where A_s is the area of the substrate under the gate, U_{tij} is the carrier generation rate per unit volume in the depletion region of the field-induced junction, and x_d is the depth of the depletion region. The quantity x_d is a unique function of V_g and is related to the space-charge capacitance (C_s) by

$$x_d = \kappa_s A_s / C_s, \tag{4}$$

where κ_s is the dielectric constant of the silicon layer. Because of (1), the value of C_s is frequency independent.[10] If the charge in the surface states does not follow the ac signal, C_s is related to C_g by[10]

$$C_g^{-1} = C_s^{-1} + C_{ox}^{-1}, \tag{5}$$

where C_{ox} is the geometrical capacitance of the insulator. Differentiating (3) and (4) with respect to V_g, and considering (2) and (5), we obtain

$$|U_{tij}(x_d)| = -A_s^2 q \kappa_s \frac{1}{C_g^2} \frac{\partial C_g}{\partial I_r}. \tag{6}$$

The generation centers in SOS could be due to the high density of dislocations caused by interfacial strain[11] and/or due to lifetime-killing impurities diffusing from sapphire into the growing film.[2] In both cases a carrier lifetime can be defined[9,10] which we write as[9]

$$\tau = n_i / 2|U_{tij}|, \tag{7}$$

where n_i is the intrinsic carrier concentration. Equations (6) and (7) are used to calculate the generation carrier lifetime from the C_g-V_g and I_r-V_g data.

Figure 2 shows the generation carrier lifetime as a function of x for different devices listed in Table I. The experimental results vary as $\exp(-ax)$ with $a \sim 10^7$ cm^{-1} and $a \sim 5\times10^6$ cm^{-1}, respectively. Using the depletion-layer approximation we determined the fixed space-charge density (N_B) from the C-V curve. The results are shown in the inset of Fig. 2. A decrease of N_B is observed which is consistent with results reported in the literature.[6]

Summarizing, we have described an experimental technique which is useful to determine the generation lifetime as a function of the distance of the insulator-semiconductor interface. This technique makes use of the gate-controlled diode and has been applied to 1-μ-thick silicon films on sapphire. It has been shown that over the region of the film investigated, the generation lifetime decreases monotonically as one moves from the SiO_2-silicon interface to the silicon-sapphire interface.

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Analytical considerations of Bragg coupling coefficients and distributed-feedback x-ray lasers in single crystals

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Expressions for the coupling coefficients characterizing Bragg x-ray scattering in single crystals are derived. These are used to obtain the threshold gain of a new type of x-ray distributed-feedback laser.

A number of investigators[1–4] have considered recently the problem of pumping atomic media to obtain laser amplification in the x-ray region. The problem of x-ray resonators for the amplifying media has also received attention.[5–7] The proposed resonance schemes involve optical paths folded into closed loops by Bragg reflections in a number of crystals.

In this paper we extend to the x-ray region the concept of distributed-feedback lasers.[8] These are lasers in which a spatially periodic perturbation provides feedback by *retro Bragg*[9] coupling between the forward- and backward-traveling waves. This feedback obviates the need for the conventional laser reflectors. The potential importance of this kind of feedback in semiconductor waveguide lasers has recently been demonstrated.[10,11]

In the x-ray region the natural periodicity of the electronic charge density in crystals provides strong Bragg coupling. The purpose of this paper is to derive expressions needed to evaluate the magnitude of this coupling and to characterize it. It is also shown that it should possible, in certain crystals, to satisfy the retro Bragg condition along a principal direction for the amplified $K\alpha_1$ radiation of one of the atomic constituents of the crystal. Such crystals when pumped above threshold should emit coherent radiation along these resonant Bragg directions.

The Bragg scattering of an incident x-ray wave of complex amplitude A_i from a set of lattice planes (hkl) is describable by means of coupled-mode equations similar to those used to describe scattering of light[12,13] from an index perturbation along the normal to the (hkl) planes:

$$\frac{dA_i}{dr_i} = \kappa_{is} A_s \exp\{+i[\mathbf{k}_i - \mathbf{k}_s - \mathbf{G}(hkl)]\cdot\mathbf{r}\}$$
$$\frac{dA_s}{dr_s} = \kappa_{si} A_i \exp\{-i[\mathbf{k}_i - \mathbf{k}_s - \mathbf{G}(hkl)]\cdot\mathbf{r}\}. \quad (1)$$

Here A_s is the scattered amplitude, $\mathbf{k}_i$ and $\mathbf{k}_s$ the incident and scattered propagation vectors, and $\mathbf{G}(hkl)$ is the reciprocal lattice vector normal to the planes (hkl). The coupling coefficient κ_{is} is proportional to the amplitude $a_{\mathbf{G}}$ of the index of refraction modulation along $\mathbf{G}(hkl)$:

$$n(\mathbf{r}) = \sum_{G(hkl)} a_{\mathbf{G}} \exp(i\mathbf{G}\cdot\mathbf{r}) \quad (2)$$

and is given by[14]

$$i\kappa_{is} = \kappa_{\mathbf{G}} = \omega\Delta n_{\mathbf{G}}/2c = \omega a_{\mathbf{G}}/c, \quad (3)$$

where ω is the x-ray radian frequency.

The Bragg condition

$$\mathbf{k}_i - \mathbf{k}_s = \mathbf{G}(hkl)$$

necessary, according to (1), for cumulative phase-matched interaction is equivalent to the conventional condition

$$2d\sin\theta = m\lambda, \quad m = 1, 2, \ldots$$

since the lattice spacing along $\mathbf{G}(hkl)$ is

$$d(hkl) = 2\pi/|G(hkl)|. \quad (4)$$

The index modulation represented by a single term in the summation of (2) is depicted, graphically, in Fig. 1.

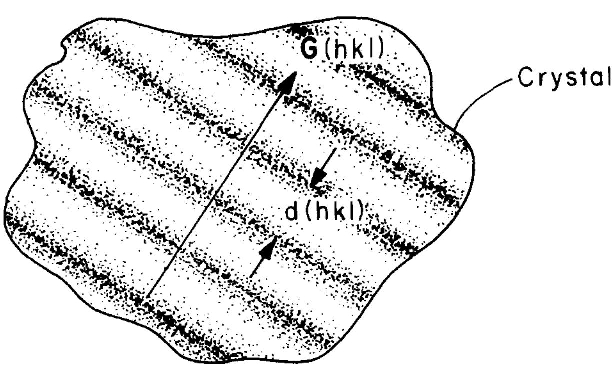

FIG. 1. The effective index modulation associated with the reciprocal lattice vector $\mathbf{G}(hkl)$.

It is thus clear that the design of x-ray components based on Bragg diffraction such as filters, reflectors, monochromators, and deflectors requires a knowledge of $a_{\mathbf{G}}$. One can then use published solutions[13] of Eqs. (1) for a variety of experimental situations.

To calculate the degree of index modulation represented by $a_{\mathbf{G}}$ we consider a crystal with a unit-cell volume V and N atoms per unit cell. The total electron density (i.e., the number of electrons in all the atomic shells, $K, L, M, \ldots,$ per unit volume) is N_0. We find by straightforward considerations, to be discussed separately, that

$$a_{\mathbf{G}} = (1/V)\int_V n(\mathbf{r})\exp(-i\mathbf{G}\cdot\mathbf{r})d^3\mathbf{r} \approx -(N_0e^2/2N\omega^2m\epsilon_0)S(hkl), \quad (5)$$

where the structure factor $S(hkl)$ is given by

$$S(hkl) = \sum_{i=1}^{N} f_i \exp[-i\mathbf{G}(hkl)\cdot\mathbf{r}_i]. \quad (6)$$

The summation is over the N atoms of the unit cell addresses $\mathbf{r}_i$. f_i is the scattering strength normalized such that $\sum_{i=1}^{N} f_i = N$. In obtaining the last two results we assumed that $n(\mathbf{r})$ is proportional to the electron charge density at r, that the electron charge density about the ith atom is of the form

$$q_i(\mathbf{r}) \propto \exp(-|\mathbf{r}-\mathbf{r}_i|^2/r_{0i}^2)$$

and that r_{0i} is small compared to the unit-cell dimensions.

To be specific let us consider the diamond or zincblende $(\bar{4}3m)$ class of crystals. For reasons that will become apparent below, this class appears to be a promising candidate for meeting the requirements of an x-ray Bragg laser.

In this class of crystals we have

$$S(100) = 0, \quad S(110) = 0, \quad (7)$$

$$S(111) = 4(1+i), \quad (8)$$

where in (8) we assumed for the sake of simplicity that the scattering strengths of the two constituent atoms are equal. Using (5) and (8) in (2) gives

$$\Delta n_{(111)}(\mathbf{r}) = -\frac{N_0e^2}{\sqrt{2}\,\omega^2 m\,\epsilon_0}\cos\left[\frac{2\pi\sqrt{3}}{a_0}\left(\xi - \frac{a_0}{8\sqrt{3}}\right)\right], \quad (9)$$

where ξ is the distance along the $\langle 111\rangle$ direction, a_0 is the dimension of the unit-cell cube. The last equation corresponds to an index modulation period of

$$d_{111} = a_0/\sqrt{3}$$

thus capable of providing Bragg feedback along the $\langle 111\rangle$ direction to a beam with a wavelength λ provided

$$a_0/\sqrt{3} = \tfrac{1}{2}\lambda. \quad (10)$$

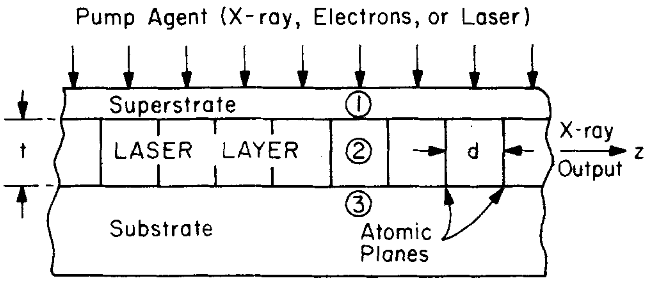

FIG. 2. An oriented single-crystal laser structure in which the atomic spacing d along the direction z satisfies the Bragg condition for retroscattering.

The zero equalities of (7) guarantee that radiation bouncing back and forth along $\langle 111\rangle$ is not scattered into the $\langle\bar{1}, 1, \bar{1}\rangle$, $\langle\bar{1}, 1, 1\rangle$, and $\langle\bar{1}, \bar{1}, 1\rangle$ directions so that a uniaxial minimal loss oscillation is possible.

As a specific example consider the use of the crystal GaP in the laser configuration sketched in Fig. 2. The inner layer 2 is an oriented single-crystal material with the $\langle 111\rangle$ direction parallel to the z axis of the figure. The upper and lower layers possess indices of refraction which are smaller than that of layer 2 so that total internal reflection can take place at the interface.[15] Layer 1 can be eliminated and one would, in that case, depend on grazing incidence from the vacuum interface.

The lattice spacing along $\langle 111\rangle$ is $d(111)=a_0/\sqrt{3}$ $=3.147$ Å. The resonant laser wavelength could be that of the $K\alpha_1$ transition in phosphorus at $\lambda=6.154$ Å. The Bragg condition (10) is thus satisfied to within 2.2%, i.e., $[d(111)-\lambda/2]/d(111)=2.2\times10^{-2}$. This discrepancy is too big to be tolerated. Two possible solutions are: (a) The control of the Bragg condition via the zig-zag angle of the rays in layer 2. For a layer thickness t approaching λ we need to modify (10) to

$$\frac{a_0}{\sqrt{3}}\left(1-\frac{\lambda^2 s^2}{8t^2}\right)=\frac{\lambda}{2}, \qquad (11)$$

where $s=1, 2, \ldots$ is a transverse mode index.[9] (b) In a mixed crystal system $Ga_{1-x}A_xP$ or $GaP_{1-x}B_x$ it should be possible to reduce the (average) value of the unit-cell dimension a_0 by controlling x.

The choice of zinc-blende-type crystals is thus justified on the basis of developed epitaxial thin-film growing techniques (GaAs, GaAlAs, ZnSe, GaP, CdTe, etc.), the "chemical" lattice-matching possibility in the mixed crystal systems, and the lack of (100) and (110) scattering.

If the crystal medium is capable of amplifying the retroscattered radiation at λ by virtue of a sufficient density of vacancies in the K shell of P, the system is that of a distributed-feedback laser.[8] The oscillation threshold gain is given by

$$\gamma_{th}=(1/L^3)(\pi/\kappa)^2, \qquad (12)$$

where L is the length of the amplifying section.

In the GaP example, $N_0=8.1\times10^{23}$ cm^{-3}, $\lambda=6.154$ Å so that from (9) and (3)

$$\Delta n(111)=1.72\times10^{-4}, \quad \kappa(111)=8.8\times10^{3}\ \text{cm}^{-1}.$$

The threshold gain constant is

$$\gamma_{th}=1.27\times10^{-1}\ \text{cm}^{-1}, \quad \text{for } L=10^{-2}\ \text{cm}$$

$$\gamma_{th}=1.27\times10^{-4}\ \text{cm}^{-1} \quad \text{for } L=10^{-1}\ \text{cm}.$$

These small numbers for the requisite gain are to be compared with the minimum value of $\gamma_{th}L\gtrsim1$ needed for the onset of superradiant narrowing or for laser action in resonant enclosures suggested to date.[5-7] Since the most difficult task in pumping x-ray lasers, however, is that of achieving the transparency condition (gain = loss), the reduced net gain of the Bragg laser compared to other proposed schemes will not lead to a drastic easing of the pumping burden. It does, however, provide a theoretical solution for the feedback problem which approaches the threshold requirement of a system with perfectly reflecting mirrors.

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