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Analytical Approximation for the Reflectivity of DH Lasers

JENS BUUS

Abstract—A simple analytical expression for the reflectivity of the cleaved facets of DH semiconductor lasers is compared with numerical results, and reasonable agreement is found. The analytical result is conveniently written in terms of waveguide parameters and thus gives a valuable tool for analysis of DH lasers.

THE simple plane wave expression for the reflectivity at the boundary between two media with different refractive indexes is frequently used in the analysis of DH lasers. For thin active layers this result is not valid; instead, the reflectivity has been calculated [1] from a numerical solution to the equation obtained from the boundary condition. This paper is often referred to, but due to the complicated equations the results are rarely used.

It turns out that the differences between the simple plane wave result (~0.32) and the numerical result (up to 0.43 for the fundamental TE mode in case of a 10 percent difference in the refractive indexes of the active and passive layers) are quite large, in particular for thin active layers, which are of great practical interest. Hence, a simple and reasonably accurate expression for the reflectivity is highly desirable.

The laser structure is considered as a simple slab with the refractive indexes of the active and passive layers being n_1 and n_2, respectively. The thickness of the active layer is d and the polarization is assumed to be parallel to the junction plane (TE mode). This structure was analyzed in a very elegant way by Lewin [2] and an approximate result for the reflectivity was found; unfortunately this work has received very little attention.

The field and power reflection coefficients are denoted ρ and R, respectively. For at the plane wave, polarized normal to the plane of incidence, we have in the case of normal incidence

on the boundary between a medium with refractive index n and air

$$\rho = \frac{n-1}{n+1}. \tag{1}$$

For the laser structure the effective refractive index is defined by

$$n_{\text{eff}} = \frac{\beta}{k} = \sqrt{bn_1^2 + (1-b)n_2^2}, \quad k = \frac{2\pi}{\lambda} \tag{2}$$

where the normalized propagation constant for the fundamental TE mode b is the solution to

$$v\sqrt{1-b} = \arctan\sqrt{\frac{b}{1-b}} \tag{3}$$

with the usual definition of the normalized frequency v, which is also referred to as the normalized thickness

$$v = k\frac{d}{2}\sqrt{n_1^2 - n_2^2}. \tag{4}$$

According to [2] we get for the fundamental TE mode

$$\rho_{\text{TE}} = \frac{n'-1}{n'+1} \tag{5}$$

with

$$n' = n_{\text{eff}}\frac{1 + (\Delta_0/2n_2^2)(n_2-1)}{1 - (\Delta_0/2n_2^2)(n_2^2 - n_2 + 1)}. \tag{6}$$

An expression for Δ_0 is given in Appendix II of [2]. It turns out that this expression can also be written in terms of waveguide parameters, introducing the confinement factor Γ:

$$\Delta_0 = \Delta\epsilon(\Gamma - b) \quad \Delta\epsilon = n_1^2 - n_2^2 \tag{7}$$

where Γ is related to b and v by

$$(\Gamma - b) = (1 - b) \frac{v\sqrt{b}}{1 + v\sqrt{b}}. \tag{8}$$

For a given laser structure (n_1, n_2, d, and wavelength λ) the value of v is found from (4), (3) is solved for b, then Δ_0 and n_{eff} can be found using (7), (8) and (2); finally, using (5) and (6) the power reflection coefficient is calculated from

$$R_{TE} = |\rho_{TE}|^2. \tag{9}$$

In case of small values of Δ_0, i.e., small values of $\Delta\epsilon$, (6) may be further approximated by neglecting a term of the order ($\Delta_0^2/2$):

$$n' \simeq n_{eff}\left(1 + \frac{\Delta_0}{2}\right) = n_{eff}\left(1 + \frac{\Delta\epsilon}{2}(\Gamma - b)\right). \tag{10}$$

For a plane wave incident under a small angle θ on the boundary between a medium and refractive index n and air it is easily verified that

$$\rho_{TE}(\theta)\,\rho_{TM}(\theta) \simeq \rho(0)^2 = \left(\frac{n-1}{n+1}\right)^2 \tag{11}$$

where TE and TM denote the respective polarization perpendicular and parallel to the plane of incidence. Assuming this relation to be valid for the slab structure under consideration, the following result is obtained for the fundamental TM mode:

$$R_{TM} = \frac{R_0^2}{R_{TE}}, R_0 = \left(\frac{n_{eff}-1}{n_{eff}+1}\right)^2. \tag{12}$$

In Fig. 1 the results from the formulas presented in the previous section are compared with the numerical results [1]. It is evident that the analytical expressions are in much better agreement with the numerical results than the plane wave formula. Particularly good agreement is found for active layer thicknesses in the range 0.05–0.2 μm where the threshold current density has its minimum value. The agreement is less satisfactory for thickness values around 0.5 μm, and it is also noted that the extrema occur for lower thickness values than found in the numerical analysis. Results for very low values of the thickness are not shown in [1]. The present analysis shows that the reflectivity for both the TE and TM cases approaches
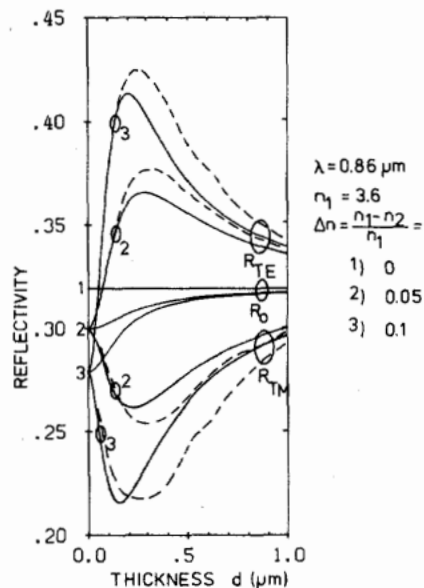

Fig. 1. Reflectivity of the fundamental mode as function of the thickness of the active layer and with the relative refractive index difference as parameter. Case 1 is the plane wave result. The broken curves are reproduced from [1].

the value obtained using n_2 in the plane wave formula; this is expected since the field in this case resembles a plane wave propagating in the passive layers.

In conclusion, an expression for the reflectivity of DH semiconductor lasers has been given using normalized waveguide parameters. The calculated results are in satisfactory agreement with results found from a numerical solution of the problem. In particular, a large deviation from the simple plane wave result for active layer thicknesses giving minimum threshold current density is accounted for. The formulas can easily be applied to laser structures with any values of wavelength and refractive indexes, i.e., DH lasers in the GaInAsP system.

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