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Analytical and Numerical investigation of natural convection in a heated cylinder using Homotopy Perturbation Method

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ABSTRACT. Homotopy Perturbation Method (HPM) has been applied to solve a nonlinear heat transfer problem. Natural convection around an isothermal horizontal cylinder was studied. Heat transfer coefficient and specific heat coefficient were assumed to be dependent on temperature. Outcomes were compared with solution of heat transfer equation with constant properties. Solutions of HPM were compared with numerical results for different cases, and variation of Nusselt number was obtained and investigated.

Keywords: homotopy perturbation method (HPM), nonlinear heat transfer, numerical Runge-Kutta Method (*NM*), natural convection, Nusselt number.

Investigação analítica e numérica de convecção natural em cilindro aquecido pelo Método de Perturbação Homotópica

RESUMO. O método de perturbação homotópica (MPH) foi aplicado para resolver um problema de transferência não linear de calor e a convecção natural cerca um cilindro horizontal isotérmico foi analisada. Os coeficientes transferência de calor e calor específico foram considerados dependentes da temperatura. Os resultados foram comparados com a solução da equação de transferência de calor com as propriedades constantes. As soluções de MPH foram comparadas com os resultados numéricos para vários casos. A variação do número de Nusselt foi obtida e investigada.

Palavras-chave: método de perturbação homotópica (MPH), transferência não linear de calor, método numérico de Runge-Kutta (NM), convecção natural, número de Nusselt.

Introduction

Many familiar heat transfer application involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipments such as power transistors, TVs and VCRs; heat transfer in electronic baseboard heaters or steam radiators; heat transfer phenomena in the refrigeration coils and power transmission lines. The fluid velocities associated with natural convection are low; typically less than 1 m/s therefore, the heat transfer coefficient encountered in natural convection are usually much lower than those encountered in forced convection (CHENG, 2009; GANGI; SADIGHI, 2007; JONES et al., 1988).

So, studying natural convection that rises in applicable engineering problems, help to manipulate better natural convection as one of the heat transfer mechanism. Cooling cylindrical fin with natural convection is a good application of this heat transfer mechanism. The boundary layer over a hot horizontal cylinder start to develop at the bottom, increasing in thickness along the circumference, and forming a rising plum at the top, as shown in Figure (1). Therefore, the local Nusselt number is highest at the bottom and lowest at the top of cylinder when the boundary layer flow remains laminar (BEJAN, 2004). Most of problems arising in heat transfer area are nonlinear and through the majority of them only a limited numbers of them have exact analytical solution so these nonlinear equations should be solved using other methods. Other methods include numerical and semi exact methods, scientists believe that the combination of these two methods can be more cost effective method and also lead to useful results.

One of the semi-exact methods is the homotopy perturbation method (HPM), which is established by by He (HE, 1999). This method has been applied by many authors to solve a wide variety of scientific and engineering problems. Esmaeilpour and Ganji (2007), Ganji et al. (2007) and Rajabi et al. (2007)

use this method and other semi exact methods to solve nonlinear heat transfer problems. Ghasemi et al. (2007) solve a nonlinear and inhomogeneous two-dimensial wave equation problem by HPM. It was shown by many authors such as Hoseinnia et al. (2008) and He (2005) that this method provides improvements over existing numerical techniques. In this paper, the mathematical model of this method is introduced and then its application in natural convection flow over a horizontal hot cylinder is studied.

The aim of this study is to consider the variation of temperature with the time in an isothermal horizontal cylinder that has been cooled with the natural convection of airflow. In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of some heat transfer equation; such methods include the HPM (ESMAEILPOUR; GANJI, 2007; RAJABI et al., 2007).

Therefore in the present work the influence of heat transfer coefficient, h, and specific heat coefficient, c, when they are variable with temperature or they are constant on isothermal cylinder and how long it takes to be cooled are studied, We find solution for these kinds of problems by HPM and compare it with numerical method (NM).

Material and methods

Analysis of the Homotopy Perturbation Method

The Homotopy perturbation method is a combination of the classical perturbation technique and Homotopy technique. To explain the basic idea of the HPM for solving nonlinear differential equations we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{1}$$

Subject to boundary condition:

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma, \tag{2}$$

where:

A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, Γ is the boundary of domain Ω and $\partial u/\partial n$ denotes differentiation along the normal drawn outwards from Ω. The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Equation (1) therefore can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \tag{3}$$

In case that the nonlinear Equation (1) has no 'small parameter', we can construct the following Homotopy:

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p(N(v) - f(r)) = 0, \tag{4}$$

where,

$$v(r,p): \ \Omega \times [0,1] \rightarrow R, \tag{5}$$

In Equation (7), $p \in [0, 1]$ is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Equation (4) can be written as a power series in p, as following:

$$v = v_0 + pv_1 + p^2 v_2 + \ldots, \tag{6}$$

And the best approximation for solution is:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \ldots, \tag{7}$$

When, Equation (4) correspond to Equation (1) and (7) becomes the approximate solution of Equation (1). Some interesting results have been attained using this method. Convergence and stability of this method is shown in (GHASEMI et al., 2007).

Description of the problem

The aim of this study is to consider the temperature variation of a small hot isothermal horizontal cylinder in Figure 1. With diameter and length of 1 cm that is being cooled with natural convection of air flow.

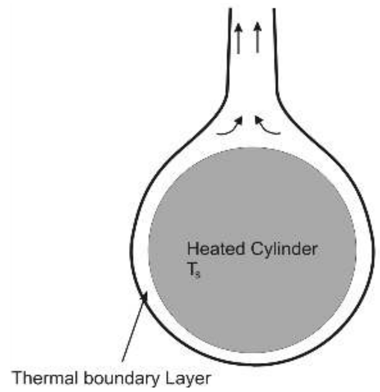

Figure 1. An isothermal horizontal cylinder.

In this article, 3 cases have been investigated which are presented in Table 1.

Table 1. Coefficients in different cases.

Case	H	c
1	Variable	variable
2	Constant	variable
3	Constant	constant

In each case parameters and the following equations have been introduced:

Case 1

In this case heat transfer coefficient, *h*, and specific heat coefficient, *c*, are variable with temperature. Equation (8) represents the heat equation of a lump system (INCCROPERA; DEWITT, 2002).

$$\rho V c\left(\frac{d}{dt}T(t)\right)+h\,A\left(T(t)-T_{\infty}\right)=0 \tag{8}$$

Which *c* is the quality of temperature dependency of specific heat on temperature.

$$c=c_0\left(1+\xi(T-T_{\infty})\right) \tag{9}$$

The average Nusselt number over the entire surface can be determined from Churchill and Chu (1975) for an isothermal horizontal cylinder:

$$Nu=\left(0.6+\frac{0.387\,Ra_d^{\;1/6}}{\left(1+\left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right)^{8/27}}\right)^2 \tag{10}$$

where Rayleigh number and heat transfer coefficient are as follow:

$$Ra_d=\frac{g\,\beta\,d^3(T-T_{\infty})}{\nu\,\alpha} \tag{11}$$

$$h=\frac{k}{d}Nu \tag{12}$$

From Equation (13), the variation of *h* against θ could be found as follow:

$$h=\frac{k}{d}\left(0.6+\frac{0.387\left(\frac{g\beta d^3}{\nu\alpha}\right)^{\frac{1}{6}}}{\left(1+\left(\frac{0.559}{pr}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}}\theta(t)^{\frac{1}{6}}\right)^2 \tag{13}$$

Substituting Equation (13) and (9) in (8), we have:

$$b\left(\frac{d}{dt}\theta(t)\right)+b\,\beta\,\theta(t)\left(\frac{d}{dt}\theta(t)\right)+0.36\,e\,\theta(t)+1.2a_0\,e\,\theta(t)^{7/6} +e\,a_0^{\;2}\,\theta(t)^{4/3}=0 \tag{14}$$

where:

$$b=\rho V c_0 \tag{15}$$

$$e=\frac{A\,k}{d} \tag{16}$$

$$a_0=\frac{0.387\left(\frac{g\,\beta\,d^3}{\nu\,\alpha}\right)^{1/6}}{\left(1+\left(\frac{0.559}{\mathrm{Pr}}\right)^{9/16}\right)^{8/27}} \tag{17}$$

Solving this equation for a real condition with the air as a cooling flow so we reach to the following coefficients and equation:

$$b=1.5 \tag{18}$$

$$e=0.000208 \tag{19}$$

$$a_0=0.0192 \tag{20}$$

$$1.57\left(\frac{d}{dt}\theta(t)\right)+0.005181\theta(t)\left(\frac{d}{dt}\theta(t)\right)+0.000074888\theta(t) +4.79232\times10^{-6}+7.667712\times10^{-8}\,\theta(t)^{4/3}=0 \tag{21}$$

Case 2

Consider heat transfer in a lumped system, Equation (8), with constant *h* and variable *c*. The specific heat coefficient varies linearly with temperature as shown in Equation (9).

Substituting Equation (9) in Equation (8), we have:

$$b\left(\frac{d}{dt}\theta(t)\right)+b\,\beta\,\theta(t)\left(\frac{d}{dt}\theta(t)\right)+f\,\theta(t)=0 \tag{22}$$

where:

$$b=\rho V c_0 \tag{23}$$

$$f=hA \tag{24}$$

For a real condition with the air as a cooling flow, we reached the following coefficients:

$$b = 1.57 \tag{25}$$

$$f = 1.13097 \times 10^{-4} \tag{26}$$

$$\beta = 0.0033 \tag{27}$$

So we have Equation (35):

$$1.57\left(\frac{d}{dt}\theta(t)\right) + 5.181 \times 10^{-3}\,\theta(t)\left(\frac{d}{dt}\theta(t)\right) + 1.13097 \times 10^{-4}\,\theta(t) = 0 \tag{28}$$

Case 3

In this case both *h* and *c* are constant.

We solve this equation for a real condition with the air as a cooling flow. So, we reach the following coefficients and equation:

$$\rho Vc = 1.57 \tag{29}$$

$$hA = 1.13097 \times 10^{-4} \tag{30}$$

$$1.57\left(\frac{d}{dt}\theta(t)\right) + 1.13097 \times 10^{-4}\,\theta(t) = 0 \tag{31}$$

Solution using Homotopy Perturbation Method

Case 1

In this section, we will apply the HPM to nonlinear ordinary differential Equation (21). According to the HPM, we can construct a homotopy of Equation (21) as follows:

$$\begin{aligned} H(\theta,p) &= (1-p)\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 7.488 \times 10^{-5}\,\theta(t)\right) \\ &+p\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 0.005181\theta(t)\left(\frac{d}{dt}\theta(t)\right) + 0.000007488\theta(t)\right. \\ &\left.+4.79232 \times 10^{-6} + 7.667712 \times 10^{-8}\,\theta(t)^{4/3}\right) \end{aligned} \tag{32}$$

$$\theta(t) = \theta_0(t) + p \cdot \theta_1(t) + p^2 \cdot \theta_2(t) \tag{33}$$

Substituting Equation (33) into Equation (32) and collect $H(\theta, p)$ and then put the coefficients of p equal to zero, we have:

$$p^0 : \; 1.57\left(\frac{d}{dt}\theta_0(t)\right) + 7.488 \times 10^{-5}\,\theta_0(t) = 0 \qquad \theta_0(0) = 100 \tag{34}$$

$$\begin{aligned} p^1 : \; & 0.005181\,\theta_0(t)\left(\frac{d}{dt}\theta_0(t)\right) + 1.57\left(\frac{d}{dt}\theta_1(t)\right) + 7.667712 \times 10^{-8}\,\theta_0(t)^{4/3} \\ & +4.79232 \times 10^{-6}\,\theta_0(t)^{7/6} = 0 \quad \theta_1(0) = 0 \end{aligned} \tag{35}$$

$$\begin{aligned} p^2 : \; & 7.488 \times 10^{-5}\theta_2(t) + 5.181 \times 10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right) + 1.57\left(\frac{d}{dt}\theta_2(t)\right) \\ & +5.181 \times 10^{-3}\theta_1(t)\left(\frac{d}{dt}\theta_0(t)\right) + 7.667712 \times 10^{-6}\theta_1(t)^{4/3} = 0 \qquad \theta_2(0) = 0 \end{aligned} \tag{36}$$

Solving Equation (34-36) with initial conditions, we have:

$$\theta_0(t) = 100\; e^{-\frac{117}{2453125}t} \tag{37}$$

$$\begin{aligned} \theta_1(t) &= -\frac{33}{2}e^{-\frac{234}{2453125}t} + \frac{48}{625} \times 10^{\frac{2}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{4}{3}} + \frac{192}{35} \times 10^{\frac{1}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{7}{6}} \\ &+\frac{33}{2} - \frac{48}{625} \times 10^{\frac{1}{3}} - \frac{192}{35} \times 10^{\frac{1}{3}} \end{aligned} \tag{38}$$

$$\theta_2(t) = 0 \tag{39}$$

$$\theta(t) = \theta_0(t) + \theta_1(t) + \theta_2(t) \tag{40}$$

So:

$$\begin{aligned} \theta(t) &= -\frac{33}{2}e^{-\frac{234}{2453125}t} + \frac{48}{625} \times 10^{\frac{2}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{4}{3}} + \frac{192}{35} \times 10^{\frac{1}{3}}\left(e^{-\frac{117}{2453125}t}\right)^{\frac{7}{6}} \\ &+\frac{33}{2} - \frac{48}{625} \times 10^{\frac{1}{3}} - \frac{192}{35} \times 10^{\frac{1}{3}} + 100\; e^{-\frac{117}{2453125}t} \end{aligned} \tag{41}$$

Case 2

By applying the HPM to nonlinear ordinary differential Equation(28) According to the HPM, we can construct a homotopy of Equation (28) as follows:

Substituting Equation (43) into Equation (42) and collect $H(\theta, p)$ and then put the coefficients of p equal zero, we have:

$$\begin{aligned} H(\theta,p) &= (1-p)\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 11.309724 \times 10^{-4}\,\theta(t)\right) \\ &+p\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 5.181 \times 10^{-3}\theta(t)\left(\frac{d}{dt}\theta(t)\right) + 11.309724 \times 10^{-4}\,\theta(t)\right) \end{aligned} \tag{42}$$

$$\theta(t) = \theta_0(t) + p \cdot \theta_1(t) + p^2 \cdot \theta_2(t) \tag{43}$$

$$p^0 : \; 1.57\left(\frac{d}{dt}\theta_0(t)\right) + 11.309724 \times 10^{-4}\,\theta_0(t) = 0 \qquad \theta_0(0) = 100 \tag{44}$$

$$\begin{aligned} p^1 & : \\ & 1.57\left(\frac{d}{dt}\theta_1(t)\right) + 11.309724 \times 10^{-4}\,\theta_1(t) + 5.181 \times 10^{-3}\,\theta_0(t)\left(\frac{d}{dt}\theta_0(t)\right) = 0 \quad \theta_1(0) = 0 \end{aligned} \tag{45}$$

$$\begin{aligned} p^2 : \; & 1.57\left(\frac{d}{dt}\theta_2(t)\right) + 11.309724 \times 10^{-4}\,\theta_2(t) - 1.309724 \times 10^{-4}\,\theta_1(t) \\ & +5.181 \times 10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right) + 1.57\left(\frac{d}{dt}\theta_2(t)\right) \\ & +5.181 \times 10^{-3}\theta_0(t)\left(\frac{d}{dt}\theta_1(t)\right) + 5.181 \times 10^{-3}\theta_1(t)\left(\frac{d}{dt}\theta_0(t)\right) = 0 \qquad \theta_2(0) = 0 \end{aligned} \tag{46}$$

Solving Equation (44-46) with initial conditions, we have:

$$\theta_0(t) = 100\ e^{-\frac{2827431}{39250000000}t} \tag{47}$$

$$\theta_1(t) = \left(-\frac{93305223}{2827432}e^{-\frac{353429}{4906250000}t} + \frac{93305223}{2827432}\right)e^{-\frac{282743}{3925000000}t} \tag{48}$$

$$\begin{aligned}\theta_2(t) = &\left(-\frac{6158143629}{282743200}e^{-\frac{282743}{39250000000}t} + \frac{26381407997213}{7994371714624}e^{-\frac{353429}{4906250000}t}\right.\\ &+\frac{26381407997213}{110976706000000000}t + \frac{26117593917239187}{1598874060181600}e^{-\frac{5654863}{39250000000}t}\\ &\left.-\frac{6228400385156744314971}{2260353840863690825560}\right)e^{-\frac{282743}{3925000000}t}\end{aligned} \tag{49}$$

So:

$$\theta(t) = \theta_0(t) + \theta_1(t) + \theta_2(t) \tag{50}$$

$$\begin{aligned}\theta(t) = &100\ e^{-\frac{2827431}{39250000000}t} + \left(-\frac{93305223}{2827432}e^{-\frac{353429}{4906250000}t} + \frac{93305223}{2827432}\right)e^{-\frac{282743}{3925000000}t}\\ &+\left(-\frac{6158143629}{282743200}e^{-\frac{282743}{39250000000}t} + \frac{26381407997213}{7994371714624}e^{-\frac{353429}{4906250000}t}\right.\\ &+\frac{26381407997213}{110976706000000000}t + \frac{26117593917239187}{1598874060181600}e^{-\frac{5654863}{39250000000}t}\\ &\left.-\frac{6228400385156744314971}{2260353840863690825560}\right)e^{-\frac{282743}{3925000000}t}\end{aligned} \tag{51}$$

Case 3

$$\begin{aligned}H(\theta,p) = &(1-p)\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 11.309724\times10^{-4}\,\theta(t)\right)\\ &+p\left(1.57\left(\frac{d}{dt}\theta(t)\right) + 11.309724\times10^{-4}\,\theta(t)\right)\end{aligned} \tag{52}$$

$$\theta(t) = \theta_0(t) + p\cdot\theta_1(t) + p^2\cdot\theta_2(t) \tag{53}$$

Substituting Equation (53) into Equation (52) and collect $H(\theta,p)$ and then put the coefficients of p equal zero, we have:

$$p^0: 1.57\left(\frac{d}{dt}\theta_0(t)\right) + 7.488\times10^{-5}\,\theta_0(t) = 0 \qquad \theta_0(0) = 100 \tag{54}$$

$$p^1: 1.57\left(\frac{d}{dt}\theta_1(t)\right) + 7.488\times10^{-5}\,\theta_1(t) = 0 \qquad \theta_1(0) = 0 \tag{55}$$

$$p^2: 1.57\left(\frac{d}{dt}\theta_2(t)\right) + 7.488\times10^{-5}\,\theta_2(t) = 0 \qquad \theta_2(0) = 0 \tag{56}$$

Solving Equation (54-56) with initial conditions, we have:

$$\theta_0(t) = 100\ e^{-\frac{2827431}{39250000000}t} \tag{57}$$

$$\theta_1(t) = 0 \tag{58}$$

$$\theta_2(t) = 0 \tag{59}$$

So we have:

$$\theta(t) = \theta_0(t) + \theta_1(t) + \theta_2(t) \tag{60}$$

$$\theta(t) = 100\ e^{-\frac{2827431}{39250000000}t} \tag{61}$$

Results and discussion

In Figure 2 the temperature distribution is compared with numerical solution. The heat transfer coefficient and heat specific is taken variable. It can be seen that there is good agreement between them.

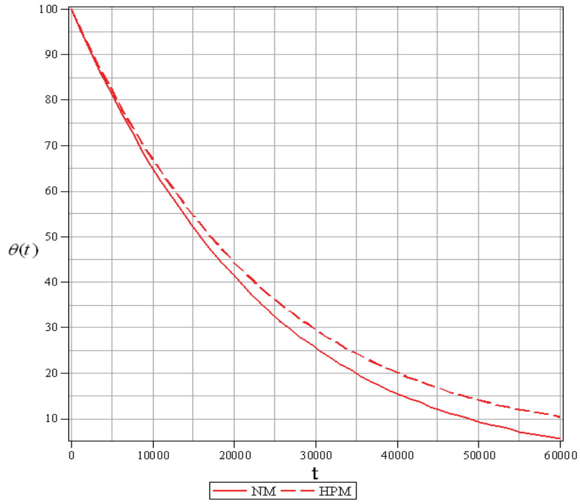

Figure 2. $\theta(t) - t$, $h \neq cte, c \neq cte$.

In Figure 3 the temperature gradient is depicted. It is increased with increasing time.

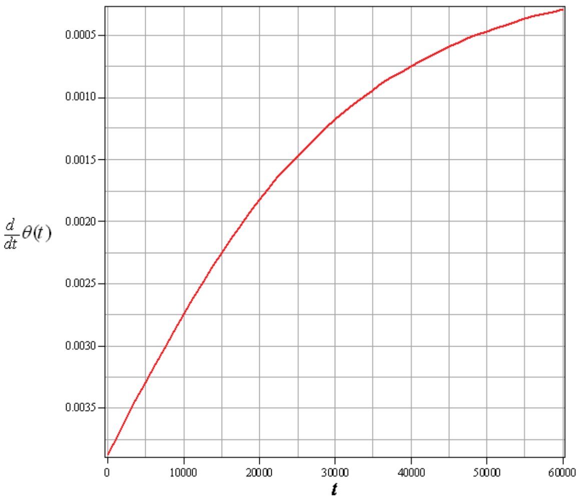

Figure 3. $\frac{d}{dt}\theta(t) - t$, $h \neq cte, c \neq cte$ ·

In Figure 4 the temperature distribution for case of constant heat transfer coefficient and variable specific heat is drawn.

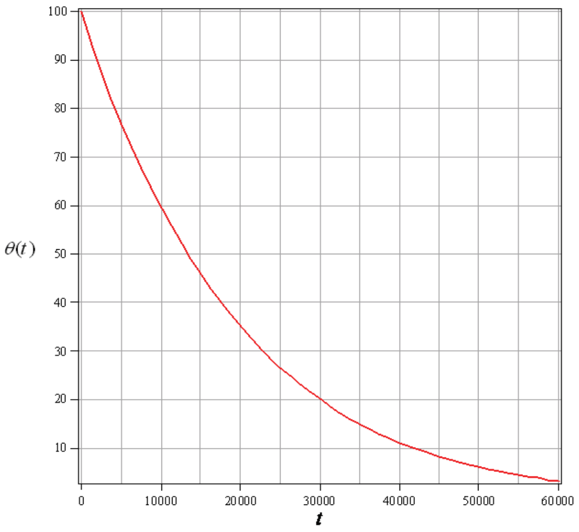

Figure 4. $\theta(t)-t, h=cte, c\neq cte$, solved by HPM.

For this case the temperature gradient is depicted also. It is depicted in Figure 5.

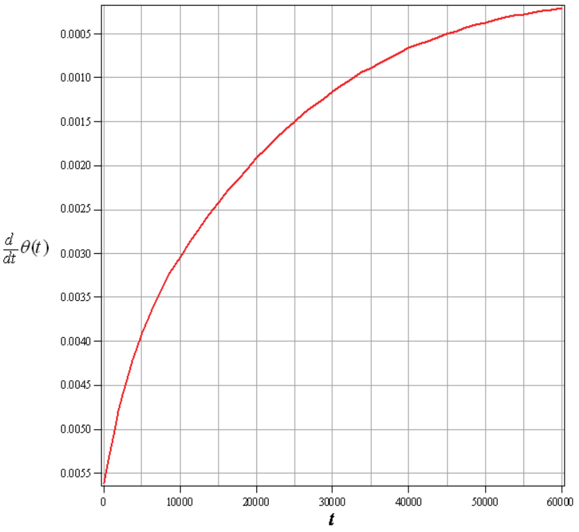

Figure 5. $\frac{d}{dt}\theta(t)-t, h=cte, c\neq cte$, solved by HPM.

In Figure 6 the temperature distribution for the case of variable heat transfer coefficient and constant specific heat with both Numerical method and HPM is depicted. With growth of time the temperature of cylinder closes to ambient temperature.

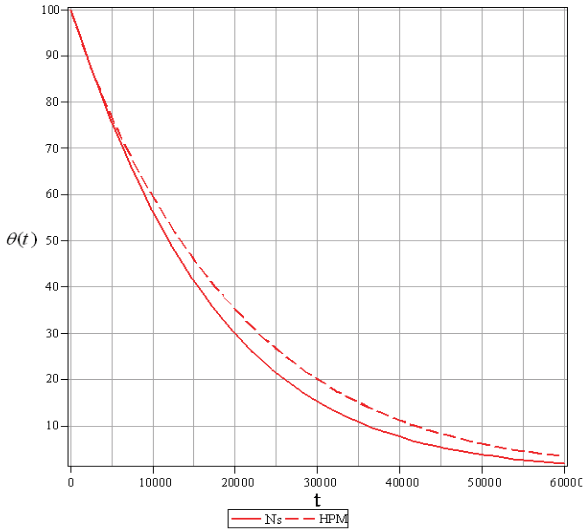

Figure 6. $\theta(t)-t, h\neq cte, c=cte$.

For this case the temperature gradient is depicted also in Figure 7.

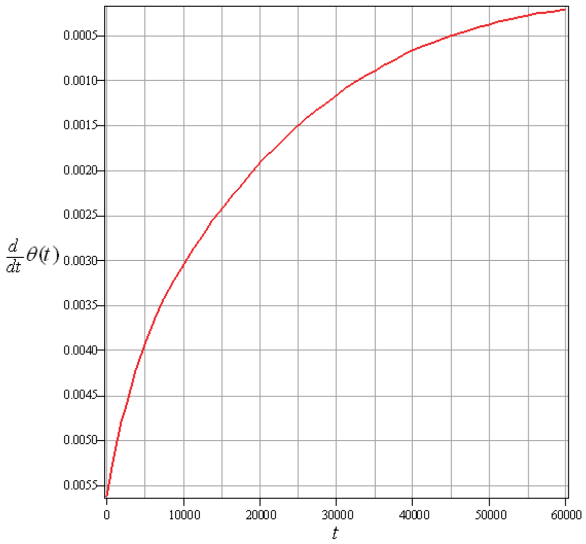

Figure7. $\frac{d}{dt}\theta(t)-t, h\neq cte, c=cte$ ·

In Figures 8 and 9 the result of constant properties and variable properties is compared .it can be seen in the case of variable properties cylinder reach the ambient temperature at a shorter time and temperature gradient tend to zero faster.

In Figures 10 and 11 the result of variable properties and case of constant heat transfer coefficient and variable specific heat are compared.

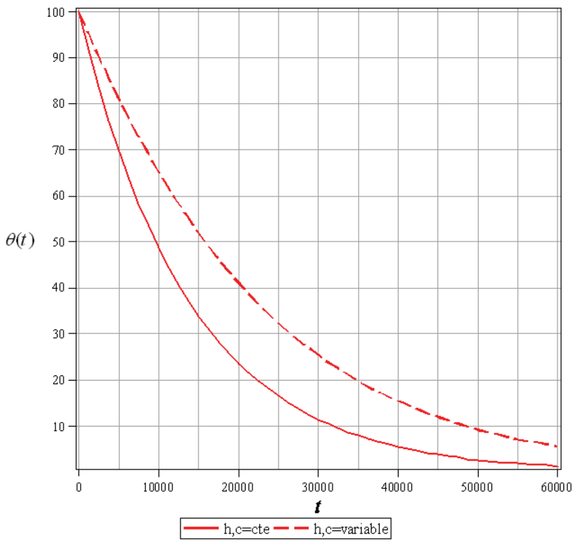

Figure 8. Compare $\theta(t)-t, h,c \neq cte\, and\, h,c = cte$.

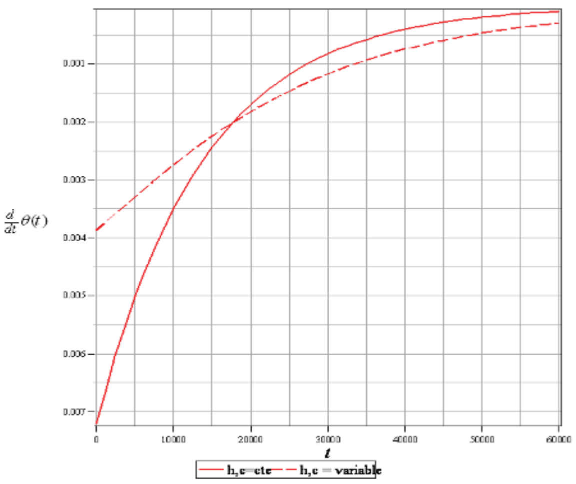

Figure 9. Compare $\frac{d}{dt}\theta(t)-t\ , h,c \neq cte\, and\, h,c = cte$.

It can be seen in the case of variable properties cylinder reach the ambient temperature at a shorter time and temperature gradient tend to zero faster.

In Figure 12 average Nusselt number in each time is shown. At the beginning time because of high gradient temperature Nusselt number is great. With growth of time Nusselt number tend to zero, because of zero temperature gradient.

For case of constant heat transfer properties the Nusselt number is zero because it related to heat transfer coefficient and thermal conductivity and they are constant Figre 13.

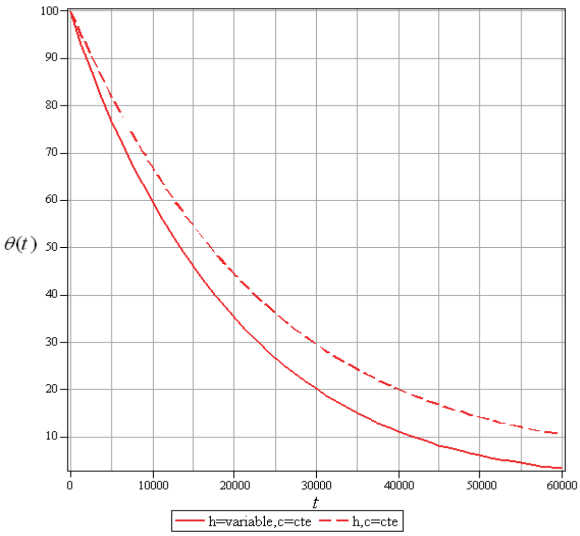

Figure 10. Compare $\theta(t)-t\ \ h,c \neq cte\, and\, h = cte, c \neq cte$.

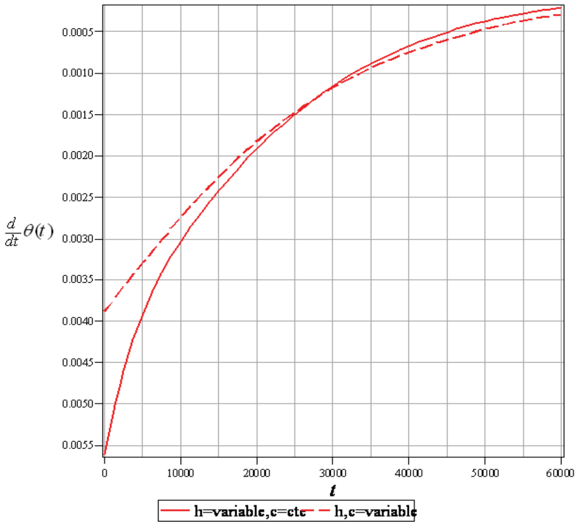

Figure 11. Compare $\frac{d}{dt}\theta(t)-t\ \ h,c \neq cte\, and\, h = cte, c \neq cte$.

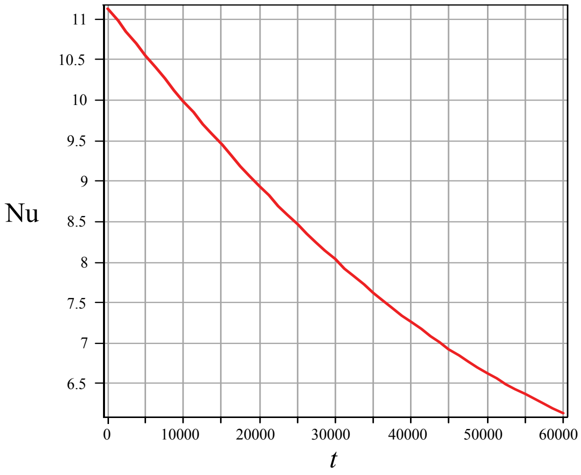

Figure 12. Nu , $h, c \neq cte$.

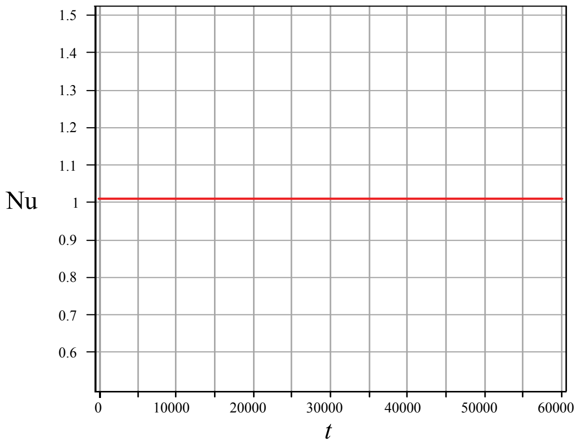

Figure 13. Nu, $h, c = cte, h \neq cte, c \neq cte$.

Nomenclature

A	Surface
c	Specific heat coefficient
d	Diameter
H	Heat transfer coefficient
HPM	Homotopy Perturbation Method
K	Thermal conductivity
NM	Numerical Runge-Kutta Method
Nu	Nusselt
P	Small parameter
Pr	Prandtl number
Ra_d	Rayleigh number
T	Temperature
T_∞	Ambient temperature
Greek symbol	
α	Thermal diffusivity
β	Thermal Expansion coefficient
ξ	Constant parameter
Θ	temperature difference
v	Kinematic Viscosity
ρ	Density

Conclusion

In the present work, natural convection flow over a hot isothermal horizontal cylinder has been analyzed. The influence of variable h and c is studied, and the nonlinear equation that is extracted by He's Homotopy Perturbation Method (HPM) is solved. These considered equations are easily solved by mentioned analytical method. Consequently, these equations are solved by the numerical method (Runge-Kutta fourth-order) using the software Maple 12® and the results of the HPM and NM are compared in Figures 2, 6, 10, 11, 12 and 13. Then effects of h and c when they are variable or constant are shown in Figures 3, 4, 5, 7, 8 and 8. Also the variations of Nusselt number are shown in Figure 14 and 15. So the following results are obtained:

(I) The effect of c is stronger than h when temperature is decreasing.

(II) With increasing *h* and decreasing *c*, the time of cooling approach will be decreased.

(III) The natural convection is not appropriate in industry that time is an important parameter.

(IV) Obtained results from case 1, 2 and 3 are approximately similar.

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