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Analytic models for the dynamics of diffuse oceanic plate boundaries

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SUMMARY

Some plates deform across zones that are many hundreds to thousands of kilometres wide, much broader than traditional boundaries such as mid-ocean ridges, deep-sea trenches and oceanic transform faults, across which most deformation is concentrated in a zone just a few kilometres wide. These wide zones of deformation, commonly referred to as diffuse plate boundaries, occur in both continental and oceanic lithosphere. Composite plates are composed of two or more rigid, or nearly rigid, component plates separated by one or more diffuse plate boundaries (Royer & Gordon 1997); such 'complete' diffuse boundaries are terminated at both ends by triple junctions (although there are other diffuse boundaries that transform into narrow boundaries). Here we consider the dynamics of complete diffuse oceanic plate boundaries by constructing simple analytical models on a flat earth and on a spherical earth assuming that the viscous force resisting deformation is described by either a linear Newtonian law or a high-exponent power law.

We investigate the observed tendency for the pole of relative motion between component plates separated by a diffuse plate boundary to lie within the diffuse boundary itself. We show that this tendency is due to geometrical effects that make it unlikely that the total torque acting between plates at a diffuse boundary could be oriented such that relative rotation occurs between the plates about a pole lying outside the boundary. This is demonstrated for both flat and spherical earth cases, assuming that resistance to strain along the diffuse boundary increases linearly with stress (Newtonian rheology). We further show that the pole of rotation is even more likely to lie in the diffuse plate boundary if the viscous force resisting deformation is described by a high-exponent power law rather than a Newtonian law.

Key words: diffuse plate boundary, East African Rift, Indo–Australian plate, plate tectonics.

1 INTRODUCTION

Much of the deformation of the Earth's surface occurs in boundary regions between any two of about 15 rigid or near-rigid plates that are in relative motion with respect to each other (Wilson 1965; McKenzie & Parker 1967; Morgan 1968). Motion between two plates may be described by their relative angular velocity about the centre of the Earth, which can be specified by a rate of rotation and an axis or pole of rotation. Most plate boundaries, including mid-ocean ridges, oceanic transform faults and the main thrust fault at trenches, are narrow, that is, all or nearly all the deformation occurs in a zone probably just a few kilometres wide. It now appears, however, that many other plate boundaries in both oceanic and continental settings are diffuse (Gordon & Stein 1992), that is, the relative velocity between plates is accommodated by deformation across a zone of deformation that is hundreds or even thousands of kilometres wide.

Royer & Gordon (1997) distinguished plates separated by diffuse plate boundaries from other types of plates. They defined 'composite' plates as those composed of two or more 'component' plates separated by one or more diffuse plate boundaries. A component plate is internally rigid or nearly so but moves relative to one or more other component plates in the same composite plate. Composite plates are bordered by narrow plate boundaries. Not all diffuse boundaries separate component plates, so we introduce the terminology 'complete

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diffuse plate boundary' to refer to a diffuse boundary that separates component plates and that terminates at triple junctions at both ends, and 'partial diffuse plate boundary' to refer to a diffuse boundary that shows a transition into a narrow plate boundary (such as the continuation of the Arctic ridge towards Asia).

The poles of rotation for motion across component plates meeting at complete diffuse plate boundaries have a tendency to lie within the boundaries themselves (Gordon 1998). This generalization holds for all the major examples of diffuse oceanic boundaries that separate composite plates into component plates: the boundary separating the North and South American plates between the Caribbean and the Mid-Atlantic Ridge (Argus 1990), the two boundaries separating the Indo–Australian plate into the Indian, Australian and Capricorn plates (Gordon *et al.* 1990; Royer & Gordon 1997), and a boundary separating the African plate into the Nubian and Somalian plate, which includes the East African Rift and continues south and southeast to the Southwest Indian Ridge (Chu & Gordon 1999). Other diffuse boundaries such as in the Western United States and between Africa and Eurasia are not of this form as these diffuse boundaries convert into a narrow boundary on at least one side (with subduction or sea-floor spreading) rather than ending at triple junctions, so that they are partial diffuse boundaries. Hereinafter we only consider complete diffuse plate boundaries (Fig. 1).

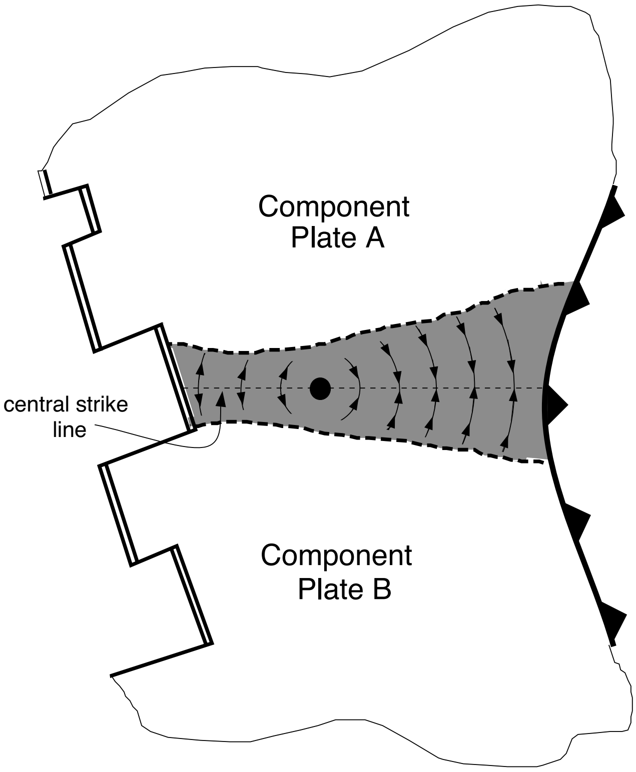

Figure 1. A cartoon example of a composite plate consisting of two component plates (A and B) separated by a diffuse plate boundary. This diffuse boundary contains the pole for the relative motion of component plate A relative to component plate B. In this case the diffuse boundary is pictured lying between a trench (on the right) and a mid-ocean ridge (on the left). The diffuse boundary is located between the dashed lines, with arrows marking extension and contraction. Also shown is a central strike line chosen for the boundary.

Although it has long been recognized that plate boundaries through continents are prone to broad bands of deformation (such as at the Andes or the Tibetan plateau), only recently have enough diffuse oceanic plate boundaries become sufficiently well documented for them to be recognized as globally significant tectonic features (Royer & Gordon 1997; Gordon 1998; Chu & Gordon 1999). Diffuse plate boundaries are associated with slow relative velocities between the component plates, with rates of 2–16 mm yr^{-1} rather than the 12–160 mm yr^{-1} velocities observed across mid-ocean ridges and the 20–100 mm yr^{-1} velocities inferred across trenches (Gordon 1998). This may be because fast motions are required to develop narrow plate boundaries, or because diffuse boundaries are more resistant to motion than narrow boundaries, or perhaps because of both of these. It is also in a sense true that the relative velocities are small because the pole lies within the boundary, which implies that even for high rates of spin the surface speeds will be low as the speed is proportional to the distance from the axis of rotation.

Building on an analysis of Martinod & Molnar (1995), Gordon (2000) estimated that the force per along-strike unit length driving deformation in diffuse oceanic plate boundaries is between 9×10^{12} and 34×10^{12} N m^{-1}. Below this level of force, oceanic lithosphere appears to be rigid or nearly rigid; above this level, the lithosphere appears to deform approximately as a power-law fluid with an exponent between ≈ 3 and ≈ 30 (Gordon 2000). The diffuse plate boundary between the North and South American component plates lies in a relatively narrow neck between the two plates, which may cause sufficient focusing of stresses to attain the level of force per unit length required to deform oceanic lithosphere. In contrast, the along-strike length of diffuse oceanic plate boundaries is larger in the Indo–Australian composite plate, where areas of stress concentration may result from a large outward push by the Tibetan plateau, by the arrangement of subducting slabs and by the focusing of stresses by collisions in the Himalayas and New Guinea (Cloetingh & Wortel 1986; Molnar *et al.* 1993; Coblentz *et al.* 1998).

Here we consider the dynamics of diffuse oceanic plate boundaries by constructing some simple analytical models, and then solving the force and torque balances in these models for the implied deformation across the boundary. The main issue that we address is why the motion between component plates separated by a diffuse oceanic plate boundary is described by a rotation pole that generally lies in the diffuse boundary itself. We find that geometrical effects are an important part of the explanation. In Section 2 we specify our key assumptions. In Section 3 we present a diffuse boundary model on a flat earth and an easily performed illustrative experiment. In Section 4 we present a diffuse boundary model on a spherical earth. In later sections we discuss the relative sizes of the components of the angular velocity between component plates, and explore the effects of non-Newtonian rheology and the implications of our analysis for the state of stress in stable plate interiors.

2 MODEL SET-UP

We consider an idealized composite plate consisting of two component plates sharing a diffuse plate boundary (Fig. 1). We assume that the component plates are rigid, but that the lithosphere of the diffuse plate boundary can be approximated by a fluid, that is, it will deform at a constant strain rate in response to a constant applied stress. The justification for this is that the diffuse plate boundary forms where the deviatoric stress within the composite plate exceeds some yield stress, fluidizing a zone within an otherwise rigid plate. However, we also assume that the boundaries are 'strong' in comparison with narrow plate boundaries (more precisely, that diffuse boundaries have high effective viscosities), as is suggested by the observation that strain rates accommodated across diffuse boundaries are at least two orders of magnitude smaller than those accommodated across narrow plate boundaries (Gordon 1998; Gordon 2000).

We consider the forces acting on each of the component plates due to the boundary. We do this by picking a 'central strike line' through the middle of the boundary and considering the traction across this line, which is then the force per unit length of the boundary on the component plates on either side. This traction is related to the local rate of strain by the constitutive relation for the material in the boundary, and acts to resist deformation. We can estimate the strain rate along the central strike line by finding the difference between the velocities of the rigid components projected onto the central strike line, and dividing this by the strike-perpendicular width of the boundary, and assuming that velocity gradients across the boundary are much more important than velocity gradients along the boundary (that is, that the boundaries are not too wide or complex). We consider two simple relationships between the local traction along the central strike line and the overall deformation of the boundary: (1) that the traction is a linear function of the difference in the velocities of the two component plates projected onto the central strike line, and (2) that the magnitude of the traction is independent of the velocity difference.

The first assumption would be approximately valid if the diffuse oceanic plate boundary deformed as a thin sheet of Newtonian viscous fluid and if the across-strike width of the deforming zone were constant. Results from several seismic profiles spanning most or all of the deforming zone in the equatorial Indian Ocean indicate that the across-strike width of the diffuse boundary between the Indian and Capricorn plates is approximately constant (Chamot-Rooke *et al.* 1993; Van Orman *et al.* 1995). Although the assumption of Newtonian rheology is unlikely to be true, it simplifies the analysis and leads to results that help in understanding the role of the geometry and rheology in controlling the location of the pole of rotation.

The second assumption would be approximately valid if the diffuse oceanic plate boundary deformed with a high-exponent power law (plastic-like) rheology. If the rheology of diffuse plate boundaries can be approximated as a power-law fluid, the Newtonian and the plastic-like cases bound the possible behaviour as they represent limiting cases ($n=1$ and $n \to \infty$) of the power-law exponent.

One reason that diffuse boundaries may behave differently from narrow boundaries is that the overall dynamics of narrow boundaries in many cases may not lead to net resistance to motion across the boundary, as the narrow boundaries may become associated with tractions that act to reinforce the motion. Plate separation at mid-ocean ridges, for example, may be enhanced via 'ridge push', and plate convergence at trenches may be enhanced via 'slab pull'.

3 DIFFUSE OCEANIC PLATE BOUNDARIES ON A FLAT EARTH

A flat earth model provides some insights into the physics of diffuse plate boundaries and may eventually be more readily testable by analogue laboratory models or numerical models than the spherical earth case. The geometry for the flat earth model is shown in Fig. 2. The position of the pole of rotation of component plate A relative to component plate B is defined to be (x_0, y_0), and we consider a central strike line for the boundary that follows a straight line along $y=0$ from $x=-L/2$ to $x=L/2$. Dynamic equilibrium requires that the force and torque exerted on each component plate through the diffuse boundary must balance the sum of the forces and torques from the other boundaries of the component plate plus mantle traction on the underside of the component plate. We can represent this balance by claiming that the boundary must provide a net force and torque on component plate B equivalent to that from some force $\mathbf{F}$ applied at some point along the central strike line of the diffuse oceanic plate boundary $\mathbf{x}_F=(D, 0)$. This equivalent force does not have to be applied at a point in the diffuse boundary (that is, it is possible for $D>L/2$ or $D<-L/2$).

If the diffuse oceanic plate boundary has a high effective viscosity (that is, depth-averaged ratio between stress and strain rate), as we have assumed, then the resultant deformation will be slow and will only slightly perturb the velocities of the component plates relative to all the other plates and the underlying mantle. Thus such deformation may have only a small

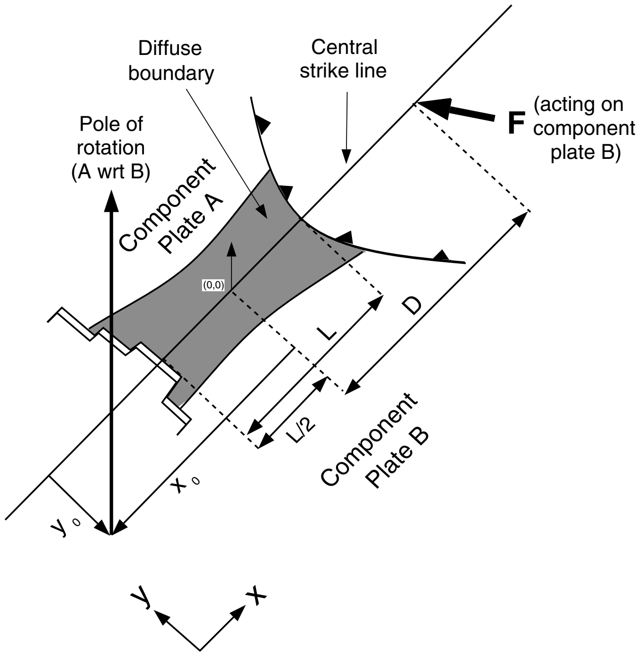

Figure 2. Geometry of a complete diffuse plate boundary model on a flat earth. The shaded region marks the boundary, and its ends are 'diffuse triple junctions', where the diffuse boundary meets other boundaries. L is the length of the boundary. The pole of rotation shown is for the motion of plate A relative to plate B. $\mathbf{F}$ is the equivalent force acting on plate B from the boundary. For mechanical equilibrium, the force and torque from $\mathbf{F}$ applied at a distance D from the middle of the boundary along the central strike line of the boundary must balance the other forces and torques on plate B from other plate boundaries and mantle traction.

effect on the forces and torques on the component plates due to other plate boundaries and mantle traction. We can therefore neglect the dependence of the equivalent force **F** and distance D on the deformation at the diffuse boundary. The following analysis still holds for boundaries with lower effective viscosities, but the interpretation is trickier as **F** and D are then themselves functions of the deformation (they are different from what they would be if there was no deformation in the boundary region).

The force balances in the x- and y-directions are

$$F_x = \alpha\omega \int_{-L/2}^{L/2} y_0 dx = \alpha\omega y_0 L\,, \tag{1}$$

$$F_y = 2\alpha\omega \int_{-L/2}^{L/2} (x - x_0) dx = -2\alpha\omega x_0 L\,, \tag{2}$$

where ω is the angular speed and α is some constant related to the resistance to deformation between the two component plates. α is approximately proportional to the viscosity and inversely proportional to the width of the diffuse oceanic plate boundary. The factor of 2 present in F_y that is not present in F_x comes from the definition of the strain rate tensor $e_{ij}=(1/2)(\partial u_i/\partial x_j+\partial u_j/\partial x_i)$, and our assumption that y gradients in the velocity are much more important than x gradients, so that F_x arises from e_{xy} and F_y from e_{yy}. The torque about the origin due to the equivalent force is

$$[\mathbf{x}_F \times \mathbf{F}]_z = DF_y\,. \tag{3}$$

This torque must be balanced by the torque that one component plate exerts on the other component plate across the diffuse oceanic plate boundary,

$$2\alpha\omega \int_{-L/2}^{L/2} x(x - x_0) dx = \frac{\alpha\omega L^3}{6}\,. \tag{4}$$

These give the solutions

$$x_0 = -\frac{L^2}{12D}\,, \tag{5}$$

$$y_0 = \frac{F_x}{F_y}\frac{L^2}{6D}\,. \tag{6}$$

Thus given F_x, F_y, D and L, one can determine the location of the pole of rotation, which is specified by x_0 and y_0. x_0 is independent of both F_x and F_y; it depends only on L, the along-strike length of the diffuse plate boundary, and on D, the distance that the force is applied from the centre of the diffuse plate boundary. In contrast, y_0 depends not only on L and D, but also on the ratio F_x/F_y, that is, it depends on the orientation of **F** but not on its magnitude. In particular, if **F** is perpendicular to the strike line, then y_0 vanishes. Eq. (5) indicates that the pole of rotation will lie inside the diffuse boundary in the along-strike (x) direction (i.e. $-L/2<x_0<L/2$) unless **F** is applied within the middle third of the diffuse boundary (i.e. $-L/6\le D\le L/6$). Physically, the further the location of **F** from the centre of the boundary, the greater the torque about the centre of the boundary and hence the more likely it will be that some cancellation of deviatoric tensional and compressional forces will be required along the boundary to satisfy simultaneously the torque balance and the force balance. x_0 and y_0 both decrease as L decreases: they scale with L multiplied by the non-dimensional number L/D. Hence, the shorter the diffuse plate boundary, the less likely it is that the pole of rotation lies outside it.

A simple desktop demonstration provides some analogous physics. Place two long, thin strips of paper alongside each other on a smooth table, then use two sharp objects to pull the strips away from each other at various points along their lengths. Unless the strips are pulled apart precisely halfway along their lengths, they will rotate as they move apart. The farther from their midpoint that the forces are applied, the larger the torques and hence the greater the rotation. If the forces are applied sufficiently far from the midpoint, the strips will overlap at one end even as they are pulled apart, with 'extension' occurring at one end of their mutual boundary and 'contraction' occurring at the other. In this example, the inertial response of the paper or the friction between the paper and the table are analogous to resistance to deformation across a diffuse boundary. If friction can be neglected (and inertial effects dominate), the location of the pole is similar to that expected in the case considered above in which the resistance is proportional to the velocity. If the inertia of the paper can be neglected (and frictional forces dominate), the case is analogous to the case considered below in which the resistance is proportional to the nth power of the relative motion for large n because the resistance to motion is independent of velocity.

That an applied force generally imparts faster spin than translation can be seen in many other everyday processes. For example, consider an elongate object floating in a tub of water (or better still a tub of viscous fluid such as honey or molasses). If a horizontal normal force is applied to the side of the object by a pen or pencil or other sharp object, the elongate floating object will spin in response to the applied force unless the force is applied precisely to the midpoint of its side.

The location of the pole of rotation in the y-direction is also restricted in this geometry unless $|F_y|\ll|F_x|$. If $D\simeq\mathcal{O}(L)$ and $F_x\lesssim F_y$, then y_0 is an order of magnitude smaller than the length of the boundary, L, unless $D=0$ (that is, **F** is applied to the precise middle of the diffuse oceanic plate boundary, the probability of which is vanishingly small). Thus the strike-slip motion across the diffuse boundary is likely to be small.

4 DIFFUSE BOUNDARIES ON A SPHERICAL EARTH

In spherical geometry, it is simpler to consider the torque balance of plates about the centre of the Earth rather than a mixture of force and torque balances. An increment of torque $\delta\mathbf{T}$ on a plate due to an increment of force $\delta\mathbf{f}$ is given by

$$\delta\mathbf{T} = \mathbf{r} \times \delta\mathbf{f}\,, \tag{7}$$

where $\mathbf{r}$ is the position vector at which the torque is applied with respect to the centre of the Earth.

For each of the two component plates that meets at a diffuse boundary, one may integrate the moments due to the forces at all of its other plate boundaries and mantle traction to find the torque, **T**, that must be matched by the torque due to the stresses in the boundary between the component plates. Similarly to the flat earth model, the assumption that the boundary has high effective viscosity implies that the torque **T** is still approximately equal to its rigid boundary value, although some deformation does take place. Otherwise, if the boundary is of low effective viscosity and allows rapid movement between the plate components, the velocities of these components with respect to the rest of the Earth will change and the torques at the other plate boundaries and in the mantle traction (and hence **T**) will be significantly different. Our simplification allows us to assume a torque **T** and then solve for the angular velocity required between the two component plates to match this torque. The following analysis still holds for boundaries with low effective viscosities, but again the interpretation is trickier as **T** then depends on the deformation.

Fig. 3 shows the geometry. We assume that the boundary roughly follows a central strike line, which lies along a great circle and defines a plane that also contains the centre of the Earth. The radial line $\theta=0$ lies in this plane and runs through the centre of the boundary. $\boldsymbol{\omega}$ will not in general be parallel to **T**, and here we show it lying within the diffuse boundary. **T**, arising as it does from the torques at other boundaries and the underlying mantle flow, would have no particular tendency to be oriented towards the diffuse boundary. ψ_T is the angle between **T** and the plane defined by the central strike line of the boundary, whereas θ_T is the angle between **T** and the line $\theta=0$ through the centre of the boundary when ψ_T is set to 0. ψ_ω and θ_ω are the similarly defined coordinates of $\boldsymbol{\omega}$. This is not a

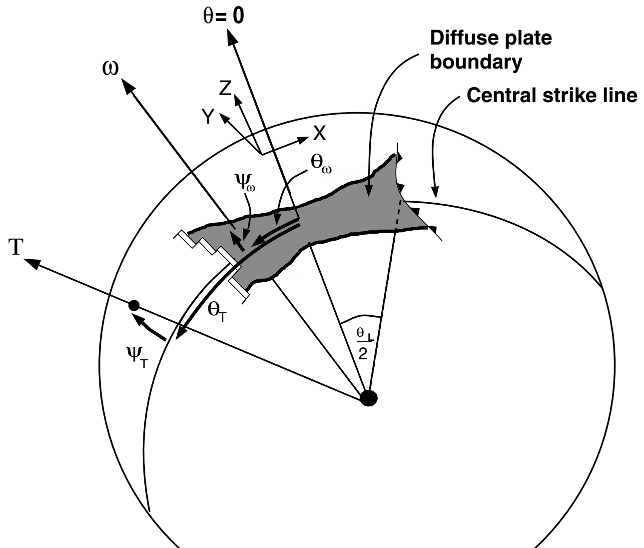

Figure 3. The geometry used in this paper for considering the torque balance of diffuse plate boundaries on a spherical earth. The line marked $\theta=0$ runs through the centre of the Earth and the centre of the boundary. A central strike line through the boundary is chosen as shown. These can then be used to define a Cartesian frame: the z-direction is parallel to the line marked $\theta=0$, the x-direction is parallel to the central strike line where it meets the line marked $\theta=0$ at the centre of the boundary and the y-direction is orthogonal. We also define an angular coordinate system: θ is the angular coordinate along the central strike line of the diffuse boundary and ψ is the angular displacement from the central strike line. The angle subtended by the boundary at the centre of the Earth (that is, the angular length of the boundary, or the angular separation of the triple junctions at each end of the boundary) is θ_L. The angular coordinate of **T** (the torque from the boundary acting on the component plate in front of the diffuse boundary) is (θ_T, ψ_T), and the angular coordinate of the pole of rotation $\boldsymbol{\omega}$ is $(\theta_\omega, \psi_\omega)$. The directions of an (x, y, z) Cartesian system oriented with z parallel to the $\theta=0$ line and x parallel to the central strike line are shown; the natural centre for this coordinate system is the Earth's centre (in this diagram it is offset for clarity).

spherical polar coordinate system but conversions are simple (see note in Appendix). We also define a Cartesian coordinate system (x, y, z) oriented with **z** parallel to the $\theta=0$ line and **x** parallel to the strike; the natural centre for this coordinate system is the Earth's centre (in Fig. 3 it is offset for clarity).

Some intuition for this system can be gained by considering the components of the torque **T** and angular velocity $\boldsymbol{\omega}$ in the Cartesian system, and assuming that each component of the resisting torque T_i arises from a differential rotation across the boundary about the corresponding axis, ω_i. The material of the diffuse plate boundary is much closer to the z-axis than to the x- or y-axes. Therefore, to balance an arbitrary but equal component of torque about each of the three axes requires much faster rotation about the z-axis than about the x-axis [by a factor of order $(\theta_L/2)\,[\int_0^{\theta_L/2}\theta^2 d\theta]^{-1}=12/\theta_L^2$ if the angular length of the diffuse boundary, θ_L, is small; the strain rate tensor consideration described earlier for the flat earth case suggests that the rotation about the y-axis will be double that for the x-axis]. In other words, if T_x, T_y and T_z have similar orders of magnitude, then $|\omega_x/\omega_z|\sim|\omega_y/2\omega_z|\sim\mathcal{O}(\theta_L^2/12)$, where θ_L is given in radians.

With the assumption that the force per unit length resisting deformation across the boundary is a linear function of the relative velocity, then following the notation of Fig. 3,

$$\mathbf{T}=\alpha\int_{-\theta_L/2}^{\theta_L/2}\mathbf{M}\mathbf{r}\times(\boldsymbol{\omega}\times\mathbf{r})d\theta \tag{8}$$

for some constant of proportionality α, where ω is the rate of relative rotation between the two component plates about the pole of rotation. $\mathbf{M}=2\mathbf{I}-\hat{\boldsymbol{\psi}}\hat{\boldsymbol{\psi}}$, and is different from the identity matrix due to the nature of the strain rate tensor as described earlier for the flat earth case. Hence

$$T_x=-2\alpha\omega r^2\cos\psi_\omega\int_{-\theta_L/2}^{\theta_L/2}\sin(\theta_\omega-\theta)\cos\theta d\theta\,, \tag{9}$$

$$T_y=-\alpha\omega r^2\sin\psi_\omega\int_{-\theta_L/2}^{\theta_L/2}d\theta\,, \tag{10}$$

$$T_z=-2\alpha\omega r^2\cos\psi_\omega\int_{-\theta_L/2}^{\theta_L/2}\sin(\theta_\omega-\theta)\sin\theta d\theta\,. \tag{11}$$

Hence, using $\tan\theta_T=-T_x/T_z$,

$$\tan\theta_\omega=\beta\tan\theta_T\,,\quad\text{where}\quad\beta=\frac{\theta_L-\sin\theta_L}{\theta_L+\sin\theta_L}\,, \tag{12}$$

where angles are given in radians. A plot of β is shown in Fig. 4.

For the diffuse oceanic plate boundaries south of India, $\theta_L\simeq30^\circ=\pi/6$ radians, indicating that $\beta\simeq0.02$. The orientation of the pole of rotation in the direction parallel to the strike only depends on the components of **T** that lie in the plane of the central strike line. Therefore, for the pole of rotation to lie outside the diffuse boundary in this direction (i.e. $|\theta_\omega|>\theta_L/2$) would require $\theta_T\simeq\pm(85^\circ-95^\circ)$, or, equivalently, $|T_x|$ to be at least 11 times as large as $|T_z|$. For the diffuse oceanic plate boundary between North America and South America, $\theta_L\simeq15^\circ=\pi/12$ radians, indicating that $\beta\simeq0.006$. For its pole of rotation to lie similarly outside the diffuse boundary in the along-strike direction would require $\theta_T\simeq\pm(87.4^\circ-92.6^\circ)$, or, equivalently, $|T_x|$ to be at least 22 times as large as $|T_z|$. The other case we consider is the diffuse plate boundary between

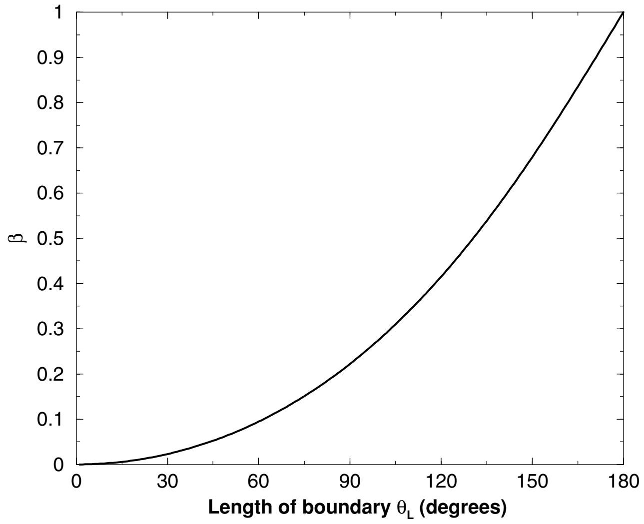

Figure 4. Variation of the parameter β defined in eq. (12) as a function of the angular length of the boundary θ_L. A small value of β implies that the displacement of the pole of rotation along the strike of the diffuse plate boundary is small: for $\theta_L \simeq 30°$, $\beta \simeq 0.025$.

Nubia and Somalia, for which θ_L is much larger, $\simeq 55°$, indicating that $\beta \simeq 0.08$. For its pole of rotation to lie similarly outside the diffuse boundary in the along-strike direction would require $\theta_T \simeq \pm(79° - 101°)$, or, equivalently, $|T_x|$ to be at least five times as large as $|T_z|$. The shaded regions of Fig. 5 indicate the conditions on θ_T and θ_L that must be met for $|\theta_\omega| > \theta_L/2$ according to eq. (12). Even for long diffuse boundaries, θ_T must be within a narrow range of values for the projection of the pole of rotation on the plane of the central strike line of the boundary to lie outside the boundary.

Eq. (7) implies that the torque arising from a resistive force at the boundary will be perpendicular to both the radius vector from the centre of the Earth and the direction of the resistive force. As Fig. 6 shows, if we temporarily neglect the contribution of T_y, the net torque on the boundary is a sum of vectors locally tangential to the Earth's surface lying in the plane defined by Fig. 3. In particular, the torques due to deviatoric compression and tension oppose each other in the direction perpendicular to the angular velocity (the x-direction) in this plane but reinforce in the direction parallel to the angular velocity (the z-direction) due to the curvature of the Earth's surface. However, locally the torques are predominantly in the x-direction of Figs 3 and 6 (as the Earth's radius of curvature is large). Therefore, unless the components of the boundary torque are such that $|T_x| \gg |T_z|$, there will in general have to be some cancellation of torques in the x-direction. This implies that there will probably be both horizontal contraction and horizontal extension in different along-strike portions of the diffuse plate boundary, or in other words that the projection of the pole of rotation on the plane lies within the boundary itself.

For the angular separation between the pole of rotation and the plane defined by the central strike line, eqs (9), (10) and (11) yield

$$\tan \psi_\omega = \gamma \tan \psi_T ,$$

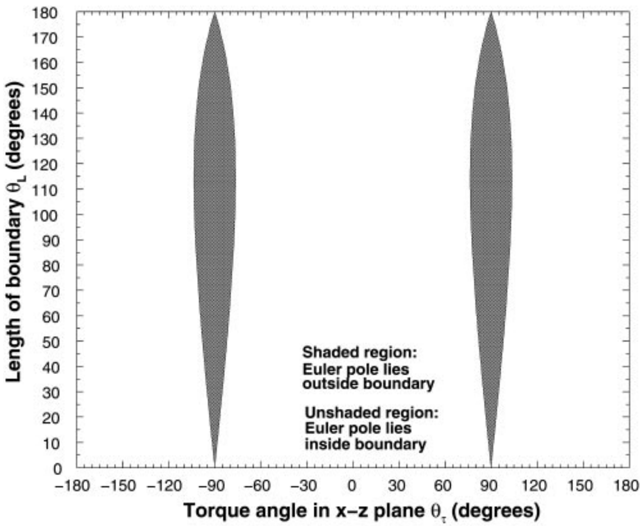

Figure 5. Parameter regimes where the pole of rotation will lie inside and outside a diffuse boundary within the x–z plane of Fig. 3, according to the length of the boundary (measured by the angle subtended by the boundary at the centre of the Earth, θ_L) and the angle θ_T at the centre of the Earth between the net torque of the boundary **T** projected into the x–z plane and the middle of the boundary (see Fig. 2). This graph illustrates the simplest case (as used to derive eq. 2) where the local resistance to deformation is proportional to the local difference in surface velocity between the two plates. For the pole of rotation to be oriented outside the boundary, θ_T and θ_L must lie *within* one of the shaded bands. For $\theta_L < 114°$, the range of such values of θ_T decreases with the length of the diffuse boundary; that is, shorter diffuse boundaries are more likely to contain their pole of rotation.

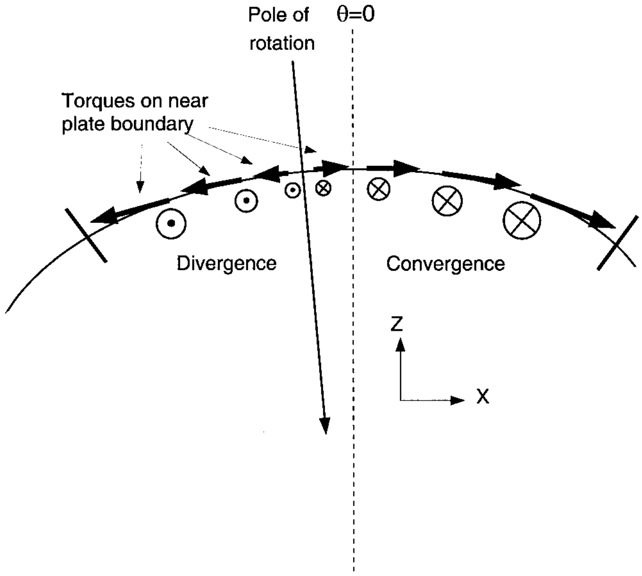

Figure 6. Cartoon of a cross-section through a diffuse plate boundary showing the relationship between convergence and divergence on a diffuse plate boundary, and the implied torques on the near plate (that is, the plate lying between the boundary and us). Circles with crosses mark convergence at the boundary (that is, the plate between us and the boundary is moving towards the boundary) and circles with dots mark divergence at the boundary. In this example the pole of rotation lies within the boundary, but is oriented downwards (implying divergence to its left and convergence to its right). There is more convergence than divergence in this example, and the net torque will be downwards and towards the right. However, the x-component of the net torque is reduced by cancellation between the convergent and divergent torques.

where

$$\gamma = \frac{1}{\theta_L}\sqrt{\frac{1}{\left(\frac{\sin^2\theta_T}{(\theta_L+\sin\theta_L)^2}+\frac{\cos^2\theta_T}{(\theta_L-\sin\theta_L)^2}\right)}}. \tag{13}$$

The displacement of the pole of rotation in the out-of-plane direction is greatly diminished as γ tends to be very small (see Fig. 7), particularly for diffuse boundaries of length $\theta_L \simeq 30°$, unless θ_T is very close to $\pm 90°$. A conceptual argument similar to that presented above for why we would expect θ_ω to be small can be constructed to explain why we would expect ψ_ω to be small.

In the limit where the radius of the Earth r is large compared to L and D from the flat earth model, then by using $L=r\theta_L$, $D=r(\pi/2-\theta_T)$, $x_0=r\theta_\omega$, $y_0=r\psi_\omega$, $F_x/F_y=-\tan\psi_T$ and appropriate small-angle approximations (where angles are in radians), eqs (12) and (13) can be converted into eqs (5) and (6) for the flat earth case.

5 THE POSITION OF THE ROTATION POLE

5.1 Off-strike position of the rotation pole (comparison of ω_x with ω_y)

A comparison of Fig. 4 ($\beta=\tan\theta_\omega/\tan\theta_T$ as a function of θ_L) with Fig. 7 ($\gamma=\tan\psi\omega/\tan\psi$ as a function of θ_L and θ_T) indicates that the factor γ in eq. (13) tends to be around twice the value of the factor β in eq. (12) unless θ_T is close to $\pm 90°$. One would therefore expect that $|\omega_y| \simeq 2|\omega_x|$ [that is, the poles of rotation would usually be less well centred in the middle of the diffuse boundary in the along-strike (x) direction than in the across-strike (y) direction if $T_y \simeq T_x$]. The origin of the factor of 2 lies in the strain tensor consideration discussed in Section 3. Observations, however, suggest the opposite: the location of the poles of rotation indicate that $|\omega_x|$ is larger than $|\omega_y|$ in the Indo–Australian diffuse plate boundaries (DeMets *et al.* 1994; Royer & Gordon 1997; Royer *et al.* 1997) and the Nubia–Somalia boundary (Chu & Gordon 1999).

The long-wavelength pattern of the orientation of stress in the Indo–Australian and African composite plates may provide some insight into this apparent difference between the theoretical and observed $|\omega_y/\omega_x|$. In both the contractional and extensional portions of the diffuse plate boundaries, the inferred orientation of maximum horizontal contractional/extensional strain (and presumably maximum/minimum horizontal deviatoric compressional stress) is fairly uniform over broad regions. Diffuse plate boundaries tend to be oriented perpendicular to the direction of the maximum/minimum horizontal deviatoric compressional stress (Gordon *et al.* 1990; Argus 1990; DeMets *et al.* 1994; Royer & Gordon 1997; Gordon 1998; Chu & Gordon 1999). The implication is that diffuse oceanic plate boundaries strike perpendicular to a principal horizontal normal stress, which in turn implies that shear stresses nearly vanish along diffuse plate boundaries. It follows that $|T_y/T_x| \ll |1$. If true, it explains why ω_y effectively vanishes across the diffuse plate boundaries and why the pole of rotation is so tightly constrained to lie near the central strike line of the diffuse plate boundary.

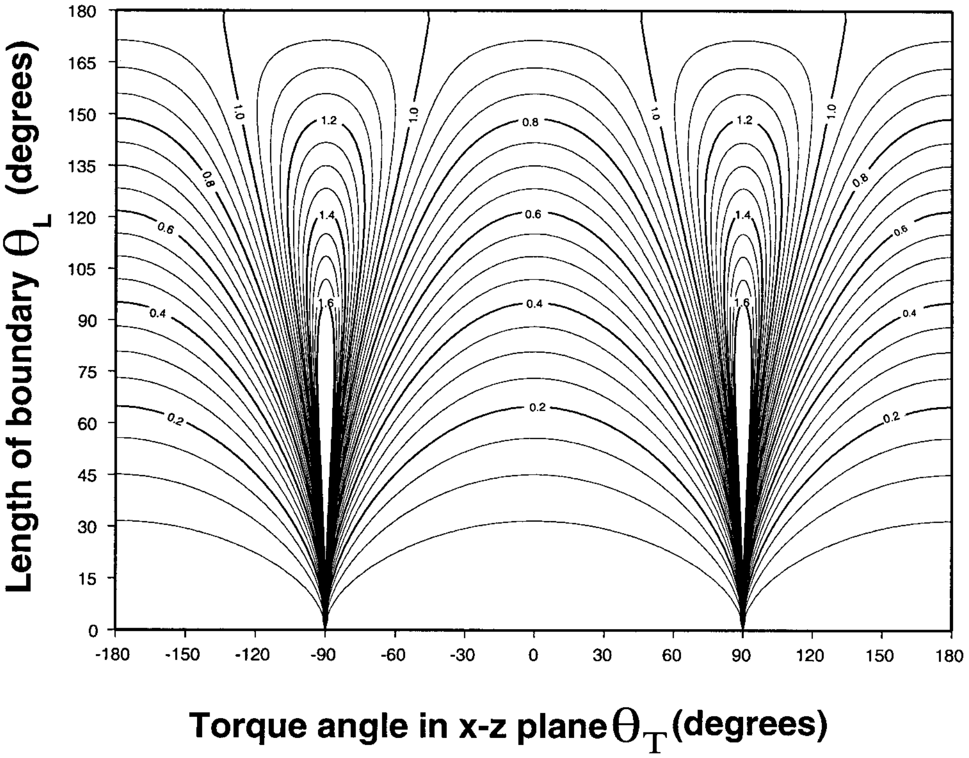

Figure 7. Values of the parameter γ as defined in eq. (13) as a function of the torque angle in the x–z plane θ_T and the angular length of the boundary θ_L. A small value of γ means that the component of torque T_y that would induce shear motion in the diffuse boundary only produces a small displacement of the pole of rotation out of the x–z plane (i.e. ω_y and ψ_ω are small). The contour interval is 0.05 and contours are shown up to $\gamma=1.6$. Except when $\theta_T \simeq 90°$, γ tends to be small ($\simeq 0.05$) for $\theta_L \simeq 30°$, which is the relevant case for the oceanic diffuse plate boundaries in the Indian Ocean.

A possible explanation for the small size of T_y is that the softening process that leads to the formation of diffuse boundaries might be associated with deviatoric tensional stresses or deviatoric compressional stresses, or both, rather than with shear stresses. If so, proto-boundaries would tend to form in an orientation such that $T_y=0$. Alternatively, the boundary may soften with time so as to be weaker in tension or compression than in shear. The former might happen if, for example, magmatic activity under the proto-boundary were important, while the latter might happen if, with sufficient deformation, buoyancy effects become important.

5.2 Along-strike position of the rotation pole (comparison of ω_x with ω_z)

As discussed in Section 4 and shown in Figs 5 and 6, geometrical factors make it very unlikely that the pole of rotation for a diffuse boundary will lie beyond the boundary in the along-strike direction: for this to happen the torque associated with the boundary would have to be oriented at an angular separation very close to 90° from the centre of the boundary. The India–Capricorn, Capricorn–Australia and Nubia–Somalia poles of rotation do lie within the respective diffuse boundary zones, but are, however, significantly displaced from the along-strike centre of their respective zones. This points to differences between the magnitudes of T_x and T_z, specifically $|T_x|$ is larger (perhaps much larger) than $|T_z|$. (The North America–South America pole of rotation may also not lie in the centre of its diffuse plate boundary, but the uncertainties are too large to confirm or deny this.) Moreover, the sense of offset of the pole of rotation from the centre of the diffuse plate boundary provides information about the sign or sense of T_x. Both Indo–Australian diffuse plate boundaries, which lie entirely within oceanic lithosphere, have longer contractional portions than extensional portions, which indicates that the sign of T_x is such that it produces deviatoric compression across the diffuse plate boundaries. Given the dominance of horizontal compression in older oceanic lithosphere worldwide (Mendiguren 1971; Sykes & Sbar 1974; Bergman & Solomon 1980; Zoback 1992), it is unsurprising that there would be a bias towards horizontal deviatoric compression in the sign of T_x. Finite element models for the stress in the Indo–Australian composite plate indicate that a combination of ridge push and concentrated collisional resistance in the Himalayas would tend to orient the horizontal compression north–south in the equatorial Indian Ocean, as is observed (Cloetingh & Wortel 1986; Coblentz *et al.* 1998).

The sign of T_z may also be expected to be predictable from seafloor age gradients, if the magnitude of the horizontal non-lithostatic compressional force per unit length increases with the age of the seafloor, as is expected from the pressure gradients predicted from simple models of cooling of oceanic lithosphere. These indicate that the sign of T_z is such that greater deviatoric compression is expected in older seafloor than in younger seafloor, which is consistent with the sign of T_z we infer for the Indo–Capricorn, Capricorn–Australia and North America–South America diffuse plate boundaries. Finite element models for the stress in the Indo–Australian composite plate also predict a west to east increase in horizontal deviatoric compression in the equatorial Indian Ocean (Cloetingh & Wortel 1986; Coblentz *et al.* 1998), consistent with the inferred sign of T_z in the India–Capricorn diffuse oceanic plate boundary.

The Nubia–Somalia diffuse plate boundary provides an interesting contrast in that most of it occurs within elevated continental lithosphere, in which the most common state of stress is horizontal deviatoric tension, not compression (Zoback 1992). In this case, we would expect the sign of T_x to have the opposite bias from the purely oceanic diffuse boundaries, as it does, given that the extensional portion of the boundary is much longer than the contractional portion of the boundary. Also, the sign of T_z is what we expect given the expected gradient in state of stress from horizontal deviatoric tension along the continental portion of the boundary to horizontal deviatoric compression in much of the oceanic portion of the boundary.

There is another possible explanation, however, for the greater length of the extensional relative to the contractional portion of the Nubia–Somalia boundary. If the boundary resists contraction more effectively than extension, then to maintain the torque balance the proportion of the boundary in extension must increase relative to the proportion in contraction: the pole of rotation would be located further towards the contractional end of the boundary, as is the case for Nubia–Somalia. Similarly, if the extensional region weakens faster than the contractional region (as might happen, for example, if the divergent end of the boundary begins to extend volcanically), then the pole would move with time towards the contractional end (see Fig. 8). If the relative strengths of the two regions vary with time (as might happen if faults are initiated in an irregular fashion) then the pole of rotation may well move around within the diffuse boundary.

6 NON-NEWTONIAN VISCOSITY

The assumption that the local resistance to motion is proportional to the relative velocity may break down in several ways. An alternative assumption is that the effective vertically averaged rheology of the lithosphere is non-Newtonian (England & McKenzie 1982; Bercovici 1995; Trompert & Hansen 1998). Here we examine whether the pole of rotation must lie close to the centre of the boundary if resistance is not proportional to relative velocity. We do so by assuming a power-law relationship between rate of deformation and the local contribution to the torque $\delta\mathbf{T}$ from an element $\delta\theta$ along the boundary,

$$|\delta\mathbf{T}|^{n-1}\delta\mathbf{T} = a\mathbf{Mr}\times\boldsymbol{\omega}\times\mathbf{r}\delta\theta\,, \tag{14}$$

where a is some constant of proportionality. This is strain rate thinning in that the effective viscosity decreases with increasing strain rate, and a linear increase in torque is associated with a greater than linear increase in rate of deformation. In the limit of large n [such that $(n-1)/n\simeq1$], which results in plastic-like behaviour (that is, the lithosphere in the diffuse oceanic plate boundary deforms only when the stress exceeds a yield stress), it can be shown for the case $\psi_\omega=0$ (no shearing across the boundary) that

$$\sin(\theta_T-\theta_\omega)=\sin\theta_T\cos(\theta_L/2) \tag{15}$$

and thence that $|\theta_\omega|<\theta_L/2$ (that is, the pole of rotation lies inside the boundary in the along-strike direction) unless θ_T is asymptotically close to 90°. In Fig. 5, the shaded regions where the pole can lie outside the boundary would be confined to two lines of infinitesimal thickness at $\pm90°$. Thus at least when ψ_T is small, strain rate thinning intensifies the confinement of the

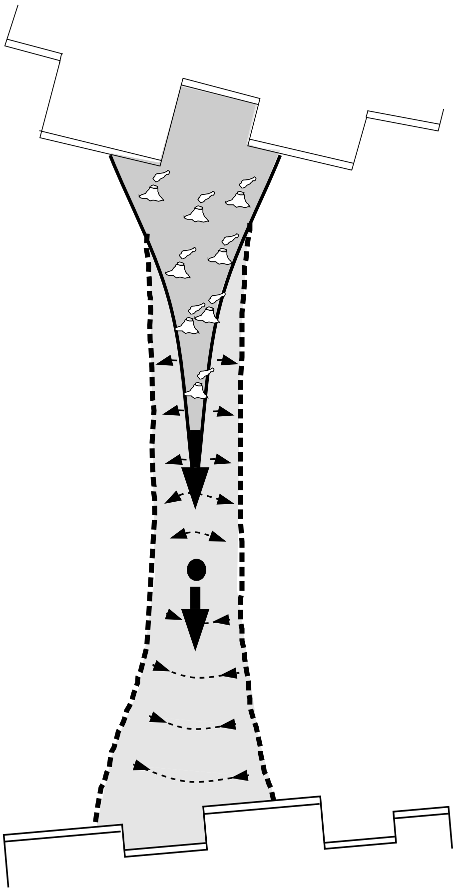

Figure 8. The evolution of the diffuse plate boundary if the end in extension begins to weaken faster than the end in compression. This weakening is shown schematically as volcanism in the upper end of the boundary (the situation between the Somalian and Nubian plates). To preserve the balance of torques, the pole of rotation moves downwards, increasing the proportion of the boundary in extension, and the rate of deformation increases, which provides a positive feedback mechanism if the extensional end of the boundary is further weakened.

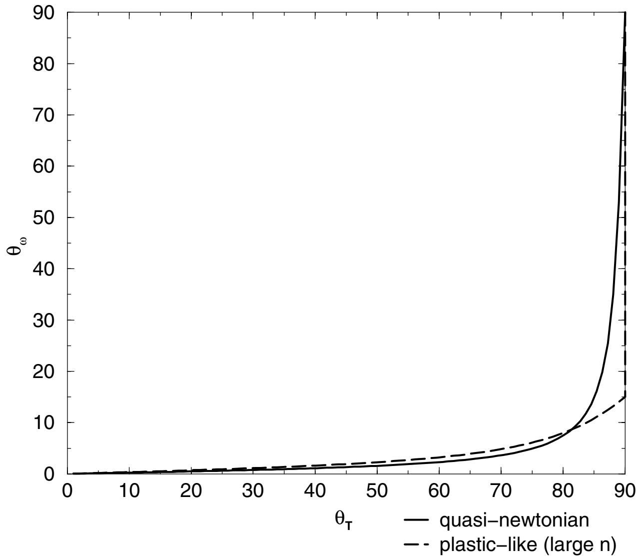

Figure 9. Comparison of θ_ω versus θ_T for the two end-member cases considered in this paper: (1) resistive force is proportional to relative plate velocity (quasi-Newtonian rheology), and (2) resistive force is proportional to the nth power of relative plate velocity when n is large (plastic-like rheology). For this example the along-strike length of the diffuse oceanic plate boundary was taken to be 30° and ψ_ω was assumed to be 0. Overall, the two curves are very similar, with important differences between them occurring only for tan $\theta_T \gg 1$ for which $\theta_\omega \leq 15°$ for the plastic-like case but $\theta_\omega \leq 90°$ for the quasi-Newtonian case.

pole to the diffuse boundary (Fig. 9). On the other hand, for $|\tan\,\theta_T| \sim \mathcal{O}(1)$ or smaller, for which $|\theta_\omega| < \theta_L/2$, θ_ω is ≈ 50 per cent larger for large n than for the Newtonian case, as is shown in the Appendix. This dependence of the location of the pole of rotation on the rheology shows the influence of the rheology on the pattern of deformation and, hence, the stress field within the boundary. For example, between the two predicted locations for the pole of rotation, the sign of the predicted deviatoric principle stress is reversed.

7 IMPLICATIONS FOR STATE OF STRESS IN STABLE PLATE INTERIORS

The arguments we have constructed for diffuse plate boundaries can be applied within plates where there is little ongoing deformation to show that it is likely that the horizontal deviatoric normal stress perpendicular to a great-circle line within a plate will change from being compressional to tensional somewhere on that line within the plate. In this case there may be more than one such transition, because the elastic deformation may be more complex than the viscous response to the relative rotation of plate components considered above. This expectation of a transition is especially true if the line runs through a narrow neck in the plate's interior. It is less likely, however, if the line remains close to a convergent or divergent plate boundary so that the torque from one plate margin, which is typically oriented along an axis $\approx 90°$ from the margin, dominates the required **T** for the line. These predictions could be tested directly should the stress within rigid portions of plates become reliably observable, or else indirectly using numerical spherical plate models.

8 DISCUSSION

If the assumption that the boundary straddles a single central strike line is loosened, it is possible for the pole of rotation to lie at a kink in the diffuse boundary, so that the boundary follows parts of two great circles that intersect at the pole and deformation at the boundary still mainly involves contraction and extension. Chu & Gordon (1999) pictured this happening on the long diffuse boundary between the Nubian and Somalian plates, although the location of the contractional arm of the diffuse boundary is unclear. Perhaps pre-existing weakness in the plate may encourage such kinking to happen. In the Nubia–Somalia case, the pole is located near the coast: the extensional region is continental and the contractional region oceanic. One may imagine that originally the boundary followed a great circle, with the pole of rotation within the continent. If the extending region softened faster than the contracting region, then as described above the pole may have moved towards the coast. Upon reaching the coast, it might then have preferentially moved south-southwest along the coast rather than into the sea if the continental plate there was intrinsically weaker than the oceanic plate (essentially, if it was easier to tear along the continent's edge than into the oceanic plate). If true, this would manifest itself as an increasing length of the extensional portion of the boundary with time to the south-southwest.

If the boundary region softens with strain but the torque **T** that it must provide to maintain the torque balance remains roughly constant, then the rate of deformation will increase. If this rate continues to increase to the point where the change in plate velocity causes the torques from the other plate boundaries and mantle traction to alter appreciably, then **T** would begin to differ significantly from the rigid boundary case and the assumption that the diffuse boundary is strong would need to be relaxed. Indeed, eventually, if prolonged and increasingly rapid deformation makes a diffuse boundary more like a conventional narrow plate boundary, the dynamics of the motion may change entirely: the development, with sufficient extension, of 'ridge push' or, with sufficient contraction, of 'slab pull', may cause the boundary to cease resisting the plate motion and even start enhancing the plate motion, adding a large torque in the x-direction of Fig. 6 so as to drive θ_T towards $\pm 90^\circ$ (and perhaps then finally driving the pole of rotation off the end of the boundary). This positive feedback mechanism might then lead to a fairly abrupt change in the torque balance of the global plate system.

9 CONCLUSIONS

Some aspects of the dynamical behaviour of diffuse plate boundaries that separate composite plates into component plates can be understood in terms of simple analytical models. Geometrical arguments show that while it is possible for the pole of rotation to lie outside the boundary, for this to happen the sum of the torques that drive the deformation (which arise from traction with the underlying mantle and forces at other plate boundaries) must have a very particular (and unlikely) orientation. While we assume a very simple boundary where resistance to deformation is either simply proportional to the rate of deformation or is independent of the rate of deformation (as expected for a power-law fluid with a high exponent), our results are qualitatively true for some more complicated cases, such as when the boundary is weaker in extension than in contraction, where the mathematics become less elegant. Relative to the linear case, non-linear failure of a diffuse plate boundary increases the likelihood that the pole of relative rotation of the two component plates will lie within the boundary.

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APPENDIX A: PARTICULAR SOLUTIONS FOR THE POLES OF ROTATION AND THE COORDINATE SYSTEM

A1 Location of pole of rotation when $n \to \infty$ and $\psi_T = 0$

To obtain eq. (15), rewrite eq. (14) in terms of unit vectors (where $\hat{\boldsymbol{\omega}}$ is a unit vector in the direction of $\boldsymbol{\omega}$),

$$|\delta\mathbf{T}|^{n-1}\delta\mathbf{T} = ar^2\omega\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}\delta\theta\,. \tag{A1}$$

$\mathbf{M}$ is defined after eq. (8) in the main text. Hence

$$|\delta\mathbf{T}| = \left[ar^2\omega|\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}|\right]^{1/n}\delta\theta \tag{A2}$$

and

$$\delta\mathbf{T} = \mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}\,\frac{(ar^2\omega)^{1/n}}{|\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}|^{n-1/n}}\,\delta\theta\,. \tag{A3}$$

If n is large so that $(n-1)/n \simeq 1$, then

$$\delta\mathbf{T} = \left(ar^2\omega\right)^{1/n}\frac{\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}}{|\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}|}\,\delta\theta\,. \tag{A4}$$

Geometrically, in the (x, y, z) coordinate system,

$$\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}} = \begin{pmatrix} 2\cos\psi_\omega\cos\theta\sin(\theta-\theta_\omega) \\ \sin\psi_\omega \\ 2\cos\psi_\omega\sin\theta\sin(\theta-\theta_\omega) \end{pmatrix}, \tag{A5}$$

$$|\mathbf{M}\hat{\mathbf{r}}\times\hat{\boldsymbol{\omega}}\times\hat{\mathbf{r}}| = \sqrt{\sin^2\psi_\omega + 4\sin^2(\theta-\theta_\omega)\cos^2\psi_\omega}\,. \tag{A6}$$

In the special case $\psi_\omega = 0$, we can use eqs (A4), (A5) and (A6) to obtain

$$\delta\mathbf{T} = \left(ar^2\omega\right)^{1/n}\begin{pmatrix}\cos\theta \\ 0 \\ \sin\theta\end{pmatrix}\frac{\theta-\theta_\omega}{|\theta-\theta_\omega|}\,\delta\theta\,. \tag{A7}$$

In this case contributions to the torque are proportional to arc elements. The total torque from the boundary is found via

$$\mathbf{T} = \int_{-\theta_L/2}^{\theta_L/2}\delta\mathbf{T}d\theta$$

$$= \left(ar^2\omega\right)^{1/n}\left[\int_{\theta_\omega}^{\theta_L/2}\begin{pmatrix}\cos\theta \\ 0 \\ \sin\theta\end{pmatrix}d\theta + \int_{\theta_\omega}^{-\theta_L/2}\begin{pmatrix}\cos\theta \\ 0 \\ \sin\theta\end{pmatrix}d\theta\right], \tag{A8}$$

$$\mathbf{T} = 2\left(ar^2\omega\right)^{1/n}\begin{pmatrix}-\sin\theta_\omega \\ 0 \\ \cos\theta_\omega - \cos\dfrac{\theta_L}{2}\end{pmatrix} \tag{A9}$$

and hence

$$\tan\theta_T = -\frac{T_x}{T_z} = \frac{\sin\theta_\omega}{\cos\theta_\omega - \cos\dfrac{\theta_L}{2}}\,, \tag{A10}$$

$$\tan\theta_T\cos\theta_\omega - \sin\theta_\omega = \tan\theta_T\cos\frac{\theta_L}{2}\,. \tag{A11}$$

Using trigonometry to rewrite the left-hand side and multiplying through by $-\cos\theta_T$, this can be converted into eq. (15).

To show that the pole of rotation cannot lie beyond the diffuse boundary in the simplified case $\psi_\omega = 0$, consider the limit where it lies on the boundary, $\theta_\omega = \theta_L/2$. Substituting this into eq. (15), we obtain $\cos\theta_T = 0$, i.e. $\theta_T \to \pi/2$, that is, the pole of rotation reaches the boundary only when the equivalent torque $\mathbf{T}$ lies on the equator of the centre of the diffuse boundary.

A2 Location of pole of rotation when $n \to \infty$ and θ_L, θ_ω, $\psi_\omega \ll 1$

In this case we allow $\psi_\omega \neq 0$ but take the limits θ_L, θ_ω, $\psi_\omega \ll 1$ and large n, and make the additional assumption that $\tan\theta_T$ and $\tan\psi_T$ are not divergent (that is, they remain finite as $\theta_L \to 0$; note that this excludes the flat earth geometry of Section 3). In this case, from eqs (A4), (A5) and (A6),

$$\delta\mathbf{T} = \delta\theta\left(ar^2\omega\right)^{1/n}\frac{1}{\sqrt{4(\theta-\theta_\omega)^2 + {\psi_\omega}^2}}\begin{pmatrix}2(\theta-\theta_\omega) \\ \psi_\omega \\ \theta(\theta-\theta_\omega)\end{pmatrix}. \tag{A12}$$

As before, we expect cancellation to occur over integration for δT_x but not δT_y or δT_z, in which case $\psi_\omega \sim \mathcal{O}(\theta(\theta-\theta_\omega))$, and it can be neglected within $\sqrt{4(\theta-\theta_\omega)^2 + {\psi_\omega}^2}$ except in a small region around $\theta = \theta_\omega$. Hence,

$$T_x = \begin{cases} -2\left(ar^2\omega\right)^{1/n}\theta_\omega & \text{for} \quad |\theta_\omega| < \dfrac{\theta_L}{2} \\ -\left(ar^2\omega\right)^{1/n}\theta_L\,\dfrac{\theta_\omega}{|\theta_\omega|} & \text{for} \quad |\theta_\omega| > \dfrac{\theta_L}{2} \end{cases}, \tag{A13}$$

$$T_z = \begin{cases} \left(ar^2\omega\right)^{1/n}\left(\dfrac{1}{4}\,\theta_L^2 - 2{\theta_\omega}^2\right) & \text{for} \quad |\theta_\omega| < \dfrac{\theta_L}{2} \\ 0 & \text{for} \quad |\theta_\omega| > \dfrac{\theta_L}{2} \end{cases}. \tag{A14}$$

Using $\tan\theta_T = -T_x/T_z$ it is then easy to derive the θ-coordinate of the pole of rotation:

$$\theta_\omega = \frac{\theta_L^2}{8}\tan\theta_T\,. \tag{A15}$$

The implied scaling $\theta_\omega \sim \mathcal{O}(\theta_L^2)$ suggests that in the limit of small θ_L, θ_ω can be neglected in the integration of δT_y, in which case

$$T_y = \left(ar^2\omega\right)^{1/n}\psi_\omega\ln\left(\frac{\theta_L}{\psi_\omega}\right), \tag{A16}$$

and this, with eq. (A13), provides an equation for the ψ coordinate of the pole of rotation,

$$\psi_\omega\ln\left(\frac{\theta_L}{\psi_\omega}\right) = \frac{\theta_L^2}{4}\,\frac{\tan\psi_T}{\cos\theta_T}\,. \tag{A17}$$

Eq. (A15) shows that θ_ω is independent of ψ_T, independent of the magnitude of $\mathbf{T}$, and is a small fraction of $\tan\theta_T$, as is also implied by Fig. 9.

It is interesting to compare these with the equations for the 'quasi-Newtonian' case in the same limit (which can be derived from eqs 12 and 13),

$$\theta_{\omega qn} = \frac{\theta_L^2}{12} \tan\theta_T \,, \tag{A18}$$

$$\psi_{\omega qn} = \frac{\theta_L^2}{6} \frac{\tan\psi_T}{\cos\theta_T} \,. \tag{A19}$$

The flat earth approximation (θ_L, θ_ω, $\psi_\omega \ll 1$, $\theta_T \simeq \pi/2$) is tricky for the large n case except when $\psi_\omega = 0$, in which case eq. (A11) and the conversions given at the end of Section 4 can be used to show that, using the notation of Section 3,

$$x_0 = D \pm \sqrt{D^2 + \frac{L^2}{4}} \,. \tag{A20}$$

The requirement that the pole of rotation lies within the boundary still holds for this limit, which means that if D is positive, the negative root should be taken, and if D is negative, the positive root should be taken.

A3 Note on the coordinate system used in this paper

The angular coordinates are not of the form (colatitude, longitude) as ψ corresponds to an angular distance along a great-circle path (the one that intersects the central strike line of the boundary at a right angle at θ). It is easy to convert this into a standard coordinate system: defining colatitude and longitude (Θ, ϕ) about the centre of the diffuse boundary, where $\phi = 0$ lies in the x-direction (strike parallel),

$$\frac{1}{r}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\cos\psi\sin\theta \\ \sin\psi \\ \cos\psi\cos\theta \end{pmatrix} = \begin{pmatrix} \cos\phi\sin\Theta \\ \sin\phi\sin\Theta \\ \cos\Theta \end{pmatrix} , \tag{A21}$$

so

$$\begin{aligned} \tan\theta &= -\tan\Theta\cos\phi \,, \\ \sin\psi &= \sin\Theta\sin\phi \,, \\ \cos\Theta &= \cos\psi\cos\theta \,, \\ \tan\phi &= -\frac{\tan\psi}{\sin\theta} \,. \end{aligned} \tag{A22}$$