FACTA UNIVERSITATIS
Series: **Architecture and Civil Engineering** Vol. 3, N^o 2, 2005, pp. 145 - 153

ANALYTICAL APPROACH FOR RESOLVING STRESS STATES AROUND ELLIPTICAL CAVITIES

UDC 624.191.8(045)=20

Dragan Lukić[1], Petar Anagnosti[2]

[1]University of Novi Sad, Faculty of Civil Engineering Subotica, Serbia and Montenegro
[2]University of Belgrade, Faculty of Civil Engineering, Belgrade, Serbia and Montenegro

Abstract. *The determination of stress states around cavities in the stressed elastic body, regardless of cavity shapes, that may be spherical, cylindrical, elliptical etc. in its analytical approach has to be based on selection of a stress function that will satisfy biharmonic equation* $\nabla^2\nabla^2\Psi = 0$, *under given boundary conditions. This paper is concerned with formulation and solution of the cited differential equation using elliptical coordinates in conformity with the cavity shape of oblong ellipsoid [1]. It is therefore considered that the formulation of the stress tensor will be done in conformity to the cited coordinates.*
The paper describes basic statements and definitions in connection to harmonic functions used for determination of stress states around cavities formed in the stressed homogeneous space. The particular attention has been paid to the use of Legendre`s functions, with definitions and derivation of recurrent formulas, that have been used for determination of stress states around an oblong ellipsoidal cavity, [1]. The paper also includes the description of procedures used in forming series based on Legendre`s functions of the first order.

Key words: *coordinates, biharmonic differential equation, stress state, stress function, harmonic functions, recurrent formulas.*

1. BIHARMONIC DIFFERENTIAL EQUATION SOLUTION BY A STRESS FUNCTION

In resolving stress states around a cavity having the oblong ellipsoidal shape, the starting point is the basic equilibrium equation in terms of displacements, derived for a stressed isotropic elastic body (neglecting the gravity). Following this approach the equilibrium equations can be formulated in a general manner by using a function of coordinates for an arbitrary point in the body, known as "stress function". Selection of stress functions that are satisfying equilibrium equations and imposed boundary conditions in either isotropic or not isotropic bodies, presents the common task for research activities in

Received February 01, 2005

this field of applied mechanics. An example of such solutions is referred to Papkovich – Neuber [1] set of stress functions, taken as the basis for determination of stress tensor coordinates in the following form:

$$\Psi = \Phi_0 + x\Phi_1 + y\Phi_2 + z\Phi_3 \tag{1}$$

that in the case of axial symmetry is reduced to:

$$\Psi = \Phi_0 + z\Phi_3 \tag{2}$$

The selected stress function used for determination of stress tensor coordinates shall satisfy biharmonic differential equation:

$$\nabla^2\nabla^2\Psi = 0 \tag{3}$$

In an analytical approach this equation shall be formulated in the coordinates selected in conformity with the geometry of the cavity around which the stress states are under investigation.

2. Biharmonic differential equation in elliptical coordinates

The shape of the cited differential equation (3) that the selected stress functions shall satisfy, depends on the expression of Laplace's operator (∇^2) in the chosen curvilinear coordinates. In the case of coordinates suited for oblong rotational ellipsoid, Laplace's operator is given in the following form:

$$\begin{aligned}\nabla^2 = (&\frac{1}{\mathrm{ch}^2 u - \cos^2\varphi}\frac{\partial}{\partial\,\mathrm{ch}\,u}\left[(\mathrm{ch}^2 u - 1)\frac{\partial}{\partial\,\mathrm{ch}\,u}\right] + \frac{1}{\mathrm{ch}^2 u - \cos^2\varphi}\frac{\partial}{\partial\cos\varphi}\\ &\left[(1-\cos^2\varphi)\frac{\partial}{\partial\cos\varphi}\right] + \frac{1}{(\mathrm{ch}^2 u - 1)(1-\cos^2\varphi)}\frac{\partial^2}{\partial\theta^2})\end{aligned} \tag{4}$$

where u, φ, θ are elliptical coordinates defined by expressions $x = L$ sh u $\sin\varphi$ $y = L$ sh u $\sin\varphi$ $\sin\theta$ $z = L$ ch u $\cos\varphi$ (L – is the focal distance along larger axis, $0 \le u \le \infty$ $0 \le \varphi \le \pi$ $0 \le \theta \le 2\pi$).

Under condition of axial symmetry state independent of the coordinate θ, the operator has the following form [2]:

$$\begin{aligned}\nabla^2 = &\frac{1}{\mathrm{ch}^2 u - \cos^2\varphi}\frac{\partial}{\partial\,\mathrm{ch}\,u}\left[(\mathrm{ch}^2 u - 1)\frac{\partial}{\partial\,\mathrm{ch}\,u}\right] +\\ &+ \frac{1}{\mathrm{ch}^2 u - \cos^2\varphi}\frac{\partial}{\partial\cos\varphi}\left[(1-\cos^2\varphi)\frac{\partial}{\partial\cos\varphi}\right]\end{aligned} \tag{5}$$

After substitution of expression (5) in equation (3), one can obtain:

$$
\begin{aligned}
&[\frac{1}{\mathrm{ch}^2 u-\cos^2\varphi}\frac{\partial}{\partial\,\mathrm{ch}\,u}\left[(\mathrm{ch}^2 u-1)\frac{\partial}{\partial\,\mathrm{ch}\,u}\right]+\\
&+\frac{1}{\mathrm{ch}^2 u-\cos^2\varphi}\frac{\partial}{\partial\cos\varphi}\left[(1-\cos^2\varphi)\frac{\partial}{\partial\cos\varphi}\right]]\\
&[\frac{1}{\mathrm{ch}^2 u-\cos^2\varphi}\frac{\partial}{\partial\,\mathrm{ch}\,u}\left[(\mathrm{ch}^2 u-1)\frac{\partial\Psi}{\partial\,\mathrm{ch}\,u}\right]+\\
&\frac{1}{\mathrm{ch}^2 u-\cos^2\varphi}\frac{\partial}{\partial\cos\varphi}\left[(1-\cos^2\varphi)\frac{\partial\Psi}{\partial\cos\varphi}\right]]\quad=0
\end{aligned}
\tag{6}
$$

Therefore the expression (6) has the form of differential equation $\nabla^2\nabla^2\Psi=0$ in elliptical coordinates, for axial symmetry case.

3. Differential equation solution in elliptical coordinates

Differential equation (6) can be commonly transformed in two equations [3] as follows:

$$\nabla^2 \mathrm{A}=0 \tag{7}$$

$$\nabla^2\Psi=\mathrm{A} \tag{8}$$

The solution of equation (7) by separation of unknowns is to be searched in the form:

$$\mathrm{A}=\mathrm{F}(\mathrm{ch}\,u)\Phi(\cos\varphi) \tag{9}$$

By application of common mathematical transformations with separation of unknowns, one can obtain two equations with independent unknowns "u" and "φ", respectively:

$$
\begin{aligned}
&[(1-\mathrm{ch}^2 u)\mathrm{F}'(\mathrm{ch}\,u)]'+[n(n+1)\ -\frac{m^2}{1-\mathrm{ch}^2 u}]\mathrm{F}(\mathrm{ch}\,u)=0\\
&[(1-\cos^2\varphi)\Phi'(\cos\varphi)]'+\ [n(n+1)-\frac{m^2}{1-\cos^2\varphi}]\Phi(\cos\varphi)=0
\end{aligned}
\tag{10}
$$

General solutions of equations (10) are known in the form:

$$
\begin{aligned}
&\mathrm{F}_{nm}(\mathrm{ch}\,u)=\mathrm{A}_{nm}\mathrm{P}_n^m(\mathrm{ch}\,u)+\mathrm{B}_{nm}\mathrm{Q}_n^m(\mathrm{ch}\,u)\\
&\Phi_{nm}(\cos\varphi)=\mathrm{C}_{nm}\mathrm{P}_n^m(\cos\varphi)+\mathrm{D}_{nm}\mathrm{Q}_n^m(\cos\varphi)
\end{aligned}
\tag{11}
$$

After insertion of (11) in (9) one can obtain the expression (9) in the form:

$$A = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [A_{nm} P_n^m(\mathrm{ch}\, u) + B_{nm} Q_n^m(\mathrm{ch}\, u)][C_{nm} P_n^m(\cos\varphi) + D_{nm} Q_n^m(\cos\varphi)] \tag{12}$$

Where: $P_n^m(\cos\varphi)$, $Q_n^m(\mathrm{ch}\, u)$ are Legendre's polynomials and functions, that

For m=0 reduce to the following form:

$$P_n(\cos\varphi),\ Q_n(\mathrm{ch}\, u)\,; \tag{13}$$

In order to satisfy the continuity of expression (12), the condition $D_{nm} = 0$ is to be fulfilled, and therefore the solution of equation (7) has the following form:

$$A = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [A_{nm} P_n^m(\mathrm{ch}\, u) + B_{nm} Q_n^m(\mathrm{ch}\, u)] \times C_{nm} P_n^m(\cos\varphi) \tag{14}$$

After insertion of the boundary condition:

$$\lim_{\mathrm{ch}\, u \to \infty} P_n^m(\mathrm{ch}\, u) = \infty; \qquad \lim_{\mathrm{ch}\, u \to \infty} Q_n^m(\mathrm{ch}\, u) = 0 \tag{15}$$

one can obtain the value of parameter $A_{nm} = 0$, and the solution of equation (7) gets the following form:

$$A = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [C_{nm} B_{nm} P_n^m(\cos\varphi)\ Q_n^m(\mathrm{ch}\, u)] \tag{16}$$

By substitution $a_{nm} = C_{nm} B_{nm}$, one can obtain the solution of equation (7) in its final form:

$$A = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [a_{nm} P_n^m(\cos\varphi)\ Q_n^m(\mathrm{ch}\, u)] \tag{17}$$

By inserting the expression (17) into equation (8) one can obtain:

$$\nabla^2 \Psi = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [a_{nm} P_n^m(\cos\varphi)\ Q_n^m(\mathrm{ch}\, u)] \tag{18}$$

After the searching its solution in the following form:

$$\Psi = \Psi_h + \Psi_p \tag{19}$$

it appears that there is a possibility to obtain two differential equations, namely:

– general solution: $$\nabla^2 \Psi_h = 0 \tag{20}$$
and
– particular solution: $$\nabla^2 \Psi_p = A \tag{21}$$

The homogeneous equation (20) can be resolved in the same manner as equation (7) and solution can be obtained in the following form:

– general solution $$\Psi_h = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [b_{nm} \mathrm{P}_n^m(\cos\varphi)\ \mathrm{Q}_n^m(\mathrm{ch}\, u)] \tag{22}$$

and

– particular solution is to be searched in the form:

$$\Psi_p = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [e_{nm} \mathrm{P}_n^m(\cos\varphi)\ \mathrm{Q}_n^m(\mathrm{ch}\, u)] \tag{23}$$

Applying differential operator (5) on Ψ_p in expression (23) and after equating the left side of equation (21), and the right side of equation (17) one can obtain the relationship between quotients a_{nm} and e_{nm}.

The final solution of equation (3) is therefore obtained in the form:

$$\Psi = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [b_{nm} \mathrm{P}_n^m(\cos\varphi)\ \mathrm{Q}_n^m(\mathrm{ch}\, u)] + \sum_{n=0}^{\infty} \sum_{m=0}^{n} [e_{nm} \mathrm{P}_n^m(\cos\varphi)\ \mathrm{Q}_n^m(\mathrm{ch}\, u)] \tag{24}$$

or,

$$\Psi = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [f_{nm} \mathrm{P}_n^m(\cos\varphi)\ \mathrm{Q}_n^m(\mathrm{ch}\, u)] \tag{25}$$

where: $f_{nm} = b_{nm} + e_{nm}$

P_n^m , and Q_n^m , are Legendre's polynomials and functions [4].

The unknown parameters f_{nm} are to be determined from boundary conditions existing on the cavity surface, that have to be given in stresses., and the expression (2) is to be rewritten as:

$$\Phi_0 = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \mathrm{A}_{nm} \mathrm{P}_n^m(\cos\varphi) \mathrm{Q}_n^m(\mathrm{ch}\, u) \quad \text{and}$$

$$z\Phi_3 = \underset{n=0\ \ m=0}{L} \mathrm{ch}\, u \cos\varphi \sum_{n=-1}^{\infty} \sum_{m=0}^{n} \mathrm{C}_{nm} \mathrm{P}_{n+1}^m(\cos\varphi) \mathrm{Q}_{n+1}^m(\mathrm{ch}\, u) \tag{26}$$

where unknown parameters A_{nm} and C_{nm} are to be determined from boundary conditions existing on the cavity surface, that have to be given in stresses.

Starting from expression (26) one can obtain analytically defined stresses $\sigma_{u,}\ \sigma_{\varphi,}\ \sigma_{\theta,}\ \tau_{u\varphi}$ by well known expressions consisting of the derivatives of functions Ψ and Φ_3. On the other hand, the stresses acting on the cavity surface are to be derived also from a given stress function "$f\,(u,\varphi,\theta)$" expressed in infinite series basing on Legendre's functions, and equated to the values resolved by stress functions Φo and Φ_3, thus forming up the final set of linear equations that relate unknown Anm and Cnm parameters to the known values of coefficients in the infinite series approximation of the boundary stresses.

4. Legendre's Polynomials and Functions

Analytical solutions of problems related to stress states determination in vicinity of elliptical, spherical and cylindrical cavities formed in the stressed homogeneous space, mostly are based on application of harmonic stress functions. The detailed description of harmonic functions and presentation of convenient types of Legendre's functions are already given in Hobson's publication [2]. In the recent work given as [1], the detailed additional analyses of the subject were presented, particularly definitions of necessary recurrent formulas, and solutions of stress states with the transformations of functions in infinite series basing on Legendre's functions of the first order.

The problems of the stress state determination in vicinity of a cavity having the shape of oblong ellipsoid with rotational (axial) symmetry can be resolved, as it has been cited previously by defining stress functions that shall satisfy the basic differential equation $\nabla^2 \nabla^2\psi = 0$. The solutions are obtainable with the help of Legendre's functions of the first and second order, defined by the following expressions:

$$\mathrm{P_n}(\cos\varphi) = \frac{1}{2^n n!}\frac{d^n}{d(\cos\varphi)^n}(\cos^2\varphi - 1)^n \tag{27}$$

$$\mathrm{P}_n^m(\cos\varphi) = (-1)^m(1-\cos^2\varphi)^{\frac{1}{2}m}\frac{d^m\mathrm{P}_n(\cos\varphi)}{d(\cos\varphi)^m} \tag{28}$$

and by use of Legendre's functions of the second order defined by expressions

$$\mathrm{Q}_n(\mathrm{ch}\,u) = \frac{1}{2}\mathrm{P}_n(\mathrm{ch}\,u)\lg\frac{\mathrm{ch}\,u - 1}{\mathrm{ch}\,u + 1} + \mathrm{W}_{n\text{-}1}(\mathrm{ch}\,u) \tag{29}$$

$$\mathrm{Q}_n^m(\mathrm{ch}\,u) = (\mathrm{ch}^2 u - 1)^{\frac{1}{2}m}\frac{d^m\mathrm{Q_n}(\mathrm{ch}\,u)}{d(\mathrm{ch}\,u)^m} \tag{30}$$

where:

$$\mathrm{P}_n(\mathrm{ch}\,u) = \frac{1}{2^n n!}\frac{d^n}{d(\mathrm{ch}\,u)^n}(\mathrm{ch}^2 u - 1)^n \tag{31}$$

and

$$\mathrm{W}_{n\text{-}1}(\mathrm{ch}\,u) = \frac{2n-1}{1n}\mathrm{P_{n\text{-}1}}(\mathrm{ch}\,u) + \frac{2n-5}{3(n-1)}\mathrm{P_{n\text{-}3}}(\mathrm{ch}\,u) + ... \tag{32}$$

The expressions (28) and (30) define associated Legendre's functions of the first and second order, respectively. It is worth to note, that Legendre's functions are frequently used due to their orthogonal properties. For the problems of stress state determinations in vicinity of the cavities, the particular suitability is stemming from the finiteness of Legendre's functions of the first order, and rapid convergence for Legendre's functions of the second order to zero values with enlarging the distance from the cavity boundary. These properties are also useful for easier control and follow up of convergence in the derived solutions. On the ground of the basic properties of Legendre's functions the derivation of recurrent formulas made possible to formulate necessary set of equations and simplification of the expressions for stress tensor. In the Hobson's paper [2] a number of recurrent formulas had been derived, and supplementary recurrent formulas were derived in the recently completed work [1], and presented as follows.

The recurrent formulas for Legendre's functions of the first order are:

$$\sin^2\varphi\, P_n^{'} = (n+1)\cos\varphi\, P_n - (n+1)\, P_{n+1}$$

$$\sin^2\varphi\, P_{n+1}^{'} = (n+1)\; P_n - (n+1)\cos\varphi\; P_{n+1}$$

$$\sin^4\varphi\, P_n^{"} = (n+1)[(2-(2+n)\sin^2\varphi)P_n - 2\cos\varphi\, P_{n+1}]$$

$$\sin^4\varphi\, P_{n+1}^{"} = (n+1)[(2\cos\varphi\, P_n - (n\sin^2\varphi + 2)\, P_{n+1}] \tag{33}$$

$$\sin\varphi\, P_n^{m+1} = -(n-m+1)P_{n+1}^m + (n+m+1)\cos\varphi\, P_n^m$$

$$\sin\varphi\, P_{n+1}^{m+1} = -(n-m+1)\cos\varphi\, P_{n+1}^m + (n+m+1)\, P_n^m$$

$$\begin{aligned}\sin^2\varphi\, P_n^{m+2} &= -2(m+1)(n-m+1)\cos\varphi\, P_{n+1}^m + \\ &\quad + (n+m+1)[2(m+1)-(m+n+2)\sin^2\varphi]\, P_n^m\end{aligned}$$

$$\begin{aligned}\sin^2\varphi\, P_{n+1}^{m+2} &= 2(m+1)(n+m+1)\cos\varphi\, P_n^m - \\ &\quad - [2(m+1)(n-m+1)+(n-m)(n-m+1)\sin^2\varphi]\, P_{n+1}^m\end{aligned} \tag{34}$$

The recurrent formulas of Legendre's functions of the second order are:

$$\mathrm{sh}^2 u\, Q_n^{'} = (n+1)(Q_{n+1} - \mathrm{ch}\, u Q_n)$$

$$\mathrm{sh}^2 u\, Q_{n+1}^{'} = (n+1)(\mathrm{ch}\, u Q_{n+1} - Q_n)$$

$$\mathrm{sh}^4 u\, Q_n^{"} = (n+1)[((n+2)sh^2 u + 2)Q_n - 2\mathrm{ch}\, u\, Q_{n+1}]$$

$$\mathrm{sh}^4 u\, Q_{n+1}^{"} = (n+1)[(n\, sh^2 u - 2)Q_{n+1} + 2\mathrm{ch}\, u\, Q_n] \tag{35}$$

$$\mathrm{sh}\, u\, Q_n^{m+1} = (n-m+1)Q_{n+1}^m - (n+m+1)\mathrm{ch}\, u\, Q_n^m$$

$$\mathrm{sh}\, u\, Q_{n+1}^{m+1} = (n-m+1)\mathrm{ch}\, u\, Q_{n+1}^m - (n+m+1)Q_n^m$$

$$\begin{aligned}\mathrm{sh}^2 u\, Q_n^{m+2} &= (n+m+1)[2m+2+(n+m+2)\mathrm{sh}^2 u]\, Q_n^m - \\ &\quad - 2(m+1)(n-m+1)\mathrm{ch}\, u\, Q_{n+1}^m\end{aligned}$$

$$\begin{aligned}\mathrm{sh}^2 u\, Q_{n+1}^{m+2} &= 2(m+1)(n+m+1)\mathrm{ch}\, u\, Q_n^m + \\ &\quad [(n-m)(n-m+1)\mathrm{sh}^2 u - (2m+2)(n-m+1)]\, Q_{n+1}^m\end{aligned} \tag{36}$$

The given recurrent formulas are to be used in forming the set of equations required for determination of unknown parameters Anm and Cnm on the basis of boundary condition function "*f*" approximated in series by Legendre's polynomials.

5. BOUNDARY STRESS APPROXIMATION IN SERIES BY LEGENDRE'S POLYNOMIALS

The Legendre's functions of the first order are commonly named as Legendre's polynomials. The boundary stress function approximation in series by Legendre's polynomials is particularly suitable for resolving the basic set of equations that is formed in such a way that one side consists of Legendre's polynomials (given in the form of series) originated from the differential equation solution, and the other side consists of boundary condition function also expressed by Legendre's polynomials.

The development of a boundary stress function "f" in series by trigonometric functions and Legendre's polynomials is given in the form:

$$f = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \mathrm{H}_{nm}\mathrm{P}_n^m(\cos\varphi)\cos m\theta \tag{37}$$

where H_{nm} parameters are to be determined on the basis of expression

$$\mathrm{H}_{nm} = \frac{(n-m)!(2n+1)!}{(n+m)!\ 2\pi\lambda_m}\int_0^{2\pi}\mathrm{d}\theta\int_0^{\pi} f\ \mathrm{P}_n^m(\cos\varphi)\cos m\theta \sin\varphi\,\mathrm{d}\varphi \tag{38}$$

with $\lambda_0 = 2\ (m=0)$, and $\lambda_m = 1\ (m \neq 0)$; [5]

In case of $m = 0$, one can obtain:

$$f = \sum_{n=0}^{\infty}\mathrm{H}_n\mathrm{P}_n(\cos\varphi) \tag{41}$$

where parameters H_n are to be determined from expression

$$\mathrm{H}_\mathrm{n} = \frac{2n+1}{2}\int_0^{\pi} f\ \ \mathrm{P}_n(\cos\varphi)\sin\varphi\,\mathrm{d}\varphi \tag{42}$$

The available literature [1] suggests the development of the function "f" as follows:

$$f = \sum_{n=0}^{\infty}\mathrm{H}_n\mathrm{P}_n^{(1)}(\cos\varphi) \tag{43}$$

where parameters H_n can be determined from expression:

$$\mathrm{H}_n = \frac{(2n+1)(n-1)!}{2\pi\,(n+1)!}\int_0^{\pi} f\ \ \mathrm{P}_n^{(1)}(\cos\varphi)\sin\varphi\ \ \mathrm{d}\varphi \tag{44}$$

while, the values of $\mathrm{P}_n^{(1)}(\cos\varphi)$ are defined by:

$$\mathrm{P}_n^{(1)}(\cos\varphi) = \frac{\mathrm{dP}_n(\cos\varphi)}{\mathrm{d}\varphi} \tag{45}$$

6. CONCLUSIONS

This paper deals with general form of differential equations for rotational symmetry stress conditions. The general solution of biharmonic equation is searched by appropriate stress functions expressed in elliptic coordinates. The solution approach is somewhat different to that where the unknown parameters in infinite series can be directly derived from boundary conditions [1]. In the presented article, the boundary conditions are defined by coordinates of stress tensor, i.e. as the solutions of partial differential equations of stress functions. On the basis of such boundary conditions, whose satisfaction is imposed, the determination procedure for obtaining values of unknown constants in the solution of differential equations defined by infinite series is described.

In this concise paper the attempt has been made to describe one valuable practical application of Legendre's functions in a research where these special functions were used for determination of stress states around cavities in a stressed body. Since the researches made by Hobson and some other contributors during the second half of 20th century, the literature related to application of these special functions is not abundant. It can be inferred that the rapid development of numerical methods based on the finite elements, and the prevalence of these types of research in the field of applied mechanics, has put the analytical methods in a secondary role, the same situation was apparent in the research activities related to special functions. However this paper contains the description of some solutions developed in the basic work [1] that may be considered as a kind of a rejuvenation of some analytical methods in applied mechanics fields. The particular attention has been paid in this paper, to demonstrate some properties of special functions, not so frequently stated in the literature, namely the properties that appeared rather useful in resolving stress states around elliptical cavities formed in the stressed elastic body.

REFERENCES

1. D. Lukić, Contribution to methods of stress state determination around cavity of Rotational ellipsoid shape, by use of elliptic coordinates, Doctor's thesis, University of Belgrade, 1998.
2. E.W. Hobson, Spherical and ellipsoidal harmonics, Cambridge, 1931.
3. D.S. Mitrinović and J.D.Keckić, Equations of mathematical physics, Belgrade, 1994.
4. D.S. Mitrinović, Special functions, Belgrade, 1990.
5. D. Kuzmanović, I. Obradović, A. Sedmak, M. Berković and D. Milošević, Methods of mathematical physics Belgrade, 1997.

MATEMATIČKE OSNOVE ODREĐIVANJA NAPONSKIH STANJA OKO ELIPTIČNIH OTVORA

Dragan Lukić, Petar Anagnosti

Određivanje naponskih stanja oko otvora predstavlja veoma složen matematički problem. Zbog toga, pri razmatranju ovog problema potrebno je najpre definisati pojedine oblasti matematičke analize koje se pri tome koriste.

Prvi deo rada razmatra rešavanje biharmonijskih diferencijalnih jednačina $\nabla^2\nabla^2\psi = 0$ *uzimajući u obzir rešenje Papkovič – Neubera [1].*

U drugom delu rada definišu se odredjene klase specijalnih harmonijskih funkcija (tipa Ležandra) kao posebno značajnih za analizu naponskih stawa.Pored toga, u radu se prikazuju rekurentne formule definisane u radu [1] kao i predstavljanje funkcija u obliku reda po Ležandrovim polinomima.