Analytical Calculation of Collapse Voltage of CMUT Membrane

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***Abstract*— Because the collapse voltage determines the operating point of the capacitive micromachined ultrasonic transducer (CMUT), it is crucial to calculate and control this parameter. One approach uses parallel plate approximation, where a parallel plate motion models the average membrane displacement. This usually yields calculated collapse voltage 25 percent higher than the actual collapse voltage. More accurate calculation involves finite element method (FEM) analysis. However, depending on the required accuracy, the computation time may require many hours.**

In this paper, we propose a fast numerical algorithm for the calculation of collapse voltage. The algorithm uses the parallel plate method to approximate the force distribution over the membrane, and then applies an analytical solution for the plate equation, loaded by the approximated force distribution. Using this method, we are able to calculate the collapse voltage in a couple of seconds, within 0.1 percent accuracy. We report on the collapse voltage calculation results using our method for four different design structures. While computation time of our method is about three orders of magnitude less than the finite element method, the percentage error of collapse voltage calculation is, nevertheless, less than four percent in all the design structures. The proposed algorithm is also suitable for the inclusion of any external force distribution on the membrane, such as atmospheric pressure.

Keywords- Capacitive micromachined ultrasonic transducer, CMUT, collapse voltage.

I. INTRODUCTION

Collapse voltage of a capacitive micromachined ultrasonic transducer (CMUT) is a critical parameter for employing the device at the optimum operating point. The operating DC bias voltage determines the performance of the transducer. It also determines the operating regime at which the device is operated, such as conventional and collapsed mode [1]. Therefore, accurate knowledge of the collapse voltage is imperative.

Initial attempts to calculate collapse voltage depended on parallel plate approximation [2]. In this approach, a piston transducer models the membrane displacement, and the displacement profile of the membrane is neglected. The piston is held over an electrostatic gap by a spring whose compliance is determined by the average spring constant of the membrane. The collapse occurs when the electrostatic force gradient overcomes the gradient of mechanical restoring force exerted by the spring. The collapse occurs as soon as the displacement reaches one third of the gap. This method usually predicts a collapse voltage 30 to 40 percent greater than the actual value.

Finite Element Method (FEM) simulation is considered the most reliable and accurate method to compute the collapse voltage. On the other hand, FEM analysis requires iteration between electrostatic and mechanical solutions. Depending on the mesh size, FEM calculation may take several hours to complete.

In this paper, we present a semi-analytical method for the calculation of membrane displacement and collapse voltage. The method depends on the known solution of the equation of motion for the membrane. We assume that the membrane is clamped at the edges. The electrostatic gap is divided into many parallel plate capacitors. The electrostatic pressure is assumed to be constant within the each segment. Our method provides reasonably accurate results in much less time than FEM analysis.

II. DESCRIPTION OF THE METHOD

The proposed method, which uses vertical segmentation of the gap, an analytical solution of the motion for the membrane, and the method of superposition and iteration, is explained in the following sections.

A. Segmentation

To find the collapse voltage and displacement profile of the membrane, the electrostatic force between the two electrodes in the CMUT structure must be known. To approximate the electrostatic force acting on the membrane, we divide the electrostatic gap into several vertical segments. Each segment is modeled as a usual parallel plate capacitor. Provided that the number of segments is large enough, the electrostatic force is assumed to be constant over the each segment. Fig. 1 shows the cross section of a circular membrane with partial metallization. As shown in this figure, the region underneath the top electrode is divided into N segments. Due to the symmetry, only half of the membrane is shown. Note that each segment is a circular ring.

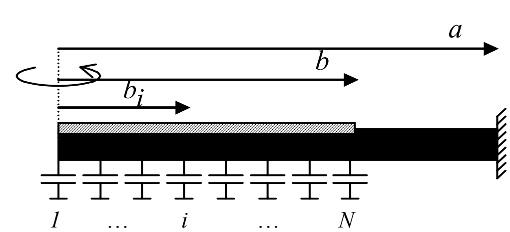

Figure 1. Method of segmentation applied on a circular membrane

The electrostatic force acting on the top electrode on each segment is given by

$$F_i = \frac{\varepsilon_0 V_{dc}{}^2 S}{2g_i^2} \tag{1}$$

where V_{dc} is the applied DC voltage between the two electrodes, S and g_i are the area and the effective gap height of the i^{th} capacitor, respectively.

This method of segmentation presents a set of electrostatic forces that, along with the analytical solution and the method of superposition, are used later on to determine the deflection profile of the membrane. In order to include the atmospheric pressure, and to ease programmatic implementation, the segmentation can extend to the whole gap. In this case, the electrostatic force will be zero over the non-metalized region of the membrane. Comparison with FEM simulations shows that a model with 100 segments over the whole membrane would be sufficient to calculate the collapse voltage accurately in a couple of seconds.

B. *Equation of motion for the membrane*

As stated before, our algorithm uses an analytical solution to the equation of motion for a membrane under electrostatic and atmospheric pressure. The governing equation for our method is a well-known equation from theory of plates. A plate is called "thin" if its thickness is at least one order of magnitude smaller than its span or diameter. If the deflection of a thin plate is small in comparison with its thickness, an acceptable approximate theory of bending of the plate can be developed. In the case of a circular plate, symmetrically loaded about the axis perpendicular to the plate through its center, the deflections are also circularly symmetric. Therefore, deflections of the points on the plate are simply a function of distance from the center of the plate, r. The resulting equation is [3]:

$$\nabla^2 D \nabla^2 w(r) = p \tag{2}$$

where w is the deflection of the plate and p is the pressure applied over the plate. D is the flexural rigidity of the plate and is given by

$$\frac{Eh^3}{12(1-\nu^2)} \tag{3}$$

where E is the modulus of elasticity of the material, h is the thickness of the plate and ν is Poisson's ratio. If flexural rigidity of the plate, D, is constant throughout the plate, the above equation becomes:

$$\nabla^4 w(r) = \frac{p}{D} \tag{4}$$

It is difficult to solve this equation for the general case in which the pressure changes continuously in the radial direction. However, having the deflections for the case of a load uniformly distributed along a concentric circle, any case of bending of a circular plate symmetrically loaded with respect to the center can be solved by using the method of superposition.

In our case, two types of pressure act on the membrane: electrostatic force and atmospheric pressure, which are both symmetrical with respect to the center of membrane. Using the method of segmentation explained above, we find a set of forces. If the number of segments is large enough, and therefore, the difference between the inner and outer radii of the ring-shape segment is infinitesimally small, each of these forces can be considered to be constant within the each segment, and can be modeled as it is applied on a concentric circle on the membrane. Therefore, we can solve the equation for the special case where there is simply a constant line load along a concentric circle on the plate. Because the mentioned governing equation is linear, we can then employ the method of superposition to find the total solution.

C. *Circular plate concentrically loaded*

As explained in the previous section, we need to solve the thin plate equation for the case of a circular plate of radius a, in which the load is uniformly distributed along a concentric circle of radius b, as shown in Fig. 2.

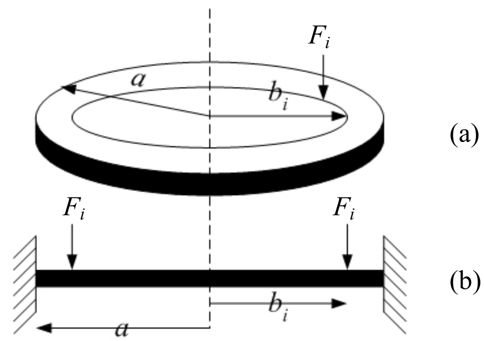

Figure 2. Circular clamped-edge plate concentrically loaded. (*a*) shows the side and top view, and (*b*) shows the side view of the plate.

Because we assume that the membrane is clamped at the edges, we must apply the clamped-edge boundary condition to the solution. The analytical closed-form solution for this problem is available [3]. The deflection profile for the outer portion of the plate is

$$w_i = \frac{F_i}{8\pi D}\left[\frac{(a^2-r^2)(a^2+b_i{}^2)}{2a^2} + (b_i{}^2+r^2)\ln\frac{r}{a}\right] \quad r > b_i \tag{5}$$

and for the inner portion,

$$w_i = \frac{F_i}{8\pi D}\left[\frac{(a^2+r^2)(a^2-b_i{}^2)}{2a^2} + (b_i{}^2+r^2)\ln\frac{b_i}{a}\right] \quad r \le b_i \tag{6}$$

Note that shearing stresses and normal pressures, acting on planes parallel to the surface of the plate, affect the bending of the plate. In the above solutions, we neglect these effects. The accuracy of these solutions depends on the thinness of the plate. In other words, the accuracy is dependent on the ratio of the thickness of the plate to the outer radius. The smaller this ratio, the more accurate is the solution.

D. Superposition and Iteration

With the analytical closed-form solution for the equation of motion of a concentrically loaded circular plate, and the linearity of the equation, we can use the method of superposition to find the total solution due to the total force distribution. We employ the analytical solution to find a closed-form solution for each of the forces, and then superimpose all the solutions.

Because the electrostatic force is dependent on the gap height, we need to iterate the solution. In the first iteration, the electrostatic force is determined over the non-deflected membrane. The atmospheric pressure is added to the electrostatic force, and with this set of forces, we calculate the first estimate of the displacement profile. The next force distribution is found by using the new deflection profile. These steps are repeated until the solution converges within an acceptable error tolerance. The displacement of the center of the membrane is used as a criterion to determine if the solution converges to a final result.

We used the geometry depicted in Fig. 3 to compare FEM and our calculation method. The membrane is under atmospheric pressure. We assumed the bottom electrode covers the surface of the substrate. In this case, the effect of parasitic capacitance is enhanced.

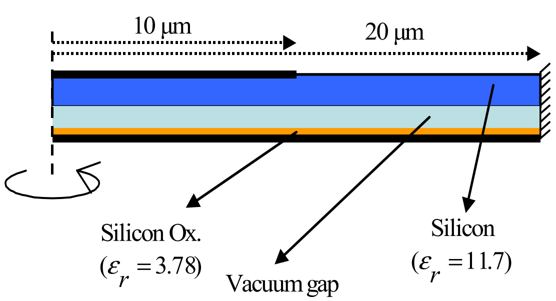

Figure 3. CMUT geometry. Top and bottom electrodes are shown by black lines and assumed to be infinitesimally thin in ANSYS calculation.

Fig. 4 compares the displacement profile of the middle plane of the membrane calculated using our method to the profile calculated using the FEM analysis. Although the computation time is significantly smaller for our algorithm, our result is analogous to the FEM simulation, within an acceptable error for practical purposes.

E. Collapse voltage

When the electrostatic force gradient overcomes the restoring mechanical force, the membrane will collapse onto the substrate.

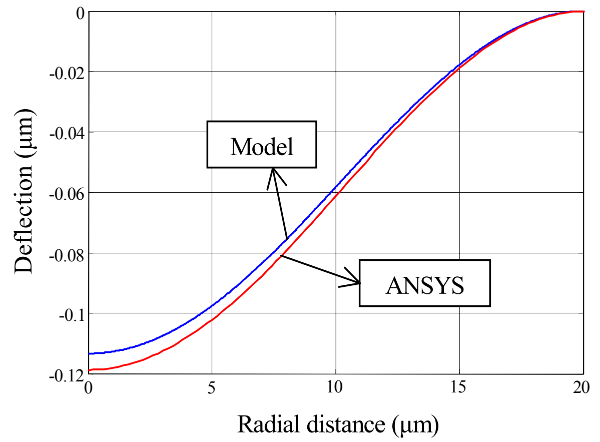

Figure 4. Displacement profile of the membrane. The bias voltage is 150V, which is less than the collapse voltage.

The collapse voltage is calculated using a binary search algorithm. As mentioned above, the displacement of the center of the membrane is used as a criterion to determine whether the solution converges or diverges. When the bias voltage is higher than the collapse voltage, the displacement of the center of the membrane diverges quickly. Fig. 5 compares the displacement of the center of the membrane versus number of iterations for different DC bias voltages.

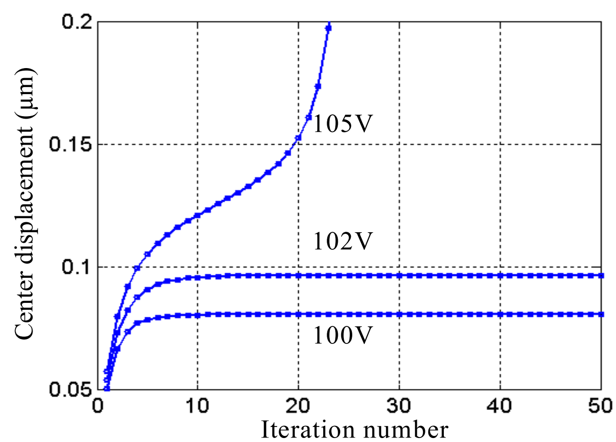

Figure 5. Center displacement as function of iteration number for various bias voltages. The collapse voltage is 104.5V.

According to these plots, in all the cases where center displacement converges to a finite value, the second derivative of displacement (with respect to the number of iterations) is always negative. For the divergence case, the second derivative of the displacement (with respect to the number of iterations) becomes zero at a point, and then positive afterwards. This observation can be used as a criterion to distinguish between convergence and divergence, and to decide whether the applied voltage is above or below the collapse voltage. Therefore, using the binary search, we can find the collapse voltage with the desired accuracy.

III. RESULTS

We compared the results of our algorithm with FEM simulation results, and found that 100 segments over the whole membrane are adequate to calculate the collapse voltage accurately. Our calculation took a few seconds; the same calculation using FEM analysis would take several hours.

Table 1 compares the result of our method to FEM simulations for four different design configurations. All of devices are made from Silicon Nitride (Si_3N_4), and have the same membrane thickness of 1µm, insulator layer thickness of 0.1µm, and gap distance of 1µm. In order to make a relatively broad comparison, we changed the radius of membrane, radius of top electrode, and location of top electrode. Location of top electrode refers to the relative location of top electrode with respect to the membrane: number "1" means that electrode is completely on top of the membrane; number "0" means that electrode has been placed underneath the membrane.

Table 1 shows that our results match the FEM simulation. For all design cases, the results obtained using our method are within 5% of those obtained with FEM analysis. However, the computation time required for our method was approximately three orders of magnitude less than for FEM analysis.

TABLE I. COMPARISON OF THE METHOD WITH ANSYS

Design parameters	Four different design structures			
	# 1	*# 2*	*# 3*	*# 4*
Radius of membrane (µm)	50	50	50	25
Radius of top electrode (µm)	50	10	50	25
Top electrode position	1	1	0	1
Our collapse voltage (volts)	166.7	352.5	131.8	637.7
FEM collapse voltage (volts)	164.8	350.3	126	630.4
Percentage error	1.15	0.64	4.6	1.16
Our computation time (sec)	~ 2	~ 2	~ 2	~ 4
FEM computation time (hours)	~ 4	~ 4	~ 4	~ 4

IV. CONCLUSION

We developed a semi-analytical method to calculate the displacement profile of a circular membrane. The same semi-analytical algorithm was employed to compute the collapse voltage of a circular CMUT membrane.

Our tool is extremely useful for the design of cMUT devices. By calculating the displacement profile, we can obtain other accurate and useful information, such as device capacitance and output pressure. . It is easy to include the atmospheric pressure or, in general, the pressure of the medium, as well as residual stresses. As shown in Table 1, the proposed algorithm is quickly calculated, and the accuracy is acceptable for most practical applications.

In future work, we will include the fringing capacitance, and also improve the accuracy of the algorithm by considering the effect of shearing stresses and lateral pressures on deflection of the membrane. We will also extend the same algorithm to any membrane shape, such as square and hexagonal, where the analytical solution exists.

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