Analytical and Numerical Modelling of Laminated Composites with Piezoelectric Elements

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ABSTRACT: We propose an accurate and efficient approach to laminated piezoelectric plates based on a refinement of elastic displacement and electric potential through the plate thickness. More precisely, the model accounts for a shearing function and a layerwise approximation for the electric potential. The layerwise approach becomes a necessity in order to accommodate electric potential at the electrode interfaces. The equations of motion for the piezoelectric composite are deduced from a variational formulation incorporating the continuity conditions at the layer interfaces by using Lagrange multipliers. Different situations are investigated among them (i) bimorph and (ii) sandwich structures for two kinds of electromechanical loads applied (density of force and electric potential) and are compared to the finite element computations performed on the 3D model. The vibration problem is also presented and the frequencies for the axial and flexural modes are obtained. At last performance and effectiveness of the model are also discussed and applications to control of the structure shape and vibration are proposed.

Key Words: laminated plates, piezoelectric composites, vibrations, piezoelectric actuators

INTRODUCTION

OWING to the successful applications of piezoelectric actuators and sensors, piezoelectric components made of laminated plates have received a particular attention in recent years. The main idea is that certain kinds of structure enable to adapt to or correct for changing operating conditions (geometry, behaviour, etc.) according to electromechanical loads. The high performance of piezoelectric composites are used for multipurpose devices or smart materials and advanced technological applications have been suggested, running from aerospace structures (shape control of large space antennas, control of vibrations) to miniature devices due to the possibility of their integration on active electronic circuits (medical apparatus, micropump, microrobots, etc.) (Tani et al., 1998).

The main objective of the present work attempts to present a consistent and efficient approach to piezoelectric composites made of laminated plate with active piezoelectric layers. Although a number of consistent and accurate approaches to piezoelectric laminated plates have been proposed (Saravanos and Heyliger, 1999), most of these models are able to accurately predict the global responses of laminated plates, especially the deflection or the induced electric charges, but they cannot provide excellent estimates of the local responses such as the through-the-thickness variations of the electromechanical quantities and the frequencies of the higher modes of vibration. Here, we propose an alternative approach which combines an equivalent single-layer representation for the mechanical displacement with a layerwise-type approximation for the electric potential. This approach becomes an interesting feature because multilayered piezoelectric structures are appropriate to accommodate multiple voltage actuator inputs and sensor outputs. Various models of piezoelectric plates have been described in the literature and they are mainly classified according to the kinematic hypothesis for approximating the through-the-thickness variations of electromechanical quantities (displacement, stresses, electric potential). The simplest approach to piezoelectric beam and plate, the so-called induced strain model accounts for effective forces and moments induced by piezoelectric actuation on beam deformation. Such models have been investigated by Wang and Rogers (1991), Pai et al. (1993) and Robbins and Reddy (1993). The equivalent single-layer approach assumes that the piezoelectric composite deforms as a single homogeneous layer, however, in the model, the Gauss law is used for the sensor function of the plate. This kind

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of approach has been reported by Lee and Moon (1989). Nevertheless, the equivalent single-layer theory becomes inadequate in some utmost situations, especially for the plate with strong variations of elastic, piezoelectric and dielectric properties. Layerwise approaches are therefore considered for which their kinematic and electric potential variations through the plate thickness is smooth enough within each layer and the continuity conditions at the layer interfaces must be fulfilled. Such a layerwise modelling has been reported in the works of Heyliger and Savaranos (1995), Carrera (1997) and Yang (1999). Some of the layerwise approaches are also dedicated to the finite element formulation (see Kim et al. 1997; Saravanos et al., 1997).

Various situations are considered such as bimorph and sandwich structures undergoing two different electromechanical loads (i) surface density of force applied to the upper face of the plate and (ii) electric potential applied to the top and bottom faces of the plate and to the layer interfaces. In order to show the quality of predictions of the present approach, the results are compared to finite element computations performed on the 3D problem. In particular, the global structural response (deflection, elongation, electric potential or charges), as well as the local response or the variation of the electromechanical states through the plate thickness are obtained and discussed.

FORMULATION FOR PIEZOELECTRIC MATERIALS

The different approaches are based on the classical piezoelectricity of which the equations are deduced from a variational formulation (Fernandes and Pouget, 2002) stated as follows

$$\delta \int_{t_1}^{t_2} \int_{\Omega} \mathcal{L}\, dv\, dt + \int_{t_1}^{t_2} \delta W\, dt = 0, \tag{1}$$

where $\mathcal{L} = K - H(S_{ij}, E_i)$ is the density of the Lagrangian functional with $K = \frac{1}{2}\rho \dot{u}_i \dot{u}_i$ the kinetic energy and $H = \frac{1}{2}\sigma_{ij}S_{ij} - \frac{1}{2}D_i E_i$ the electric enthalpy density function. In the formulation, ρ is the mass density, u_i is the elastic displacement (the dot denotes the time derivative), $S_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ is the linear part of the strain tensor, E_i is the electric field vector, σ_{ij} are the components of the stress tensor and D_i represents the electric displacement or induction vector. The last integral in Equation (1) is the variation of works of the prescribed mechanical and electric loads on the domain boundary $\partial\Omega$ and they are given by

$$\delta W = \int_{\partial\Omega} (T_i \delta u_i + Q \delta \Phi)\, ds. \tag{2}$$

In Equation (2), T is the surface traction and Q is the surface density of electric charge per unit of area applied to the domain boundary. The scalar variable Φ is the electric potential such as $E_i = -\Phi_{,i}$ (quasi electrostatic approximation). The equations of motion are deduced from the variational formulation Equation (1) and are given by

$$\sigma_{ij,j} = \rho \ddot{u}_i, \quad D_{i,i} = 0, \tag{3}$$

along with the boundary conditions for the applied electromechanical loads on the boundary domain, namely $\sigma_{ij}n_j = T_i$ on $\partial\Omega_\sigma$, $u_i = \bar{u}_i$ on $\partial\Omega_u$ and $D_i n_i = Q$ on $\partial\Omega_D$, $\Phi = \bar{\Phi}$ on $\partial\Omega_\Phi$ ($\partial\Omega = \partial\Omega_\sigma \cup \partial\Omega_u$ and $\partial\Omega = \partial\Omega_D \cup \partial\Omega_\Phi$ with $\partial\Omega_\sigma \cap \partial\Omega_u = \partial\Omega_D \cap \partial\Omega_\Phi = \emptyset$). The equations of motion are completed by constitutive equations for piezoelectric materials and they take the following matrix form

$$\begin{bmatrix} \sigma \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^E & -\mathbf{e}^T \\ \mathbf{e} & \varepsilon^S \end{bmatrix} \begin{bmatrix} \mathbf{S} \\ \mathbf{E} \end{bmatrix}, \tag{4}$$

where $\mathbf{C}^E$, $\mathbf{e}$ and ε^S are the fourth-order tensor of elasticity coefficients measured at a constant electric field, third-order tensor of piezoelectric coefficients and second-order tensor of the dielectric permittivity measured at a constant strain, respectively (Ikeda, 1996).

APPROXIMATION OF THE ELECTROMECHANICAL FIELDS

The geometrical setting of the laminated plate are depicted in Figure 1. We consider an expansion of the displacements and electric potential as a series of the thickness coordinate and they are of the form (Fernandes and Pouget, 2002)

$$\begin{cases} u_\alpha = U_\alpha - z w_{,\alpha} + f(z)\gamma_\alpha, & \alpha \in \{1,2\}, \\ u_3 = w, & \\ \phi^{(\ell)} = \phi_0^{(\ell)} + z_\ell \phi_1^{(\ell)} + P_\ell(z_\ell)\phi_2^{(\ell)} + g(z)\phi_3^{(\ell)}. & \end{cases} \tag{5}$$

With $\ell \in \{1, \cdots, N\}$ is the layer number. In Equation (5) z_ℓ is the thickness coordinate with respect to the mid-plane of the ℓth layer (see Figure 1). In Equation (5), U_α is the middle plane displacement component, w is the deflection and γ_α represents the shearing function (Touratier, 1991). All the functions are defined at the mid-plane coordinate $(x, y, 0)$. The functions of the thickness coordinate are choosen as follows (Fernandes and Pouget, 2001, 2002)

$$P_\ell(z_\ell) = z_\ell^2 - \left(\frac{h_\ell}{2}\right)^2, \quad f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right),$$
$$g(z) = \frac{h}{\pi}\cos\left(\frac{\pi z}{h}\right), \tag{6}$$

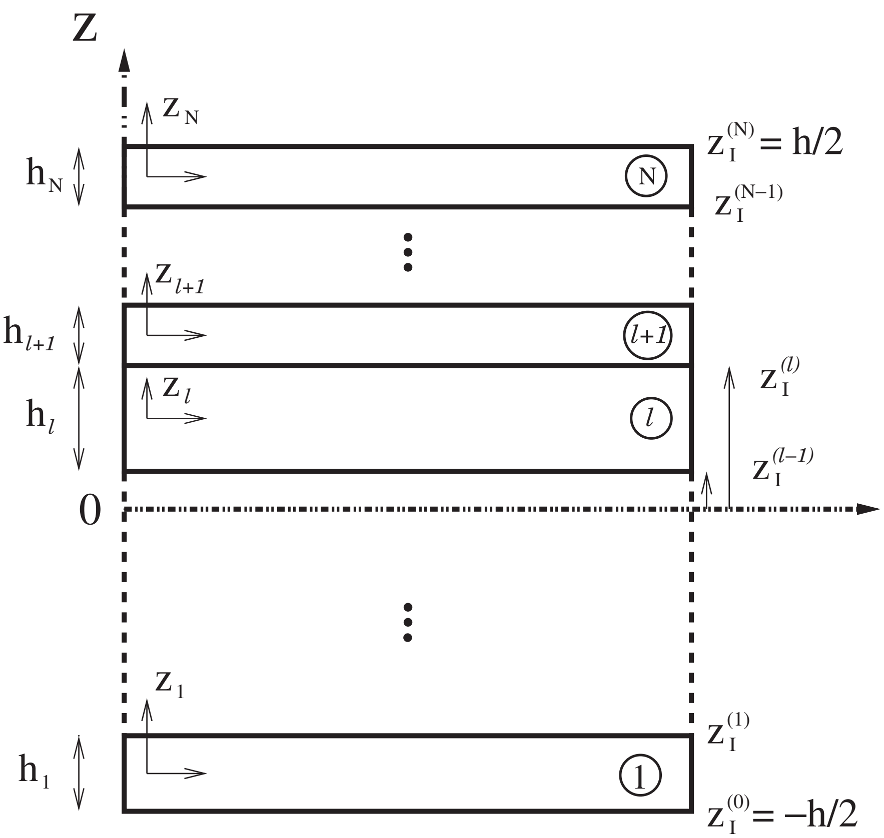

Figure 1. *Piezoelectric multilayered plate.*

where h is the plate thickness and h_ℓ is the thickness of the ℓth layer and $z_\ell \in [-h_\ell/2, h_\ell/2]$.

However, the continuity of the electric potential as well as the normal component of the electric induction must be satisfied at $z = z_I^{(\ell)}$ ($\ell \in \{1, \ldots, N-1\}$). The conditions read as

$$\begin{cases} \mathcal{A}_\ell = \phi^{(\ell+1)}(x,y,-h_{\ell+1}/2) - \phi^{(\ell)}(x,y,+h_\ell/2) = 0\,, \\ \mathcal{B}_\ell = D_3^{(\ell+1)}(x,y,-h_{\ell+1}/2) - D_3^{(\ell)}(x,y,+h_\ell/2) = 0\,, \end{cases} \tag{7}$$

where the normal component of the electric induction of the ℓth layer is calculated from the constitutive equation Equation (4). The condition Equation $(7)_2$ must be satisfied if no electric potential is applied at $z = z_I^{(\ell)}$, otherwise the condition of continuity Equation $(7)_2$ is not considered and it is replaced by the jump condition $[[D_3]]_{z=z_I^{(\ell)}} = Q_\ell$ where Q_ℓ is the output surface density of electric charge at the interface $z = z_I^{(\ell)}$. In order to account for the additional conditions Equation (7), Lagrange multipliers are introduced (Fernandes and Pouget, 2002).

EQUATIONS OF THE PIEZOELECTRIC PLATES

The variational principle stated in "Formulation for Piezoelectric Materials" is considered by using the approximation Equation (5) and taking the conditions in Equation (7) into account. The variational formulation for the piezoelectric laminated plate can be recast into the following form

$$\int_{t_1}^{t_2} (\delta K - \delta U + \delta W_1 + \delta W_2 + \delta \Lambda)\, dt = 0. \tag{8}$$

In Equation (8), the first term is the variation of the kinetic energy not given here (see Fernandes and Pouget (2002) for details). The variation of the works of internal forces takes on the form

$$\delta U = \int_\Sigma \Bigg\{ N_{\alpha\beta}(\delta U_\alpha)_{,\beta} - M_{\alpha\beta}(\delta w)_{,\alpha\beta} + \hat{M}_{\alpha\beta}(\delta\gamma_\alpha)_{,\beta} + \hat{Q}_\alpha \delta\gamma_\alpha + \sum_{\ell=1}^{N} \left[\sum_{m=0}^{3} D_\alpha^{(m)(\ell)} \delta\phi_{m,\alpha}^{(\ell)} + \sum_{k=1}^{3} D_3^{(k)(\ell)} \delta\phi_k^{(\ell)} \right] \Bigg\}\, da. \tag{9}$$

where we have introduced the following stress and electric charge resultants

$$\left(N_{\alpha\beta}, M_{\alpha\beta}, \hat{M}_{\alpha\beta}\right) = \sum_{\ell=1}^{N} \int_{z_I^{(\ell-1)}}^{z_I^{(\ell)}} (1, z, f(z)) \sigma_{\alpha\beta}^{(\ell)}\, dz\ , \tag{10}$$

$$\hat{Q}_\alpha = \sum_{\ell=1}^{N} \int_{z_I^{(\ell-1)}}^{z_I^{(\ell)}} f'(z)\sigma_{\alpha 3}^{(\ell)}\, dz, \tag{11}$$

$$\begin{aligned}&\left(D_\alpha^{(0)(\ell)}, D_\alpha^{(1)(\ell)}, D_\alpha^{(2)(\ell)}, D_\alpha^{(3)(\ell)}\right)\\ &\quad = \int_{z_I^{(\ell-1)}}^{z_I^{(\ell)}} (1, z_\ell, P_\ell(z_\ell), g(z))D_\alpha^{(\ell)}\, dz\ ,\end{aligned} \tag{12}$$

$$\left(D_3^{(1)(\ell)}, D_3^{(2)(\ell)}, D_3^{(3)(\ell)}\right) = \int_{z_I^{(\ell-1)}}^{z_I^{(\ell)}} \left(1, P'_\ell(z_\ell), g'(z)\right)D_3^{(\ell)}\, dz, \tag{13}$$

where the superscript 'prime' denotes the derivative with respect to z. The work of electromechanical loads applied to the plate boundary can be split into the sum of works of loads applied to the top and bottom faces of the plate and those applied along the plate contour; we write

$$\delta W_1 = \int_\Sigma \left\{ f_\alpha \delta U_\alpha - p\delta w + \hat{m}_\alpha \delta\gamma_\alpha + \sum_{\ell=1}^{N}\sum_{m=0}^{3} q_m^{(\ell)} \delta\phi_m^{(\ell)} \right\} dS\ , \tag{14}$$

$$\begin{aligned}\delta W_2 = &\int_{\mathcal{C}} \left\{ F_\alpha \delta U_\alpha + T\delta w + C_\alpha \delta\gamma_\alpha - M_f (\delta w)_{,n} \right\} ds\\ &- \sum_p Z_p \delta w_p.\end{aligned} \tag{15}$$

In Equation (15), f_α and p are the densities of force per unit area, $\hat{m}_\alpha$ is a surface moment density. The resultants applied to the plate contour F_α and T are densities of force per unit of length, M_f and C_α are moment densities per unit of length and Z_p are transverse forces applied at angular points of the boundary contour $\mathcal{C}$ of the plate. Finally, $(\delta w)_{,n}$ is the derivative of the variation δw with respect to the normal direction to the boundary contour. The last term in Equation (8) is the variation of work due to Lagrangian multipliers and it is written as

$$\delta\Lambda = \sum_{\ell=1}^{N-1} \int_\Sigma \delta[\mu_\ell \mathcal{A}_\ell + \nu_\ell \mathcal{B}_\ell]\, da, \tag{16}$$

The formulation Equation (8) can be modified in order to introduce the case where an electric potential is applied to an electrode interface (see Fernandes and Pouget (2002) for more details).

On collecting all the variations Equations (9)–(16) in the variational formulation Equation (8), we arrive at the set of equations of motion.

$$\begin{cases} \mathcal{N}_{\alpha\beta,\beta} + f_\alpha = \Gamma_\alpha^{(u)}\ , \\ \mathcal{M}_{\alpha\beta,\alpha\beta} - p = \Gamma^{(w)}, \\ \hat{\mathcal{M}}_{\alpha\beta,\beta} - \hat{Q}_\alpha + \hat{m}_\alpha = \Gamma_\alpha^{(\gamma)}, \\ D_{\alpha,\alpha}^{(m)(\ell)} - \mathcal{D}_3^{(m)(\ell)} + q_m^{(\ell)} = 0\ , \\ \quad \ell \in \{1, \cdots, N\} \quad \text{and} \quad m \in \{0,1,2,3\}\ , \end{cases} \tag{17}$$

where $\mathcal{N}_{\alpha\beta}$, $\mathcal{M}_{\alpha\beta}$, $\hat{\mathcal{M}}_{\alpha\beta}$ and $\mathcal{D}_3^{(m)(\ell)}$ are the generalized resultants altered by the Lagrange multipliers (see Fernandes and Pouget (2002)). The associated boundary conditions on the plate contour are also deduced from the variational formulation, but they are not given for the sake of consistency.

It is worth noting that if an electric potential is applied to the top and bottom faces of the plate, the variations of the unknown functions $\phi_0^{(1)}$ and $\phi_0^{(N)}$ are no longer arbitrary, accordingly Equation $(17)_4$ for $m=0$ and $\ell = 1$ or N disappears.

CONSTITUTIVE EQUATIONS FOR PIEZOELECTRIC PLATES

The constitutive laws for the generalized resultants defined by Equations (10)–(13) can be obtained by using the constitutive equations for piezoelectric materials with an orthotropic symmetry (see Equation (4)). The constitutive equations can be written in a matrix form

$$\begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{bmatrix} = \begin{bmatrix} \mathbb{M} & \mathbb{O} \\ \mathbb{O} & \mathbb{L} \end{bmatrix} \begin{bmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \end{bmatrix} \tag{18}$$

where $[\mathbf{R}_1]$ and $[\mathbf{R}_2]$ are vectors of $3(N+3)$ and $2(4N+1)$ dimensions respectively, containing the resultant components, $[\mathcal{C}_1]$ and $[\mathcal{C}_2]$ are vectors whose components are the generalized deformations, electric fields and potentials. In addition, $[\mathbb{M}]$ and $[\mathbb{L}]$ are $3(N+3) \times 3(N+3)$ and $2(4N+1) \times 2(4N+1)$ matrices respectively. They consist of block matrices depending on the material constants and geometrical parameters of the layers. We do not give the analytical form of the different block matrices, for more details see Fernandes and Pouget (2002).

NUMERICAL RESULTS

Piezoelectric Plate under Cylindrical Bending

An interesting and simple situation is a plate simply supported under cylindrical bending. We suppose there are no shear traction ($f_\alpha = 0$) and no surface moment density ($\hat{m}_\alpha = 0$) applied to the plate faces. The electromechanical load functions can be written as Fourier series

$$(p(x), V(x)) = \sum_{n=1}^{\infty}(S_n, V_n)\sin(\lambda_n x), \qquad (19)$$

with $\lambda_n = n\pi/L$ where S_n and V_n are Fourier coefficients for uniform applied surface densities of force S_0 and electric potential V_0. A solution to the equations of motion (17) satistfying the cylindrical bending boundary conditions are looked for as Fourier series. The Fourier coefficients of the unknown functions satisfy a set of linear algebraic equations of the form

$$\mathbb{A}_n \mathbf{X}_n = \mathbf{B}_n,$$

for each n, where $\mathbb{A}_n$ is a square matrix of $6N+1$ order in the case of open circuit and of $6N-1$ order in closed-circuit conditions. The vector $\mathbf{X}_n$ contains the Fourier coefficients $\mathbf{X}_n = \{U_n, W_n, \Gamma_n, \Phi^{(\ell)}_{m,n}, M^{(r)}_n, N^{(r)}_n\}$ with $m \in \{0,1,2,3\}, \ell \in \{1,\ldots,N\}, r \in \{1,\ldots,N-1\}$. The vector $\mathbf{B}_n$ contains the applied fields as a function of the Fourier factors S_n and V_n. The matrix $\mathbb{A}_n$ and vector $\mathbf{B}_n$ depend on λ_n, the layer geometry and material constants.

Results and Comparisons

In this section a collection of benchmark tests is proposed for the piezoelectric bimorph and sandwich structures. Two kinds of electromechanical loads are considered (i) sensor function with a surface force density applied to the top face and (ii) actuator function with an electric potential applied to the top and bottom faces of the plate and eventually at the layer interfaces. Furthermore, with the view of testing the capability of the present modelling, the results are compared to those provided by finite element formulation. The material coefficients of the piezoelectric layers made of PZT ceramics and composites are given in Tables 1 and Table 2 respectively.

PIEZOELECTRIC BIMORPH STRUCTURE

In this case both the piezoelectric layers are made of identical materials, however the piezo-active axes are in opposite direction along the z-axis (see Figure 2). The study includes only two Lagrangian multipliers in order to ensure the continuity conditions on the electric potential and normal component of the electric induction.

Surface density force applied to the top face

The numerical results and comparisons to FE computations are presented in Figure 3. The induced electric potential at $x = L/2$ exhibiting an asymmetric profile is shown in Figure 3(a). The transverse shear stress σ_{13} at $x = L/4$ is plotted in Figure 3(b). It is observed, the continuity of the shear stress through the interface between both the piezoelectric layers. The estimate of the electromechanical responses in comparison to FE computations leads to an error less than 0.1%.

Table 1. Independent elastic, piezoelectric and dielectric constants of piezoelectric materials (transversally isotropic symmetry).

	C^E_{11} (GPa)	C^E_{12}	C^E_{33}	C^E_{13}	C^E_{44}	e_{31}(C/m^2)	e_{33}	e_{15}	ϵ^S_{11}(nF/m)	ϵ^S_{33}
PZT-4	139	77.8	115	74.3	25.6	−5.2	15.1	12.7	13.06	11.51

Table 2. Independent elastic, piezoelectric and dielectric constants of a composite made of graphite fibres along the x-direction in epoxy matrix.

	C^E_{11}(GPa)	C^E_{12}	C^E_{22}	C^E_{23}	C^E_{55}	e_{31}(C/m^2)	e_{33}	e_{15}	ϵ^S_{11}(nF/m)	ϵ^S_{33}
Composite	134.86	5.1563	14.352	7.1329	5.654	0	0	0	0.031	0.0266

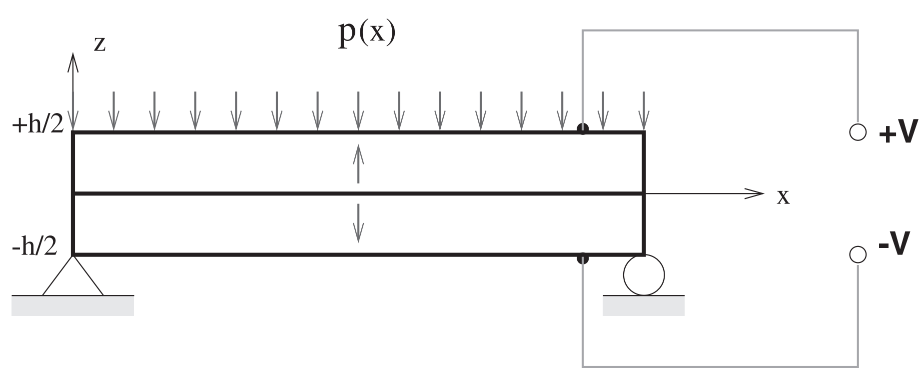

Figure 2. *Piezoelectric bimorph setting.*

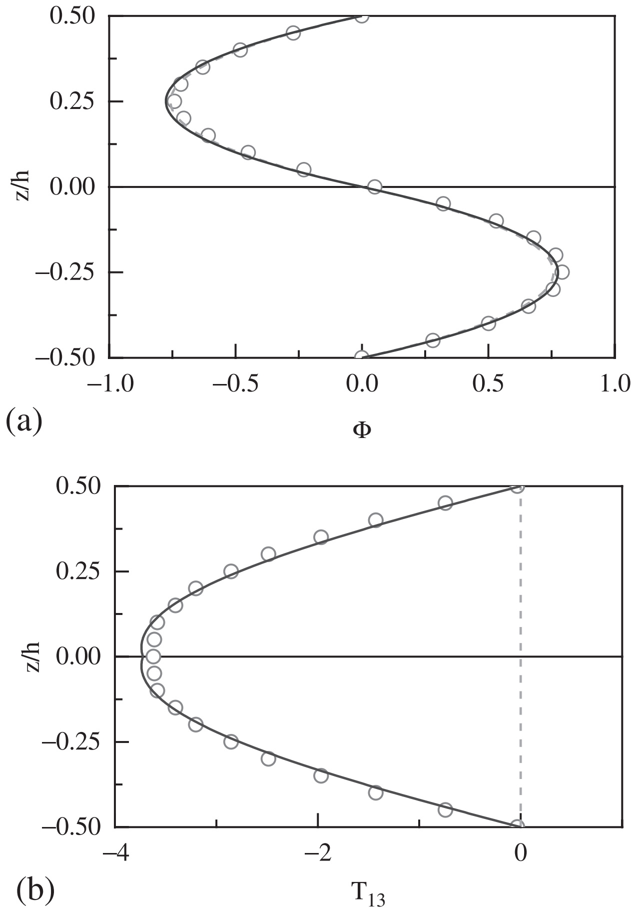

Figure 3. *Force density applied on the top face of a piezoelectric bimorph in closed circuit for L/h = 10 (present model : solid line, simplified model : dash line, FEM : small circle).*

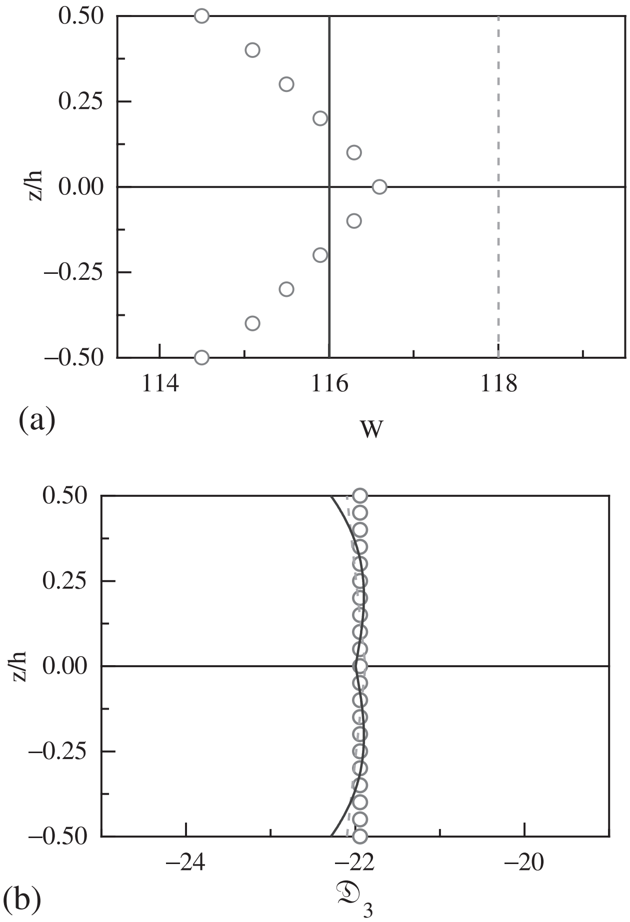

Figure 4. *Electric potential applied to a piezoelectric bimorph for L/h = 10 (present model : solid line, simplified model : dash line, FEM : small circle).*

Applied electric potential

In this situation, when an electric potential is then applied to the top and bottom faces of the bimorph, one layer elongates while the other one shrinks, resulting in a global bending of the whole plate. The results of flexural displacement w at the plate centre is displayed in Figure 4(a). The normal component of the electric induction at the plate centre is plotted in Figure 4(b), it is almost constant through the plate thickness. From the practical point of view, an applied electric potential of the order of 100 V produces a deflection of the order of 30 μm for $L/h = 50$.

PIEZOELECTRIC SANDWICH PLATE

We consider a laminated plate made of composite material (graphite fibres aligned in the x-direction of an epoxy matrix) for the central layer sandwiched with two piezoelectric PZT layers. The top and bottom faces of both the piezoelectric layers are re-covered with metallic electrodes as shown in Figure 5.

Surface density force applied to the top face

The sandwich plate is then subjected to a surface density of force normal to the top face. The flexural displacement at $x = L/2$ is presented in Figure 6(a). We note that the error in predicting the maximum value of the deflection, at the plate centre, in comparison to the FE results is excellent and it is less than 5% for $L/h = 10$. The discrepancy increases for the simplified model (no shear correction, based on the Love–Kirchhoff theory of elastic thin plate) and it is about 16%. Figure 6(b) provides the profile of the induced electric potential at $x = L/2$ within the piezoeletric layers. All these results demonstrate the effectiveness of the present model.

Applied electric potential

The through-the-thickness distribution of the flexural displacement at the plate centre and axial stress are presented in Figure 7(a) and (b) respectively. The axial stress exhibits very clearly jumps at the layer interfaces. The comparisons to the FE computations are satisfactory and less than 2% for $L/h = 10$. The piezoelectric sandwich produces a maximum deflection of the order of 90 μm for an applied electric potential of 100 V for $L/h = 50$. Such a sandwich piezoelectric structure is a good candidate to design versatile and precise actuators (Zhang and Sun, 1996; Benjeddou et al., 2000).

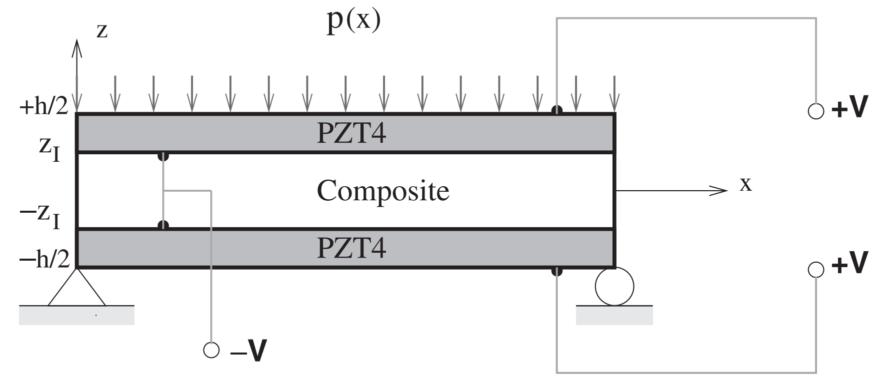

Figure 5. *Piezoelectric sandwich plate with intermediate electrodes.*

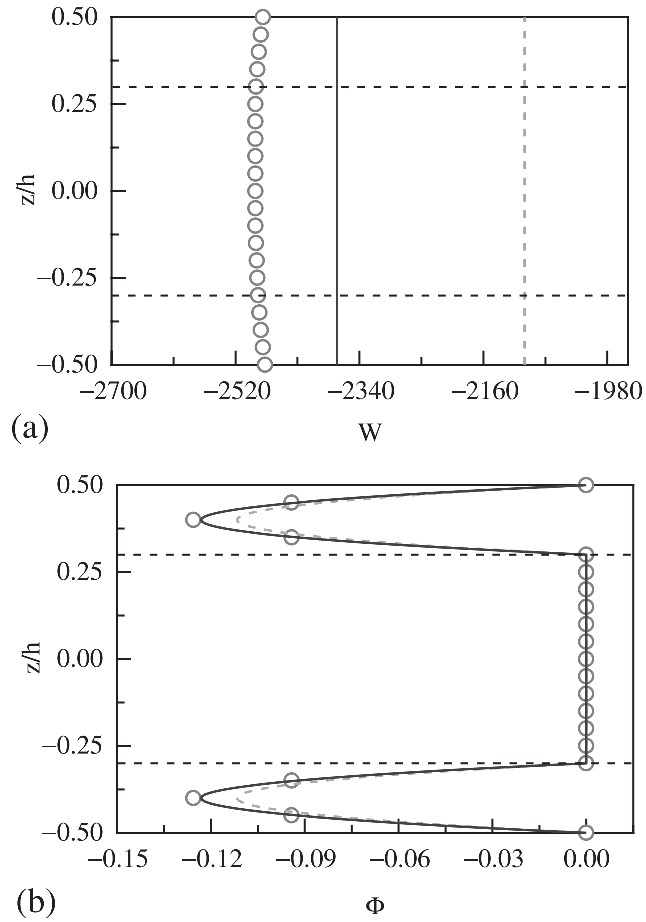

Figure 6. *Force density applied on the top face of a piezoelectric sandwich plate in closed circuit for L/h = 10 (present model : solid line, simplified model : dash line, FEM : small circle).*

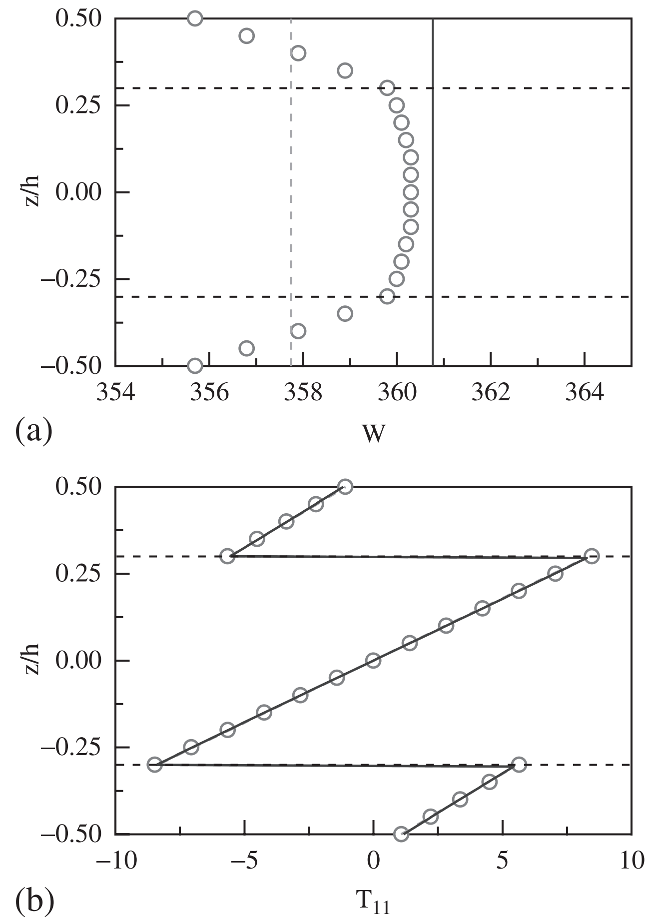

Figure 7. *Electric potential applied to a piezoelectric sandwich plate for L/h = 10 (present model : solid line, simplified model : dash line, FEM : small circle).*

VIBRATION OF PIEZOELECTRIC LAMINATED PLATES

We consider, here, dynamical processes for the piezoelectric plate based on the present approach. We propose the prediction of modal frequencies of piezoelectric plates for the closed-circuit condition ($\phi = 0$) on the top and bottom faces. The knowledge of frequencies of vibration modes of such piezoelectric composites plays an important role in the vibration control of structures. In particular, piezoelectric elements can be used as components of a passive damping device, thereby avoiding complex control and feedback systems (Haggod and Von Flotow, 1991; Gaudenzi et al., 2000). The solution to the piezoelectric plate equations Equation (17) depends on time by introducing the factor $e^{i\omega t}$ in the Fourier series of the unknown functions. Now, the Fourier coefficients of the solution are searched for by solving the homogeneous set of linear equations of the form

$$\mathbb{A}_n(\Omega_n)\mathbf{X}_n = \mathbf{0}, \tag{20}$$

for the free vibration problem. The subscript n holds for the mode number. The boundary conditions are those of the cylindrical bending as in the static case. The numerical results and comparisons to the FE computations are given in Table 3 for the piezoelectric bimorph in closed-circuit with $L/h = 10$. According to the numerical results, the plate has a series of natural bending vibration frequencies (the first seven modes are given in Table 3). In addition, there are axial modes of vibration. It is clear that there is very tiny difference between the results of the present model and those of the FE computations. Nevertheless, the simplified model based on the kinematic assumption of Love–Kirchhoff plate theory provides inaccurate values, especially for higher modes the error is greater than 30%. The prediction of the modal frequencies for the present modelling remains within an error of 5%. Finally, the frequencies of the vibration modes in the case of a sandwich plate for $L/h = 10$ are listed in Table 4. Here, the present approach is definitely better than the simplified model for the first seven flexural modes. The study of vibration modes of the piezoelectric composites demonstrates the crucial role played by the shear correction and the layerwise approach of the electric potential in the prediction of the frequencies of the bending modes. Such an investigation of modal frequencies of vibration is a first step in controlling vibration of elastic structures.

Table 3. Modal frequencies for the piezoelectric bimorph (L/h = 10).

Frequencies (Hz) – L/h = 10					
Modes	**FE**	**Present Model**	**Error (%)**	**Simplified Model**	**Error (%)**
Flex. $n = 1$	15747	15769	0.1	16030	1.8
Flex. $n = 2$	59370	59677	0.5	63338	6.3
Flex. $n = 3$	122994	124291	1	139721	12
Flex. $n = 4$	199046	202511	1.7	241909	17.7
Flex. $n = 5$	282019	289352	2.5	366039	22.9
Flex. $n = 6$	368241	381771	3.5	508113	27.5
Flex. $n = 7$	455253	478014	4.8	664352	31.5
Axial $n = 1$	188372	188599	0.1	188599	0.1

Table 4. Modal frequencies for piezoelectric sandwich plate (L/h = 10).

Frequencies (Hz) – L/h = 10					
Modes	**FE**	**Present Model**	**Error (%)**	**Simplified Model**	**Error (%)**
Flex. $n=1$	29530	29570	0.1	29869	1.1
Flex. $n=2$	112624	113295	0.6	117537	4.2
Flex. $n=3$	236707	239584	1.2	257639	8.1
Flex. $n=4$	388585	396150	1.9	442512	12.2
Flex. $n=5$	557423	573241	2.7	663571	16
Flex. $n=6$	734569	764062	3.9	912474	19.5
Flex. $n=7$	912254	964119	5.4	1181874	22.8
Axial $n=1$	324061	324220	0.05	324220	0.05

CONCLUSION

In the present study, we have been interested in the modelling of piezoelectric composites and their electromechanical responses to static and dynamic loads. The present study is devoted to a 2D approach to composite plates made of piezoelectric and non-piezoelectric but elastic layers. We have proposed an accurate and efficient approximation for the elastic displacement including the shear correction function and a layerwise modelling for the electric potential. Such an approach can incorporate the local electromechanical response of the individual layer and becomes a necessity when an electric potential is applied to metallic interfaces between layers. The shear function has a beneficial influence on the accuracy of the results, especially, for computing frequencies of vibration modes. The numerical results and comparisons to FE simulations demonstrate the efficiency of the model and its capability to accurately predict the local and global responses. A number of numerical examples have been presented for piezoelectric laminated structures of particular interest such as piezoelectric bimorph and sandwich structures. The numerical tests provide excellent agreement with those coming from FE computations with errors in estimating within a range of 1–2%. Nevertheless, the limitation of the present modelling concerns the prediction of the transverse shear stress (except for the bimorph). Therefore a full layerwise approach is required and further studies will be devoted to layerwise approach based on mixed variational formulation (Carrera, 1996). Extensions of the present study to the piezoelectric elements bonded to an elastic structure are being examined.

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