(0.5+0.886 n_{3}-0.944 n_{3}^{2}+0.861 n_{3}^{3}-0.32 \eta_{3}^{4}\right) \\
& f_{\left(n_{4}\right)}=\left(1.0-1.47 n_{4}+0.18 n_{4}^{2}+0.518 \eta_{4}^{3}-0.184 n_{4}^{4}\right)  \tag{Iv-7}\\
& f_{\left(n_{5}\right)}=\left(1.0-1.077 n_{5}+0.508 n_{5}^{2}-0.186 n_{5}^{3}+0.0368 \eta_{5}^{4}\right) .
\end{align*}
$$

From the consideration of the Prandtl's mixing length theory, and becau'se the velocity profiles for layer 1-3 (fig. IV-25) have been found "similar," the following equation can be derived for the growth of layer 1-3,

$$
\begin{equation*}
\frac{d}{d x}\left(y_{0}-y_{3}\right)=c_{w_{1}} \frac{1-\frac{u_{3}}{u_{e}}}{1+\frac{u_{3}}{u_{e}}} \tag{IV-4}
\end{equation*}
$$

where

$$
\begin{aligned}
y_{0}= & \text { distance in layer } 1-3 \text { above the airfoil surface such that } u\left(y_{0}\right)= \\
& u\left(y_{0}\right)=u_{3}+0.75\left(u_{e}-u_{3}\right) \\
y_{3}= & \text { value of } y \text { in layer } 1-3 \text { where the velocity is minimum } \\
u_{3}= & \text { velocity at } y=y_{3} \\
c_{w_{1}}= & \text { constant found from experimental data } \\
\approx & 0.216
\end{aligned}
$$

An attempt was made to determine the similarity or a one parameter velocity profile family for layer 4-0 of figure IV-25. However, experimental data obtained with the presently used total and static pressure probes gave considerable scatter within layer 4-0, and hence it was not possible to investigate such parameters for the flow within this layer. The use of a laser velocimeter for the investigation of the flow in the separated flow region looks most promising because of the absence of any disturbing flow measuring device in the flow and ability of the laser to accurately measure the velocity close to the wall. Moreover, it is required to know the trends of variation of the shear stress profile and accurate static pressure profiles across the separated flow region for the development of separated flow predictions by the multi-layer method. A laser velocimeter can be used to obtain both types of data and verify these quantities by direct and indirect measurement techniques.
IV. 2 Correlative, Comparative, and Investigative Analysis for the Viscous Flow in the Wake of Blunt and Sharp T.E. Airfoils

Detailed measurements of the velocity and static pressure profiles were obtained in the wake of the present blunt and sharp trailing edge GA(W)-1 airfoils. These measurements were obtained with the use of forward and reverse total pressure probes and circular disc-type static pressure probes. Velocity and static pressure profiles in the wake were measured for these airfoils at angles of attack $\alpha=0^{\circ}, 6^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$. The above measurements were obtained at several chordwise locations in the wake for each of the above angles of attack. This was done for the purpose of constructing a physical wake flow model and functional relations between fundamental wake flow parameters for blunt and sharp trailing edge airfoils. This section shows the measured data for velocity and static pressure profiles, the computed chordwise variations of the profile drag by the use of the above measurements, and the investigation of fundamental wake flow parameters and parametric relationships for blunt and sharp trailing edge airfoils.
IV.2.1 Wake flow velocity profiles for sharp and blunt T.E. airfoils. Measured velocity profiles in the wake of the present blunt trailing edge airfoil at several chordwise locations are shown in figures IV-27(a) through 27 (d) for $\alpha=0^{\circ}, 6^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$, respectively. Whereas corresponding measurements in the wake of the sharp trailing edge $G A(W)-1$ airfoil are shown in figures $\operatorname{IV}-28(a)$ through $28(c)$ for $\alpha=0^{\circ}, 6^{\circ}$, and $10.3^{\circ}$, respectively. These velocity profile plots show, in the case of both blunt and sharp trailing edge airfoils, that near the trailing edge the velocity defect is maximum and the wake boundary layer thickness has a lower value. For a constant angle of attack the velocity defect decreases and the wake boundary layer thickness increases as the chordwise distance, downstream of the airfoil trailing edge, increases in the wake. As the angle of attack is increased, the magnitude of minimum velocity at the same chordwise location in the wake decreases and the wake boundary layer thickness increases. The reverse flow velocity profiles were not detected in the wake of the blunt trailing edge GA(W)-1 airfoil for a distance as close as 0.5 inches from the trailing edge and for angles of attack up to $10.3^{\circ}$. The absence of reverse flow velocity profiles in the wake of the present blunt-base $G A(W)-1$ airfoil in the vicinity of the trailing edge; for angles of attack less than $10^{\circ}$, is quite surprising and represents contradiction to the flow phenomena behind a bluff body in a channel flow. When the angle of attack for the present blunt-base $G A(W)-1$ airfoil is increased to $14.4^{\circ}$ typical flow phenomena in the airfoil wake, similar to that encountered for bluff bodies in a channel flow, can be observed in figure $1 \mathrm{~V}-27(\mathrm{~d})$. This figure shows that the width of the reverse flow velocity or circulatory zone (corresponding to Region 1 of figure II-1) is the largest near the trailing edge and this width decreases with the increasing chordwise distance aft of the trailing edge. This is in contrast to the separated boundary layer development on the upper surface of the airfoil where, as was discussed in the previous section, the width of the circulating or reverse flow velocity zone and the absolute magnitude of the reverse velocity increases in the downstream direction. The point of confluence, which corresponds to the zero thickness of the circulatory zone or the location where the magnitude of negative velocity in the reverse velocity
zone has increased to the value zero, is located at $X^{1 / C} \cong 0.15$ as can be seen in figure IV-27(d), This figure also shows that the wake flow downstream of the point of confluence, i.e, in Region IV of figure II-1, is qualitatively similar to the flow in the wake behind the sharp trailing edge airfoil.

The comparison of the velocity profiles in the wake of blunt and sharp trailing edge GA(W)-1 airfoils is shown in figures IV-29(a) through 29(d) for angles of attack $\alpha=0^{\circ}, 6^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$, respectively. It is seen from these figures that the present sharp trailing edge airfoil has a larger momentum deficit than the blunt trailing edge airfoil for $\alpha=0^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$, whereas for $\alpha=6^{\circ}$ both these airfoils have approximately the same momentum deficit. This would imply that the total drag coefficient $\mathrm{CD}_{\mathrm{t}} \mathrm{tatal}$ would be higher for the present sharp trailing edge airfoil than for the blunt-base airfoil at the same angle of attack if the drag due to static pressure in the wake is identical for both cases. However, in general the same momentum deficit in the wake velocity profile does not imply the same total airfoil drag because the drag due to non-zero pressure coefficients $C_{p}$ in the wake contributes significantly to the airfoil total drag coefficient. The comparison of the velocity profiles between the present blunt trailing edge GA(W)-1 airfoil (trailing edge thickness $=2$ percent of chord) with the blunt trailing edge $G A(W)-1$ airfoil of reference 7 is shown in figure IV-30 for approximately the same chordwise location. The momentum deficit for the wake velocity profiles for the present blunt-base airfoil is higher at both $\alpha=10^{\circ}$ and $14.4^{\circ}$ as seen from this figure. However, the plots of the total drag coefficient shown in figure IV-31 show the higher $C_{D_{t o t a l}}$ for the GA(W)-1 airfoil of reference 7 than for the present airfoilat the corresponding angles of attack. This can be attributed to the higher contribution due to wake static pressures to the total drag of the $G A(W)-1$ airfoil of reference 7. The values of drag coefficient shown in figure $\operatorname{IV}-31$ for present sharp and blunt GA(W)-1 airfoils have been obtained from figures IV-35 and IV-36; the drag coefficients for the 0.7 percent thick trailing edge $G A(W)-1$ were obtained from reference 13 for a $30 \%$ chord nested flap GA(W)-1 airfoil.
IV.2.2 Static pressure profiles in the wake of blunt and sharp T.E.
airfoils. - Examples of typical measurements of static pressure profiles in the wake of the blunt-base GA(W)-1 airfoil are shown in figures IV-32(a) and IV-32(b) for angles of attack of $\alpha=0^{\circ}$ and 14.4ㅇ, respectively. Corresponding measurements of velocity profiles are also shown in these figures. Static pressure profiles shown in figure IV-32(a) at different chordwise locations for an angle of attack of $0^{\circ}$ correspond to the situation when the flow in the blunt-base airfoil wake has forward velocity profiles, whereas the profiles of figure IV-32(b) correspond to the situation where mixed wake flow velocity profiles (i.e. velocity profiles with both forward and reverse flow conditions) exist near the trailing edge but change to velocity profiles with positive velocities at farther distances from the trailing edge. Measurements of static pressures shown in figure IV-32(a) indicate that the static pressure is not constant across the wake but has an approximate parabolic variation. Static pressure data shown in figure IV-32(b) for the wake flow with the mixed velocity profiles also indicate the varying static pressure across the wake flow boundary layer. However, the shape of the static pressure profiles is quite different for the mixed velocity flow than those shown in figure

IV-32(a). When the mixed flow changes to a completely forward velocity wake flow at the farthest distance from the trailing edge, parabolic shaped static pressure profiles, similar to those of figure IV-32(a), result as indicated in figure $\operatorname{IV}-32(b)$ for $X^{\prime} / C=0.25$. Figures $\operatorname{IV}-32(a)$ and $32(b)$ also indicate higher value of $C_{p}$ at the lower edge of the wake than the upper edge of the wake for an airfoill positive lift coefficient. Additionally, for $X^{\prime \prime} / C$ locations in the wake, for the forward flow velocity profile, the maximum value of static pressure occurs in the vicinity of the $y$ location corresponding to the minimum velocity point.
IV.2.3 Analysis of profile drag from measurements of flow in the airfoil wake. - Figures IV-33(a) through IV-33(d) show experimental data for the variations of momentum thickness and form factor in the wake of the present blunt trailing edge airfoil at angles of attack of $\alpha=0^{\circ}, 6^{\circ}, 10.3^{\circ}$, and $14.4^{\circ}$, respectively. The measured variations of momentum thickness and form factor in the present sharp trailing edge airfoil wake are shown in figures IV-34(a) through IV-34(c). For angles of attack below $\alpha=10.3^{\circ}$, momentum thickness and form factor decrease monotonically in the wake downstream of the trailing edges for both sharp and blunt-base airfoils. However, at an $\alpha=14.4^{\circ}$ for the blunt-base airfoil the momentum thickness first increases up to an X/C of 0,35 in the wake and then decreases continuously. The reason for the initial increase in the momentum thickness in this case can be attributed to the presence of the extensive region of the circulatory flow (Region l) up to $X / C=0.15$.

Figures IV-35 and IV-36 show the variation of the measured drag coefficients in the wake of the present sharp and blunt trailing edge airfoils at several angles of attack. The plots of the drag coefficients shown in these figures were calculated from the measured velocity and static pressure profiles at several distances in the airfoil wake. The two components of the total drag coefficients, which are shown in figures IV-35 and IV-36, are the drag due to the momentum deficit in the wake and the drag due to pressure which is different from the free stream pressure. These are defined as,
$C_{D_{q}}=$ component of total drag coefficient due to momentum deficit in the wake

$$
\begin{equation*}
=2 \int_{-y_{\ell}}^{y_{u}} \frac{u}{U_{\infty}}\left(1-\frac{u}{U_{\infty}}\right) d(y / C) \tag{1v-5}
\end{equation*}
$$

where

$$
\begin{aligned}
u & =\text { local velocity at any chordwise distance in the airfoil wake } \\
u_{\infty} & =\text { freestream velocity } \\
c & =\text { airfoil chord }
\end{aligned}
$$

$$
\left.\begin{array}{rl}
y_{u}= & \text { upper edge of the wake boundary layer where total pressure has } \\
& \text { become constant and equal to freestream total pressure }
\end{array}\right\}
$$

and
$C_{D_{p}}=$ component of total drag coefficient due to non-zero static pressure

$$
\begin{align*}
& =2 \int_{-y_{\ell}}^{y_{u}}\left(1-C_{p(y)}\right) d(y / c)-2 \int_{-y_{\ell}}^{y_{u}}\left(\left(1-c_{p(y)}\right)^{\frac{1}{2}}\right) d(y / C)  \tag{IV-6}\\
& C_{D_{\text {total }}}=C_{D_{q}}+c_{D_{p}} \tag{Iv-7}
\end{align*}
$$

Few interesting phenomena can be observed from the figures IV-35 and IV-36. The total drag coefficient $C_{D_{\text {total }}}$, calculated as above by the use of equation IV-7, is found to be varying with respect to chordwise distance from the trailing edge rather than having a constant value; these variations are higher at higher angles of attack. The drag due to momentum deficit, $C_{D_{q}}$ is always higher than the $C_{D_{t o t a l}}$ and the difference between $C_{D_{q}}$ and $C_{D_{\text {total }}}$ generally decreases as the distance from the airfoil trailing edge in the wake increases. The contribution to the total drag due to the finite value of the static pressure coefficient in the wake, i.e. the drag coefficient component $C_{D_{p}}$, has been found to be of negative sign for various chordwise locations in the wake for several angles of attack. The magnitude of $C_{D}$ may vary from $10 \%$ of the value of $C_{D_{\text {total }}}$ to as much as $100 \%$ depending upon (1) angle of attack, (2) presence or absence of boundary layer separation on airfoil surfaces, (3) sharp trailing edge airfoil or airfoil with finite trailing edge thickness, and (4) the chordwise distance in the wake from the airfoil trailing edge. The above observations point at the necessity of accurate measurements of static pressures in the wake for the purpose of determining the true drag coefficient of the airfoil section in the wind tunnel.

The effect of trailing edge thickness for a $G A(W)-1$ airfoil on the important aerodynamic characteristic L/D is illustrated in Figure IV-37. This figure shows curves of $L / D$ versus trailing edge thickness for several lift coefficients. For low values of lift coefficients ( $C_{L} \leqslant 0.75$ ), the curves of L/D vs. trailing edge thickness have a concave parabolic shape which becomes a straight line for $C_{L}=1.0$. For high values of lift coefficients ( $C_{L} \geqslant 1.3$ a convex parabolic shape is obtained. The results shown in figure IV-37 point out that there is an optimum value of trailing edge thickness for which the maximum values of L/D is obtained and this optimum value of the trailing edge thickness is dependent upon the value of the desired lift coefficient.
IV.2.4 Velocity profile similarity in the wake flow, - Comparisons of velocity profile similarity, between the flows in the wake of blunt and sharp trailing edge airfoils in Region IV (i,e, at distances far from the trailing edge) are shown in figures iV-38(a) through IV-38(c). Velocity profile similarity comparisons for blunt and sharp T.E. airfoils are shown in these figures for angles of attack $\alpha=0^{\circ}, 6^{\circ}$, and $14.4^{\circ}$ and for approximately the same chordwise location in the wake. Plots of experimental data shown in these figures indicate that the points for blunt as well as sharp trailing edge airfoils align themselves nicely on the well-defined curve at the same chordwise location for the various angles of attack. This suggests that in Region IV local dynamic similarity exists between the flows in the wake of blunt and sharp trailing edge airfoils. This being the case, the solution for the entire viscous flow in the wake of a blunt-base airfoil can be accomplished by first solving the set of differential equations in Region 1 up to the point of confluence. By doing so, the initial conditions at the point of confluence (which is analogous to the trailing edge of the sharp trailing edge airfoil), such as velocity and static pressure profiles, can be calculated. Flow in Region IV can then be calculated by the already developed method (ref. 1) for the solution of the flow in the wake of sharp trailing edge airfoils. The drag of the blunt base airfoil can thus be calculated by first making computations for the circulatory flow in Region $I$ (fig. $11-1$ ) and then for the flow in Region IV up to sufficient distance behind the airfoil trailing edge where the static pressure has reached the freestream value. Figure IV-38(d) shows the "mean or average" similarity curve for the velocity profile in Region 1 of the blunt trailing edge airfoils and the "mean" similarity curve for the velocity profiles in the wake of blunt and sharp trailing edge single-component airfoils in Region IV is shown in figure IV-38(e). The similarity curve for Region I can be approximated by the analytic expression,

$$
\begin{equation*}
P_{(\xi)}=\frac{1}{2}\left\{1+\cos \left(\xi^{0: 65} \cdot \frac{\pi}{2}\right)\right\} \tag{1v-8}
\end{equation*}
$$

An analytical expression for the curve which fits the experimental data of figure IV-38(e) is given by

$$
\begin{equation*}
P_{1(\xi)}=\frac{1}{2}\left\{1+\cos \left(\xi^{0.9} \cdot \frac{\pi}{2}\right)\right\} \tag{Iv-9}
\end{equation*}
$$

IV.2.5 Variation of wake boundary layer stretch parameter, integral area functions and growth rate functions. - The detailed observation of the experimental data in the wake of blunt and sharp trailing edge airfoils indicates that the shape of the similarity curves is not absolutely constant but varies with respect to such parameters as the ratio of the minimum velocity in the wake profile to the edge velocity, $U_{w} / U_{e l}$ or $U_{w} / U_{e_{u}}$, wake boundary layer thickness, and the ratio of upper or lower wake thickness to the total thickness of the wake boundary layers at a given chordwise location in the wake. In order to take into account this variation of velocity profile similarity function, and hence to develop improvements in the physical and mathematical model for computations of wake flow behind airfoils, the effects of this variation were studied on important wake flow parameters. Figure

IV-39 shows the effect of variation of the similarity function on the wake boundary layer stretch parameter. This stretch parameter is defined as that value of the similarity variable as the similarity function approaches the value of zero. Thus referring to figure IV-39,

$$
\begin{equation*}
\operatorname{lmt}_{y \rightarrow y_{u}} P(\xi)=\frac{u_{e u}-u}{u_{e u}-u_{w}}=0 \quad \underset{y \rightarrow y_{u}}{\xi}=K_{u}=\frac{y-y_{2}}{y_{2 c}-y_{2}}=\frac{y_{u}-y_{2}}{y_{2 c}-y_{2}} \tag{IV-10}
\end{equation*}
$$

The variation of the above defined stretch factor is plotted in figure IV-39 as a function of the velocity ratio ( $U_{w} / U_{e}$ ) and the ratio of the wake boundary layer heights. The functional relationship for the wake boundary layer stretch factor, in terms of dimensionless variable quantities for the wake boundary layer stretch factor, in terms of dimensionless variable quantities for the wake boundary layer flow parameter, derived from a least square curve fit of experimental data is given by:

$$
\begin{equation*}
Y_{1}=3.05-0.335 x_{1}-3.56 x_{1}^{2}+5.008 x_{1}^{3}-2.16 x_{1}^{4} \tag{IV-11}
\end{equation*}
$$

where

$$
\begin{aligned}
& Y_{1}=\frac{K_{u}}{\left(1+R_{1 u}\right) 0.25} \quad \text { or } \quad \frac{K_{\ell}}{\left(1+R_{1 \ell}\right) 0^{0.25}} \\
& x_{1}=R_{l u}\left(1+\frac{\delta_{u}}{\delta_{u}+\delta_{l}}\right)^{0.5} \text { or } R_{i \ell}\left(1+\frac{\delta_{\ell}}{\delta_{u}+\delta_{\ell}}\right)^{0.5} \\
& R_{1 u}=\frac{U_{w}}{U_{e u}} ; \quad R_{1 \ell}=\frac{U_{w}}{U_{e \ell}} \\
& \delta_{u}=y_{2 c}-y_{2} ; \quad \delta_{\ell}=y_{2}-y_{l c} .
\end{aligned}
$$

Theoretical equations in Section ll contain such terms as

$$
\int_{0}^{K} P(\xi) d \xi, \quad \int_{0}^{K} P_{(\xi)}^{2} d \xi
$$

and several combinations formed for these terms. For the solution of equations in Region IV during the studies conducted in reference 1 , values of the above integrals were assumed constant. The present investigation revealed that the velocity profiles in the wake behind the blunt and sharp trailing edge airfoils are not strictly similar. This would imply that the values of the above integrals are not constant but depend upon local values of certain dependent variables, such as value of the ratio and the wake boundary layer thicknesses $U_{w} / U_{e}$. As the wake boundary layer stretch parameter is a function of the abovementioned dependent variables, the functional dependence and relationship of
integrals

$$
\int_{0}^{K} P_{(\xi)} d \xi \quad \text { and } \quad \int_{0}^{K} P_{(\xi)}^{2} d \xi
$$

was established in terms of the stretch factor. Figure IV-40 shows the plots of experimental data for these integrals expressed as the function of the stretch factor K. It can be seen that the experimental data for the integrals align themselves on a well-defined curve when expressed as a function of $K$. A least-squares curve fit of these integrals is given by the following expressions,

$$
\begin{align*}
& \int_{0}^{K} P(\xi) d \xi=1.225-.3215 K+.1348 K^{2}-.0142 K^{3}  \tag{IV-12}\\
& \int_{0}^{K} P_{(\xi)}^{2} d \xi=0.6183+.2349 K-.1221 K^{2}+.01762 K^{3}
\end{align*}
$$

It should be pointed out that the assumption of the "mean or average" similarity of the velocity profiles in the wake flow makes it possible to derive a set of ordinary differential equations for the solution of the viscous flow in Regions I and IV. The second order effects, due to the deviation in the velocity profile similarity, for the purpose of improving the results of theoretical wake flow computations can be accomplished by the use of auxiliary equations such as (IV-11) and (IV-12).

The growth rate equation for the wake boundary layer can be derived based on the assumption of the "average or mean" velocity profile similarity. This equation can then be modified to take into account the deviations due to departure from the exact similarity conditions. If we make use of the hypothesis that the total or substantive derivative of the width $\delta_{u}$ (fig. IV-41) is proportional to the $y$ component of the fluctuating velocity $v^{\prime}$, then we can write

$$
\begin{aligned}
& \frac{D \delta u}{D t} \sim v^{\prime} \\
& u \frac{\partial \delta u}{\partial x}+v \frac{\partial \delta u}{\partial y}+\frac{\partial \delta u}{\partial t} \sim v^{\prime} \\
& \quad 0^{\downarrow} \quad 0^{\dagger} \text { for steady flow } \\
& \text { for 2-D flow } \\
& u \frac{\partial \delta y}{\partial x} \sim v^{\prime}
\end{aligned}
$$

Further, by making use of the Prandtl's mixing length hypothesis for $v^{\circledR}$ and making use of the mean velocity for the upper half of the wake, the above equation can be modified as

$$
\begin{equation*}
\left(\frac{U_{e u}+U_{w}}{2}\right) \quad \frac{d \delta_{u}}{d x} \sim \ell \frac{d u}{d y} \sim \frac{\ell}{K u} \frac{U_{e u}-U_{w}}{\delta u} \tag{IV-13}
\end{equation*}
$$

Now, if some "average" similarity in the velocity profile is assumed for the upper half wake boundary layer, then this assumption implies the similarity in local geometrical dimension for the wake flow considered. Thus

$$
\begin{equation*}
\ell=\text { mixing layer }=R_{4 u} \cdot K u \cdot \delta u \tag{IV-14}
\end{equation*}
$$

Substituting equation (IV-14) into equation (IV-13) gives

$$
\begin{equation*}
\frac{d \delta u}{d x}=R_{4 u} \frac{U_{e u}-U_{w}}{U_{e u}+U_{w}} \tag{1v-15}
\end{equation*}
$$

If the experimental data indicated the "true" velocity profile similarity, which is independent of the velocity ratio and wake boundary layer thickness ratio, then $R_{4 u}$ would have a constant value. However, due to deviations from "true" similarity conditions, the second order effects can be accounted for by the use of the functional relationship for $R_{4 u}$ in terms of dependent variables in the wake flow boundary layer. The groups of dimensionless parameters for functional dependence and the experimental data are shown in figure IV-41. The functional relationship for the variation of $R_{4 U}$ is given by the following least square curve fit
where
or

$$
Y_{2}=R_{4 u}\left(1+R_{1 \ell}\right)^{2}\left(1.1+\frac{\delta_{u}-\delta_{\ell}}{\delta_{u}+\delta_{\ell}}\right)^{2.3}
$$

$$
x_{2}=R_{1 u}\left(1+\frac{\delta_{\ell}}{\delta_{u}+\delta_{\ell}}\right)^{0.8}
$$

or

$$
\begin{aligned}
& =R_{1 \ell}\left(1+\frac{\delta_{u}}{\delta_{u}+\delta_{\ell}}\right)^{0.8} \\
R_{1 u} & =\frac{U_{w}}{U_{e u}} ; \quad R_{1 \ell}=\frac{U_{w}}{U_{e \ell}}
\end{aligned}
$$

$\delta_{u}, \delta_{\ell} \rightarrow$ as shown in figure IV-41
Subscript $u \rightarrow$ upper wake
Subscript $\ell \rightarrow$ lower wake.
IV.2.6 Generalized parameters for the pressure distribution in the wake of blunt and sharp T.E. airfoils. - Figure IV-42(a) shows the plot of dimensionless pressure distribution on the locus of minimum velocity in Region 1 behind the blunt trailing edge airfoil. The parameter for the nondimensional pressure is defined as

$$
\begin{equation*}
\gamma=\frac{1}{1-C_{P_{T E}}} \cdot\left(C_{P_{\mathrm{CR}}(x)}-C_{P_{T E}}\right) \tag{IV-17}
\end{equation*}
$$

where

$$
\begin{aligned}
& { }^{C_{P_{C R}}(x)}=\begin{array}{l}
\text { static pressure coefficient along the locus of the minimum } \\
\text { velocity in Region } \mid \text { behind the blunt trailing edge }
\end{array} \\
& { }^{C_{P} P_{U E}}=\begin{array}{l}
\text { pressure coefficient on the blunt-base airfoil at the } \\
\text { trailing edge on the upper surface }
\end{array}
\end{aligned}
$$

The dimensionless pressure coefficient $\gamma$ is plotted versus the distance along the wake Region 1 normalized with respect to the length $L_{c}$ of Region 1 . The least-square curve fit for $\gamma$ vs $x / L_{c}$ is given by the expression

$$
\begin{equation*}
\gamma=0.4927\left(\frac{x^{\prime}}{L_{c}}\right)-0.2541\left(\frac{x^{\prime}}{L_{c}}\right)^{2} \tag{IV-18}
\end{equation*}
$$

Figure IV-42(b) shows the generalized parametric representation for the pressure distribution in Region IV of blunt and sharp trailing edge airfoils. The parameters for this universal pressure distribution were derived from consideration of the flow behind a backward facing step and from physical reasoning. By the choice of a properly transformed $X$-coordinate, experimental points for the static pressure coefficient on the locus of minimum velocity, for both blunt and sharp trailing edge airfoils, align themselves quite well on a single curve. This fact further illustrates that the flow in Region IV is similar for both blunt and the sharp trailing edge airfoils. The functional relationship between the parameters for the pressure along the locus of minimum velocity in the airfoil wake in Region IV, for both sharp and blunt T.E. airfoils, is given by

$$
\begin{equation*}
\frac{{ }^{C} P_{u_{\min }(x)}}{C_{P_{c}}}=0.89 e^{-0.255 \beta}+\left(0.11-0.42 \beta+0.33 \beta^{2}\right) e^{-2 \beta} \tag{IV-19}
\end{equation*}
$$

where

$$
\begin{aligned}
{ }^{{ }^{P_{U_{m i n}}(x)}} & = \\
& \text { velocity in the wake in Region } N
\end{aligned}
$$

$$
=c_{P_{c}}\left(\frac{x^{\prime}}{c}\right)\left(\frac{c}{\delta_{1}{ }^{*}}\right)
$$

$$
\begin{aligned}
\delta_{1} *= & \text { displacement thickness at the point of confluence for blunt } \\
& \text { trailing edge, or } \\
= & \text { sum of displacement thickness on the upper and lower surface } \\
& \text { at the trailing edge of the sharp trailing edge airfoil } \\
X^{\prime}= & \text { distance along chord line from the point of confluence for } \\
= & \text { blunt trailing edge airfoils, or } \\
= & \text { distance fromthe trailing edge for sharp trailing edge } \\
& \text { airfoils } \\
C= & \text { airfoil chord }
\end{aligned}
$$

Figures IV-42(c) and IV-42(d) show the generalized parametric relationships for the purpose of determining the variation of static pressure coefficients at the edges of the boundary layer and at the half-velocity points. This information is useful in the present method for the evaluation of the integrals and derivatives such as

$$
\int_{y_{5}}^{y_{8}} c_{p} d y, \quad \int_{y_{1}}^{y_{4}} c_{p} d y, \quad \frac{d C_{P_{y_{8}}}}{d x}
$$

etc., which appear in the theoretical equations of Section 11 for the solution of the flow in the wake behind an arbitrarily shaped blunt or sharp trailing edge airfoil. The auxiliary equations, developed by the use of dimensional analyses and experimental data, for the variation of $C_{p}$ on the upper and lower edges as well as half velocity points in the wake boundary layer are given by

$$
\begin{aligned}
& C_{p e_{u}}=C_{P_{u_{\text {min }}}}\left[0.85 e^{-0.1 X_{u}}+\left(0.15-0.06 x_{l}+0.21 X_{u}\right) e^{-X_{l}}\right] \\
& \text { or } \ell
\end{aligned}
$$

where

$$
\begin{aligned}
X_{u} & =C_{P_{u_{m i n}}} \frac{\delta_{u}}{\delta_{T}} \frac{C}{\delta_{T}} \quad \text { or } \\
\delta & =\text { thickness of the wake layer between minimum velocity and the } \\
& \text { half velocity point } \\
\delta_{T} & =\delta_{u}+\delta_{\ell} \\
C_{P_{u_{m i n}}} & =\text { static pressure coefficient on the locus of minimum velocity } \frac{\delta_{T}}{\delta_{T}} \\
C & =\text { airfoil chord }
\end{aligned}
$$

Subscript u $u$ upper half of wake boundary layer
$\rightarrow$ lower half of wake boundary layer


Figure IV-1 (a). Comparison of Experimental Pressure Distributions Between * the Present Sharp and Blunt Trailing Edge GA(W)-1 Airfoil at an Angle of Attack of 0.0



Figure IV-1(c). Comparison of Experimental Pressure Distributions Between the Present Sharp and Blunt Trailing Edge GA(W)-1 Airfoil at an Angle of Attack of $10.3^{\circ}$


Figure IV-1(d). Comparison of Experimental Pressure Distributions Between the Present Sharp and Blunt Trailing Edge $G A(W)-1$ Airfoil at an Angle of Attack of $14.4^{\circ}$






Figure IV-3(b). Comparison of Experimental Pressure Distributions on Blunt Trailing Edge GA(W)-1 Airfoils of Present
Experiment, Refs. 7 and 8 , at $\alpha=14.4$.



Figure IV-4(b). Comparison of Experimental Pressure Distributions on Present Sharp Trailing Edge $G A(W)-1$ Airfoil and Blunt Trailing Edge $G A(W)-1$ Airfoils of Refs. 7 and 8 at $\alpha=14.4$


Present Sharp T.E. GA(W)-1 Airfoil
Complications with Theoretical Viscous Pressure Distribution as Boundary Condition
Upper Surface $\alpha=0$
© Experimental Data, $\theta / \mathrm{C}$
Q Experimental Data, $\delta * / \mathrm{C}$
© Experimental Data, $H$


Figure 5.b) - Boundary Layer Development on the Upper Surface of Present Sharp T.E. GA(W)-1 Airfoil at $\alpha=0.0$,
and Comparison with Experiments


Figure 5(c). Boundary Layer Development on the Upper Surface of
Present Sharp T.E. GA(W)-1 Airfoil at $\alpha=0.0$, and Comparison with Experiments

$$
\begin{gathered}
\text { Present Blunt T.E. } G A(W)-1 \text { Airfoil } \\
M_{\infty}=0.184, R_{N}=2 \times 10^{6} \\
\alpha=0.0
\end{gathered}
$$

———TTheoretical Viscid Solution
© Theoretical Inviscid Solution
© Present Experiment


Figure IV-6. Comparison of Theoretical Viscous and Potential Solution with Experimental Data on Blunt Trailing Edge GA(W)-1 Airfoil


Figure $1 V-7$. $S / C$ versus $X / C$ for Blunt Trailing $E d g e ~ G A(W)-1$ Airfoil


Figure IV-8(a). Comparison of Theoretical Viscous Pressure Distribution with Experimenta] Data on Sharp Trailing Edge $G A(W)-1$ Airfoil at an Angle of Attack of $6^{\circ}$


Figure IV-8(b). Boundary Layer Development on the Upper Surface of Present Sharp Trailing Edge Airfoil at $\alpha=6.0$, and Comparison with Experiments

Present Blunt T.E. GA(W)-1 Airfoil
Angle of Attack $\alpha=6^{\circ} ; M_{\infty}=0.184 ; R_{N}=2 \times 10^{6}$
$\cdots-x$ Theoretical Viscous Solution at $\alpha=6^{\circ}$
$\Delta \Delta$ Theoretical Inviscid Solution at $\alpha=6^{\circ}$
$\odot$ Present Experimental Data on Blunt T.E. $\mathrm{GA}(\mathrm{W})-1$ Airfoil at $\alpha=6^{\circ}$

Figure IV-9(a). Comparison of Theoretical Viscous and Inviscid Solution with the Experimental Pressure Distribution on Blunt Trailing Edge GA(W)-1 Airfoil at an Angle of Attack of $6^{\circ}$


Figure IV-9(b). Boundary Layer Computations on the Upper Surface of Present Blunt T.E. GA(W)-1 Airfoil at $\alpha=6^{\circ}$ and Comparison with Experimental Data


Figure IV-9(c). Boundary Layer Computations on the Upper Surface of Present Blunt T.E. GA(W)-1 Airfoil at $\alpha=6^{\circ}$ and Comparison with Experimental Data.


Figure IV-9(d). Boundary Layer Computations on the Upper Surface of Present Blunt T.E. GA(W)-1 Airfoil at $\alpha=6^{\circ}$ and Comparison with Experimental Data


Figure IV-10. Comparison of Theoretical Viscous Pressure Distribution with Experimental Data on Sharp Trailing Edge GA(W)-1 Airfoil at an Angle of Attack of 10.3


Figure IV-11(a). Comparison of Theoretical Viscous and Inviscid Solutions with Experimental Pressure Distribution on Blunt Trailing Edge $G A(W)-1$ Airfoil at an Angle of Attack of $10.3^{\circ}$


Figure IV-11(b). Boundary Layer Development on the Upper Surface of Present Blunt Trailing Edge $G A(W)-1$ Airfoil at $\alpha=10.3$, and Comparison with Experiments


Figure IV-11(b). Concluded.


Figure IV-11(c). Boundary Layer Computations with Inviscid Pressure Distribution for Blunt Base $G A(W)-1$ Airfoil and Comparison with Experimental Data at $\alpha=10.3^{\circ}$
$\alpha=10.3$
Present Blunt T.E. GA(W)-1 Airfoil

- B.L. Computations with


Figure $\operatorname{IV}-11(c)$. Concluded.


Figure IV-12(a). Comparison of Theoretical Viscous and Inviscid Pressure Distributions with Experimental Data on Sharp Trailing Edge $G A(W)-1$ Airfoil at an Angle of Attack of $14.4^{\circ}$


Figure $\operatorname{IV}-12(b)$. Comparison of Theoretical Viscous and Potential Pressure Distributions


Figure IV-12(c). Boundary Layer Development on the Upper Surface of Present Blunt T.E. GA(W)-1 Airfoil at $\alpha=14.4$ and Comparison With Experiments.


Figure IV-12(c). Continued.


Figure IV-12(d). Results or Computations of Boundary Layer with Inviscid. Pressure Distribution for Present Blunt T.E, GA(W)-1 Airfoil at $\alpha=14.4^{\circ}$ and Comparison with Experiments.


Figure IV-12(d). Continued.

. $x / C$
Figure IV-13(a): Comparison of Experimental Pressure Distribution on Blunt T.E. GA(W)-1 Airfoil; With Theoretical Viscid and Inviscid Distribution.


Figure IV-13(b). Results of Computations of Boundary Layer ising Experimental Pressure Distribution and Experimental Boundary Layer Measurements at $\alpha=16.0^{\circ}$.


Rigure IV-13(b). Continued.


Figure IV-13(c). Results of Boundary Layer Computations with Inviscid Pressure Distribution and Experimental Boundary Layer Data for Blunt Base Airfoil at $\alpha=16,0^{\circ}$.


Figure $\operatorname{IV}-13(\mathrm{c})$. Continued.

Experimental Data are for $\mathrm{GA}(\mathrm{W})-1$ Airfoil of Reference 8 with Trailing Edge Thickness of 0.7 percent.


Figure IV-14(b). Comparison Between Theoretical Viscous Pressure Distribution and Data of Ref. 8.


Fịgüre $1 \mathrm{~V}-14(\mathrm{c})$, Continued.


Figure IV-15(a). Comparison of Present Experimental Data for Boundary Layer Integral Properties with the Data of Reference 7 at $\alpha=10.3^{\circ}$.


Figure IV-15(a). Continued


Figure IV-15(b). Comparison of Present Experimental Data for Boundary Layer Integral Properties with the Data of Reference 7 at $\alpha=14.4^{\circ}$.


Figure IV-15(b). Continued.


Figure IV-16, Calibration Curve for the Wall Shear


Figure IV-17. Comparison of Various Calibrations for Wall Shear Stress Measurements


Figure IV-18. Schematic lllustration of the Rator Blade Calibration Parameters for Equivalent Pitot Measurements.
© $\mathrm{C}_{\mathrm{f}}$ By Calibration of Ref. 11
$\Delta \mathrm{C}_{\mathrm{f}}$ By Calibration of Ref. 12
——Theoretical $\mathrm{C}_{\mathrm{f}}$ by Ref. 6


Figure IV-19. Comparison of Experimental and Theoretical Skin Friction Distribution on the Surface of a Sharp T.E. GA(W)-1 Airfoil.


Figure IV-19(c). Continued


Figure IV-20(a). Velocity Profile Comparison Between the Present Data and of Reference 7 in the Attached Flow Region Over the Upper Surface of Blunt Base. GA $(W)-1$ Airfoil


Figure IV-20 (b). Velocity Profile Comparison Between the Present Data and of Reference 7 in Approaching Separation Region Over the Upper Surface of Blunt Base GA(W)-1 Airfoil
$\odot$ Present Data; AData of Reference 7


Figure IV-20(c). Velocity Profile Comparison Between the Present Data and of Reference 7 in the Separated Flow Region over the Upper Surface of Blunt Base GA(W)-1 Airfoil.
$\alpha=10.3$
$x / c=0.48$


Figure IV-21. Variations of Static Pressure Profiles in the Three Regions and Corresponding Velocity Profile
$\alpha=6.0$
$x / c=0.64$


Figure IV-21, Continued


Figure IV-21. Continued
$\stackrel{\rightharpoonup}{w}$


Figure IV-21. Continued


Figure $\mathrm{IV}-21$. Continued


Figure IV-21. Continued


Figure IV-21. Continued


Figure IV-22. Comparison of Static Pressure and Velocity Profiles, Between the Present Data and of Reference 7 in Three Flow Regions


Figure IV-23(a). Experimental Data for the Chordwise Variations. of Static Pressure Profiles at $\alpha=6^{\circ}$


Figure IV-23(b). Experimental Data for the Chordwise Variations of Static Pressure Profiles at $\alpha=10.3^{\circ}$
$\alpha=14.4$, Present Blunt T.E. $G A(W)-1$ Airfoil


Figure IV-23(c). Experimental Data for the Chordwise Variations of Static Pressure Profiles at $\alpha=14.4^{\circ}$

-igure IV-23(d). Experimental Data for the Chordwise Variations of Static Pressure Profiles at $\alpha=16^{\circ}$


Figure IV-24. Velocity Profile Measurements of Attached Flow and Incipient Separated Flow on the Surface of a Blunt Trailing Edge GA(W)-1 Airfoil at $\alpha=14.4^{\circ}$



Figure $\operatorname{IV}-24$. Continued

## (Separated Flow)



Figure IV-25. Physical Flow Model for Separated Viscous Flow Over Airfoils Exhibiting Trailing Edge Stall
Velocity Profile Similarity Curve for Layer $1-3$
By the Use of Parameter 1



Figüre IV-26(b). Velocity Profile Similarity Curve for Layer 1-3 By the Use of Parameter 2


Figure IV-26(c). Velocity Profile Similarity Curve for Layer 1-3 by the Use of Parameter 3


Figure. IV-26(d). Velocity Profile Similarity Curve for Layer 1-3 By the Use of Parameter 4

$$
\begin{aligned}
& Y_{375}=\text { Distance } Y \text { in Layer } 1-3 \text { when } u=u_{3}+0.75\left(u_{e}-u_{3}\right) \\
& f\left(n_{5}\right)=1.0-1.077 \eta_{5}+0.508 \eta_{5}^{2}-0.186 \eta_{5}^{3}+.0368 n_{5}^{4}
\end{aligned}
$$



Figure IV-26(e). Velocity Profile Similarity Curve for Layer 1-3 by the Use of Parameter 5


Figure IV-27(a). Experimental Velocity Profiles in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=0$.


Figure IV-27(b). Experimental Velocity Profiles in the Wake of Blunt T.E. $\mathrm{GA}(\mathrm{W})-1$ Airfoil at $\alpha=6^{\circ}$

Present Blunt T, E. Airfoil
$\alpha=10.3^{\circ}, C_{\text {Ref }}=27.94 \mathrm{cms}$


Figure $\operatorname{IV}-27(c)$. Experimental Velocity Profiles in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=10.3^{\circ}$


Figure $\operatorname{IV}-27(d)$. Experimental Velocity Profiles in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=14.4^{\circ}$

## Present Blunt T.E. Airfoil




Fibure IV-27(d). Continued


Figure IV-28(a). Experimental Velocity Profiles in the Wake of Sharp T.E. GA(W)-1 Airfoil at $\alpha=0$


Figure IV-28(b). Experimental Velocity Profiles in the Wake of Sharp T.E. GA(W)-1 Airfoil at $\alpha=6^{\circ}$

Present Sharp T.E. Airfoil.

$$
\alpha=10.3, \mathrm{C}_{\text {Ref }}=27.94 \mathrm{cms}
$$



Figure IV-28(c). Experimental Velocity Profiles in the Wake


Figure $\operatorname{IV}-29(a)$. Comparison of the Velocity Profiles in the Wake of Sharp and Blunt T.E. GA(W)-1 Airfoils at $\alpha=0^{\circ}$.
$\alpha=6.0^{\circ} ; C_{\text {Ref }}=27.94 \mathrm{~cm}$
■ Present Sharp T.E. GA(W)-1 Airfoil; © Present Blunt T.E. GA(W)-1 Airfoil


Figure $\operatorname{IV}-29(b)$. Comparison of Velocity Profiles in the Wake of Sharp and Blunt T.E. GA(W)-1 Airfoils at $\alpha=6^{\circ}$.

$\square^{\prime}$ Present Sharp T.E. $G A(W)-1$ Airfoil; OPresent Blunt T.E. GA(W)-1 Airfoil


Figure IV-29(d). Comparison of Velocity Profiles in the Wake of Sharp and Blunt T.E. $G A(W)-1$ Airfoils at $\alpha=14.4^{\circ}$.


Figure IV-30(a). Comparison of Velocity Profiles Between the Present Blunt T.E. GA(W)-1 Airfoil and of Reference 7.


Figure $I V-30(\mathrm{~b}) . \quad$ Continued.


Figure $\operatorname{VV-31.} C_{D_{\text {total }}}$ Versus Angle of Attack for $G A(W)-1$ Airfoils of Varying Trailing Edge Thickness Ratios


Figure IV-32(a). Measured Static Pressure Profile in the Wake of Blunt Base GA(W)-1 Airfoil at Angle of Attack of Zero Degree..


Figure IV-32(a). Continued.

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Figure IV-32(b). Measured Static Pressure Profile in the Wake of Blunt Base $G A(W)-1$ Airfoil at an Angle of Attack of $14.4^{\circ}$.


Figure IV-32(b). Continued.


Figure IV-32(b). Continued.


Figure IV-33(a). Momentum Thickness and Form Factor Distribution in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=0$.


Figure $1 V-33(b)$. Momentum Thickness and Form Factor Distribution in the Wake of Blunt T.E. $G A(W)-1$ Airfoil at $\alpha=6^{\circ}$.


Figure IV-33(c). Momentum Thickness and Form Factor Distribution in the Wake of Biunt T.E. GA(W)-1 Airfoil at $\alpha=10.3^{\circ}$.


Figure $\operatorname{lV}-33(\mathrm{~d})$. Momentum Thickness and Form Factor Distribution in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=14.4^{\circ}$.


Figure IV-34(a). Momentum Thickness and Form Factor Distribution in the Wake of Sharp T.E. GA(W)-1 Airfoil at $\alpha=0^{\circ}$.

Present Sharp T.E. Airfoil; $\alpha=6^{\circ} ; C=27.94 \mathrm{~cm}$


Figure IV-34(b). Momentum Thickness and Form Factor Distribution in the Wake of Sharp T.E. $G A(W)-1$ Airfoil at $\alpha=6^{\circ}$.


Figure IV-34(c). Momentum Thickness and Form Factor Distribution in the Wake of Blunt T.E. GA(W)-1 Airfoil at $\alpha=10.3^{\circ}$.


Figure IV-35(a). Variation of the Measured Drag Coefficient in the Wake of Present Blunt Trailing Edge Airfoil at $\alpha=0^{\circ}$.

Present Blunt T.E. GA(W)-1 Airfoil; $\alpha=6^{\circ} ; \mathrm{C}=16.11 \mathrm{~cm}$


Figure IV-35(b). Variation of the Measured Drag Coefficient in the Wake of Present Blunt T.E. Airfoil at $\alpha=6^{\circ}$.


Figure IV-35(c). Variation in the Measured Drag Coefficient in the Wake of Present Blunt T.E. Airfoil at $\alpha=10.3^{\circ}$.

Present Blunt T.E. GA(W)-1 Airfoil; $\alpha=14.4 ; C=26.11 \mathrm{~cm}$
Drag Due to Momentum Deficit in Wake, $C_{D_{q}}$ Q Drag Due to Non-Zero Pressure in the Wake © Total Drag Coefficient; Total Drag Coefficient for Sharp T.E. Airfoil


Figure IV-35(d). Variation in the Measured Drag Coefficient in the Wake of Present Blunt T.E. Airfoil at $\alpha=14.4^{\circ}$.

Present Sharp T.E. GA(W)-1 Airfoil; $\alpha=0.0 ; C=27.44 \mathrm{~cm}$


Figure $\operatorname{IV}$-36(a), Variation of the Measured Drag Coefficient in the Wake of Present Sharp T.E, Airfoil at $\alpha=0^{\circ}$.

Present Sharp T.E. GA (W)-1 Airfoil; $\alpha=6^{\circ} ; \mathrm{C}=27.94 \mathrm{~cm}$
. $\oint$ Drag Due to Momentum Deficit in Wake, $C_{D_{q}}$ $\odot$ Drag Due to Non-Zero $C_{p}$ in the Wake, $C_{D_{p}}$ © Total Drag Coefficient $C_{D_{\text {total }}}$


Figure IV-36(b). Variation of the Measured Drag Coefficient in the Wake of Present Sharp T.E. Airfoil at $\alpha=6^{\circ}$.

Present Sharp T.E. GA(W)-1 Airfoil; $\alpha=10.3$; C -27.94 cm


Figure IV-36(c). Variation in the Measured Drag Coefficient in the Wake of Present Blunt T.E. Airfoil at $\alpha=10.3^{\circ}$.


Figure IV-37. L/D Versus Trailing Edge Thickness for $G A(W)-1$ Airfoil.


Figure $\operatorname{IV}$-38(a). Comparison of Velocity Profile Similarity in the Wake (Region IV) of Sharp and Blunt T.E. GA(W)-1 Airfoil.


Figure IV-38(b). Comparison of Velocity Profile Similarity in the Wake (Region IV) of Sharp and Blunt T.E. GA(W)-1 Airfoil.
$\alpha=14.4$


Figure IV-38(c). Comparison of Velocity Profile Similarity in the Wake (Region IV) of Sharp and Blunt T.E. GA(W)-1 Airfoil.


Figure IV-38(d). Similarity Curve for the Velocity Profiles in the Wake (Region 1) of the Blunt Base Airfoils

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Figure IV-38(e). Similarity Curve for the Velocity Profiles in the Wake (Region IV) of the Sharp and Blunt Base Airfoils


Figure IV-39. Parameter Relationship for the Wake Boundary Layer Stretch Function


Figure IV-40. Velocity Profile Similarity Integral Versus Stretch Function for Wake Behind Blunt and Sharp T.E. Airfoils


Figure IV-41. Generalized Parametric Relationship for Growth Rate Function of the Viscous Flow in the Wake of Sharp and Blunt T.E. Alrfoils


Figure IV-42(a). Variation of $\mathrm{C}_{\mathrm{P}_{\mathrm{min}}}$ in the Region I Behind the Blunt T:E: Airfoils:


Figure IV-42(b). Variation of CpUmin on the Locw of Minimum Velocity in Region IV of the Wake Flow.


Figure IV-42(c). Variation of $C p$ at the Edge of Wake Boundary Layer.


Figure IV-42(d). Variation of $C_{p}$ at Half Velocity Point in the Wake Boundary Layer.

## v. CONCLUSIONS AND RECOMMENDATIONS

From the theoretical and experimental studies presented in this report, the following conclusions and recommendations for future studies can be made.

## V. 1 Conclusions

(1) Experimental measurements indicated significant differences in the pressure distributions for $G A(W)-1$ airfoils having trailing edge thickness values of 0.0 percent (sharp trailing edge), 0.7 percent and 2.0 percent of chord at a constant angle of attack. These effects are most noticeable at low angles of attack, and tend to diminish as the angle of attack is increased, and the thickness of the separated boundary layer becomes large with respect to trailing edge thickness.
(2) The agreement between experimental data and theoretical viscous computations for pressure distributions on the $G A(W)-1$ airfoils with trailing edge thicknesses of 0.0 percent and 0.7 percent is quite good at angles of attack up to that corresponding to incipient separation at the trailing edge. However, when the trailing edge thickness is increased to approximately 2 percent of chord, the agreement between theoretical viscous pressure distribution and the experimental data suffers at angles of attack below incipient separation. The disagreement becomes more noticeable as the angles of attack are increased beyond the point of incipient separation.
(3) Even in the presence of separation, the agreement between theoretical boundary layer computations by the methods of reference 5 and the experimental measurements is quite good up to the separation point on the airfoil surface when the experimental pressure distribution is used to establish the boundary conditions. On the other hand, when the theoretically converged pressure distribution is used, the boundary layer development is accurately predicted only in the absence of separation. Reasonably good success has been achieved, however, in identifying both the point of separation and the boundary layer growth to that point by using an empirically modified boundary layer model in conjunction with the distribution. This has been verified for several GA(W)-1 and other airfoils having both sharp and blunt trailing edges.
(4) The present measurements of boundary layer quantities on the airfoil surface have been obtained by the use of forward and reverse total pressure probes and a disc type of static pressure probe. These measurements have provided sufficient qualitative information to develop a physical flow model of the turbulent separated flow boundary layer. However, considerable differences in experimental boundary layer velocity profiles in the separated flow region exist between the present measurements and the measurements of reference 7. For example, the magnitude of the negative velocity in the separated flow region and the thickness of the reverse velocity flow layer are much smaller for the data of reference 7 than that indicated by the present data under similar conditions. Measurements of wall shear gave acceptable
results in the attached flow region, but were not meaningful in the separated flow region.
(5) Parameters governing the similarity of velocity profiles in the outer layer of the separated flow region have been successfully investigated, and a similarity function has been established with the help of experimental data. An attempt was made to determine the similarity of one parameter family of the velocity profiles for the inner-most layer of the separated flow region. However, experimental data obtained with the presently used total and static pressure probes gave considerable scatter in the inner layer, and it was not possible to effectively investigate such parameters for the flow within this layer. The use of laser velocimeter measurements for the inner layers of the separated flow region looks most promising because the highly-sensitive, low-velocity flow field is not disturbed.
(6) Experimental data for the velocity profile similarity comparison in the wake at large distances from the airfoil trailing edge (Wake Region IV) align themselves nicely on a well-defined curve for both sharp and blunt trailing edge airfoils. This suggests that local dynamic similarity exists for the flow in this region and that the methodology previously developed in reference 1 for sharp trailing edge airfoils is applicable to the case of blunt trailing edges. Thus, solution of the entire viscous flow field in the wake of blunt-base airfoils and, hence, the computations of the associated profile drag, can be accomplished by first solving the new set of blunt-base differential equations in wake Region 1 (up to the point of confluence) and then continuing with the previously developed method and computer program subroutines for wake flow solutions in Region IV.
(7) The value of the profile drag coefficient, obtained by the measurement of total and static pressures in the airfoil wake at a constant angle of attack, was found to vary as a function of chordwise distance from the trailing edge, rather than being constant. The variations are higher at the higher angles of attack and when flow separation is present on the airfoil surface. Static pressure is a highly critical parameter in this computation of profile drag and, therefore, inaccuracies in static pressure measurement (at very low levels of $\Delta P$ ) are responsible for the observed variations. It becomes obvious, therefore, that the disc-type static pressure probes used in this program do not give the required accuracy, and that further investigation of alternate probe designs and/or the use of indirect methods will be required to determine true values of static pressure in the flow.
(8) Using the experimental results from reference 13 for $G A(W)-1$ airfoils with different trailing edge thicknesses, it is found that there is an optimum value of trailing edge thickness which gives the highest value of ML/D or L/D. This optimum value of trailing edge thickness varies as a function of airfoil lift coefficient.
(9) Detailed observation of the flow in the wake of blunt and sharp trailing edge airfoils indicates that wake velocity profiles are not strictly "similar." The shape of the similarity curve depends upon such parameters as ratio of the minimum velocity to the edge velocity in the wake, wake boundary
layer thickness, and the ratio of the upper and lower wake thicknesses to the total thickness of the wake boundary layer at a given chordwise location behind the airfoil. To account for this second-order effect and hence, to improve the accuracy of the physical and mathematical models, auxiliary equations have been developed to quantify the relationships between velocity profile variations and the various wake flow parameters such as growth rate functions, integral area function, and wake boundary layer parameter.

## V. 2 Recommendations

(1) To establish the validity of the physical and mathematical model for the flow in Region $l$ of the wake of a blunt trailing edge airfoil, a numerical scheme and computer program subroutines must be developed to solve the set of theoretical differential equations defined during the present study. Such computer program subroutines can be used effectively to conduct parametric studies to define optimum airfoil shape and trailing edge thickness for any given application.
(2) Additional experimental studies will be necessary to acquire valid quantitative data in regions of separated flow. The use of a noninterfering measurement device such as the laser velocimeter is recommended, particularly for measurements in the inner layers of the separated flow. Further, because small inaccuracies in wake static pressure measurement have a profound effect on integrated wake momentum (and profile drag), new techniques for direct or indirect measurement of wake static pressure must be developed.
(3) A physical flow model for the separated boundary layer on the surface of an airfoil has been developed with the help of experimental data required during the present study. The next logical steps are to develop the equations and computerized numerical schemes for the computation of characteristic separated boundary layer quantities for airfoils which exhibit trailing edge separation.
(4) The theoretical methods, developed during the present study and in reference 1 for single-component airfoils having blunt and sharp trailing edges, are valid conceptually for the computation of profile drag for the more important case of multi-component airfoil sections. The validity of the theoretical approach for sharp trailing edge single-component airfoils was established in reference 1 , and the ground work for prediction of the profile drag of single-component blunt-base airfoils has been laid out during the present studies. It is recommended that this overall approach be extended to the computations of profile drag of two component airfoils with sharp and/or blunt trailing edges.

## vi. REFERENCES

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[^0]:    Measurements of flow in the wake behind the blunt and sharp trailing edge, as well as on the upper surface of the airfoils, both in the attached and separated flow boundary layer region, were performed by the use of specially designed instrumentation. The schematics of the probe assembly - which includes forward and total pressure probes, disc-type static pressure probe and the hot-wire anemometer is shown in figure 111-3. The photograph of the probe assembly in the test section of the wind tunnel, which shows the model, probe

