Analysis on the Redundancy of Wireless Sensor Networks

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ABSTRACT

Wireless sensor networks consist of a large number of tiny sensors that have only limited energy supply. One of the major challenges in constructing such networks is to maintain long network lifetime as well as sufficient sensing area. To achieve this goal, a broadly-used method is to turn off redundant sensors. In this paper, the problem of estimating redundant sensing areas among neighbouring wireless sensors is analysed. We present an interesting observation concerning the minimum and maximum number of neighbours that are required to provide complete redundancy and introduce simple methods to estimate the degree of redundancy without the knowledge of location or directional information. We also provide tight upper and lower bounds on the probability of complete redundancy and on the average partial redundancy. With random sensor deployment, our analysis shows that partial redundancy is more realistic for real applications, as complete redundancy is expensive, requiring up to 11 neighbouring sensors to provide a 90 percent chance of complete redundancy. Our results can be utilised in designing effective sensor scheduling algorithms to reduce energy consumption and in the mean time maintain a reasonable sensing area.

Categories and Subject Descriptors

C.2 [**Computer-Communication Networks**]: Miscellaneous; F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Geometrical problems and computations*

General Terms

Reliability

Keywords

Wireless Sensor Networks, Coverage, Redundancy Analysis

1. INTRODUCTION

Recent progress in communication technology and the emerging field of low cost, reliable, and MEMS (MicroElectroMechanical System) based sensors has resulted in numerous promising applications, in which the physical world can be observed and influenced through wireless sensor networks. Such networks consist of sensors that monitor surrounding environment, carry out simple calculation, and communicate with each other through short-range radio transmission. Compared with traditional sensors, the sensors in wireless sensor networks could be extremely small, as tiny as a cubic millimetre [1, 3, 18]. This miniaturization effort dramatically reduces energy consumption and also makes network installation very convenient: the sensors could be simply dropped in place without any pre-determination of positions. When clustered together, these sensors automatically create highly flexible, low-power networks with applications ranging from building control system to smart entertainment devices that adjust audio and video quality based on their surroundings.

While the application potential of wireless sensor networks is limitless, the construction of such networks is extremely challenging. One of the main challenges is to maintain long network lifetime as well as sufficient sensing area. Although sensors cost low power in general, their energy supply is still very difficult. Sensors are usually deployed densely (high up to 20 nodes/m^3 [12]), causing problems of scalability, redundancy, and radio channel contention.

On the one hand the high density of sensors wastes a lot of energy, but on the other hand it provides much room and many opportunities for us to design energy efficient protocols. A broadly-used strategy for reducing energy consumption in wireless sensor networks is to turn off redundant sensors by scheduling sensor nodes to work alternatively based on some heuristic schemes [16, 21]. The heuristic of utilising nodes' redundancy has also been used in wireless ad hoc networks [2, 7, 8, 19] and usually depends on location or directional information such as that obtained with the Global Positioning System (GPS) or the directional antenna technology. The energy cost and system complexity involved in obtaining geometric information, however, may compromise the effectiveness of the proposed solutions as a whole. Reducing energy cost without introducing complexity in other parts of the system is still a very hard problem.

As a commonly used strategy to reduce energy cost, turning off sensors may generate blind points and consequently, reduces the network's coverage range. For a given deployment area, we refer the blind points as the regions that can-

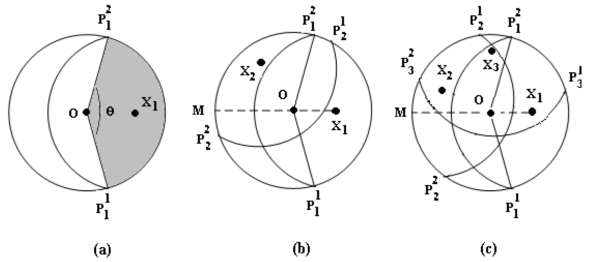

Figure 1: Illustration of Theorem 1 and sponsored sector

not be monitored by any sensors. A sensor network provides the maximum sensing coverage when all sensors are powered on. Nevertheless, keeping all sensors on-duty will waste a lot of energy and thus reduces network lifetime. Since most applications may not require the maximal sensing coverage, it is critical to provide good heuristics for turning off sensors without degrading sensing coverage significantly in a statistical sense.

In this paper, we propose a mathematical model to describe the redundancy in randomly deployed sensor networks. Based on theoretical analysis, we present simple formulae to estimate the probability that a sensor is completely redundant and to estimate the average partial redundancy. We base our analysis on random deployment since this deployment strategy is easy and cheap [17]. In addition, the analysis on the redundancy problem for other deployment methods with regular topology might be easy, since the locations of sensors are pre-determined. Also, some analytical results [11] based on grids are available.

Although the estimation of redundancy has been studied by simulation [7], to the best of our knowledge, there is no paper presenting a thorough analysis for such an estimation. Our analytical results will benefit the research in wireless sensor networks by providing simple formulae to estimate sensor redundancy. They can be utilised in designing deployment-aware scheduling scheme to save energy consumption. Different sensor deployment strategies can cause very different network topology, and thus different degrees of sensor redundancy. The knowledge for sensor deployment, however, is usually available in advance. For instance, it is easy to know the number of sensors and how the sensors are dispatched for a particular application. Unlike methods based on geometric information, scheduling algorithms solely based on deployment knowledge do not require additional energy cost to obtain location information and present a promising research direction. We expect our work could stimulate more research along this direction.

The rest of the paper is organized as follows. In Section 2, we present preliminary definitions and a mathematical model for the analysis of sensor redundancy. In Section 3, we give bounds on the number of neighbours required to provide completely redundant coverage. We calculate the probability that a sensor is completely redundant and analyse the average partial redundancy in Section 4 and Section 5 respectively. Numerical results are provided in Section 6. We introduce related work in Section 7 and present further work in Section 8.

2. PRELIMINARIES

In this paper, we base our analysis on a commonly used deployment strategy: random deployment. We assume a sensor's sensing range is a circular area centred at this sensor with a radius of R. Also, we assume all sensors lie on a 2-dimensional plane. The analytical results in this paper, however, can be easily extended into 3-dimensional space. In addition, all sensors are supposed to have the same sensing range and no two sensors can be deployed exactly at a same location in the 2-dimensional plane.

To facilitate later discussion, we introduce the following definitions:

Definition 1: Neighbour [16]. The 1-hop neighbour set of sensor i is defined as

$N(i) = \{j \in \aleph \mid d(i,j) \leq r, j \neq i\}$

where $\aleph$ represents the sensor set in the deployment region, $d(i,j)$ denotes the distance between sensor i and sensor j, and r is the radius of the sensing range. If sensor m is not sensor i's 1-hop neighbour but is within the sensing ranges of sensor i's 1-hop neighbours, sensor m is called a 2-hop neighbour of sensor i.

Definition 2: completely and partially redundant sensor. Let C_i be the sensing area of sensor i. If $\bigcup_{j \in N(i)} C_j \supseteq C_i$, we call sensor i a completely redundant sensor, since sensor i's sensing area can be covered by its 1-hop neighbours completely. If $\bigcup_{j \in N(i)} C_j$ can cover only part of C_i, we call sensor i a partially redundant sensor.

Note that the sensing range might be completely independent of the radio transmission range because different hardware is involved. We assume that a sensor has mechanisms to know how many sensors are within its sensing range. Such mechanisms could be direct radio transmission if sensing range is smaller than radio transmission range, or other techniques such as using acoustic signals. If sensing range is much larger than radio transmission range, turning off sensors based on sensing range might result in network partition. Handling this issue, however, is not related to the analysis in this paper and will be our future work on designing a lightweight deployment-aware sensor scheduling scheme.

In this paper, we only consider a sensor's 1-hop neighbours, even if a partially redundant sensor may be completely covered by its 1-hop plus 2-hop neighbours. Keeping 2-hop neighbourhood information needs larger storage cost and will make the calculation and analysis more difficult. High cost of memory and calculation will waste energy and violate our original research motivation in investigating the redundancy problem: reducing energy consumption and enlarging network lifetime.

Without causing confusion, in the rest of the paper, neighbours will be referred to 1-hop neighbours only.

Definition 3: Sponsored sector and effective angle [16]. Suppose node O and node X_1 are neighbours. As shown in Fig. 1(a), the sector, bounded by radius OP_1^1, radius OP_1^2, and the inner arc $\widehat{P_1^1 P_1^2}$, is called the sponsored sector by node X_1 to node O, and is denoted as $\mathrm{S}_{X_1 \to O}$. P_1^1 and P_1^2 are two intersections of O's and X_1's sensing areas and are arranged in the counterclockwise order. At the rest of this paper, P_1^1 and P_1^2 are also called the first and the second intersection point of nodes O and X_1 respectively. The centre angle of the sponsored sector is called effective angle of $S_{X_1 \to O}$ and is denoted as $\theta_{X_1 \to O}$. It is easy to see that $120^0 \leq \theta_{X_1 \to O} < 180^0$.

The following notations will be used:

1. $C = C(O, R)$, the circular sensing area of node O with radius R;
2. $C_i = C_i(X_i, R), 1 \leq i \leq n$, the circular sensing area with radius R and centred at X_i, where X_i is a neighbour of node O.
3. $X_i, 1 \leq i \leq n$ are random points distributed inside C independently and uniformly;
4. For each X_i, we associate a triple (P_i^1, P_i^2, θ_i) where (a) (P_i^1, P_i^2) are the two intersections between C and C_i, and are arranged in the counterclockwise order; (b) $\frac{2}{3}\pi \leq \theta_i < \pi$ is the effective angle ($\angle P_i^1 O P_i^2$).

If the Quality of Service (QoS) of sensor networks is defined as the percentage of a given deployment region that can be monitored, the best QoS can be achieved if all sensors are on-duty. Turning off completely redundant sensors will save energy consumption without degrading QoS. Nevertheless, without accurate geometric information, it is very difficult to check whether a sensor is completely redundant. Therefore, it is critical to propose good heuristics based on which turning off sensors would not degrade QoS largely. In the following sections, we will analyse the sensor redundancy problem and present simple estimation on QoS reduction when sensors are powered off.

3. BOUNDS ON NEIGHBOUR SET

In this section, we present an interesting observation: given a sensor i, if its sensing area, C_i, can be covered by its neighbours, then at least three neighbours and at most five neighbours are needed to cover C_i. Note that there are no two sensors can be at the same location.

Lemma 1: $\bigcup_{j \in N(i)} S_{j \to i} \supseteq C_i$ iff $\bigcup_{j \in N(i)} (C_i \cap C_j) \supseteq C_i$.

Proof: (1) If $\bigcup_{j \in N(i)} S_{j \to i} \supseteq C_i$ then $\bigcup_{j \in N(i)} (C_i \cap C_j) \supseteq C_i$.

The proof of this part is presented in [16]. It is obvious since for any $j \in N(i)$, $(C_i \cap C_j) \supseteq S_{j \to i}$.

(2) If $\bigcup_{j \in N(i)} (C_i \cap C_j) \supseteq C_i$ then $\bigcup_{j \in N(i)} S_{j \to i} \supseteq C_i$.

Without losing generality, suppose C is covered by C_1, C_2, ... C_n. First, let's check how C_1, ..., C_n could cover C. Fig. 1(b) shows the relative positions of C and C_1, where P_1^1 and P_1^2 are two intersections of C and C_1, and O and X_1 are the two centers respectively. Assume M is the intersection of line X_1O and the outer arc $\widehat{P_1^1 P_1^2}$ along C. Since $120^0 \leq \angle P_1^1 O P_1^2 < 180^0$, we need at least two additional sensing areas to cover the outer arc of $\widehat{P_1^1 P_1^2}$ along C because any effective angle is smaller than 180^0. Obviously, $\angle P_1^1 OM \leq 120^0$ and $\angle P_1^2 OM \leq 120^0$.

At most two neighbours are enough to cover the inner arc $\widehat{P_1^2 M}$. As shown in Fig. 1(b), if there is a neighbour, say X_2, whose sensing range covers points P_1^2, O, M and thus $\widehat{P_1^2 M}$, then only one neighbour is enough. If such a neighbour does not exist, in the counterclockwise order, we choose the neighbour (denoted as X_2 in Fig. 1(c)) whose first intersection with C falls on the inner arc $\widehat{P_1^2 M}$ and is the **nearest** to P_1^2, and the neighbour (denoted as X_3 in Fig. 1(c)) whose second intersection with C falls on the inner arc $\widehat{P_1^2 M}$ and is the **nearest** to M. As shown in Fig. 1(c), these two sensing areas, C_2 and C_3, will cover the inner arc $\widehat{P_1^2 M}$, because otherwise there is a gap in the inner arc $\widehat{P_1^2 M}$ that cannot be covered by any neighbours. Obviously, C_2 also covers M, and C_3 also covers P_1^2 since $\angle P_1^2 OM \leq 120^0$.

For the same reason, At most two neighbours are enough to cover the inner arc $\widehat{P_1^1 M}$.

Finally, we can get a complete coverage of C by at most five sponsored sectors. Therefore, $\bigcup_{j \in N(0)} S_{j \to 0} \supseteq C$ holds. □

From the above proof, we can obtain the following theorem:

Theorem 1: If $\bigcup_{j \in N(i)} (C_i \cap C_j) \supseteq C_i$, then there must be a subset of $N(i)$, denoted as $N'(i)$, such that $\bigcup_{j \in N'(i)} (C_i \cap C_j) \supseteq C_i$ and $3 \leq |N'(i)| \leq 5$, where $|N'(i)|$ denotes the number of elements in $N'(i)$.

From Theorem 1, if a sensor is a completely redundant sensor, then at least three neighbours are needed and at most five neighbours are enough. Without accurate location information, however, it is very hard to decide whether a sensor is a completely redundant sensor. Unfortunately, it is undesirable to obtain and maintain accurate location information in wireless sensor networks due to stringent resource constraints.

In the next section, we will estimate the probability that a sensor is completely redundant for the random deployment approach. Our analysis does not rely on any location or directional information.

4. ANALYSIS ON THE COMPLETE REDUNDANCY PROBLEM

In [17], three commonly used deployment strategies are studied: random deployment, regular deployment, and planned deployment. In the random deployment method, sensors are distributed with a uniformly distribution within the field. In the regular deployment method, sensors are placed in regular geometric topology such as a grid. In the planned deployment method, the sensors are placed with higher density in areas where the phenomenon is concentrated. In the planned deployment strategy, although sensors are deployed with a non-uniform density in the whole deployment area, in a small range, sensors are approximately placed randomly. In this sense, the following analytical results of random deployment are applicable to planned deployment. In this paper, we do not analyse regular deployment since it may be hard to organize a large number of sensors in regular geometric topology and the analysis is trivial since the locations of sensors are pre-determined.

In this section, we answer the following question: what is the probability that a sensor is a completely redundant sensor with the random deployment strategy? Using the notations defined in Section 2, the problem is described as: given C and its n neighbouring $C_i, 1 \leq i \leq n$, if A is the event that C is fully covered by $C_i, 1 \leq i \leq n$, what is $Pr\{A\}$?

Lemma 2: For each i, if A_i is the event that there is no other neighbour X_j such that the point P_i^1 is on the arc

$\widehat{P_j^1 P_j^2}, \forall j \neq i$, then $A = \bigcap_{i=1}^{n} \overline{A_i}$ where $\overline{A_i}$ is the complement of A_i.

Proof: We claim that A is true if and only if each point on the edge of the circle C is covered. The "only if" part is obvious because the edge is part of the circle. For the "if" part, we can show by direct comparing the distances that if a point P on the edge of C is covered by a neighbour, then all the points on the segment OP are also covered by the same neighbour. Based on the above claim, we only need to prove that $\bigcap_{i=1}^{n} \overline{A_i}$ is true if and only if each points on the edge of C is covered. This is obvious from the definition of A_i's. □

The intuitive meaning of Lemma 2 is that C is fully covered by its n neighbouring C_i's $(1 \leq i \leq n)$ if and only if the first intersection point of C_i $(1 \leq i \leq n)$ and C is covered by a C_j $(j \neq i)$. Otherwise, there will be a gap along the edge of C that cannot be covered by any C_i $(1 \leq i \leq n)$. In other words, C_i's $(1 \leq i \leq n)$ must overlap in order to fully cover C.

The theorem below gives the probability that a sensor is completely covered by its n random neighbours:

Theorem 2:

$$Pr\{A\} = 1 - nPr\{A_1\} + \frac{n(n-1)}{2} Pr\{A_1 A_2\}.$$

Proof: From the proof of Lemma 1, we can easily see that it is impossible to have three or more gaps along the edge of C, since all effective angles are not smaller than 120^0. Based on Lemma 2 and the fact that for any $i \neq j \neq k, Pr\{A_i A_j A_k\} = 0$ (it is impossible to have three gaps along the edge of C as stated above), we have

$$\begin{aligned}
&Pr\{A\} = Pr\{\bigcap_{i=1}^{n} \overline{A_i}\} \\
&= 1 - Pr\{\bigcup_{i=1}^{n} A_i\} \\
&= 1 - (\sum_{i=1}^{n} Pr\{A_i\} - \sum_{i<j} Pr\{A_i A_j\} \\
&+ \sum_{i<j<k} Pr\{A_i A_j A_k\} + \cdots + (-1)^{n-1} Pr\{A_1 A_2 \cdots A_n\} \\
&= 1 - \sum_{i=1}^{n} Pr\{A_i\} + \sum_{i<j} Pr\{A_i A_j\} - 0 + \cdots - (-1)^{n-1} 0 \\
&= 1 - \sum_{i=1}^{n} Pr\{A_i\} + \sum_{i<j} Pr\{A_i A_j\}.
\end{aligned}$$

The theorem then follows from the fact that X_i are independently and identically distributed. □

We now calculate the probability of A_1 and $A_1 A_2$ to get lower and upper bounds on the probability of A.

Theorem 3:

$$Pr\{A_1\} = (\frac{1}{3} + \frac{\sqrt{3}}{2\pi})^{n-1} \approx 0.609^{n-1}.$$

Proof: As shown in Fig. 2(a), U is the intersection area between C and a circle with the radius R and centred at P_1^1. Then the event A_1 happens if and only if the rest of the neighbours $X_2, \cdots, X_n$ all fall outside of U. Otherwise, P_1^1 will be covered by a X_i $(2 \leq i \leq n)$. Therefore,

$$\begin{aligned}
&Pr\{A_1\} = Pr\{X_2,\ X_3, \cdots,\ X_n\ fall\ outside\ of\ U\} \\
&= (\frac{C-U}{C})^{n-1} \\
&= (\frac{\pi R^2 - (\frac{1}{3}\pi R^2 + 2(\frac{\pi R^2}{6} - \frac{\sqrt{3}}{4} R^2))}{\pi R^2})^{n-1} \\
&= (\frac{1}{3} + \frac{\sqrt{3}}{2\pi})^{n-1} \\
&\approx 0.609^{n-1} \quad \square
\end{aligned}$$

Next, we consider the probability of the event $A_1 A_2$, that is, the event that both C_1 and C_2 have gaps at their counterclockwise front.

Theorem 4:

$$Pr\{A_1 A_2\} \leq (0.276)^{n-1}.$$

Proof: As shown in Fig. 2(b), M is the second intersection (in the counterclockwise order) between C and the circle centred at P_1^2 with radius R, and N is the first intersection (in the counterclockwise order) between C and the circle centred at P_1^1 with radius R. If A_1 and A_2 both happen, X_2 must be in the shadowed area, W_1. Otherwise, either P_1^1 will be covered by C_2 or P_2^1 will be covered by C_1.

Fixing X_1 and X_2, let W_2 be the region which any $X_i, i \geq 3$ must be in for both A_1 and A_2 to happen, i.e., the shadowed region in Fig.2 (c). We have

$$Pr\{A_1 A_2\} =$$

$$(\frac{1}{\pi R^2})^n \left(\int_C \int_{W_1} \left(\int_{W_2} \cdots \int_{W_2} dX_3 \cdots dX_n \right) dX_2 dX_1 \right).$$

Let

$$\Delta = (\frac{1}{\pi R^2})^{n-2} \int_{W_2} \cdots \int_{W_2} dX_3 \cdots dX_n.$$

In Fig. 2(c), M and N are the intersections between C and the circle centred at P_1^1 with radius R, and J and K are the intersections between C and the circle centred at P_2^1 with radius R. Let $\alpha = \angle JON$ and $\beta = \angle MOK$. Since the effective angles $\angle JOK$ and $\angle NOM$ are no less than 120^0, $\alpha + \beta \leq 120^0$. Thus, we can assume that $\alpha \geq 60^0$ and $\beta < 60^0$ without losing generality. It can be calculated that the area of the shadowed region inside the sector $\widehat{JON}$ is $[\frac{\alpha}{2\pi} - (\frac{1}{3} - \frac{\sqrt{3}}{2\pi})]\pi R^2$, and the area of the shadowed region inside the sector $\widehat{MOK}$ is less than or equal to $\frac{1}{2\pi}(\frac{2\pi}{3} - \alpha)\pi R^2$. It follows that the total area of the shadowed region W_2 is less than or equal to $\frac{\sqrt{3}}{2\pi}\pi R^2$. Therefore,

$$\Delta \leq (\frac{\sqrt{3}}{2\pi})^{n-2} \approx 0.276^{n-2}.$$

Since it is easy to see that

$$(\frac{1}{\pi R^2}) \int_{W_1} dX_2 = 1 - (\frac{2}{3} + (\frac{1}{3} - \frac{\sqrt{3}}{2\pi})) = \frac{\sqrt{3}}{2\pi},$$

the result follows. □

Based on the above results, we have

Corollary 1:

$$1 - n0.609^{n-1} \leq Pr\{A\} \leq 1 - n0.609^{n-1} + \varepsilon$$

where $\varepsilon = \frac{n(n-1)}{2}(0.276)^{n-1}$.

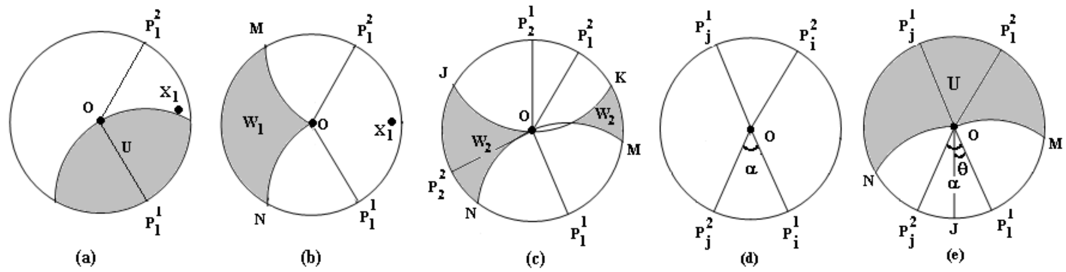

Figure 2: Illustration of Theorem 3, Theorem 4, and Theorem 5.

5. ANALYSIS ON THE PARTIAL REDUNDANCY PROBLEM

Here we consider the average coverage of C by n random neighbours. Let S denote the area that are not covered by the random neighbours and $E[S]$ be the expectation.

Theorem 5:

$$E[S] \leq \pi R^2 0.609^n + \frac{1}{2}R^2 n 0.06^n[(-1)^n \frac{4}{3}\pi - \frac{2\pi}{n}]$$

Proof: Consider the sponsor sectors of all the neighbours $\{P_i^1 O P_i^2, 1 \leq i \leq n\}$. For each i, let area I_i be the gap area (not covered) in the counterclockwise direction of X_i's sponsor region. Two cases need to be considered: (1)P_i^1 is in another neighbour's sponsor region; or (2)P_i^1 is not in any other neighbours' sponsor region. In the case (1), there is no gap in the counterclockwise front of the sponsor region X_i, and thus $I_i = 0$. For the case (2), let P_j^2 be the point closest to P_i^1 in the counterclockwise direction and $\alpha = \angle P_j^2 O P_i^1$, as shown in Fig. 2(d). Note that I_i is smaller than the area of the sector $P_j^2 O P_i^1$, because part of the sector $P_j^2 O P_i^1$ can be covered by C_i and C_j.

Let $I = \sum_{i=1}^{n} I_i$, we have $S \leq I$ and thus $E[S] \leq E[I]$. In the following we estimate $E[I]$. Since $E[I] = \sum_{i=1}^{n} E[I_i]$ and the neighbours are independently and identically distributed, we only need to estimate

$$E[I_1] = \int_{X_1} (\int_{X_2,\cdots,X_n} I_1 dX_2 \cdots dX_n) dX_1.$$

Let $\Delta(X_1) = \int_{X_2,\cdots,X_n} I_1 dX_2 \cdots dX_n$, we have

$$\Delta(X_1) \leq \int_{X_2,\cdots,X_n} \pi R^2 \frac{\alpha}{2\pi} dX_2 \cdots dX_n = \frac{R^2}{2} E[\alpha | X_1].$$

Given X_1, we now consider the conditional probability distribution of the angle α. As shown in Fig. 2(e), for any given $\theta > 0$, let J be a point on the outer arc $\widehat{P_1^1 P_1^2}$ such that $\angle JOP_1^1 = \theta$, M be the second intersection (in the counterclockwise order) between the circle C and the circle centered at P_1^1 with radius R, and N be the first intersection (in the counterclockwise order) between the circle C and the circle centered at J with radius R.

Note that $\alpha \geq \theta$ if and only if all the neighbours $X_2, \cdots, X_n$ are in the shadowed region U in Fig. 2(d), since otherwise either P_1^1 or J will be covered by a X_i $(2 \leq i \leq n)$. Calculations show that

$$Pr\{\alpha \geq \theta\} = (\frac{1}{3} + \frac{\sqrt{3}}{2\pi} - \frac{\theta}{2\pi})^{n-1}.$$

It follows that the distribution function of α is

$$F(\theta) = 1 - (\frac{1}{3} + \frac{\sqrt{3}}{2\pi} - \frac{\theta}{2\pi})^{n-1}.$$

Given X_1, α can range from 0 to $\frac{4}{3}\pi$ since the smallest effective angle of X_1 could be $\frac{2}{3}\pi$. Therefore, we have

$$\begin{aligned}
&E[\alpha|X_1] = \int_0^{\frac{4}{3}\pi} \theta dF(\theta) \\
&= \frac{4}{3}\pi[1 - (\frac{1}{3} + \frac{\sqrt{3}}{2\pi} - \frac{2}{3})^{n-1}] - \\
&\int_0^{\frac{4}{3}\pi} [1 - (\frac{1}{3} + \frac{\sqrt{3}}{2\pi} - \frac{\theta}{2\pi})^{n-1}] d\theta \\
&= \frac{4}{3}\pi[-(\frac{\sqrt{3}}{2\pi} - \frac{1}{3})^{n-1}] + \int_0^{\frac{4}{3}\pi} (\frac{1}{3} + \frac{\sqrt{3}}{2\pi} - \frac{\theta}{2\pi})^{n-1} d\theta \\
&= \frac{2\pi}{n} 0.609^n + (-1)^n \frac{4}{3} 0.06^{n-1}\pi.
\end{aligned}$$

This finishes the proof. □

Based on Theorem 5, it is easy to calculate expectation of the percentage of covered area.

Corollary 2: The percentage of a sensor's sensing area that is covered by its n random neighbours is not smaller than

$$1 - 0.609^n - \frac{1}{2} n 0.06^n [(-1)^n \frac{4}{3} - \frac{2}{n}]$$

6. NUMERICAL RESULTS

Table 1 shows the numerical results calculated using Corollary 1 and Corollary 2. For complete redundancy, the results are discouraging to engineers: a sensor requires about 11 neighbours to get a 90% probability of being a complete redundant sensor. In other words, providing complete redundancy is expensive. Fortunately, since it is hard to guarantee full coverage for a given deployment area even if all sensors are on-duty, small sensing holes are not likely to influence the effectiveness of sensor networks and are acceptable for most application scenarios. Our analytical results illustrate that if we relax our requirement a bit, the network density can be reduced dramatically, resulting in huge savings, especially for large-scale sensor networks. For instance, if we only require a sensor's 90% sensing area to be covered by its neighbours, 5 neighbours are necessary. This result is very similar to the phenomenon observed with simulation in [7]: if a wireless node has more than 4 neighbours, above 90 percent of its transmission range can be covered by its neighbours.

Number of neighbours	Probability of complete redundancy	Percentage of the redundant area (≥)
4	9.56%- 22.24%	86.24%
5	31.22%- 37.01%	91.62%
6	49.74%- 52.13%	94.90%
7	64.29%- 65.21%	96.89%
8	75.15%- 75.49%	98.11%
9	82.97%- 83.09%	98.85%
10	88.48%- 88.52%	99.30%
11	≈ 92.28%	99.57%
12	≈ 94.87%	99.74%
13	≈ 96.62%	99.84%
14	≈ 97.78%	99.90%

Table 1: Redundancy with different numbers of neighbours

7. RELATED WORK

Reducing energy consumption is a very challenging task for large-scale wireless sensor networks. This problem has been dealt with from different system layers. While tremendous efforts have been made to design hardware architecture with low power consumption [9], a lot of work [5, 16, 21] focuses on energy efficient protocols and algorithms, such as energy efficient routing, topology control, and node scheduling. Most work on energy efficient protocols tries to reduce the number of active nodes and relies on the assumption that location information or directional information is available to each sensor.

In [21], Ye et al. proposed a probing-based density control algorithm that utilises geometric information to derive redundancy and allows redundant nodes to fall asleep. As only a subset of nodes is in working mode, the protocol scales well for large sensor networks. Working nodes may run out of energy or be damaged. To solve this problem, a sleeping node wakes up periodically to probe its neighbourhood and transits to working mode if there is no working node within its neighbourhood. In [16], a node scheduling scheme was proposed to turn off redundant sensors. Directional information was used to decide redundancy by checking whether a node's sensing range has already been covered by its neighbours. In addition, a back-off based self-scheduling scheme was presented to avoid generating possible blind points of coverage when several neighbouring sensors try to fall asleep simultaneously.

The same research strategy for reducing energy consumption has been used for wireless ad hoc networks. Although energy efficient protocols for wireless ad hoc networks are mainly on detecting communication instead of sensing redundancy, the features of the handled problem remain the same. In [2], an algorithm was proposed to turn off nodes based on the necessity for maintaining connectivity. In [19], a Geographical Adaptive Fidelity (GAF) algorithm was proposed to divide the network area into grids. By maintaining one active node each grid, GAF reduces energy consumption for large ad hoc networks significantly.

Another highly related research topic is on deciding number of neighbours to maintain network connectivity. In [20], Xue et al. analysed this problem for random networks and presented the lower bound on the number of neighbours so that the overall network is connected. Their research provides new constraints on turning off sensors.

Location information has been broadly used in algorithms for saving network bandwidth and reducing energy cost. Techniques for location discovery [6] and location utilisation [10] have been investigated. Based on location information, several papers [4, 7, 8, 15] proposed algorithms to alleviate the broadcast storm problem in mobile ad hoc networks. Broadcast operation is fundamental for routing protocols in mobile ad hoc networks, since most on-demand routing protocols rely on broadcasting route request messages to search for effective paths. Broadcast, however, is costly and can cause severe message collision and channel contention, especially for large size networks. Based on simulation, Ni et al. [7] observed that when a node has more than four neighbours, the benefit for that node to rebroadcast a message is very small. Based on this observation, they proposed approaches to reduce the frequency of broadcast. In [8], Peng and Lu introduced a protocol in which a node does not rebroadcast a message if all its neighbours have been covered by previous transmissions. They also presented criteria to check whether a node's neighbours have been covered based on two-hop neighbourhood information. Location information was utilised to reduce the range of message broadcast in [4]. In [15], Sun and Lai utilised nodes' location information to calculate optimal local cover set for broadcast in ad hoc networks. When nodes in a node S's local cover set re-transmit, they should cover all nodes within 2 hops away from S. Stojmenovic [14] has presented a comprehensive survey on location-based routing and pointed out that obtaining and managing location information could be very expensive and solving this problem alone could be very challenging.

In the literature, there has been work on covering a circle by arcs. Early in 1982, Siegel and Holst [13] studied the problem of covering a circle with random arcs of random sizes and established a complicated integral representation for the coverage probability. The problem discussed in this paper is different from that in [13] which considers arcs only. Our study on the average size of partial covering addresses a more general problem and uses quite different techniques. Our approach to establish the coverage bounds makes use of the unique features of the problem and is novel.

8. CONCLUSION AND FURTHER WORK

In this paper, we analyse the redundancy problem for wireless sensor networks. We provide an easy and relatively accurate estimation about the degree of redundancy without the knowledge of location or directional information. Although the estimation of redundancy has been studied by simulation in [7], to the best of our knowledge, there is no paper presenting a theoretical analysis for such an estimation. We present an interesting observation concerning the minimum and maximum number of neighbours that are required to provide complete redundancy. We also provide tight upper and lower bounds on the probability of complete redundancy and on the average partial redundancy. The numerical results of our analysis are consistent with the phenomena observed by simulations in [7].

Our results will benefit the research on wireless sensor networks by providing simple estimation of redundancy for designing new energy efficient protocols. Our further work includes analysing the redundancy problem under different node distribution and sensing coverage models. We are having built a unified, lightweight deployment-aware sensor scheduling scheme, which integrates the results of this paper and [20].

Finally, we want to mention that although the sensor coverage model in this paper is also used by other papers [16, 21], it may be unrealistic for concrete sensor technology which usually takes into account the sensor's orientation, angular aperture, time-varying properties of detections, etc. In this sense, the analytical results of this paper are only suitable for the situation where each node can persistently perceive phenomena within its surrounding circular area. Since it is very difficult to provide a general model that can capture the behaviour of different (magnetic, optical, thermal, mechanical, etc.) sensors, we are trying to expand our analysis to particular sensor technology and concrete application scenarios.

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9. REFERENCES

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci. A survey on sensor networks. *IEEE Communications Magazine*, 40(8):102–114, August 2002.

[2] B. Chen, K. Jamieson, H. Balakrishnan, and R. Morris. Span: An energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks. In *Proceedings of the seventh annual international conference on Mobile computing and networking (MobiCom 01)*, pages 85–96. Rome, Italy, July 2001.

[3] J. Kahn, R. Katz, and K. Pister. Next century challenges: Mobile networking for smart dust. In *Proceedings of the fifth annual international conference on Mobile computing and networking (MobiCom 99)*, pages 263–270. Seattle, USA, August 1999.

[4] Y. Ko and N. H. Vaidya. Location-aided routing (lar) mobile ad hoc networks. In *Proceedings of the fourth annual international conference on Mobile computing and networking (MobiCom 98)*, pages 66–75. Dallas, USA, October 1998.

[5] L. Li and J. Halpern. Minimum energy mobile wireless networks revisited. In *Proceedings of IEEE International Conference on Communications (ICC)*, pages 278–283. Helsinki, Finland, June 2001.

[6] A. Nasipuri and K. Li. A directionality based location discovery scheme for wireless sensor networks. In *Proceedings of First International Workshop on Wireless Sensor Networks and Applications (WSNA'02)*, pages 105–111. Atlanta, USA, September 2002.

[7] S. Ni, Y. Tseng, Y. Chen, and J. Sheu. The broadcast storm problem in a mobile ad hoc network. In *Proceedings of the fifth annual international conference on Mobile computing and networking (MobiCom 99)*, pages 151–162. Seattle, USA, August 1999.

[8] W. Peng and X. Lu. On the reduction of broadcast redundancy in mobile ad hoc networks. In *Proceedings of the first ACM international symposium on Mobile ad Hoc networking and computing (MobiHoc 00)*, pages 129–130. Boston, USA, August 2000.

[9] A. Porret, T. Melly, C. Enz, and E. Vittoz. A low-power low-voltage transceiver architecture suitable for wireless distributed sensor networks. In *IEEE International Symposium on Circuits and Systems (CD-ROM Proceedings)*, Geneva, Switzerland, May 2000.

[10] S. Ratnasamy, B. Karp, L. Yin, F. Yu, D. Estrin, R. Govindan, and S. Shenker. GHT: a geographic hash table for data-centric storage. In *Proceedings of First International Workshop on Wireless Sensor Networks and Applications (WSNA'02)*, pages 78–87. Atlanta, USA, September 2002.

[11] S. Shakkottai, R. Srikant, and N. Shroff. Unreliable sensor grids: Coverage, connectivity and diameter. In *Proceedings of IEEE Infocom 2003*. San Francisco, USA, March 2003.

[12] E. Shih, S. Cho, N. Ickes, R. Min, A. Sinha, A. Wang, and A. Chandrakasan. Physical layer driven protocol and algorithm design for energy-efficient wireless sensor networks. In *Proceedings of the seventh annual international conference on Mobile computing and networking (MobiCom 01)*, pages 272–287. Rome, Italy, July 2001.

[13] A. Siegel and L. Holst. Covering the circle with random arcs of random sizes. *Journal of Applied Probability*, 19:373–381, 1982.

[14] I. Stojmenovic. Position based routing in ad hoc networks. *IEEE Communications Magazine*, 40(7):128–134, July 2002.

[15] M. Sun and T. Lai. Computing optimal local cover set for broadcast in ad hoc networks. In *Proceedings of IEEE International Conference on Communications (ICC)*, pages 3291–3295. New York, USA, April 2002.

[16] D. Tian and N. Georganas. A coverage-preserved node scheduling scheme for large wireless sensor networks. In *Proceedings of First International Workshop on Wireless Sensor Networks and Applications (WSNA'02)*, pages 32–41. Atlanta, USA, September 2002.

[17] S. Tilak, N. Abu-Ghazaleh, and W. Heinzelman. Infrastructure tradeoffs for sensor networks. In *Proceedings of First International Workshop on Wireless Sensor Networks and Applications (WSNA'02)*, pages 49–57. Atlanta, USA, September 2002.

[18] B. Warneke, M. Last, B. Leibowitz, and K. Pister. Smart dust: Communicating with a cubic-millimeter computer. *IEEE Computer Magazine*, 34(1):44–51, January 2001.

[19] Y. Xu, J. Heidemann, and D. Estrin. Geography-informed energy conservation for ad hoc routing. In *Proceedings of the seventh annual international conference on Mobile computing and networking (MobiCom 01)*, pages 70–84. Rome, Italy, July 2001.

[20] F. Xue and P. Kumar. The number of neighbours needed for connectivity of wireless networks. To

appear in *Wireless Networks*.

[21] F. Ye, G. Zhong, S. Lu, and L. Zhang. Peas: A robust energy conserving protocol for long-lived sensor networks. In *Proceedings of ICDCS 2003*. Rhode Island, USA, May 2003.