Analytic expression for triple-point electron emission from an ideal edge

Levi Schächter[a)]
Electrical Engineering Department, Technion–Israel Institute of Technology, Haifa 32000, Israel

(Received 24 September 1997; accepted for publication 25 November 1997)

The electric field in the vicinity of a metallic edge attached to a dielectric half-space is calculated analytically. The resulting electric field is used to evaluate the current emitted from the edge using the Fowler-Nordheim formula. It is shown analytically that the emitted current is proportional to the dielectric coefficient of the material.
[S0003-6951(98)02904-0]

The ability to control electron emission from metallic surfaces is crucial in many electronic devices either if the purpose is to generate an electron beam as is the case of microwave vacuum tubes or flat displays or in order to avoid arcing or RF breakdown. In the present letter we present a simple *analytic* result which indicates that the current emitted via field emission from a metallic edge attached to a dielectric half-space is proportional to the dielectric coefficient of the material. This is particularly interesting bearing in mind that dielectric coefficients of more than 10^3 are available (e.g., ferro-electric or piezo-electric materials). The present analysis is based on elementary *electrostatic* considerations[1] and it is shown that the presence of the dielectric causes an enhancement in the curvature of the potential which in turn leads to an increase in the local electric field and consequently an increase in the current density. For evaluation of the total current, the Fowler-Nordheim relation has been used and it is shown that for sufficiently high electric fields, the result can be expressed in terms of an analytic function.

Consider a sharp edge of angle α attached to a dielectric (ϵ_r) half-space as illustrated in Fig. 1. We shall assume that the radius of curvature of the edge is much smaller than the longitudinal dimension (Δ_z) of the edge such that we may assume that the system is infinite in the z direction. Furthermore, on the edge the electrostatic potential is assumed to be zero therefore we may write the following solution for the Laplace equation

$$\Phi(r,\phi)=\begin{cases} A_1 \sin[\nu(\phi-\pi+\alpha)]r^{\nu} & 0<\phi<\pi-\alpha \\ A_2 \sin[\nu(\phi+\pi)]r^{\nu} & -\pi<\phi<0\,. \end{cases} \tag{1}$$

This solution satisfies the boundary conditions at $\phi=\pi-\alpha$ and $\phi=-\pi$. The curvature (ν) of the field is determined by imposing the boundary conditions for Φ and D_ϕ at $\phi=0$. From the two resulting equations we can establish the expression which determines the variation of ν as a function of α and ϵ_r; it reads

$$\epsilon_r \tan[\nu(\pi-\alpha)]=-\tan(\pi\nu). \tag{2}$$

Solution of this expression determines the general behavior of the potential in the vicinity of the edge. A typical distribution is illustrated in Fig. 1 for $\alpha=\pi/6$ and $\epsilon_r=300$.

At the limit of the very high dielectric coefficient ($\epsilon_r\rightarrow\infty$) the curvature approaches the value $\nu\rightarrow 0.5$. For the other extreme ($\epsilon_r=1$), the solution of (2) has an analytic form which reads

$$\nu=\frac{1}{2}\,\frac{1}{1-(\alpha/2\pi)}. \tag{3}$$

Figure 2 illustrates the variation of the curvature (ν) as a function of the dielectric coefficient (ϵ_r) between the two limits mentioned above.

The curvature of the potential determines the electric field which in turn controls the surface charge distribution on the metallic edge. Let us now calculate the charge stored on both sides of the edge on a strip which is Δ_z long and its width is R. The choice of radius (R) is arbitrary but it is tacitly assumed that it is much larger than the radius of curvature of the edge and much smaller than the distance to the anode. On the top strip of the edge the total charge is given by $Q_{\mathrm{top}}=-\Delta_z\int_0^R dr D_\phi(r,\phi=\pi-\alpha)=\Delta_z\epsilon_0 A_1 R^{\nu}$ whereas on the bottom strip $Q_{\mathrm{bottom}}=-\Delta_z\epsilon_0\epsilon_r A_2 R^{\nu}$. The electrostatic potential also enables us to calculate the total electrostatic energy (W_{E}) stored in a partial ($\pi-\alpha>\phi>-\pi$) cylinder of radius R and length Δ_z. This energy enables us to define the capacitance of the system according to $C\equiv Q_{\mathrm{total}}^2/2W_E$ or explicitly as

$$C=\epsilon_0\Delta_z\frac{2\{\sin(\nu\pi)+\epsilon_r\sin[\nu(\pi-\alpha)]\}^2}{(\pi-\alpha)\sin^2(\nu\pi)+\pi\epsilon_r\sin^2[\nu(\pi-\alpha)]}, \tag{4}$$

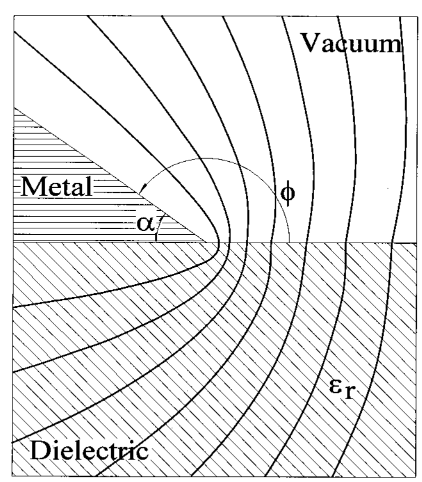

FIG. 1. Schematic of the triple-point system and the associated constant potential lines.

[a)]Electronic mail: levi@ee.technion.ac.il

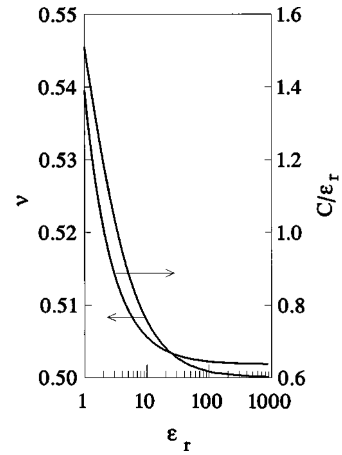

FIG. 2. The ''dispersion'' relation between the curvature parameter ν and the dielectric coefficient ϵ_r. The second curve illustrates the correlation between the nonlinear characteristic of the capacitance and ν; $\alpha=\pi/6$.

where we used the definition of the total charge on both sides of the edge i.e., $Q_{\mathrm{total}}=Q_{\mathrm{top}}+Q_{\mathrm{bottom}}$. In zero order this capacitance is *linear* in ϵ_r; however if we plot the ratio $C' \equiv C/\epsilon_r\epsilon_0\Delta_z$ (see Fig. 2) we find that the curvature parameter (ν) determines also the deviation from linearity of the capacitance. With the capacitance established we may define an *effective* voltage as $V_{\mathrm{eff}}\equiv Q_{\mathrm{total}}/C$ hence

$$V_{\mathrm{eff}}=\frac{1}{2}A_1R^{\nu}\frac{(\pi-\alpha)\sin^2(\nu\pi)+\pi\epsilon_r\sin^2[\nu(\pi-\alpha)]}{\sin(\nu\pi)\{\sin(\nu\pi)+\epsilon_r\sin[\nu(\pi-\alpha)]\}}. \tag{5}$$

We shall next examine the current emitted from the top strip of the edge assuming either a constant voltage (V_{eff}) or a constant charge (Q_{top}).

According to Fowler-Nordheim, the current density emitted via field emission is given by $J\simeq k_1E^2e^{-k_2/E}$ where $k_1=1.54\times10^{-6}/W$ [A/V^2], $k_2=6.83\times10^7W^{1.5}$ [V/m] and W is the work function of the metal. Using the analytic expressions for the electric field which can be derived from Eq. (1), we can determine the current emitted from the top surface; the latter is given by

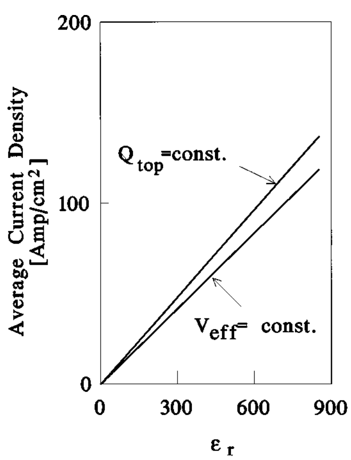

FIG. 3. Average current density as a function of the dielectric coefficient ϵ_r for constant charge on the top of the metallic edge Q_{top} and constant voltage V_{eff}; $\alpha=\pi/6$.

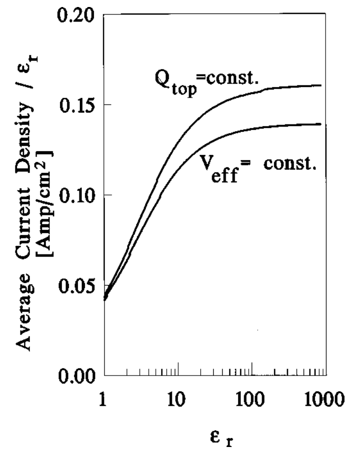

FIG. 4. Average current density divided by ϵ_r as a function of the dielectric coefficient ϵ_r for constant charge on the top of the metallic edge Q_{top} and constant voltage V_{eff}; $\alpha=\pi/6$.

$$I=I_0\frac{\nu^2}{2\nu-1}\left[e^{-a_0/\nu}+\frac{a_0}{\nu}\int_0^1 \mathrm{d}y\, y^{(2\nu-1)/(1-\nu)}e^{-a_0y/\nu}\right], \tag{6}$$

where $I_0=\Delta_zRk_1E_{\mathrm{eff}}^2$, $a_0=k_2/E_{\mathrm{eff}}$, and $E_{\mathrm{eff}}=Q_{\mathrm{top}}/\epsilon_0R\Delta_z$. This expression indicates that the emitted current is inversely proportional to $2\nu-1$ and bearing in mind that the curvature parameter (ν) approaches 0.5 as ϵ_r tends to infinity, we may expect the current to increase as ϵ_r increases. This result is confirmed in Fig. 3 where we plot the average current density defined as I/Δ_zR. In fact Eq. (6) indicates that to a good approximation ($a_0\ll1$) the current is *linear* in ϵ_r. A closer look at the variation of the current is revealed in Fig. 4 where we plot the same average current density but divided by ϵ_r and, as in the case of the capacitance, we observe that the curvature parameter (ν) affects the total current emitted by the metal. For these results it was tacitly assumed that the charge on the top side of the edge is constant ($Q_{\mathrm{top}}=$ const); the parameters are as follows: $\Delta_z=1$ cm, $R=1$ mm, $\alpha=\pi/6$, $W=4.5$ eV (tungsten) and it is assumed that the surface charge density is 0.5×10^{-6} C/cm^2 thus $Q_{\mathrm{top}}=0.5\times10^{-6}\Delta_zR$.

A similar behavior is observed when V_{eff} is assumed to be constant; the current in this case is given by

$$I(\nu)=I_1\frac{\chi^2(\nu)}{2\nu-1}\Bigg[e^{-b_0/\chi(\nu)} + \frac{b_0}{\chi(\nu)}\int_0^1 \mathrm{d}y\, y^{(2\nu-1)/(1-\nu)}e^{-b_0y/\chi(\nu)}\Bigg], \tag{7}$$

where $I_1=(\Delta_zR)k_1(V_{\mathrm{eff}}/R)^2$, $b_0=k_2/(V_{\mathrm{eff}}/R)$ and

$$\chi(\nu)=\frac{2\nu\sin(\nu\pi)\{\sin(\nu\pi)+\epsilon_r\sin[\nu(\pi-\alpha)]\}}{(\pi-\alpha)\sin^2(\nu\pi)+\pi\epsilon_r\sin^2[\nu(\pi-\alpha)]}. \tag{8}$$

Figures 3 and 4 illustrate the same aspects discussed previously for the case when $V_{\mathrm{eff}}=80$ V.

In either one of the cases, if the electric field is sufficiently high ($k_2<<E$) such that $e^{-k_2/E}\approx1$, then the expression for the current has a simple form

$$I_Q \equiv I_{Q_{\mathrm{top}}=\mathrm{const}}(\nu) = I_0 \frac{\nu^2}{2\nu-1},$$
$$I_V \equiv I_{V_{\mathrm{eff}}=\mathrm{const}}(\nu) = I_1 \frac{\chi^2(\nu)}{2\nu-1}. \tag{9}$$

At the two extremes ($\epsilon_r=1$ and $\epsilon_r \gg 1$) and assuming $\alpha \ll 2\pi$, these expressions can be further simplified to read

$$\epsilon_r = 1: \quad I_Q = I_0 \left(\frac{\pi}{2\alpha}\right), \quad I_V = I_1 \left(\frac{2}{\pi\alpha}\right),$$
$$\epsilon_r \gg 1: \quad I_Q \simeq I_0 \left(\frac{\pi}{2\alpha}\right)\left(\frac{\epsilon_r}{2}\right), \quad I_V \simeq I_1 \left(\frac{2}{\pi\alpha}\right)\frac{\epsilon_r}{2}; \tag{10}$$

thus the effective enhancement parameter for $\epsilon_r \gg 1$ is given by $\beta \simeq \sqrt{\epsilon_r/2}$.

The result presented above is subject to several tacit assumptions we have made in the process of the analysis: first, we have used the Fowler-Nordheim formula in the vicinity of the edge although it has been developed for a planar geometry. This clearly sets limits on the radius of curvature of the tip — it should be much larger than a quantum level radius of curvature ($\sim$1–5 nm) and much smaller than macroscopic curvature $\sim$1–5 μm. Second, the effect of the emitted charge on the electrostatic potential is ignored, therefore no direct conclusion regarding the dynamics of the electrons can be drawn from this analysis; furthermore no conclusion about the limiting (Child–Langmuir) current in this kind of geometry can be deduced from the present study. Third, the metal was assumed to be characterized by an infinite conductivity which implies that, although the current density close to the edge can be very high, the current flow is not associated with a temperature increase — which in turn may lead to volatilization of the tip. The voltage drop associated with finite conductivity is also ignored.

In conclusion, the *analytic* expressions in (10) clearly reveal that for large dielectric coefficients ($\epsilon_r \gg 1$) the current emitted via field emission is proportional to the dielectric coefficient of the material. This result has several implications:

(i) it indicates that an edge attached to a material of high dielectric coefficient has the potential of generating currents which are by a few orders of magnitude more intense than in case of the same edge without the dielectric.

(ii) In the past is had been suggested that dielectric ''islands'' may have substantial contribution to electrons emission via whiskers which generate micro-channels of current which find their way from the metallic surface *through* the dielectric to the vacuum. Their contribution was shown by Latham[2] to have a similar form to Fowler-Nordheim. The present model indicates that ''circumference'' of these islands may have a substantial contribution and not only the bulk material.

(iii) This ''simple'' triple-point emission implies that if dielectric islands develop adjacent to micro-protrusions, the current enhancement at the cathode is determined not only by the geometric parameters[3–7] but also by the dielectric coefficient.

(iv) The model discussed by Mesyats[8] to describe the emission from metal-dielectric cathodes relies on plasma formation, which is perfectly justified in high voltage devices. Here we show that substantial current densities can be generated by ''regular'' Fowler-Nordheim emission without plasma being involved; such a mechanism may be of importance in low voltage systems.

(v) Finally, it seems possible to modulate the current generated at the metallic surface by enforcing temporal variations[9–11] on the dielectric coefficient of the medium.

This study was supported by the United States Department of Energy, by the Bi-National Science Foundation United States–Israel, and by Technion VPR Fund–Henri Gutwirth Promotion of Research Fund.

[1] J. D. Jackson, *Classical Electrodynamics*, 2nd ed. (Wiley, New York, 1975), p. 75.
[2] R. V. Latham, Vacuum **32**, 137 (1982).
[3] T. J. Lewis, J. Appl. Phys. **26**, 1405 (1955).
[4] P. A. Chatterton, Proc. Phys. Soc. London **88**, 231 (1966).
[5] G. N. Fursei and P. N. Vorontsov-Vel'yaminov, Sov. Phys. Tech. Phys. **12**, 1370 (1968).
[6] G. A. Frall, J. Appl. Phys. **41**, 563 (1970).
[7] B. L. Henson, J. Appl. Phys. **55**, 158 (1984).
[8] G. A. Mesyats, IEEE Trans. Dielectr. Electr. Insul. **2**, 272 (1995).
[9] J. D. Ivers, L. Schächter, J. A. Nation, G. S. Kerslick, and R. Advani, J. Appl. Phys. **73**, 2667 (1993).
[10] L. Schächter, J. D. Ivers, G. S. Kerslick, and J. A. Nation, J. Appl. Phys. **73**, 8097 (1993).
[11] T. C. Cavazos, W. L. Wilbands, C. B. Fleddermann, and D. A. Shiffler, Appl. Phys. Lett. **65**, 2612 (1994).