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Erratum

Erratum to: Analytic Quasi-Perodic Cocycles with Singularities and the Lyapunov Exponent of Extended Harper's Model⋆

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We correct the statement of Theorem 2.5 in [2], which generalizes Avila's "quantization of the acceleration" [1] from analytic $SL(2, \mathbb{C})$-cocycles to arbitrary non-singular cocycles. All notation is kept precisely as in the paper.

Correctly stated, the theorem should read as follows:

Theorem 1 (Quantization of the acceleration). *Consider an analytic cocycle* (β, D) *where* β *is irrational and* $\det D(x)$ *is bounded away from zero on a strip* $\mathbb{T}_\delta$. *For* $|\epsilon| \leq \delta$, $\omega(\beta, D; \epsilon) \in \frac{1}{2}\mathbb{Z}$. *Moreover, for (non-singular) Jacobi cocycles,* $\omega(\beta, A^E; \epsilon) \in \mathbb{Z}$.

Thus, even though in general the acceleration is only half-integer valued, for Jacobi cocycles[1]

$$A^E(x) := \begin{pmatrix} E - v(x) & -\overline{c}(x-\beta) \\ c(x) & 0 \end{pmatrix}, \tag{1}$$

the object of interest in the paper, one still has integer-valued acceleration. In particular, the correction of Theorem 2.5 does not have any effect on the conclusions for extended Harper's model.

[1] We mention that despite the appearance of the complex conjugate in the Jacobi operator, the resulting Jacobi cocycle can still be realized as an analytic $M(2, \mathbb{C})$-valued function on $\mathbb{T}$: Given $c \in \mathcal{C}^\omega(\mathbb{T}; \mathbb{C})$, $c(x) =: C(\mathrm{e}^{2\pi i x})$, we simply "re-interpret" the complex-conjugate of c as $\overline{c}(x) := C^\sharp(\mathrm{e}^{2\pi i x})$ with $C^\sharp(z) := \overline{C(\frac{1}{\overline{z}})}$, corresponding to a reflection of C on the unit circle. Note that for $x \in \mathbb{T}$ this has no effect since $\overline{c}(x) = \overline{c(x)}$. It was pointed out to us by some readers, that our notation $\overline{c}(x)$ may be confusing at first glance, which is why we comment on it here.

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The error resulted from accidentally dropping a factor of $\frac{1}{2}$ on one side of (2.13), which correctly reads

$$\frac{1}{2}\int_{\mathbb{T}} \log|\det(D(x+i\epsilon)|\mathrm{d}x = \frac{1}{2}L\left(\beta, \begin{pmatrix} \det D_\epsilon(x) & 0 \\ 0 & \det D_\epsilon(x) \end{pmatrix}\right) . \tag{2}$$

It is shown in Lemma 2.6 of [2] that the integral on the left hand side of (2) takes values in $2\pi\mathbb{Z}$ which, taking into account the factor of $\frac{1}{2}$ in (2), yields $\omega(\beta, D; \epsilon) \in \frac{1}{2} \in \mathbb{Z}$, as now stated in Theorem 1.

Integer-valued acceleration for Jacobi cocycles only boils down to the following observation:

$$\frac{1}{2}\int_{\mathbb{T}} \log|\det A^E(x+i\epsilon)|\mathrm{d}x = \frac{1}{2}\left(I_\epsilon(c) + I_{-\epsilon}(c)\right) = I(c), \tag{3}$$

where $I_\epsilon(c) := \int_{\mathbb{T}} \log|c(x+i\epsilon)|\mathrm{d}x$. The second equality in (3) uses that by Lemma 2.6 (i),

$$I_\epsilon(c) = I(c) + 2\pi\epsilon N, \tag{4}$$

for some $N \in \mathbb{Z}$ and $|\epsilon| \leq \delta$.

More generally, one can claim integer-valued acceleration for any non-singular cocycle (β, D), where $\sqrt{\det D}$ can be defined as a *one*-periodic holomorphic function as opposed to *two*-periodicity, which always holds due to Lemma 2.3. This is a consequence of (2.12) and the following simple fact which may be considered an amendment to Lemma 2.3:

Fact 1. *Let* $f \in \mathcal{C}_\delta^\omega(\mathbb{T}; \mathbb{C})$ *with* $\min_{|\mathrm{Im}\, z| \leq \delta} |f(z)| > 0$. *If*

$$\frac{1}{2\pi}\left(\mathrm{D}_+ \int_{\mathbb{T}} \log|f(x+i\epsilon)|\mathrm{d}x\right)\Big|_{\epsilon=0} \in 2\mathbb{Z}\,, \tag{5}$$

there exists $g \in \mathcal{C}_\delta^\omega(\mathbb{T}; \mathbb{C})$ *satisfying* $g^2 = f$.

Proof. If f is a trigonometric polynomial, the claim follows directly from the form of g, explicitly constructed in the proof of Lemma 2.3, Step 1, therein. For general analytic f, approximating by trigonometric polynomials $f_n \to f$ on $\mathbb{T}_\delta$, the same arguments as in the proof of Lemma 2.6 (i) imply that $\left(D_+ \int_{\mathbb{T}} \log|f_n(x+i\epsilon)|\mathrm{d}x\right)\big|_{\epsilon=0}$ eventually stabilizes to its limit $\left(D_+ \int_{\mathbb{T}} \log|f(x+i\epsilon)|\mathrm{d}x\right)\big|_{\epsilon=0}$. Since, $\sqrt{f_n} \to \sqrt{f}$, we conclude the statement as claimed. □

Finally, for the sake of completeness, we would like to correct two minor misprints. Equation (2.22) of course only holds at $\epsilon = 0$; for $\epsilon \neq 0$, in agreement with (3) and (4),

$$L(\beta, (A_\epsilon^E)') = L(\beta, B_\epsilon^E) + I_\epsilon(c) - I(c) = L(\beta, B_\epsilon^E) + 2\pi N\epsilon\ . \tag{6}$$

Similarly, the second equality in (2.4) only holds at $\epsilon = 0$.

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References

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