Analysis on the Influence of Backlash and Motor Input Voltage in Geared Servo System

J. H. Baek, Y. K. Kwak, and S. H. Kim

Abstract— **This paper analyzes how the input voltage of the motor and the magnitude of the total backlash of a servo system with a gear reducer affect the frequency response characteristic of the servo system. The bandwidth of the system is defined as the anti-resonance frequency, which appears in the frequency response characteristic. It is found that the amount of system's bandwidth reduction due to the backlash changes greatly with motor input voltage. It is also shown that when the motor input voltage is infinite, the servo system has a bandwidth of the system that does not have any backlash. Through this work, it has become possible to determine the maximum permissible magnitude of total backlash to satisfy the desired bandwidth for a servo system with a gear reducer.**

***Index Terms*— Anti-resonance, Backlash, Bandwidth, Resonance**

I. Introduction

INTELLIGENT systems such as small, autonomous vehicles, unmanned airplanes, and guided missiles require rapid responsiveness and outstanding adaptability to environments. Such requirements need automatic servo devices with fast responses. Up to the present time, servo systems with gear reducers have been widely used in fields including autonomous vehicles and guided missiles where there are limitations on installment space and weight.

The size and weight of installed servo systems on flying objects such as guided missiles are very important as increases in the weight of the loaded servo system shortens the distance that the object can fly on a constant amount of fuel. Therefore, when the weight of the servo system increases the amount of fuel should also be increased to maintain the same flying distance. However, since the increased amount of fuel also increases the weight of a guided missile, the weight increase of the servo system has a significant effect on the performance of a guided missile.

The velocity control bandwidth and responsiveness of servo systems are greatly limited by the anti-resonance and resonance frequency appearing in the motor angular velocity output to the motor torque input [1], [2]. The anti-resonance frequency is defined as the servo system's velocity control bandwidth, so it is necessary to increase the anti-resonance frequency of the system for the servo system to have an even faster tracking and responsiveness [1]. For this purpose, we needs the research that aims towards estimating the bandwidth of a servo system with a gear reducer at the design stage and expanding the bandwidth at the design revised stage. In related research, Rue reported that the stiffness of driving linkage such as a shaft greatly affected the bandwidth of the system [3]. Dhaouadi et al. studied that the backlash affected the anti-resonance and resonance frequencies of the system [4]. Jang and Oh verified that the increase of backlash magnitude made the bandwidth reduced in the experiment [5]. These works showed that the bandwidth reduction of the system due to backlash is small.

In order to expand the bandwidth without increasing the weight of the system, Bigley et al. used the optimal control technique or state equalization control technique to minimize or eliminate the effect of the anti-resonance frequency on the servo system [1], [2], [6], [7]. However, the optimal control technique requires precise modeling, and the state equalization control technique needs very large motor output when the moment of inertia of load is large, making it difficult to manufacture the system.

In this paper, it is shown that the backlash has a significant effect on the bandwidth of the system according to the amplitude of input voltage of the motor. It is also found that a mechanical way to expand the bandwidth without increasing the weight of the system is to reduce the magnitude of backlash. Therefore, the effect of backlash magnitude will be analyzed in the change of motor's input voltage.

II. Theoretical approach

A. Modeling

Fig. 1(a) is a schematic diagram of a servo system with a gear reducer installed on a guided missile, the segment gear 2 on the fixed shaft does not rotate, and the entire portion of the shaded area in Fig. 1(a), pinion 2, the rotating shaft, gear 1, pinion 1, the motor, and the bearing rotate around the $\overline{OO'}$ axis with the rotation of the motor. It is assumed that the bearings in each rotating shaft support each shaft without clearance by preload.

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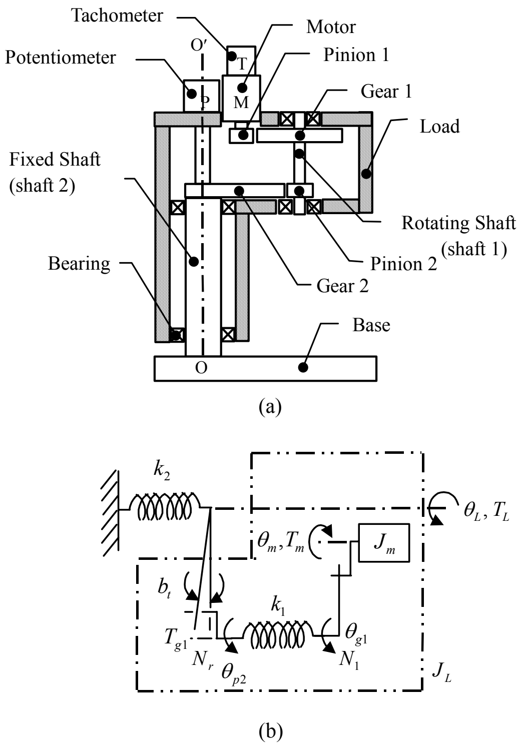


Fig. 1. The structure and model of servo system (a) The structure of servo system (b) The model of servo system

The damping effect of each component can be modeled by Yang and Sun's method [8]. However, It is assumed that the damping effect can be neglected because the considered system consists of stiff components with short length and high stiffness. The effect of viscous friction is also assumed to be negligible because it can be considered using resonant Q factor, if necessary [1], [9]. Since it is in a servo system with a two-stage gear reducer, if pinion 1 is fixed, the total backlash b_t of the system, measured at the rotating axis $\overline{OO'}$ of the load, can be expressed as

$$b_t = b_2 + \frac{1}{N_r} b_1 \tag{1}$$

where b_1 is the angular backlash magnitude expressed in degrees, obtained by measuring the backlash between pinion 1 and gear 1; b_2 is the angular backlash amount expressed in degrees, obtained by measuring the backlash between pinion 2 and gear 2; and N_r is the revolution gear ratio between pinion and gear 2. The total backlash was measured using potentiometer shown in Fig. 1(a). Because the backlash b_1 decreases with the gear ratio N_r, the backlash b_2 has a dominant effect on the system [5], [10]. Therefore, it is assumed that the total backlash b_t, measured at the final load stage, exists at the location of backlash b_2. Because the moments of inertia of gear 1, the rotating shaft, and pinion 2 are very small compared to the magnitude of the moments of inertia of the motor and load, these moments of inertia were assumed to be negligible and only the torsion stiffness was considered. The portion enclosed by a double dot line is the rotating part of Fig. 1(a), and the model of this is shown in Fig. 1(b). The equivalent torsion stiffness k_1 in the Fig. 1(b) can be represented as

$$k_1 = \frac{k_{g1} k_{s1}}{(k_{g1} + k_{s1})} \tag{2}$$

where k_1 is the equivalent torsion stiffness of gear 1 and the rotating shaft $(N \cdot m/rad)$; k_{g1} is the torsion stiffness due to tooth stiffness between pinion 1 and gear 1 $(N \cdot m/rad)$; and k_{s1} is the torsion stiffness of the rotating shaft $(N \cdot m/rad)$. The torsion stiffness k_{g1} is given as follows [10]:

$$k_{g1} = \frac{d_{g1}^2 E_{p1} Z_{p1} E_{g1} Z_{g1}}{4(E_{p1} Z_{p1} + E_{g1} Z_{g1})} t_{g1} \tag{3}$$

where d_{g1} is the pitch circle diameter of gear 1 (m); E_{p1} and E_{g1} are the moduli of elasticity of pinion 1 and gear 1, respectively (N/m^2); Z_{p1} and Z_{g1} are the elasticity deformation factors of pinion 1 and gear 1, respectively; and t_{g1} is the tooth face of gear 1 (m). And, the elasticity deformation factor is also given as follows [10]:

$$Z_i = \frac{y_i}{0.242 + 7.25\, y_i} \tag{4}$$

where y_i is the Lewis form factor of gear i. Also, the torsion stiffness k_{s1} of the rotating shaft used in (2) can be calculated as

$$k_{s1} = \frac{\pi G_{s1} d_{s1}^4}{32 L_{s1}} \tag{5}$$

where G_{s1} is the shear modulus of elasticity of the rotating shaft (N/m^2), d_{s1} is the diameter of the rotating shaft (m), and L_{s1} is the length of the rotating shaft between gear 1 and pinion 2 (m). In the same way, the equivalent torsion stiffness between gear 2 and the fixed shaft can also be written as

$$k_2 = \frac{k_{g2} k_{s2}}{(k_{g2} + k_{s2})} \tag{6}$$

where k_2 is the equivalent torsion stiffness of gear 2 and the fixed shaft $(N \cdot m/rad)$, k_{g2} is the torsion stiffness due to tooth stiffness between pinion 2 and gear 2, and k_{s2} is the torsion stiffness of the fixed shaft. At this time, the torsion stiffnesses, k_{g2} and k_{S2}, can be obtained by equations like (3) and (5).

B. *Equations of Motion*

In this study, a permanent magnetic field type DC motor with a tachometer was used. The electrical equations for this motor are given as follows [11]:

$$V_m = L_a \frac{di_a}{dt} + R_m i_a + k_b \dot{\theta}_m \tag{7a}$$

$$T_m = k_t i_a \tag{7b}$$

$$V_t = k_{ts}\dot{\theta}_m \tag{7c}$$

where V_m is the motor input voltage (V), L_a is the inductance of the motor's armature (H), i_a is the current of the motor's armature (A), R_m is the resistance of the motor's armature (Ω), k_b is the back emf constant $(V \cdot s/rad)$, θ_m is the angle of rotation of the motor pinion (rad), T_m is the motor torque $(N \cdot m)$, k_t is the motor torque constant $(N \cdot m/A)$, V_t is the output voltage of the tachometer (V), and k_{ts} is the tachometer sensitivity $(V \cdot s/rad)$. The equation of motion for the motor can be written as

$$J_m \ddot{\theta}_m = T_m - \frac{T_{g1}}{N_1} - T_{f,m} sign(\dot{\theta}_m) \tag{8}$$

where J_m is the moment of inertia of the motor rotor $(kg \cdot m^2)$, T_{g1} is the torque exerted on both ends of the equivalent torsion spring $(N \cdot m)$, N_1 is the gear ratio between pinion 1 and gear 1, $T_{f,m}$ is the motor's static friction torque $(N \cdot m)$, and $sign(\cdot)$ is the sign of the value inside ( ). The relation between the angle θ_m of rotation of pinion 1 and the angle θ_{g1} of rotation of gear 1 can be expressed as

$$\theta_{g1} = \frac{\theta_m}{N_1} \tag{9}$$

where θ_{g1} is the angle of rotation of gear 1 (rad). The torque T_{g1} can be represented as

$$T_{g1} = k_1\left(\frac{\theta_m}{N_1} - \theta_{p2}\right) \tag{10}$$

where θ_{p2} is the angle of rotation of pinion 2 (rad). Then, the relation between the torque T_{g1} and the torque T_L of the load can be represented as

$$T_{g1} = \frac{T_L}{N_r} \tag{11}$$

where T_L is the torque of the load $(N \cdot m)$, N_r is the revolution gear ratio between pinion 2 and gear 2 $(N_r = N_2 + 1)$, and N_2 is the ratio of the pitch circle diameter between pinion 2 and gear 2 $(N_2 = d_{g2}/d_{p2})$. From the relation between (10) and (11), the angle θ_{p2} of rotation of pinion 2 can be obtained as follows:

$$\theta_{p2} = \frac{\theta_m}{N_1} - \frac{T_L}{k_1 N_r} \tag{12}$$

Due to the backlash between pinion 2 and gear 2, the torque T_L of the load can be expressed as follows:

$$T_L = \begin{Bmatrix} k_2(\theta_d - \delta) & , & \theta_d > \delta \\ 0 & , & |\theta_d| < \delta \\ k_2(\theta_d + \delta) & , & \theta_d < -\delta \end{Bmatrix} \tag{13}$$

where θ_d is the angular transmission error $(\theta_d = \theta_{p2}/N_r - \theta_L)$ (rad), θ_L is the angle of rotation of the load (rad), and δ is 1/2 of the magnitude of total backlash b_t measured at the final load stage when pinion 1 is fixed ($\delta = \pi\, b_t/360^\circ$) (rad). Here, the dead-zone model was used for the backlash model [12], [13]. The equation of motion for the load can be represented as

$$J_L \ddot{\theta}_L = T_L - T_{f,L} sign(\dot{\theta}_L) \tag{14}$$

where J_L is the moment of inertia of load $(kg \cdot m^2)$ and $T_{f,L}$ is the load's static friction torque $(N \cdot m)$.

C. Transfer Function Analysis

When the backlash δ, the friction torques $T_{f,m}$, and $T_{f,L}$ of the equations of motion derived in section II.B are assumed to be 0, the servo system with a gear reducer becomes a linear system. In this case, the transfer function of motor angular velocity $\dot{\theta}_m$ to the motor torque T_m can be obtained as

$$\frac{\dot{\theta}_m(s)}{T_m(s)} = \frac{N^2\left(J_L s^2 + k_{eq}\right)}{s\left[J_L N^2 J_m s^2 + k_{eq}(N^2 J_m + J_L)\right]} \tag{15}$$

where N is the total gear ratio $(N = N_1 N_r)$ and k_{eq} is the total equivalent torsion stiffness of the system. Then, the total equivalent torsion stiffness can be expressed as

$$k_{eq} = \frac{N_r^2 k_1 k_2}{N_r^2 k_1 + k_2} \tag{16}$$

The frequency corresponding to the zero and pole of (15) is called anti-resonance frequency and resonance frequency, respectively, and they can be written as

$$f_{AR} = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{J_L}} \tag{17a}$$

and

$$f_R = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{N^2 J_m J_L/(N^2 J_m + J_L)}} \tag{17b}$$

where f_{AR} is the anti-resonance frequency (Hz) and f_R is the resonance frequency (Hz).

From (17a) and (17b), it can be seen that the anti-resonance and resonance frequencies of a servo system with a gear reducer are determined by the gear reducer's structural stiffness, gear ratio, and the moments of inertia of the motor and load, when backlash and friction torques were assumed to be 0.

D. The Effect of Backlash Increase

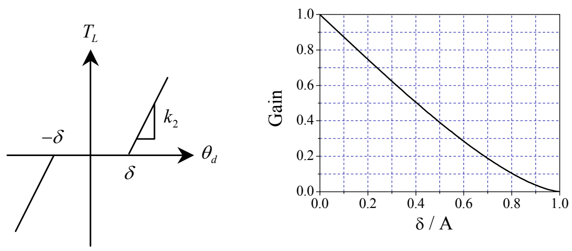
The model's describing function gain can be obtained by applying the describing function method on the backlash model as shown in Fig. 2(a). The normalized describing function gain is given as follows [12](this is also presented in Fig. 2(b).):

$$N(A) = 1 - \frac{2}{\pi}\left[\sin^{-1}\left(\frac{\delta}{A}\right) + \frac{\delta}{A}\sqrt{1-\left(\frac{\delta}{A}\right)^2}\right] \tag{18}$$

where $N(A)$ is the normalized describing function gain and A is the amplitude of the input sinusoidal signal. The backlash has a gain between 0 and 1, and the increase in backlash magnitude causes a decrease in the equivalent torsion stiffness k_2 of the system. The effective equivalent torsion stiffness reduced by the backlash can be expressed as

$$k_{2,eff} = N(A)\,k_2 \tag{19}$$

where $k_{2,eff}$ is the effective equivalent torsion stiffness $(N \cdot m)$.



(a) (b)

Fig. 2. The model and describing function gain of backlash (a) Backlash model using dead-zone model (b) The describing function gain of dead-zone model

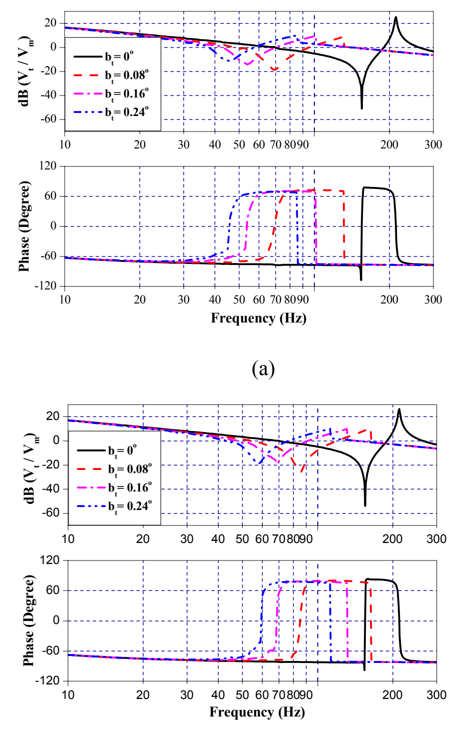
In the end, the backlash increase of the gear reducer inside the servo system reduces the effective equivalent torsion stiffness. Therefore, the anti-resonance and resonance frequencies of the system are reduced.

III. Simulations

A. The Effect of the amplitude of Motor Input Voltage

The simulations were performed by Matlab Simulink. The sinusoidal voltage was supplied to the motor from 10 Hz to 300 Hz. The motor's angular velocity response with respect to each excited frequency was sampled with the time interval of 0.5 ms, and the system's gain and phase were found by the frequency analysis of the obtained motor's angular velocity data. The frequency response characteristics of the servo system obtained in this way are shown in Fig. 3(a) and Fig. 3(b).

In the case of Fig. 3(b), the amount of decrease of the anti-resonance and resonance points when the magnitude of total backlash is 0.08° are smaller than the case in Fig. 3(a), and the amount of decrease of the anti-resonance and resonance points due to additional backlash increase is smaller than in Fig. 3(a). Because the describing function gain of the backlash model is a function of not only the magnitude of backlash but also the amplitude of the input sinusoidal signal, the angular transmission error is increased according to the increase in motor velocity due to the increase of the motor input voltage. In conclusion, the gain of the describing function is increased in spite of the same amount of backlash. Therefore, for the same amount of backlash, Fig. 3(b), which has a larger motor input voltage than Fig. 3(a), has higher anti-resonance and resonance frequencies. The reason why there is a significant reduction of the anti-resonance and resonance frequencies when the magnitude of the backlash is 0.08° is because, when the backlash is 0, the frequencies are not affected by the amplitude of motor input voltage, however, when the backlash is not 0, they are affected by the motor's input voltage. Therefore, when analyzing the effect of backlash on frequency response characteristic, the maximum input voltage of the motor is very important. When the amount of total backlash is 0.08°, the change in anti-resonance and resonance frequencies due to the increase in the motor's input voltage is presented in Fig. 4, and when the motor input voltage is increased infinitely, it can be seen that the system has a bandwidth of the system that does not have any backlash. However, since there is a limit to the maximum input voltage of the motors, the motor's maximum voltage used in this study was limited to 17.9 V_{pk}.



(b)

Fig. 3. The Bode diagram V_t/V_m of gear reduction servo system (a) V_m = 10.0 V_{pk} (b) V_m = 17.9 V_{pk}

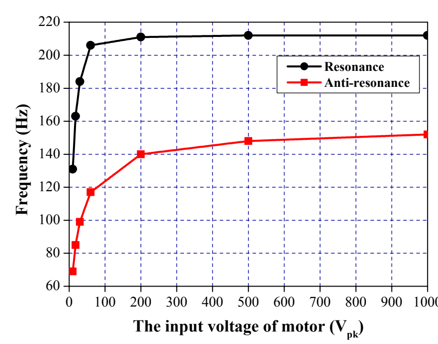


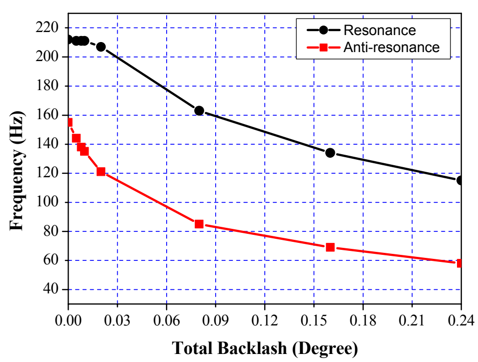
Fig. 4. The change of anti-resonance & resonance frequencies according to the input voltage of motor (b_t = 0.08°)

B. *The Effect of Backlash Magnitude*

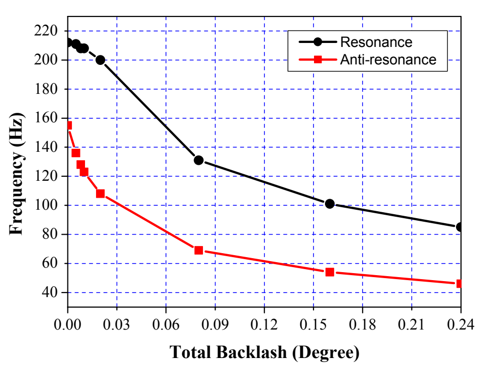
This section looks into changes in the anti-resonance and resonance frequencies of servo systems with a gear reducer, which are caused by the change in the gear reducer's backlash magnitude when the maximum input voltages of the motor are 10.0 V_{pk} and 17.9 V_{pk}. Both Fig. 5(a) and Fig. 5(b) show that the system's anti-resonance and resonance frequencies decrease as backlash increases. The decrease rate of the anti-resonance and resonance frequencies seen in Fig. 5(b), which has smaller motor input voltage, is much greater than the decrease rate seen in Fig. 5(a), and also, when the magnitude of total backlash is equal, the anti-resonance and resonance frequencies shown in Fig. 5(b) are much smaller than those shown in Fig. 5(a).

When the motor of the servo system is determined and consequently the maximum input voltage is limited, it can be seen that the adequate method to expand the system's bandwidth without increasing the system's weight is to reduce the magnitude of the system's backlash.

There are two ways to reduce the magnitude of backlash in the system. One is to reduce a distance between the shaft of a pinion and that of a gear, the other is to use a precision manufacturing machine [14].



(a)



(b)

Fig. 5. Anti-resonance & resonance according to backlash variation (a) V_m = 17.9 V_{pk} (b) V_m = 10.0 V_{pk}

C. *Determining the Motor's Input Voltage and Backlash Magnitude*

This section deals with the change in anti-resonance frequency caused by the magnitude of backlash and the input voltage of the motor. The results are illustrated in Fig. 6. In the case that the servo system has anti-resonance frequency, that is bandwidth that the designer intends, the input voltage of the motor and the magnitude of backlash can be determined by Fig. 6. For example, when the servo system with a gear reducer has a bandwidth of over 100 Hz and a motor with a maximum voltage of 10 V is used, the maximum allowable backlash magnitude on the system would be 0.03°. However, if a motor with a maximum input voltage of 30.0 V is used to reduce the costs of processing and manufacturing, the allowable backlash magnitude would be 0.08°.

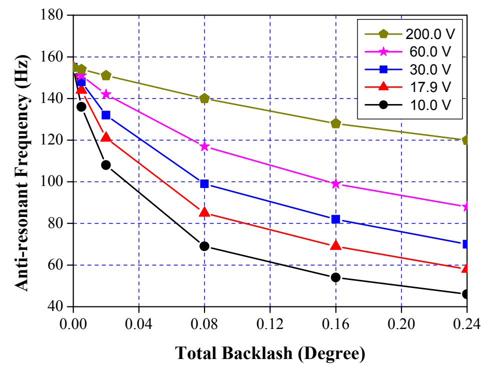


Fig. 6. Anti-resonance frequency according to the backlash and the input voltage of motor

From this, it can be seen that the maximum allowable backlash magnitude in the system depends on the maximum input voltage of the motor, and the adequate backlash magnitude and the required motor voltage can be determined from Fig. 6. However, it is expected that the value of anti-resonance frequency shown in Fig. 6 would be reduced in an experiment because it is obtained under the assumption that the effects of damping and viscous friction are negligible. Therefore, in order to find the anti-resonant frequency more accurately, the effects should be considered by calculating the frequency reduction ratio due to the resonant Q factor. The detailed contents for this consideration are in [1] and [9].

IV. Conclusion

In this study, the change in the system's bandwidth due to the change in the amplitude of the input voltage of the motor and the magnitude of backlash has been investigated.

It has been found that when backlash exists, the effect of the motor's input voltage on bandwidth is very significant. It has also been shown that, when analyzing the bandwidth of the servo system with a gear reducer, one should consider the maximum input voltage of the motor as the backlash reduction amounts required to expand the bandwidth of the servo system differs depending on the that maximum input voltage. Also, it has become possible through this study to determine the amount of maximum allowable backlash to satisfy the bandwidth requirement of the system if the motor has already been determined.

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