Analytical and Experimental Evaluation of Nonlinear Viscoelastic-Viscoplastic Composite Laminates under Creep, Creep-Recovery, Relaxation and Ramp Loading

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Abstract. A numerical procedure to predict long term laminate properties of fibre reinforced composite materials was developed. In the procedure, we extended the classical laminate theory to include time related response of composite materials for membrane and flexural loading. The material response, dependent on the stress history, was modelled using the Schapery single integral equation. The integrals were handled by an approximate method that uses the Prony's series and only requires the storing of the current stress and some internal strain components. An efficient semi-direct time-integration scheme, providing a stable integration process, was derived to be included in the numerical procedure. Comparisons of theoretical results were made with experiments conducted on composite materials under creep/creep-recovery, relaxation and ramp loading.

Keywords: viscoelasticity, viscoplasticity, cyclic mechanical loading, durability, ramp loading, relaxation, stress/strain rate dependent

Introduction

Most plastics and polymer based composite systems exhibit strong hereditary type non-linear viscoelastic behavior leading any stress-strain analysis of these materials into a very demanding experience.

The single integral formulations, such as the Schapery nonlinear viscoelastic theory, though simple in form, have been found to be mathematically too complex to be used in an engineering analysis, due to the presence of Volterra-type integrals. Tuttle and Brinson (1986) presented a closed-form solution to an arbitrary number of discrete steps in stress based upon the Schapery theory. This solution, based on hereditary nature, requires that all stresses at each time step are stored and reused for all subsequent time steps. Furthermore, each additional time step requires the hereditary integral to be recalculated from the initial starting time. Therefore, the total solution time grows geometrically with the number of time steps which ultimately limits them, regardless the size of the step or the speed of the computer. An efficient method to handle the constitutive integral equation of Schapery theory is presented. The method avoids storing the stress history and computes the Volterra-type integral by replacing it with numerical equivalent ordinary integrals.

The thermodynamic theory developed by Schapery (1969) gives the possibility to express material properties in terms of either stress or strain.

Although the behavior of several materials is consistent with one or the other representation, Schapery showed that there is no basic reason to expect that a given material will agree with both representations unless it is linearly viscoelastic. The constitutive equation in terms of strain, the well-known Schapery equation, is restricted to small strains by the underlying thermodynamic theory. That relation cannot be applied when structural changes are closely related to the strain level. In contrast, the constitutive equation in terms of stresses is not restricted to small strains.

Notwithstanding Findley (1989), among others, stated that since creep and stress relaxation behaviors are two aspects of the time sensitive mechanical behavior of materials, one behavior should be predictable if the other behavior is known. For linear viscoelastic materials, the relationship between them and the interconversion of creep and stress relaxation functions is relatively simple. For nonlinear viscoelastic materials, Findley developed a procedure to predict the stress relaxation from given creep behavior regardless of the method used to determine the kernel functions. The description of this procedure, given below, follows the development suggested by Findley (1989). Consider a creep test performed under a prescribed stress program $\sigma = \sigma(t)$, for which the corresponding creep strain response is $\epsilon = \epsilon(t)$. Then in a relaxation test performed under a prescribed axial strain program equal to the strain response $\epsilon = \epsilon(t)$ resulting from the prescribed creep test program, the observed stress response in the relaxation test would be equal to the prescribed stress program $\sigma = \sigma(t)$ in the creep test. Consider now a variable-stress creep program where $\sigma = \sigma(t)$, for which the resulting strain response is a constant strain

$\epsilon = \epsilon_0$. Then from the previous conclusion the stress relaxation response, at a prescribed constant strain $\epsilon = \epsilon_0 H(t)$, will be $\sigma = \sigma(t)$

In this paper the interconversion of creep and stress relaxation function of a nonlinear viscoelastic-viscoplastic material is discussed. It is shown that the proposed method, to handle the constitutive integral equations of Schapery theory, can easily be used to predict the stress relaxation and the rate-dependent stress/strain behavior. Good agreement between computed results and the experimental data was observed for the T300/5208 and IM7/5260 composites.

Analytical-model Description

The predictions for multiple-angle laminates can be obtained using the classical laminate theory. The constitutive models are defined at the ply level, which is a "mesomechanic" approach. Let the plane of a single ply be defined by the 1-2 coordinate system, where the 1 and 2 axes are parallel and perpendicular to the fibres, respectively. Consider a ply subjected to a plane stress state $\{\sigma_{11}, \sigma_{22}, \tau_{12}\}$ that may change with time. The total strains induced within the ply are given by

$$\left\{ \begin{array}{c} \epsilon_{11}(t) \\ \epsilon_{22}(t) \\ \gamma_{12}(t) \end{array} \right\} = \left[\begin{array}{ccc} S_{11} & S_{12} & 0 \\ S_{12} & S_{22}(t) & 0 \\ 0 & 0 & S_{66}(t) \end{array} \right] \left\{ \begin{array}{c} \sigma_1(t) \\ \sigma_{22}(t) \\ \tau_{12}(t) \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ \epsilon_{22}(t) \\ \gamma_{12}(t) \end{array} \right\}_{VP} \quad (1)$$

where the subscript VP denotes the viscoplastic component of the strain. We assume that the fibre-dominated compliance terms S_{11} and $S_{12} (= S_{21})$ are time-independent, while the matrix dominated compliance terms $S_{22}(t)$ and $S_{66}(t)$ are time-dependent.

The total strains associated with the matrix-dominated compliances S_{22} and S_{66} were modelled using the modified Schapery theory to include the viscoplastic behavior, firstly presented by Tuttle (1993). In this constitutive model, the viscoelastic and viscoplastic strains refer to the strains which do not occur instantaneously upon application of a stress but rather developed with the passage of time. Further, viscoelastic strains can be completely recovered if all stress are relieved and sufficient time passes. However the viscoplastic strains cannot be recovered; they are permanent and irrecoverable, even if all stress are removed. The viscoelastic strains are predicted using the Schapery model, while viscoplastic strains are predicted using the functional employed by Zapas and Crissman (1984). According to this, the total ply strains induced by an arbitrary stress history are given by

$$\left\{\begin{array}{c}\epsilon_{11}(t)\\ \epsilon_{22}(t)\\ \gamma_{12}(t)\end{array}\right\} = \left[\begin{array}{ccc}S_{11} & S_{12} & 0\\ S_{12} & g_{0,22}\cdot S_{0,22} & 0\\ 0 & 0 & g_{0,66}\cdot S_{0,66}\end{array}\right]\left\{\begin{array}{c}\sigma_1(t)\\ \sigma_{22}(t)\\ \tau_{12}(t)\end{array}\right\} + \left\{\begin{array}{c}0\\ g_{1,66}\int_0^t \Delta S_{22}(\psi-\psi')\dfrac{d\left(g_{2,22}\sigma_{22}(\tau)\right)}{d\tau}d\tau\\ g_{2,66}\int_0^t \Delta S_{66}(\psi-\psi')\dfrac{d\left(g_{2,66}\tau_{12}(\tau)\right)}{d\tau}d\tau\end{array}\right\} + \left\{\begin{array}{c}0\\ \epsilon_{22}(t)\\ \gamma_{12}(t)\end{array}\right\}_{VP} \quad (2)$$

with

$$\left\{ \begin{array}{c} 0 \\ \epsilon_{22}(t) \\ \gamma_{12}(t) \end{array} \right\}_{VP} = \left\{ \begin{array}{c} 0 \\ \left[C_{22} \int_0^t (\sigma_{22}(\tau))^{N_{22}} d\tau \right]^{n_{22}} \\ \left[C_{66} \int_0^t (\tau_{12}(\tau))^{N_{66}} d\tau \right]^{n_{66}} \end{array} \right\} \tag{3}$$

where C_{22}, N_{22}, n_{22} and C_{66}, N_{66}, n_{66} are stress-independent but temperature dependent material properties. The kernels $\Delta S_{22}(t)$ and $\Delta S_{66}(t)$ are the transverse and shear elastic compliance, respectively, with the correspondent reduced times Ψ and Ψ' given by

$$\Psi = \int_0^t \frac{d\tau'}{a_{\sigma,22}}, \Psi' = \int_0^\tau \frac{d\tau'}{a_{\sigma,22}} \quad , \tag{4}$$

and

$$\Psi = \int_0^t \frac{d\tau'}{a_{\sigma,66}}, \Psi' = \int_0^\tau \frac{d\tau'}{a_{\sigma,66}} \quad , \tag{5}$$

where $g_{0,22}, g_{1,22}, g_{2,22}, a_{\sigma,22}$ and $g_{0,66}, g_{1,66}, g_{2,66}, a_{\sigma,66}$ are stress-dependent nonlinearizing parameters. The common parameter for the transverse and shear nonlinear compliance is the octahedral shear stress in the matrix that is a function of matrix transverse stress and matrix shear stress. A more detailed explanation for the use of the octahedral shear stress parameter is given by Schapery (1969).

In order to eliminate the Volterra-type integrals, the transverse and shear compliance are expressed using Prony series, as Henriksen (1984), Gramoll et. al.(1989) and Czyz (1990) have already discussed.

$$\begin{cases} \Delta S_{22}(t) = \sum\limits_{i=1}^{\infty} S_{i,22}\left(1 - e^{-\lambda_{i,22} t}\right) \\ \Delta S_{66}(t) = \sum\limits_{i=1}^{\infty} S_{i,66}\left(1 - e^{-\lambda_{i,66} t}\right) \end{cases} , \tag{6}$$

where $S_{i,22}, \lambda_{i,22}, S_{i,66}, \lambda_{i,66}$ are linear viscoelastic parameters for the transverse and shear compliance, respectively. For example, substituting (6) into (2), for the transverse strain, we obtain

$$\epsilon_{22}(t) = g_{0,22} \cdot S_{0,22} \cdot \sigma_{22}(t) + g_{1,22} \sum_{i=1}^{\infty} \epsilon_{i,22}(t) + \epsilon_{vp,22}(t) \quad , \tag{7}$$

where

$$\epsilon_{i,22}(t) \quad = \quad S_{i,22} \int_0^t \left(1 - e^{-\lambda_{i,22}(\psi - \psi\prime)}\right) \frac{d\left(g_{2,22} \cdot \sigma_{22}\right)}{d\tau} d\tau \quad . \tag{8}$$

After some transformations, equation (8), gives the relation between the internal strain $\epsilon_{i,22}(t_{j+1})$ and $\epsilon_{i,22}(t_j)$. Let us assume a linear variation during $t_j \leq t \leq t_{\ j+1}$ for the stress, σ_{22} , as

$$\frac{d\sigma_{22}}{dt} \quad = \quad \frac{\sigma_{22}(t_{j+1}) - \sigma_{22}(t_j)}{\Delta t} \quad . \tag{9}$$

Let also assume that the function $\tilde{g}_{i,22}(\sigma)$ can be approximated by a linear variation such as:

$$\frac{d\tilde{g}_{i,22}(\sigma)}{dt} \cong \frac{\tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - \tilde{g}_{i,22}(\sigma_{22}(t_j))}{\Delta t} \quad , \tag{10}$$

where

$$\tilde{g}_{i,22}(\sigma) = S_{i,22} \cdot g_{2,22} \cdot \sigma_{22} \quad . \tag{11}$$

Substituting Equation (11) into Equation (8) we obtain

$$\epsilon_{i,22}(t_j) = \tilde{g}_{i,22}(\sigma_{22}(t_j)) - e^{-\lambda_{i,22}\theta(t_j)} \int_0^{t_j} e^{-\lambda_{i,22}\theta(\tau)} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau \quad , \tag{12}$$

where $\theta(t) = \int_0^t \frac{d\tau'}{a_{\sigma,22}} \quad .$

In order to calculate $\epsilon_{i,22}(t_{j+1})$ in terms of $\epsilon_{i,22}(t_j)$. we obtain

$$\begin{aligned} \epsilon_{i,22}(t_{j+1}) &= \tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(t_j)]} \left[\tilde{g}_{i,22}(\sigma_{22}(t_j)) - \epsilon_{i,22}(t_j)\right] - \\ & \int_{t_j}^{t_{j+1}} e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(\tau)]} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau \quad . \end{aligned} \tag{13}$$

The integral of Equation (13) can be solved using the approximation function (10) as:

$$\begin{aligned} \int_{t_j}^{t_{j+1}} e^{-\lambda_{i,22}[\theta(t_{j+1})-\theta(\tau)]} \frac{d\tilde{g}_{i,22}(\sigma)}{d\tau} d\tau &= \left(\frac{\tilde{g}_{i,22}(\sigma_{22}(t_{j+1})) - \tilde{g}_{i,22}(\sigma_{22}(t_j))}{\Delta t}\right) \cdot \\ & \left(\frac{a_{\sigma,22}(\sigma_{22}(t_{j+1}))}{\lambda_{i,22}}\right) \cdot \\ & \left(1 - e^{-\lambda_{i,22}\frac{\Delta t}{a_{\sigma,22}(\sigma_{22}(t_{j+1}))}}\right) \quad , \end{aligned} \tag{14}$$

where $\Delta t = t_{j+1} - t_j$ and the following approximation was used

$$\begin{aligned} -\lambda_{i,22}[\theta(t_{j+1}) - \theta(\tau)] &= -\lambda_{i,22} \int_{\tau}^{t_{j+1}} \frac{d\tau'}{a_{\sigma,22}(\sigma_{22})} \\ &\cong \frac{\lambda_{i,22}}{a_{\sigma,22}(\sigma_{22}(t_{j+1}))}(t_{j+1} - \tau) \quad . \end{aligned} \tag{15}$$

Substituting Equation (14) into Equation (13) we obtain

$$\begin{aligned}\epsilon_{i,22}\left(t_{j+1}\right) &= e^{-\eta_i \Delta t} \cdot \epsilon_{i,22}\left(t_j\right) + \\ &\left[1 - \frac{1}{\eta_i \Delta t}\left(1 - e^{-\eta_i \Delta t}\right)\right] \cdot \tilde{g}_i\left(\sigma_{22}\left(t_{j+1}\right)\right) + \\ &\left[\frac{1}{\eta_i \Delta t}\left(1 - e^{-\eta_i \Delta t}\right) - e^{-\eta_i \Delta t}\right] \cdot \tilde{g}_i\left(\sigma_{22}\left(t_j\right)\right) \quad , \qquad (16)\end{aligned}$$

where

$$\eta_i = \frac{\lambda_{i,22}}{a_{\sigma,22}\left(\sigma_{22}\left(t_{j+1}\right)\right)} \quad . \qquad (17)$$

Tuttle (1993) presented the solution of the viscoplastic term assuming a linear variation in stress over every time step as

$$\begin{aligned}\epsilon_{22}\left(t_{j+1}\right)_{VP} &= \left[C_{22}\int_0^{t_j}\left(\sigma_{22}(\psi)\right)^{N_{22}} d\psi + C_{22}\int_{t_j}^{t_{j+1}}\left(\sigma_{22}(\psi)\right)^{N_{22}} d\psi\right]^{n_{22}} \\ &= \left[\beta\left(t_{j-1}\right) + C_{22}\cdot\Omega\cdot\left(t_{j+1}-t_j\right)\right]^{n_{22}} \quad , \qquad (18)\end{aligned}$$

with

$$\Omega = \frac{\left(\sigma_{22}\left(t_{j+1}\right)\right)^{N_{22}+1} - \left(\sigma_{22}\left(t_j\right)\right)^{N_{22}+1}}{\left(N_{22}+1\right)\cdot\left(\sigma_{22}\left(t_{j+1}\right) - \sigma_{22}\left(t_j\right)\right)} \quad , \qquad (19)$$

where $\beta(t_j)$ is given by recursion:

$$\beta\left(t_j\right) = \beta\left(t_{j-1}\right) + C_{22}\cdot\Omega\cdot\left(t_j - t_{j-1}\right) \quad . \qquad (20)$$

The total strain for each layer, given by Equation (2), can then be written as

$$\{\epsilon(t_{j+1})\} = [S_{elast}]\{\sigma(t_{j+1})\} + \{R(t_{j+1})\} \quad , \tag{21}$$

where

$$\{R(t_{j+1})\} = \left\{ \begin{array}{c} 0 \\ g_{1,22}\sum_{i=1}^{n}\varepsilon_{i,2}(t_{j+1}) + \varepsilon_{vp,2}(t_{j+1}) \\ g_{1,66}\sum_{i=1}^{n}\varepsilon_{i,6}(t_{j+1}) + \varepsilon_{vp,6}(t_{j+1}) \end{array} \right\} \tag{22}$$

The vector of Equation (22) contains the viscoelastic and viscoplastic deformations as given by Equation (2). This vector is calculated recursively using the formulas given by Equations (16) and (18) since they can equally be applied to transverse and shear deformations.

The objective is to determine the behavior of multidirectional composites knowing the stress-strain relationships for unidirectional plies. Therefore a modified laminate plate theory for laminates subjected to in-plane loads $\{N\}$ and out-plane-loads $\{M\}$ was developed. The subscript (x, y, z) represents the off-axis coordinate directions (laminate or global coordinates).

Assuming that the ply strain is linear in the thickness coordinate z, the total strain of the k^{th} ply in the global coordinate system is given by

$$\{\epsilon(t_{j+1})\}_k = \left\{ \begin{array}{c} \epsilon_x^0(t_{j+1}) \\ \epsilon_y^0(t_{j+1}) \\ \gamma_{xy}^0(t_{j+1}) \end{array} \right\} + z_k \left\{ \begin{array}{c} k_x(t_{j+1}) \\ k_y(t_{j+1}) \\ k_{xy}(t_{j+1}) \end{array} \right\} \quad , \tag{23}$$

where $\{\epsilon^0\}$ is the in-plane laminate strain vector, $\{k\}$ is the laminate curvature vector and z_k represents the z coordinate of the k^{th} ply. Taking into account the in-plane loads and moments acting on the laminate, the equilibrium equations in global coordinates are:

$$\left\{ \begin{array}{l} \displaystyle\int_{-h/2}^{h/2} \{\sigma\} \; dz = \{N\} \\ \displaystyle\int_{-h/2}^{h/2} \{\sigma\} \; z \; dz = \{M\} \end{array} \right. \quad , \tag{24}$$

where h represents the total laminate thickness. If we assume a constant stress at each ply, which is not true for flexural loads, using Equations (21), (23) and (24) we obtain a system of equations that allow us to determine the in-plane laminate strain, the laminate curvature and the stress state in each ply as:

$$\left\{ \begin{array}{l} \{\epsilon^0\} + \{k\} \, z_1 - [S_{elast}]_1 \, \{\sigma\}_1 = \{R\}_1 \\ \{\epsilon^0\} + \{k\} \, z_2 - [S_{elast}]_2 \, \{\sigma\}_2 = \{R\}_2 \\ \quad \vdots \\ \{\epsilon^0\} + \{k\} \, z_p - [S_{elast}]_p \, \{\sigma\}_p = \{R\}_p \\ \{\sigma\}_1 \, z_1 + \{\sigma\}_2 \, z_2 + \cdots + \{\sigma\}_p \, z_p = \{N\} \\ \{\sigma\}_1 \, h_1 z_1 + \{\sigma\}_2 \, h_2 z_2 + \cdots + \{\sigma\}_p \, h_p z_p = \{M\} \quad , \end{array} \right. \tag{25}$$

where the subscripts indicate the ply number. Since the vector $\{R\}_k$ depends on the present stress state of the k^{th} ply, an iterative procedure for each time step must be used until the stress state converges.

In order to avoid large systems of equations the ply stress state given by Equation (21) can be used in Equation (24) to obtain the following condensed system of equations as:

$$[Se]\left\{ \begin{array}{c} \{\epsilon^0\} \\ \{k\} \end{array} \right\} = \left\{ \begin{array}{c} \{N\} \\ \{M\} \end{array} \right\} + \left\{ \begin{array}{c} \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} \{R\}_k \, z_k \\ \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} \{R\}_k \, z_k h_k \end{array} \right\} \quad , \tag{26}$$

where

$$[Se] \quad = \quad \left[\begin{array}{cc} \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} z_k & \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} z_k h_k \\ \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} z_k h_k & \sum\limits_{k=1}^{p} [S_{elast}]_k^{-1} \left(z_k^2 h_k + \frac{h_k^3}{12} \right) \end{array} \right] \quad , \tag{27}$$

and h_k represents the thickness of k^{th} ply The previous formulation permit us to solve the creep problem. If a restriction is imposed, like prescribing an in-plane strain or a curvature, then the relaxation problem must be solved. In a creep test the loads are imposed and the in-plane strain and curvature are determined. In a relaxation test the in-plane strain and/or curvature are imposed and the resulting loads are to be determined. The present method can also be used to solve these type of problems. For example, if the k_x curvature is prescribed, then the related flexural load M_x is to be determined. The problem is solved by exchanging M_x with k_x in Equation (25), resulting in the following system of equations:

$$\left\{ \begin{array}{l} \{\epsilon^0\} + \left\{ \begin{array}{c} 0 \\ k_y \\ k_{xy} \end{array} \right\} z_1 - [S_{elast}]_1 \{\sigma\}_1 = \{R\}_1 - \left\{ \begin{array}{c} k_x \\ 0 \\ 0 \end{array} \right\} z_1 \\ \vdots \\ \{\epsilon^0\} + \left\{ \begin{array}{c} 0 \\ k_y \\ k_{xy} \end{array} \right\} z_p - [S_{elast}]_p \{\sigma\}_p = \{R\}_p - \left\{ \begin{array}{c} k_x \\ 0 \\ 0 \end{array} \right\} z_p \\ \{\sigma\}_1 z_1 + \{\sigma\}_2 z_2 + \cdots + \{\sigma\}_p z_p = \{N\} \\ \left\{ \begin{array}{c} M_x \\ 0 \\ 0 \end{array} \right\} + \{\sigma\}_1 h_1 z_1 + \{\sigma\}_2 h_2 z_2 + \cdots + \{\sigma\}_p h_p z_p = \left\{ \begin{array}{c} 0 \\ M_y \\ M_{xy} \end{array} \right\} . \end{array} \right. \tag{28}$$

The argument for the existing solution was already discussed before. In strict mathematical terms, each restriction implies an inversion of the correspondent nonlinear integral equation and numerical methods are the only way to obtain the solution. We can conclude that this formulation allows us to solve all sort of problems related with creep, relaxation and rate dependent stress/strain behavior for in-plane and flexural loads. The method described here was developed in a FORTRAN computer program called LAMFLU, Guedes (1997).

Materials Testing

Two different materials, the IM7/5260 graphite/bismaleimide composite and the T300/5208 graphite-epoxy composite, were used to test the procedure presented here.

Edward Wu (1982, 1983a, 1983b) published experimental data for the T300/5208 composite under creep/creep-recovery, ramp loading and multiple relaxation and therefore it is accessible to the technical community. The scope of that program was to provide a data base that could be used to characterize overall matrix-dominated time-dependent deformation and time-dependent strength. The specimens were [$\pm 45^o$] laminates tested in tension. The nonlinear viscoelastic model developed by Schapery was selected to study the time dependent behavior of the composites. The material properties were obtained through the creep/creep-recovery data at four different shear stress levels of 22.15, 32.30, 43.60 and 48.95MPa at room temperature, (Guedes, 1997). Along with the viscoelastic behavior, it was observed a viscoplastic behavior well modelled by the viscoplastic functional employed by Zapas and Crissman (1984) and later by Tuttle (1993). The linear viscoelastic compliance was modelled by an exponential series expansion, the Prony series. The calculation of the unrecoverable strain, i.e. viscoplastic strain, was accomplished by extrapolation of an exponential series expansion (Guedes, 1997).

Gates (1992) and Tuttle (1993) characterized the IM7/5260 composite in two different ways. Gates (1992) used an extension of an elastic/viscoplastic model for an orthotropic material to predict the rate-dependent stress/strain behavior of polymer matrix composites. Experimental procedures to provide data necessary to generate material constants were described. For the elastic/plastic and elastic/viscoplastic material constants, Gates executed off-axis tests on [15^o], [30^o] and [40^o] specimens. In this study four temperatures 23^o, 70^o, 125^o and 200^oC were selected and the strain rates were between $50\mu\varepsilon/s$ and $200\mu\varepsilon/s$. On the other hand Tuttle (1993) used the modified viscoelastic/viscoplastic nonlinear Schapery model to predict the strain vs. time response of multiple-angle composite subjected to cyclic thermomechanical loading over long times. The material properties for the accelerated characterization scheme were obtained from $10 hour$ creep/creep-recovery tests at 93^o, 121^o and $149^o C$ on $[0^o]$, $[90^o]$ and $[\pm 45^o]$ laminates. At every test temperature, the specimens were tested at stress levels ranging from $7.7 MPa$ to $46.27 MPa$.

Results and Discussion

Three types of tests were used during this study. Namely creep/creep-recovery loading history, constant strain rate and multiple-step relaxation tests.

Creep tests

Edward Wu (1983b) subjected T300/5208 $[\pm\ 45^o]$ laminates to cyclic creep/creep-recovery tests at room temperature. The static transverse tensile strength value for the T300/5208 laminate found in literature was around $68 MPa$. In Figures 1 and 2 shear strain for the test data and the LAMFLU predictions are plotted together for two different tests. The experimental recovery data for cycle 7, in Figure 1 and for cycle 1, in Figure 2, were not available. The creep tests were performed at 72% and 69%, respectively, of the ultimate shear stress level. The specimens were tested for a total time of $10176 hours$, representing approximately $14 months$.

A very good agreement between the prediction and measured data over the entire tests was obtained for the modified Schapery model with the viscoplastic component, (Guedes, 1997).

In Figure 2 one can observe that the material deformed in an asymptotic form, meaning a behavior change from glassy to rubbery state. This also signifies that the viscoplastic simple power law model will fail at longer time range due to its inherent monotonic increasing character.

Relaxation tests

Edward Wu (1983a) subjected T300/5208 [$\pm 45^o$] laminates to multiple-step relaxation tests at room temperature.

The LAMFLU procedure was used to compute the relaxation shear stress for different constant shear strains extending them into nonlinear range. The resulting relaxation behavior predicted from the creep data showed a good agreement with the actual shear stress relaxation behavior as shown in Figure 3.

The multiple-step relaxation test consisted of applying and maintaining a step-displacement while the load relaxed. After the load relaxation approaches the asymptotic level, another step-displacement was superimposed on the previous step and again maintained while the load relaxed. The specimens were loaded by displacement conditions. The displacement rate for each step was nominally constant at $0.02 cm/s$ and the holding time was approximately $50 hours$.

In Figure 4 the relaxation shear stress data for a multiple-step relaxation test is plotted together with the LAMFLU predictions. Good agreement between the test and the predicted data was observed. From the third step the test data showed higher peaks than the ones predicted by the LAMFLU at the beginning of each step-displacement, but the relaxed shear stresses still stay very close.

Constant rate strain tests

For the ramp loading tests, where the strain rate was imposed, comparisons with predictions were made for the stress and plastic strain. The plastic strain was calculated as:

$$\epsilon^{plastic} = \epsilon - \sigma \cdot E \quad , \tag{29}$$

where E represents the initial modulus in the elastic region. This formulation (Tsotsis, 1993) suggests that the deviation from linearity is a result of plastic deformation, but in fact it embodies both viscoelastic and viscoplastic deformations.

Edward Wu (1982) subjected T300/5208 $[\pm 45^o]$ laminates to multiple-step relaxation tests at room temperature. The specimens were loaded by displacements conditions and the displacement rates were nominally constant. Because it was inappropriate to interpret the data as constant strain rate, the reported records of the actual strain-time histories were used in LAMFLU program. In Figures 5, 6 and 7 test data and respective plastic shear strain is plotted for three different shear strain rates of $0.125\mu\varepsilon/s$, $12.5\mu\varepsilon/s$ and $1250\mu\varepsilon/s$, respectively together with the LAMFLU predictions. Comparisons between the experimental results and the curves based on the theoretical relationship show an excellent agreement at both high and low strain rates.

The strain rate-dependent behavior of a IM7/5260 composite was compared with two theoretical models. As outlined before, Gates (1992) characterized the nonlinear rate-dependent behavior of a IM7/5260 composite using an elastic/viscoplastic model and Tuttle (1993, 1995) characterized the time-dependent behavior of a IM7/5260 composite using the modified Schapery nonlinear viscoelastic model to include the viscoplastic term. In Figures 8 and 9 the stress relaxation and the plastic strain predictions for the Gates model are plotted together with the Tuttle-LAMFLU predictions for the IM7/5260 [25^o] laminates at a temperature of 125^oC for two different strain rates. As one can observe, predictions of both models are in a very good agreement for both strain rates.

Concluding remarks

A numerical method to handle the constitutive integral equation of the Schapery theory was developed. The procedure can be used to predict the creep, the stress relaxation and the rate-dependent stress/strain behavior of composite laminates. A good agreement between computed and experimental results or reference data was observed in the presented examples.

Partially, these good results could be related to the simplicity of the nonlinear viscoelastic behavior of T300/5208 [$\pm 45^o$] laminates due essentially to simple shearing in the principal material directions of the ply. Due to the 45^oangle-ply lay-up, the shear stress that is tangent to the fibres in each unidirectional ply is equal to $\sigma/2$ and the associated shearing strain is $(1+\nu)\varepsilon$, where σ, ε and ν are the efective, axial stress, axial strain and Poisson's ratio of the laminate. The ply stress normal to the fibres is relatively small due to the high fibre modulus.

The plastic modified Schapery model proposed by Tuttle (1993) appear to be a general model at least for the limited stress or strain states and histories used in this study.

Acknowledgments

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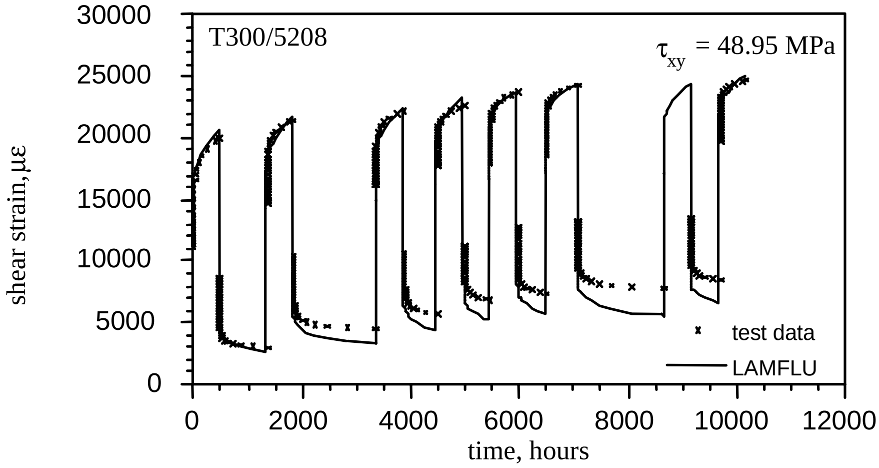

Figure 1: Test data and LAMFLU prediction shear strain of a laminate subjected to a cyclic creep/creep-recovery test at room temperature.

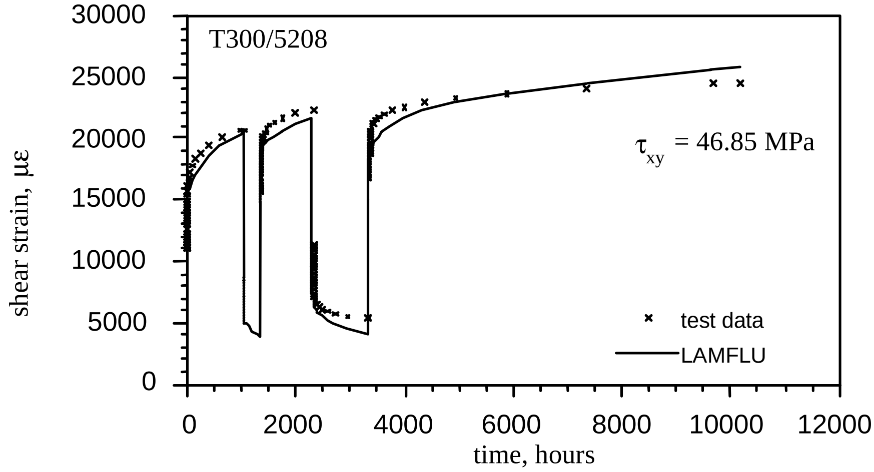

Figure 2: Test data and LAMFLU prediction shear strain of a laminate subjected to a cyclic creep/creep-recovery test at room temperature.

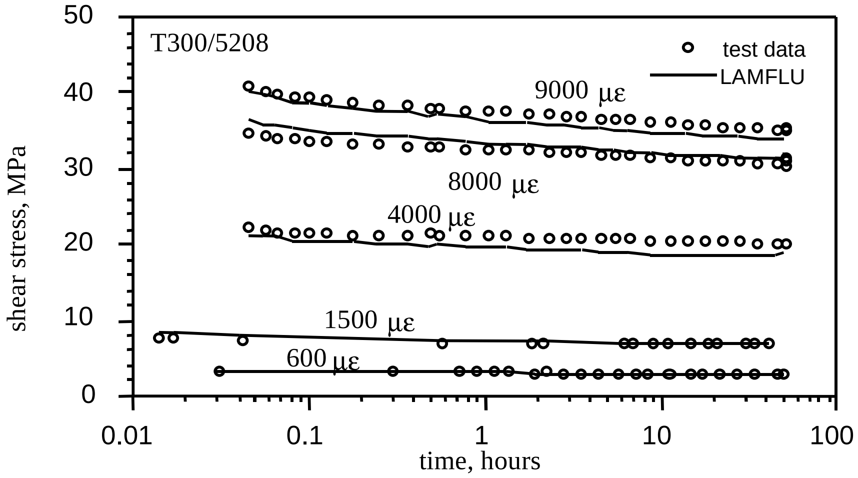

Figure 3: Shear stress relaxation of T300/5208 [$\pm 45^o$] laminates under constant shear strain at room temperature, and predicted with LAMFLU.

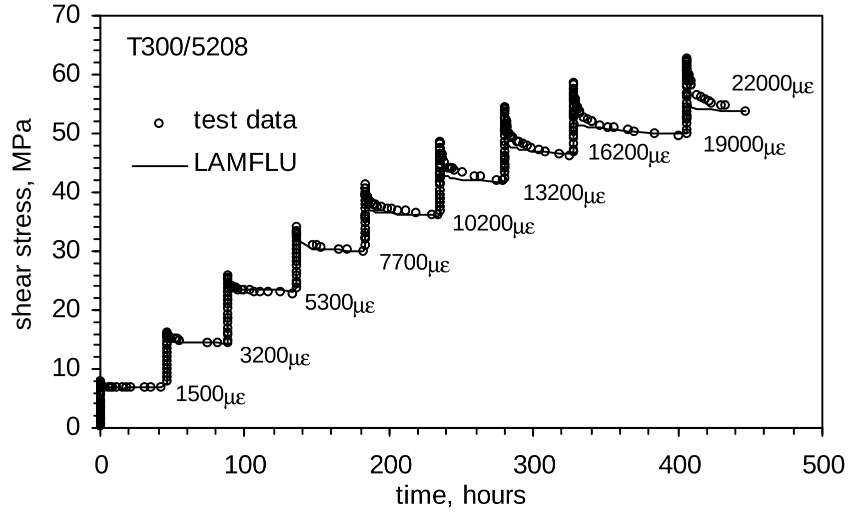

Figure 4: Multiple-step relaxation test of a T300/5208 [$\pm 45^o$] laminate at room temperature, with those predicted with LAMFLU.

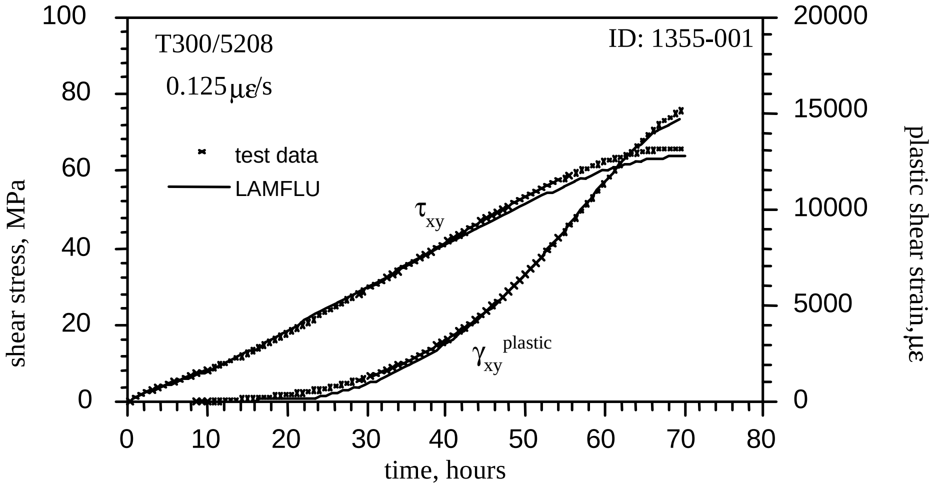

Figure 5: Constant shear strain rate test of a T300/5208 [$\pm 45^o$] laminate at room temperature, with those predicted with LAMFLU.

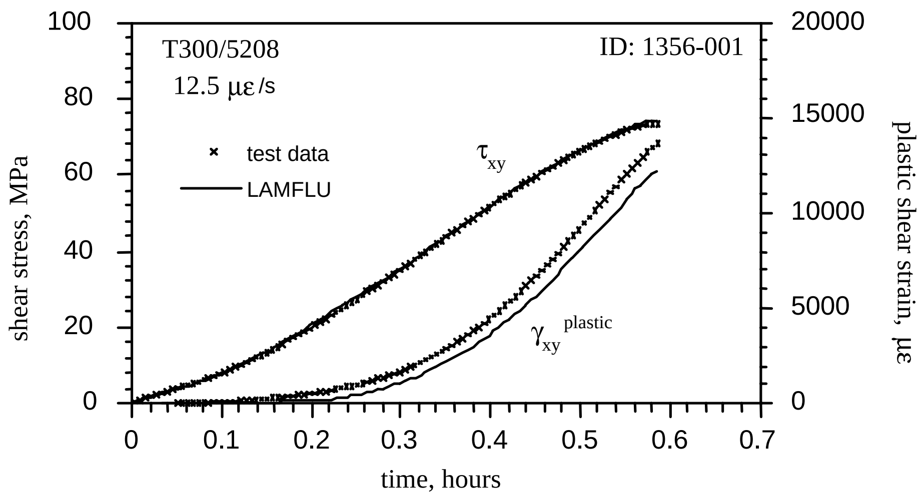

Figure 6: Constant shear strain rate test of T300/5208 [$\pm 45^o$] laminate at room temperature, with those predicted with LAMFLU.

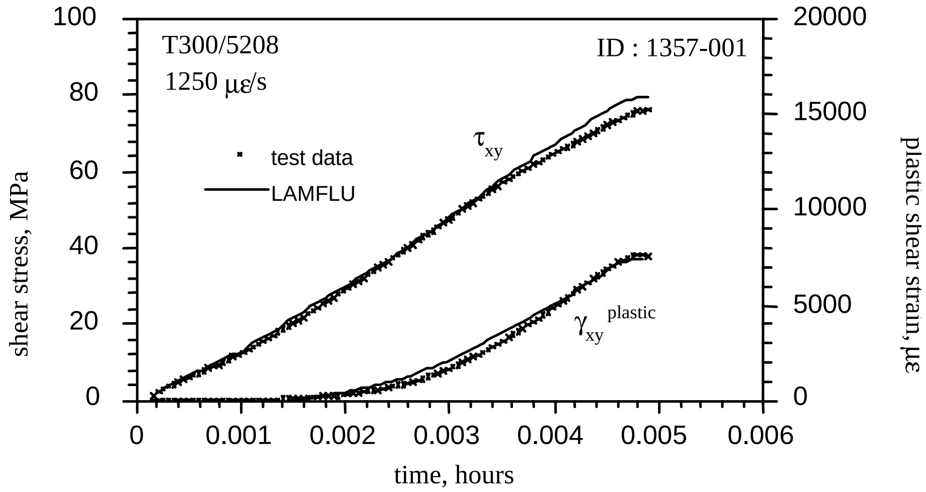

Figure 7: Constant shear strain rate test of T300/5208 [$\pm 45^o$] laminate at room temperature, with those predicted with LAMFLU.

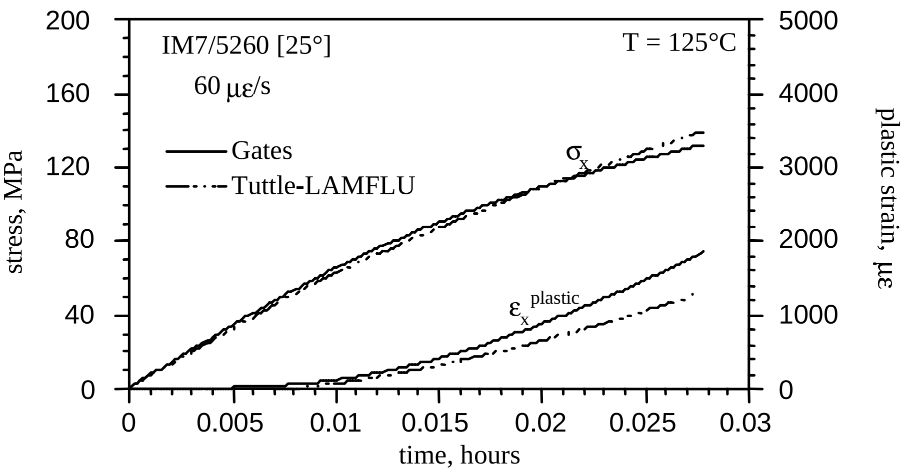

Figure 8: Stress prediction for the Gates and the Tuttle-LAMFLU models for a constant strain rate test simulation of a IM7/5260 [25^o] laminate at 125^oC.

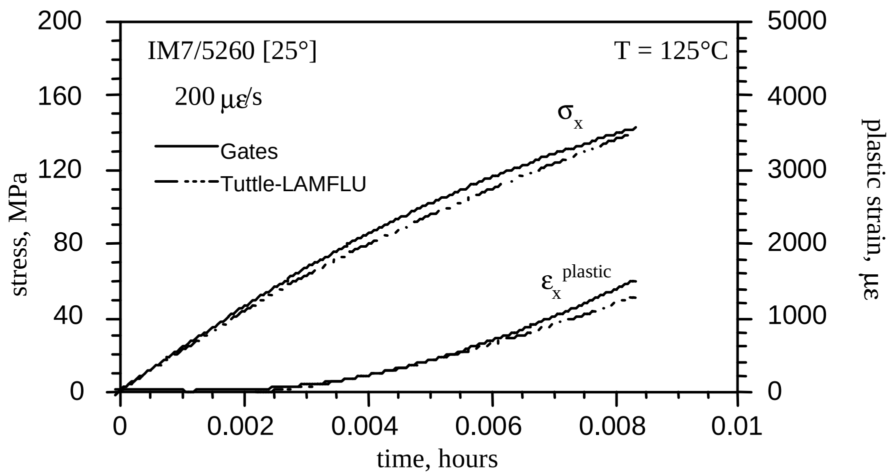

Figure 9: Stress prediction for the Gates and the Tuttle-LAMFLU models for a constant strain rate test simulation of a IM7/5260 [25^o] laminate at 125^oC.