Transactions on Ecology and the Environment vol 3, © 1994 WIT Press, www.witpress.com, ISSN 1743-3541

Analytical and numerical model for a nozzle particle collector

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Abstract

The objective of this paper is to present an analytical and numerical model for the fluid streamlines and particle trajectories inside a Particle Nozzle Collector. This work is part of a main research project that is being developed by the Technology Center of the Federal University of Alagoas (Brazil). The final objective of this research is the design and the construction of a solid particle Nozzle Collector. To this end, it is necessary to simulate mathematically the pathlines of those particles collected from the atmosphere, so that one can relate the collector's efficiency taking into account the various physical and geometrical parameters, such as diameter, velocity, inertia, concentration, etc. This first part of the research deals with streamlines of the fluid flow, where a methodology was developed using the complex variable analysis and numerical methods to obtain the particle trajectories, for a non-transient and two dimensional flow. Results are obtained taking the inertial parameter as the main cause for the particle collection.

1 Introduction

Dust and smoke are removed in many types of air-cleaning equipment by means of impaction of the small particles on collectors. For particles in the micron and submicron ranges of sizes, inertial, interception, electrostatic attraction, and settling are the primary mechanisms by which collection takes place on surface in a aerosol stream. In the case of inertia, a solid particle carried along by the air stream on approaching an obstruction tends to follow the stream but may strike the obstruction because of its inertia. This work has as final aim the design and construction of a divergent nozzle collector of solid particle in the atmosphere, phenomenon very important specially in the field of atmospheric pollution. The remotion of this aerosols, contaminated or not, has received special attention lately due to its average particle diameter between 0.01 and 10 μm, aerosols separators have been shown not so efficient when deal with particles in this size of range. To this end, an investigation of the mechanism of such particles was undertaking, using a method involving dimensionless parameters so that we can determine the efficiency of impaction for the collector. Impingement of aerosol jets of various forms has been used for many years as means of sampling aerosols and determining particle size. According to Ranz & Wong [8], May [12] was the first to report quantitative experimental information on the performance of impactors and made a dimensional analysis of the variables involved in their operation. As reported by Ranz & Wong [8], in general the limitation of the impinger for collecting aerosol particles in the submicron range of size has not been determined, and an adequate theoretical analysis of the size separation has not been developed.

Also, in their paper, Ranz & Wong mentioned the work done on the impaction of aerosol particles on body collectors, such as cylinders and spheres, by Albrecht [1], Landhal & Hermann [4], Langmuir & Blodgett [5] and Sell [10], were primarily of theoretical nature and considered inertia as the only mechanism of collection.

Ranz & Wong [8] developed a mathematical statement of the problem of impaction, at the same time realistic and rigorous and carries out limiting solutions and order of magnitude calculations which establish the nature and importance of the various mechanisms of impaction. Peters, Fan & Swedney [7] used a more complete model by incorporating other parameters besides inertia.

In the first part of the work, we try to visualize the fluid flow streamlines in the collector, by using stream function of an incompressible, non-transient and irrotational fluid flow, and some of the solid particle trajectories.

2 Methodology

The process for the Nozzle collector was based on the spatial separation of the components of a gaseous mixture in a expanding supersonic jet stream. The separating effect of the jet may be explained qualitatively by the fact the streamlines of an expanding jet are curved and the mixture is subject to the action of a centrifugal acceleration. The nozzle was first used in the separation of uranium isotopes for enrichment, based on the difference between the velocities with which $^{235}UF_6$ and $^{238}UF_6$ molecules migrate through porous membranes [2].

Actually, as show in figure 1, the collector to be constructed has the shape of cylinders, non-concentric and symmetrically curved in relation to an axis. The minor cylinder has to have a smaller radius (r_o) than the radius (R_o) of the bigger cylinder to allow the entrance of the aerosols

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jet stream. The distance between the centers, b, the radius ro and R_o as well as the others parameters that will affect the efficiency of the nozzle collector, will be determinate in a next step of this research.

To develop a mathematical model related to the aerosols collection in a atmospheric suspension, it is necessary to know the nature and importance of the several parameters affecting the impact mechanism. As a initial step, we will try to visualize the fluid particles streamlines (through the stream function, ψ) inside the collector, using the analytical technique for solution of the differential equations and complex variables.

2.1 The stream function

The stream function of a fluid flow is an exact solution to the continuity equation when only two independent spatial variables are involved in the flow. By assuming an irrotational, two-dimensional, incompressible and non-transient flow, and defining the stream function, ψ, as

$$V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \tag{1}$$

$$V_\theta = -\frac{\partial \psi}{\partial \mathrm{r}} \tag{2}$$

the continuity equation becomes, see White [11],

$$\frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{r}\frac{\partial \psi}{\partial \mathrm{r}}\right) + \frac{1}{\mathrm{r}^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0 \tag{3}$$

For the solution of eqn [3] was used a special linear fractional transformation [3], in the complex plane. So one obtain two concentric circles from two other non-concentric (nozzle collector geometry) as shown in figures 2 and 3, respectively. Once obtained the cylindrical symmetry for the collector, one can conclude that the stream function does not depend on θ, and then ψ can be written as

$$\psi(\mathrm{r}) = A \, \ln(\mathrm{r}) + B \tag{4}$$

where r is a non-dimensional parameter ($\mathrm{r} = r/R_c$), A and B are arbitrary constants which are found by the arbitrary boundary conditions, that is,

$$\psi(\mathrm{r} = 1) = 0 \tag{5}$$

$$\psi(\mathrm{r} = \mathrm{r}_o / \mathrm{R}_c) = \psi_o \tag{6}$$

The transformation used in this work allowed that the stream function inside the collector be found. Mathematically, this transformation maps the domain *D* related to the region inside of the collector, onto the annulus *D'*, as shown in figures 2 and 3.

By rotation and change of scale we can assume that the outer circle in figure 2 is $|Z| = 1$ and the inner circle *C* has its center at some point $x_o > 0$ [6].

The radius ρ of *C* satisfies $\rho + x_o < 1$. We shall map $|Z| < 1$ onto $|W| < 1$ in such way that image *C** of *C* is a circle centered at the origin. Thus *D* will be mapped onto the annulus *D'* by equation below

$$W = \frac{Z - \alpha}{\alpha Z - 1}; \qquad -1 < \alpha < 1 \tag{7}$$

and it remains only to choose α. Since α is real, conjugate points are mapped into conjugate points, and *C** is bisected by the real axis in the W plane. This shows that the center *C* does not map into the center of *C**. However, the diameter

$$x_o - \rho \leq x \leq x_o + \rho \tag{8}$$

does map into a diameter $u_1 \leq u \leq u_2$ of *C** and hence the center W_o of *C** satisfies $2W_o = u_1 + u_2$. That is,

$$2\, W_o = \frac{(x-\rho)-\alpha}{\alpha\,(x-\rho)-1} + \frac{(x+\rho-\alpha)}{\alpha\,(x+\rho)-1} \tag{9}$$

setting $W_o = 0$ leads to the quadratic equation

$$\alpha^2 x_o - \alpha\,(1 + x_o^2 - \rho^2) + x_o = 0 \tag{10}$$

which has distinct real roots if $1 + x_o^2 - \rho^2 > 2\, x_o$ This holds because $1 - x_o > \rho$. The product of roots is 1, so one satisfies $|\alpha| < 1$. Using the roots in eqn [7] gives the desired transformation.

2.2 The fluid velocity

By using the coordinates system shown in figures 2 and 3, we can rewrite eqn [4] as

$$\psi(r) = \psi(\xi, \eta) = \frac{\psi_o}{\ln r_o} \sqrt{\ln(\xi^2 + \eta^2)} \tag{11}$$

These coordinates (ξ,η) are obtained through the transformation given by eqn [7] :

$$x = \rho \cos \beta \tag{12}$$

$$y = \rho \text{ sen } \beta \tag{13}$$

as well,

$$\xi = r \cos \theta \tag{14}$$
$$\eta = r \text{ sen } \beta \tag{15}$$

From eqns [1] and [2] we can get

$$V_{\rho} = \frac{\psi_o}{\rho\left(\xi^2 + \eta^2\right) \ln r_o} \left[\xi \frac{\partial \xi}{\partial \beta} + \eta \frac{\partial \eta}{\partial \beta} \right] \tag{16}$$

$$V_{\beta} = \frac{\psi_o}{\left(\xi^2 + \eta^2\right) \ln r_o} \left[\xi \frac{\partial \xi}{\partial \rho} + \eta \frac{\partial \eta}{\partial \rho} \right] \tag{17}$$

where

$\frac{\partial \xi}{\partial \beta}$, $\frac{\partial \eta}{\partial \beta}$, $\frac{\partial \xi}{\partial \rho}$ and $\frac{\partial \eta}{\partial \rho}$ can be founded from the results of eqn [7].

2.3 The solid particle trajectory

A solid small particle, carried along with his stream, whose motion will not be identical with the fluid because it is subject to several forces which tend to impact it on the surface of the collector [8], can have his movement equation given by,

$$-\frac{(\vec{u} - \vec{v})}{Z_d} = \frac{d(m\vec{u})}{dt} \tag{18}$$

where $\vec{u}$, $\vec{v}$ are the vectors velocities of the solid particle and the fluid respectively, Z_d is the solid particle mobility which is function only of the particle diameter and the physical properties of the air, and m is the solid particle mass. The term on the left-hand side represents the fluid force resistance opposing the relative movement of the particle through the fluid, and the term on the right-hand side represents the force required to accelerate the particle.

After some vectorial and algebraic manipulations and using the cartesian coordinate system, we can get the pair of ordinary differential equation for the solid particle trajectories,

$$mZ_d \frac{d^2 X}{dt^2} + \frac{dX}{dt} - V_x(x,y) = 0 \tag{19}$$

$$mZ_d \frac{d^2 Y}{dt^2} + \frac{dY}{dt} - V_y(x,y) = 0 \tag{20}$$

where *X, Y* are the solid particle coordinates, and V_x , V_y the fluid flow velocities in cartesian coordinate.

3 Results and conclusions

In order to visualize the fluid trajectories, a computer program was elaborated taking as input the minor non-dimensional radius ρ_0 and the non-dimensional coordinate x_0, origin of the minor circle. Figure 4 shows a graphic output where ten streamlines and a solid particle trajectory are shown. The results obtained in this first part of work, will be of crucial importance on choosing the best collector geometry. In a next step of this work, from the fluid velocities, we will simulate the path of some solid particles inside the collector in the fluid suspension, and we will analyze the behavior of both to relate the various parameters that affect the remotion of such particles. Thus, we can define the efficiency for the nozzle collector.

References

[1] Albrecht, F., *Physik. Z.*,Vol.32, pp 48, (1931).
[2] Becker, E.W., Bier, K., Bier, W., Schutte, R., & Seidel, D., Separation Nozzle Process, *Angew. Chem. International*, Vol.6, pp 507, (1967).
[3] Churchill, R.V., Brown ,J.W. & Verhey, R.F., *Complex Variables and Applications*, ed. Mc Graw Hill, (1967).
[4] Landhal, H.D., & Herrmann, R.G., *J. Colliod Sci.*, Vol.4, pp 103, (1949).
[5] Langmuir, I. & Beodgett, K.B., *General Eletric Research Laboratory*, Schenectady NY. - Rept. RL - 225, (1944-45).
[6] Levinson, N. & Redheffer, R.M., *Complex Variables*, ed. Holden-Day, (1970).
[7] Peters, M.H., Liang-Shin Fan & Swedney, T.L., Simulation of Particulate Removal in Gas-Solid Fluid zed Beds, *AIChE Journal*, Vol.28, pp 39, (1982).
[8] Ranz, W.E. & Wong, J.B., Impaction of Dust and Smoke Particles, *Industrial and Engineering Chemistry*, Vol.44, pp 1371, (1952).
[9] Sabersky, R.H., Acosta & A.J., Hauptmamnn, E.G., *Fluid Flow · A First Course in Fluid Mechanics*, ed. Mcmillan, (1971).
[10] Sell, W., *Forsch. Gebiete Ingenieurw.*, Vol.2, pp 347, (1931).
[11] White, F.M., *Viscous Fluid Flow*, ed. Mc Graw Hill, (1974).
[12] May, K.R. *Journal of Science Instruments*, Vol.22, pp 187, (1945).

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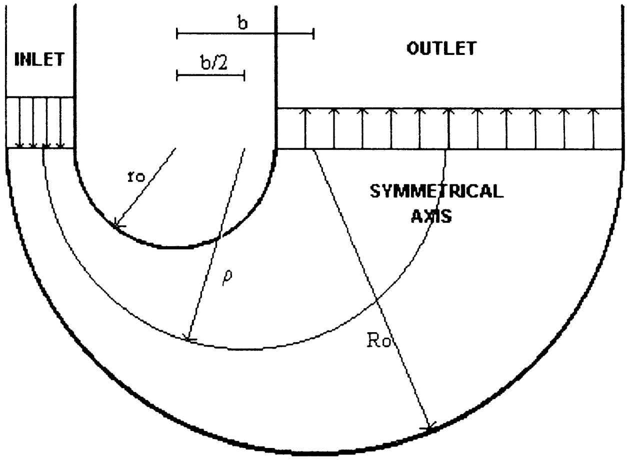

Figure 1: Nozzle Collector Geometry.

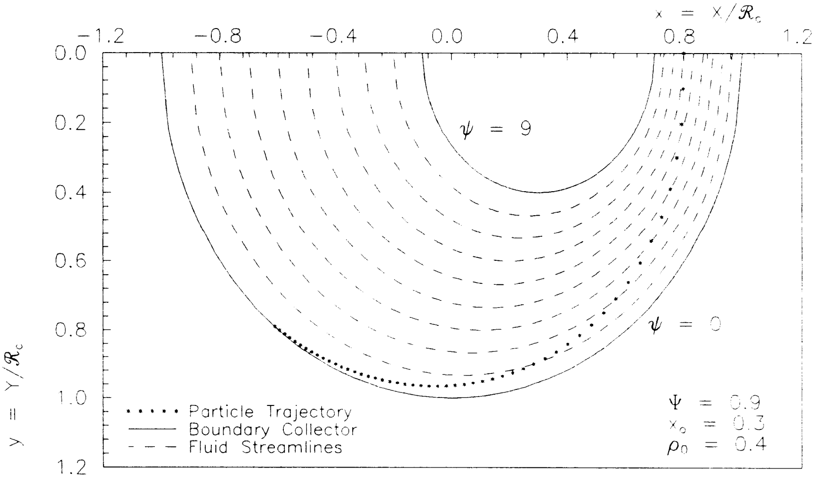

Figure 4: Graphic Output with ten streamlines and a solid particle trajectory.

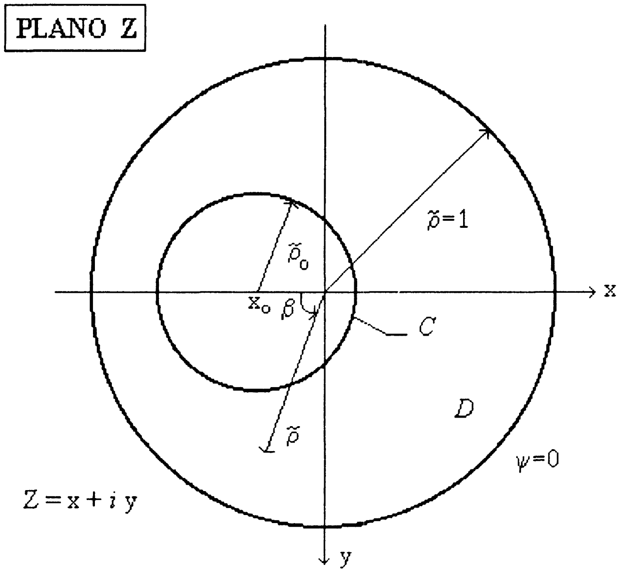

Figure 2: Collector Geometry to be Transformed.

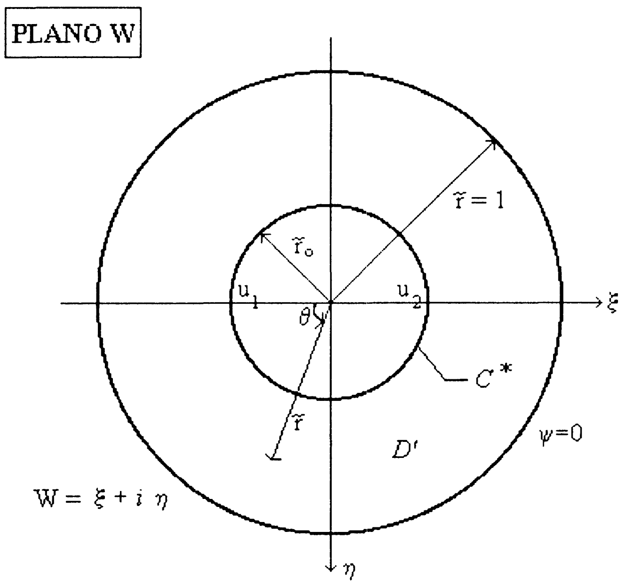

Figure 3: Cylindrical Collector new Geometry.