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Analytic Solutions of the Kadomtsev-Petviashvili Equation with Power Law Nonlinearity Using the Sine-Cosine Method

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Abstract In this paper, a sine-cosine method is used to construct many periodic and solitary wave solutions to Kadomtsev-Petviashvili equation with power law nonlinearity. Many new families of exact traveling wave solutions of the Kadomtsev-Petviashvili equation with power law nonlinearity are successfully obtained.

Keywords Sine-Cosine Method, Kadomtsev-Petviashvili Equation, Periodic Solution

1. Introduction

Nonlinear partial differential equations (NPDEs) are widely used to describe complex phenomena in various fields of science, especially in physics. Therefore solving nonlinear problems plays an important role in nonlinear sciences. Many effective methods of obtaining explicit solutions of NPDEs have been presented such as the tanh-method[1-3],the extended tanh method[4-6], the sine–cosine method[7-10], the homogeneous balance method[11], homotopy analysis method[12-18], the F-expa nsion method[19],three-wave method[20-22], extended homoclinic test approach[23-25], the (G'/G)-expansion method[26] and the exp-function method[27-30].

In this paper, by means of the Sine-cosine method, we will obtain some analytic solutions for the Kadomtsev-Petviashvili equation with power law nonlinearity. In the following section we have a brief review on the Sine-cosine method and in Section 3 and 4 , we apply the Sine-cosine method to obtain analytic solutions of the Kadomtsev-Petviashvili equation with power law nonlinearity. Finally, the paper is concluded in Section 5.

2. The Sine-cosine method

1. We introduce the wave variable $\xi = x - ct$ into the PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots) = 0 \tag{1}$$

where $u(x,t)$ is traveling wave solution. This enables us to use the following changes:

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \ \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \ldots \tag{2}$$

One can immediately reduce the nonlinear PDE (1) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{3}$$

The ordinary differential equation (3) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form[8]

$$u(\xi) = \begin{cases} \lambda sin^{\beta}(\mu\xi), & |\xi| \le \dfrac{\pi}{\mu}, \\ 0 & otherwise, \end{cases} \tag{4}$$

or in the form

$$u(\xi) = \begin{cases} \lambda cos^{\beta}(\mu\xi), & |\xi| \le \dfrac{\pi}{2\mu}, \\ 0 & otherwise, \end{cases} \tag{5}$$

where λ, μ and $\beta \neq 0$ are parameters that will be determined, μ and c are the wave number and the wave speed respectively. We use

$$\begin{aligned} & u(\xi) = \lambda sin^{\beta}(\mu\xi), \\ & u^n(\xi) = \lambda^n sin^{n\beta}(\mu\xi), \\ & (u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi), \\ & (u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) \\ & \qquad + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \end{aligned} \tag{6}$$

and the derivatives of (5) becoms

$$\begin{aligned} & u(\xi) = \lambda cos^{\beta}(\mu\xi), \\ & u^n(\xi) = \lambda^n cos^{n\beta}(\mu\xi), \\ & (u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi), \\ & (u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) \\ & \qquad + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \end{aligned} \tag{7}$$

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and so on for other derivatives.

3.We substitute (6) or (7) into the reduced equation obtained above in (3), balance the terms of the cosine functions when (7) is used, or balance the terms of the sine functions when (6) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms whit same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns μ,β and λ. We obtained all possible value of the parameters μ,β and λ [7].

3. The (1+2)-Dimensional KP Equation with Power Law Nonlinearity

The dimensionless form of the (1+2)-dimensional KP equation, with power law nonlinearity, that is going to be studied in this paper is given by[31]

$$(u_t+au^n u_x+u_{xxx})_x+bu_{yy}=0. \tag{8}$$

Here in Eq. (8), a and b are real valued constants. After that we use the transformation

$$u(x,y,t)=\varphi(\xi) \quad , \quad \xi=x+y-ct \tag{9}$$

where c is constant. There for the Eq. (8) converts to

$$(-c\varphi'+a\varphi^n\varphi'+\varphi''')'+b\varphi''=0, \tag{10}$$

where by integrating twice we obtain

$$(b-c)\varphi+\frac{a}{n+1}\varphi^{n+1}+\varphi''=0, \tag{11}$$

substituting (4) into (11) gives

$$\begin{aligned}&(b-c)\lambda sin^{\beta}(\mu\xi)+\frac{a}{n+1}\lambda^{(n+1)}sin^{(n+1)\beta}(\mu\xi)\\&-\mu^2\beta^2\lambda\sin^{\beta}(\mu\xi)+\mu^2\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi))=0\end{aligned} \tag{12}$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$\begin{aligned}&(\beta-1)\neq 0,\\&\beta-2=(n+1)\beta,\\&(b-c)\lambda-\mu^2\beta^2\lambda=0,\\&\lambda\mu^2\beta\big(\beta-1\big)+\frac{a}{n+1}\lambda^{(n+1)}=0\end{aligned} \tag{13}$$

Solving the system (13) yields

$$\beta=-\frac{2}{n},\mu=\frac{1}{2}\sqrt{b-c},\ \lambda=\left[\frac{(n+2)(n+1)(c-b)}{2a}\right]^{\frac{1}{n}} \tag{14}$$

where c is a free parameter. Hence, for $b>c$, the following periodic solutions

$$u(\xi)=\left\{\frac{(n+2)(n+1)(c-b)}{2a}\csc^2\left[\frac{\sqrt{b-c}}{2}(\xi)\right]\right\}^{\frac{1}{n}} \tag{15}$$

where $0<\frac{\sqrt{b-c}}{2}(\xi)<\pi$, and

$$u(\xi)=\left\{\frac{(n+2)(n+1)(c-b)}{2a}\sec^2\left[\frac{\sqrt{b-c}}{2}(\xi)\right]\right\}^{\frac{1}{n}} \tag{16}$$

where $|\frac{\sqrt{b-c}}{2}(\xi)|<\frac{\pi}{2}$.

However, for $c>b$, the following periodic solutions

$$u(\xi)=\left\{\frac{(n+2)(n+1)(b-c)}{2a}csch^2\left[\frac{\sqrt{c-b}}{2}(\xi)\right]\right\}^{\frac{1}{n}},$$

$$u(\xi)=\left\{\frac{(n+2)(n+1)(c-b)}{2a}sech^2\left[\frac{\sqrt{b-c}}{2}(\xi)\right]\right\}^{\frac{1}{n}}.$$

4. The (1+3)-Dimensional KP Equation with Power Law Nonlinearity

The dimensionless form of the (1+3)-dimensional KP equation, with power law nonlinearity, that is going to be studied in this paper is given by[32]

$$(u_t+au^n u_x+u_{xxx})_x+bu_{yy}+cu_{zz}=0 \tag{17}$$

Here in Eq. (17), a,b and c are real valued constants. After that we use the transformation

$$u(x,y,t)=\varphi(\xi) \quad , \quad \xi=x+y+z-mt \tag{18}$$

where m is constant. There for the Eq. (17) converts to

$$(-m\varphi'+a\varphi^n\varphi'+\varphi''')'+(b+c)\varphi''=0 \tag{19}$$

where by integrating twice we obtain

$$(b+c-m)\varphi+\frac{a}{n+1}\varphi^{n+1}+\varphi''=0 \tag{20}$$

substituting (4) into (20) gives

$$\begin{aligned}&(b+c-m)\lambda sin^{\beta}(\mu\xi)+\frac{a}{n+1}\lambda^{(n+1)}sin^{(n+1)\beta}(\mu\xi)\\&-\mu^2\beta^2\lambda sin^{\beta}(\mu\xi)+\mu^2\lambda\beta(\beta-1)sin^{\beta-2}(\mu\xi))=0\end{aligned} \tag{21}$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$\begin{aligned}&(\beta-1)\neq 0,\\&\beta-2=(n+1)\beta,\\&(b+c-m)\lambda-\mu^2\beta^2\lambda=0,\\&\lambda\mu^2\beta\big(\beta-1\big)+\frac{a}{n+1}\lambda^{(n+1)}=0\end{aligned} \tag{22}$$

solving the system (22) yields

$$\begin{aligned}&\beta=-\frac{2}{n} \quad , \quad \mu=\frac{1}{2}\sqrt{b+c-m} \quad ,\\&\lambda=\left[\frac{(n+2)(n+1)(m-b-c)}{2a}\right]^{\frac{1}{n}},\end{aligned} \tag{23}$$

where m is a free parameter. Hence, for $b+c>m$, the following periodic solutions

$$\begin{aligned}&u(\xi)=\\&\left\{\frac{(n+2)(n+1)(m-b-c)}{2a}\csc^2\left[\frac{\sqrt{b+c-m}}{2}(\xi)\right]\right\}^{\frac{1}{n}},\end{aligned} \tag{24}$$

where $0<\frac{\sqrt{b+c-m}}{2}(\xi)<\pi$, and

$$u(\xi)=\left\{\frac{(n+2)(n+1)(m-b-c)}{2a}\sec^2\left[\frac{\sqrt{b+c-m}}{2}(\xi)\right]\right\}^{\frac{1}{n}} \quad (25)$$

where $\left|\frac{\sqrt{b+c-m}}{2}(\xi)\right|<\frac{\pi}{2}$.

However, for $b+c<m$, the following periodic solutions

$$u(\xi)=\left\{\frac{(n+2)(n+1)(b+c-m)}{2a}csch^2\left[\frac{\sqrt{m-b-c}}{2}(\xi)\right]\right\}^{\frac{1}{n}},$$

$$u(\xi)=\left\{\frac{(n+2)(n+1)(m-b-c)}{2a}sech^2\left[\frac{\sqrt{b+c-m}}{2}(\xi)\right]\right\}^{\frac{1}{n}}.$$

5. Conclusions

In this paper, by using the sine-cosine method, we obtained some new explicit formulas of solutions for the generalized (1+2)-dimensional and the generalized (1+3)-dimensional KP equations. Those solutions were similar to the solutions obtained in other paper. The study reveals the power of the method.

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