

ANALYTIC REPRESENTATION OF UPPER
ATMOSPHERE DENSITIES BASED ON JACCHIA'S STATIC
DIFFUSION MODELS

James C. G. Walker

Institute for Space Studies
Goddard Space Flight Center, NASA
New York, New York

~~Available for sale and
NASA only.~~

\$ _____
(S) \$ _____
copy (HC) 1.00
Microfiche (MF) .50

ff 853 July 65

N66-22215

FACILITY FORM 602

(ACCESSION NUMBER) <u>8</u>	(THRU) <u>1</u>
(PAGES) <u>TMX 50450</u>	(CODE) <u>13</u>
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

XERO
COPY

XERO
COPY

XERO
COPY

X
C

In a recent paper, Jacchia (1964) has presented a set of models of the upper atmosphere designed to reproduce the satellite drag density data while requiring the minimum amount of tabulation. He uses an empirical expression for the temperature profile,

$$T(z) = T_{\infty} - (T_{\infty} - T_{120}) \exp \left[-s(z - 120) \right], \quad (1)$$

where T_{120} is the temperature at 120 km, T_{∞} is the exospheric temperature, z is the altitude in kilometers, and s is an analytic function of T_{∞} . Jacchia presents analytic expressions for T_{∞} which represent the solar cycle, short period solar activity, semi-annual, diurnal, and geomagnetic activity variations of atmospheric density. The expressions for T_{∞} and s have been determined so that the temperature profile (1) yields atmospheric densities in agreement with satellite drag data (Jacchia and Slowey, 1963) when the diffusive equilibrium equations for N_2 , O_2 , O , and He are integrated numerically using a single set of boundary conditions.

I wish to call attention to a minor modification of Jacchia's procedure which eliminates the numerical integration of the diffusive equilibrium equations, yielding a completely analytic representation of upper atmosphere densities.

If (1) is replaced by the very similar expression,

$$T(z) = T_{\infty} - (T_{\infty} - T_{120}) \exp \left[- \sigma \zeta \right], \quad (2)$$

where ζ is the geopotential altitude,

$$\zeta = \frac{(z - 120)(R + 120)}{R + z}, \quad R = 6356.77 \text{ km}, \quad (3)$$

the diffusive equilibrium equation can be integrated directly to

give

$$n(i|z) = n(i|120) \left(\frac{1 - a}{1 - ae^{-\sigma \zeta}} \right)^{1+\alpha+\gamma} \exp \left[- \sigma \gamma \zeta \right], \quad (4)$$

where $n(i|z)$ is the number density of constituent i at altitude z ,

$$a = \frac{T_{\infty} - T_{120}}{T_{\infty}}, \quad (5)$$

α is the thermal diffusion coefficient (equal to -0.4 for He and zero for N_2 , O_2 , and O), and

$$\gamma = \frac{m_i \varepsilon_{120}}{\sigma k T_\infty} \quad (6)$$

where m_i is the molecular mass of constituent i , k is Boltzmann's constant, and

$$\varepsilon_{120} = 980.665 \left(1 + \frac{120}{R}\right)^{-2} = 944.655 \text{ cm sec}^{-2} \quad (7)$$

is the acceleration of gravity at 120 km. This relationship (4) was derived by Bates (1959; cf. Stein and Walker, 1965) without allowance for thermal diffusion.

If

$$\sigma = s + \frac{1}{R+120} = s + 0.00015, \quad (8)$$

the temperatures given by (1) and (2) are identical at the altitude

where

$$s(z - 120) = \sigma \zeta = 1 \quad (9)$$

as well as at 120 km and at infinity. The fractional difference between the two temperatures is given approximately by

$$\frac{\Delta T}{T} = \frac{a(z - 120)}{(R+120) \left[e^{s(z - 120) - a} \right]} \left[1 - s(z - 120) \right] \quad (10)$$

and is less than one per cent at all altitudes for $T_{\infty} = 2000\text{K}$.

The fractional difference is even smaller for smaller values of T_{∞} . Thus analytic density profiles given by (4) will differ negligibly from profiles obtained by numerical integration using Jacchia's temperature profile (1). The analytic representation is easier to compute than the representation involving numerical integration.

Acknowledgements. This work was supported by a Research Associate-
ship of the National Academy of Sciences-National Research Council.

REFERENCES

Bates, D. R., 1959: Some problems concerning the terrestrial atmosphere above about the 100 km level. Proc. Roy. Soc. London, A253, 451-462.

Jacchia, L. G., 1964: Static diffusion models of the upper atmosphere with empirical temperature profiles. Res. in Space Sci., Smithsonian Astrophys. Obs., Spec. Rpt. 170, 53 pp.

Jacchia, L. G., and J. Slowey, 1963: Accurate drag determinations for eight artificial satellites; atmospheric densities and temperatures. Smithsonian Contrib. Astrophys., 8, 1-99.

Stein, J. A., and J. C. G. Walker, 1965: Models of the upper atmosphere for a wide range of boundary conditions. J. Atmospheric Sci., 22, 11-17.