Progress In Electromagnetics Research, PIER 79, 119–136, 2008

ANALYTICAL INVESTIGATION AND EVALUATION OF PULSE BROADENING FACTOR PROPAGATING THROUGH NONLINEAR OPTICAL FIBERS (TRADITIONAL AND OPTIMUM DISPERSION COMPENSATED FIBERS)

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Abstract—In this paper, analytical relation for pulse width evolution and broadening in fiber systems using the Volterra series transfer function (VSTF) in linear and nonlinear cases are derived. This evaluation is done for traditional and optimum dispersion compensated fibers. Effects of group velocity dispersion (GVD) and self-phase modulation (SPM) are taken into account. It is shown that the analytical formulation can be applied to design and analysis the long hauls practical systems, and is helpful in understanding the pulse distortion caused by the interaction between SPM and GVD. The proposed relations are extracted analytically and for the first time pulse broadening factor in general case is derived.

1. INTRODUCTION

High-speed communications and computations are the main industrial and academic demands. Optical method is one of interesting alternatives for doing this purpose. Optical fiber is a physical medium for realization of optical communications which is the main method applied for high speed data communications. The main drawbacks of this physical medium are loss, dispersion and nonlinear effects.

Fiber nonlinearity has a great influence on the performance of long-haul optical fiber systems. Recently long transmission distance can be easily achieved by using erbium-doped fiber amplifiers [1, 2] and conventional dispersion compensators. Numerical methods have been widely used to study the fiber nonlinear effects because analytical method for analysis of the pulse propagation cannot be obtained except for the special cases of soliton transmission and the Volterra series approach. Numerical simulations provide accurate results, analytical expressions offer great advantages in estimation of various parameters based on their influence on system performance. The pulse broadening factor $T(z)/T_0$ defined as the ratio of optical output width to input pulse width, is widely used to evaluate the performance of intensity modulation direct-detection (IM/DD) systems. A closed-form analytical formula for $T(z)/T_0$ in a linear dispersive regime of fiber is given in [3]. Furthermore, approximate formulas with good accuracy for $T(z)/T_0$ in a uniform (fiber parameters do not vary along fiber length), lossless and nonlinear fiber with small positive group velocity dispersion (GVD) were derived in [4, 5]. Formulas for $T(z)/T_0$ in [6] and [7] are based on the idea that the effect of self-phase modulation (SPM) can be approximately represented by an effective intensity-dependent phase shift or frequency chirp of input pulses. Here, the expression of the effective phase shift should be determined first. However, how to rigorously derive this expression is not very clear, especially for chained optical amplifier systems with axially varying parameters of fiber dispersion including dispersion management, nonequal amplifier spacing and nonequal amplifier output power. In the paper, analytical formulas for pulse width evolution in optical communication systems are derived using the Volterra series transfer function [8]. It has been considered that the system may have axially nonuniform parameters and the fiber dispersion is not necessarily weak, hence the resulting formula is able to operate in a wide range of applications with satisfactory accuracy. Recently dispersion and nonlinear effects in optical fibers have been studied extensively [9–28]. Although different aspects of fibers have been considered but a low concentration on designing of the fiber using analytical formulation has been done. An expression for the effective phase shift caused by the SPM has been derived which is identical to one proposed by [7].

Organization of this paper is as follows.

In Section 2, the nonlinear Schrödinger equation governing the evolution of pulse in the dispersive and nonlinear fiber is given. This equation is solved by the Volterra series expansion. Simulation results are presented in Section 3. Finally the paper ends with a short conclusion.

2. MATHEMATICAL MODELING

In this section mathematical principle for pulse propagating through nonlinear fiber optic based on the Volterra series is presented. Usually for pulse propagation the nonlinear Schrödinger equation (NLS) is used which can be given as follows (Eq. (1)).

$$\frac{\partial A(t,z)}{\partial z} = -\frac{\alpha}{2}A(t,z) - \frac{j}{2}\beta_2(z)\frac{\partial^2 A(t,z)}{\partial t^2} + j\gamma\left|A(t,z)\right|^2 A(t,z) \quad (1)$$

The Fourier transform of Eq. (1) is:

$$\begin{aligned}\frac{\partial A(\omega,z)}{\partial z} &= \left[-\frac{\alpha}{2} + j\frac{\omega^2}{2}\beta_2(z)\right]A(\omega,z)\\ &\quad + \frac{j\gamma}{4\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A(\omega_1,z)A^*(\omega_2,z)A\left(\omega-\omega_1+\omega_2,z\right)d\omega_1 d\omega_2, \quad (2)\end{aligned}$$

where α, β_2, γ and A are fiber loss, the group velocity dispersion, nonlinear coefficient (Kerr coefficient) and amplitude of the electric field respectively.

Based on the Volterra series [7–9] the following solution can be applied for Eq. (2).

$$\begin{aligned}A(\omega,z) &= H_1(\omega,z)A(\omega,0)\\ &\quad + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H_3\left(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z\right)A(\omega_1,0)\\ &\quad\quad A^*\left(\omega_2,0\right)A\left(\omega-\omega_1+\omega_2,0\right)d\omega_1 d\omega_2, \quad (3)\end{aligned}$$

where H_1 and H_3 are the first and the third order Kernels of the Volterra series. After substituting the proposed solution into Eq. (2)

the following differential equations for the kernels are obtained.

$$\frac{\partial H_1(\omega,z)}{\partial z} = G_1(\omega,z)H_1(\omega,z), \tag{4}$$

$$\begin{aligned}\frac{\partial H_3}{\partial z}(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z) &= G_1(\omega,z)H_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z)\\ &\quad+G_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2)H_1(\omega_1,z)\\ &\quad H_1^*(\omega_2,z)H_1(\omega-\omega_1+\omega_2,z), \end{aligned}\tag{5}$$

where $G_1(\omega,z) = -\frac{\alpha}{2} + j\frac{\omega^2}{2}\beta_2(z)$ and $G_3(\omega_1,\,\omega_2,\,\omega-\omega_1+\omega_2) = j\left(1+\frac{\omega}{\omega_0}\right)[a_0 + Q_R S_R(\omega_1-\omega_2)]$ is the fiber nonlinear kernel [7–9].

The first order differential equations (4) and (5) can be solved using conventional techniques in this field. For this purpose the following factor of integration is defined.

$$\lambda = e^{\int_0^z\left(-\frac{\alpha}{2}+j\frac{\omega^2}{2}\beta_2(z)\right)dz} = e^{\left[\frac{\alpha}{2}z-j\frac{\omega^2}{2}\int_0^z\beta_2(z)dz\right]} \tag{6}$$

Considering the following boundary conditions first and third order kernels can be obtained as follows.

$$\begin{aligned} H_1(\omega,0) &= 1\\ H_3(\omega_1,\omega_2,\omega-\omega_1-\omega_2,0) &= 0 \end{aligned}$$

$$H_1(\omega,z) = e^{\left[-\frac{\alpha}{2}z+j\frac{\omega^2}{2}\int_0^z\beta_2(z)dz\right]} \tag{7}$$

$$\begin{aligned} H_3(\omega_1,\omega_2,\omega-\omega_1+\omega_2,z) &= e^{\left[-\frac{\alpha}{2}z+j\frac{\omega^2}{2}\int_0^z\beta_2(z)dz\right]}\\ &\quad\times\frac{j\gamma}{4\pi^2}\int_0^z e^{\left[-\alpha z+j\left(\omega_1^2-\omega(\omega_1-\omega_2)-\omega_1\omega_2\right)\int_0^z\beta_2(z)dz\right]}dz \end{aligned}\tag{8}$$

Therefore, the solution of the NLS equation in frequency domain is given as:

$$\begin{aligned} A(\omega,z) &= e^{\left[-\frac{\alpha}{2}z+j\frac{\omega^2}{2}\int_0^z\beta_2(z)dz\right]}A(\omega,0) + e^{\left[-\frac{\alpha}{2}z+j\frac{\omega^2}{2}\int_0^z\beta_2(z)dz\right]}\\ &\quad\times\frac{j\gamma}{4\pi^2}\int_0^z e^{-\alpha z}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{\left[j\left(\omega_1^2-\omega(\omega_1-\omega_2)-\omega_1\omega_2\right)\int_0^z\beta_2(z)dz\right]}\right.\\ &\quad\left.\times A(\omega_1,0)A^*(\omega_2,0)A(\omega-\omega_1+\omega_2,0)d\omega_1 d\omega_2\right\}dz \end{aligned}\tag{9}$$

Now, we consider a Gaussian pulse profile as an example:

$$A(t,0) = a_0 e^{\left(-\frac{t^2}{T_0^2}\right)}$$

The Fourier Transform of the Gaussian pulse is:

$$A(\omega,0) = A_0 e^{\left(-\frac{\omega^2 T_0^2}{2}\right)},$$

where $A_0 = \sqrt{2\pi} T_0 a_0$.

Therefore, the solution of the NLS equation is:

$$\begin{aligned} A(\omega,z) &= e^{\left[-\frac{\alpha_0}{2} z + j\frac{\omega^2}{2}\int_0^z \beta_2(z)dz\right]} A_0 e^{\left(-\frac{\omega^2 T_0^2}{2}\right)} + e^{\left[-\frac{\alpha_0}{2} z + j\frac{\omega^2}{2}\int_0^z \beta_2(z)dz\right]} \\ &\quad \times \frac{j\gamma}{4\pi^2}\int_0^z e^{-\alpha z}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{\left[j\left(\omega_1^2-\omega(\omega_1-\omega_2)-\omega_1\omega_2\right)\int_0^z \beta_2(z)dz\right]}\right. \\ &\quad \left.\times A_0^3 e^{\left(-\frac{\omega_1^2 T_0^2}{2}\right)} e^{\left(-\frac{\omega_2^2 T_0^2}{2}\right)} e^{\left(-\frac{(\omega-\omega_1-\omega_2)^2 T_0^2}{2}\right)} d\omega_1 d\omega_2\right\} dz \end{aligned}$$

For simplicity the following definition is introduced.

$$B \triangleq \int_0^z \beta_2(z)dz \tag{10}$$

For the Gaussian input pulse the intensity is given by:

$$\begin{aligned} I(z) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{\left[j\left(\omega_1^2-\omega(\omega_1-\omega_2)-\omega_1\omega_2\right)B\right]} \\ &\quad \times e^{\left(-\frac{\omega_1^2 T_0^2}{2}\right)} e^{\left(-\frac{\omega_2^2 T_0^2}{2}\right)} e^{\left(-\frac{(\omega-\omega_1-\omega_2)^2 T_0^2}{2}\right)} d\omega_1 d\omega_2 \end{aligned}$$

After some mathematical simplification the final form is derived as follows.

$$I(z) = \frac{2\pi\sigma_0^2\, e^{\left(-\frac{\omega^2}{2\sigma_0^2}\right)}}{\sqrt{\sigma_0^4 B + 3 - j\,6\sigma_0^2 B}}\, e^{\left[-\frac{j\,80\sigma_0^6 B^3 - 112\sigma_0^4 B^2 - j\,48\sigma_0^2 B + 16}{j\,16\sigma_0^8 B^3 + 80\sigma_0^6 B^2 + j\,144\sigma_0^4 B - 48\sigma_0^2}\,\omega^2\right]}$$

where $\sigma_0 = \frac{1}{T_0}$.

Using mathematical tools, amplitude of the electric field in frequency domain is obtained as follows.

$$A(\omega,z) = A_0 e^{\left(-\frac{\alpha}{2}z\right)} e^{\left(-\frac{1-j\sigma_0^2 B}{2\sigma_0^2}\omega^2\right)} + e^{\left(-\frac{\alpha}{2}z+j\frac{\omega^2}{2}B\right)} \frac{j\gamma}{4\pi^2} A_0^3 \int_0^z e^{-\alpha z} I(z)dz \tag{11}$$

Now, for simplicity we consider three cases.

A- Linear case ($\gamma = 0$)

B- Nonlinear case ($\beta_2(z) = 0$)

C- Linear and Nonlinear case simultaneously

A- Linear case ($\gamma = 0$),

In this case effect of dispersion is only considered. Considering Eq. (11) and inserting $\gamma = 0$, the following field amplitude is obtained.

$$A(\omega,z) = A_0 e^{\left(-\frac{\alpha}{2}z\right)} e^{\left(-\frac{\left[1-j\sigma_0^2\int_0^z \beta_2(z)dz\right]T_0^2}{2}\omega^2\right)} \tag{12}$$

Using the inverse Fourier transform the following field in time domain is derived.

$$A(\omega,z) = A_0 e^{\left(-\frac{\alpha}{2}z\right)} e^{\left(-\frac{t^2}{2\left[1-j\sigma_0^2\int_0^z \beta_2(z)dz\right]T_0^2}\right)}$$

Using standard methods for calculation of root mean square the width of propagating pulse at arbitrary distance can be calculated as follows. For this purpose the variance of electric field is defined as follows.

$$Var = \left\langle A^2 \right\rangle - \langle A \rangle^2 \tag{13}$$

Also, in the following average value in general case is given.

$$A^p = \frac{\int_{-\infty}^{+\infty} t^p |A(t,z)|^2 dt}{\int_{-\infty}^{+\infty} |A(t,z)|^2 dt}$$

where P is integer number.

For this case after mathematical simplification variance of the electric field in linear case and finally the pulse broadening factor in position dependence compensated fibers are obtained.

$$\begin{aligned}
Num &= \int_{-\infty}^{+\infty} t^2 \left|A\left(t,z\right)\right|^2 dt = a_0^2 e^{-\alpha z} \int_{-\infty}^{+\infty} t^2 e^{\left(-\frac{t^2}{T_0^2\left(1+T_0^4 B^2\right)}\right)} dt \\
&= \frac{\sqrt{\pi}}{2} a_0^2 e^{-\alpha z} T_0^3 \left(1+T_0^{-4}B^2\right)^{\frac{3}{2}} \\
Den &= \int_{-\infty}^{+\infty} \left|A\left(t,z\right)\right|^2 dt = a_0^2 e^{-\alpha z} \int_{-\infty}^{+\infty} e^{\left(-\frac{t^2}{T_0^2\left(1+T_0^4 B^2\right)}\right)} dt \\
&= \sqrt{\pi} a_0^2 e^{-\alpha z} T_0 \left(1+T_0^{-4}B^2\right)^{\frac{1}{2}} \\
Var(z) &= \frac{Num}{Den} = \frac{T_0^2}{2}\left(1+T_0^{-4}B^2\right) \qquad (14) \\
Var(z) &= \frac{T(z)}{\sqrt{2}} \\
\frac{T(z)}{T_0} &= \left[1+\frac{1}{T_0^4}\left(\int_0^z \beta_2(z)dz\right)^2\right]^{\frac{1}{2}} \qquad (15)
\end{aligned}$$

B- Nonlinear case ($\beta_2(z) = 0$)

In this case only the effect of nonlinear term is considered. Considering Eq. (11) with $\beta_2(z) = 0$, the following field amplitude is obtained.

$$A(\omega,z) = A_0 e^{-\frac{\alpha}{2}z} e^{-\frac{T_0^2}{2}\omega^2} + \frac{j\gamma}{4\pi^2} e^{-\frac{\alpha}{2}z} L_{eff} \frac{2\pi}{\sqrt{3}T_0^2} A_0^3 e^{-\frac{T_0^2}{6}\omega^2} \qquad (16)$$

Using inverse Fourier transform the following form is obtained in time domain.

$$A(t,z) = a_0 e^{-\frac{\alpha}{2}z}\left[e^{-\frac{t^2}{2T_0^2}} + \frac{j\gamma L_{eff} a_0^2}{\sqrt{3}} e^{-\frac{t^2}{6T_0^2}}\right]$$

After some mathematical calculations the following relation for variance and then for pulse broadening factor is obtained.

$$Num = \int_{-\infty}^{+\infty} t^2 \left|A\left(t,z\right)\right|^2 dt = a_0^2 e^{-\alpha z} \int_{-\infty}^{+\infty} t^2 \left[e^{-\frac{t^2}{T_0^2}} + \frac{\gamma^2 L_{eff}^2 a_0^4}{3} e^{-\frac{t^2}{3T_0^2}}\right] dt$$

$$= a_0^2 e^{-\alpha z}\frac{\sqrt{\pi}}{2}T_0^3\left[1+\frac{\gamma^2 L_{eff}^2 a_0^4}{9\sqrt{3}}\right]$$

$$Den = \int_{-\infty}^{+\infty}|A(t,z)|^2\,dt = a_0^2 e^{-\alpha z}\int_{-\infty}^{+\infty}\left[e^{-\frac{t^2}{T_0^2}}+\frac{\gamma^2 L_{eff}^2 a_0^4}{3}e^{-\frac{t^2}{3T_0^2}}\right]dt$$

$$= a_0^2 e^{-\alpha z}\sqrt{\pi}T_0\left[1+\frac{\gamma^2 L_{eff}^2 a_0^4}{3\sqrt{3}}\right]$$

$$Var(z) = \frac{Num}{Den} = \frac{T_0^2}{2}\left[\frac{1+\dfrac{\gamma^2 L_{eff}^2 a_0^4}{9\sqrt{3}}}{1+\dfrac{\gamma^2 L_{eff}^2 a_0^4}{3\sqrt{3}}}\right] \tag{17}$$

$$\frac{T(z)}{T_0} = \left[\frac{1+\dfrac{1}{9\sqrt{3}}\phi_{\max}^2}{1+\dfrac{1}{3\sqrt{3}}\phi_{\max}^2}\right]^{\frac{1}{2}} \tag{18}$$

where $\phi_{\max}\triangleq\gamma P_0 L_{eff}$.

C- Linear and nonlinear case simultaneously

In this case linear and nonlinear effects simultaneously are considered. The following electric field is used according to analytical solution of the NLS equation.

$$A(\omega,z) = A_0 e^{-\frac{\alpha}{2}z}\left[e^{-\frac{\left(1-j\sigma_0^2 B\right)}{2\sigma_0^2}\omega^2}+\frac{j\gamma A_0^2}{2\pi}f(z)e^{-\frac{\left(11-j\sigma_0^2 B\right)}{2\sigma_0^2}\omega^2}\right], \tag{19}$$

where $f(z)=\int\limits_0^z \frac{e^{-\alpha\zeta}}{\int_0^z \beta_2(\zeta)d\zeta}d\zeta$.

Using inverse Fourier transform the time dependent electric field is obtained.

$$A(t,z) = a_0\,e^{-\frac{\alpha}{2}z}\left[e^{-\frac{\sigma_0^2}{2\left(1-j\sigma_0^2 B\right)}t^2}+j\gamma a_0^2 T_0^2 f(z)e^{-\frac{\sigma_0^2}{2\left(11-j\sigma_0^2 B\right)}t^2}\right]$$

Also, the variance and consequently the pulse broadening factor using some simplification assumption are given. It has been assumed that

$$\sigma_0^2 B \gg 1$$

and

$$e^{-\dfrac{6\sigma_0^2\left(11+\sigma_0^4B^2\right)}{\left(11+\sigma_0^4B^2\right)^2+100\sigma_0^4B^2}t^2} \approx e^{-\frac{6}{\sigma_0^2B^2}t^2}$$

$$\cos\left(\frac{10\sigma_0^2B}{\left(11+\sigma_0^4B^2\right)^2+100\sigma_0^4B^2}\right) \approx 1$$

$$|A(t,z)|^2 = a_0^2e^{-\alpha z}\left[\begin{array}{l} e^{-\frac{\sigma_0^2}{1+\sigma_0^4B^2}t^2}+\gamma^2a_0^4T_0^4f^2(z)e^{-\frac{11\sigma_0^2}{121+\sigma_0^4B^2}t^2} \\ +2\gamma\, a_0^2T_0^2\, f\,(z)\; e^{-\frac{6}{\sigma_0^2B^2}t^2} \end{array}\right]$$

$$Num = a_0^2e^{-\alpha z}\int_{-\infty}^{+\infty}t^2\left[\begin{array}{l} e^{-\frac{\sigma_0^2}{1+\sigma_0^4B^2}t^2}+\gamma^2a_0^4T_0^4f^2(z)e^{-\frac{11\sigma_0^2}{121+\sigma_0^4B^2}t^2} \\ +2\gamma a_0^2T_0^2f(z)e^{-\frac{6}{\sigma_0^2B^2}t^2} \end{array}\right]dt$$

$$Den = a_0^2e^{-\alpha z}\int_{-\infty}^{+\infty}\left[\begin{array}{l} e^{-\frac{\sigma_0^2}{1+\sigma_0^4B^2}t^2}+\gamma^2a_0^4T_0^4f^2(z)e^{-\frac{11\sigma_0^2}{121+\sigma_0^4B^2}t^2} \\ +2\gamma a_0^2T_0^2f(z)e^{-\frac{6}{\sigma_0^2B^2}t^2} \end{array}\right]dt$$

$$Var(z) = \frac{Num}{Den}$$

$$= \frac{T_0^2}{2}\left[\frac{\left(1+\sigma_0^4B^2\right)^{\frac{3}{2}}+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{11\sqrt{11}}\left(121+\sigma_0^4B^2\right)^{\frac{3}{2}}+4\gamma a_0^2\sigma_0^4f(z)B^3}{\left(1+\sigma_0^4B^2\right)^{\frac{1}{2}}+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{\sqrt{11}}\left(121+\sigma_0^4B^2\right)^{\frac{1}{2}}+2\gamma a_0^2f(z)B^3}\right] \tag{20}$$

$$\frac{T(z)}{T_0} = \left[\frac{\left(1+T_0^{-4}B^2\right)^{\frac{3}{2}}+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{11\sqrt{11}}\left(121+T_0^{-4}B^2\right)^{\frac{3}{2}}+4\gamma a_0^2T_0^{-4}f(z)B^3}{\left(1+T_0^{-4}B^2\right)^{\frac{1}{2}}+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{\sqrt{11}}\left(121+T_0^{-4}B^2\right)^{\frac{1}{2}}+2\gamma a_0^2f(z)B}\right]^{\frac{1}{2}} \tag{21}$$

To obtain simpler relation, one may accept $T_0^{-2}B \gg 1$, then the following pulse broadening factor is obtained.

$$\frac{T(z)}{T_0} = T_0^{-2}B\left[\frac{1+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{11\sqrt{11}}+4\gamma a_0^2T_0^2f(z)}{1+\dfrac{\gamma^2a_0^4T_0^4f^2(z)}{\sqrt{11}}+2\gamma a_0^2T_0^2f(z)}\right]^{\frac{1}{2}} \tag{22}$$

The derived analytical relations in this section are useful for study of the pulse propagation in high power or multi-channel communication cases such as wavelength division multiplexing systems and networks. In the next section simulated results are presented and discussed.

3. SIMULATION RESULTS

In this section based on derived relations in previous section, we present some simulations about the pulse broadening factor for light propagation through nonlinear traditional fiber optic, partially and optimum dispersion compensated as well.

Based on the Volterra series analytical solution of the NLS equation for light propagating through optical fiber for same output pulse as input one the following group velocity dispersion is required. On the other hand the following curve is optimum GVD for pulse propagation without any perturbation in shape. The following curve is non-periodic solution of the NLS equation for optimum operation (output pulse is exactly the input one).

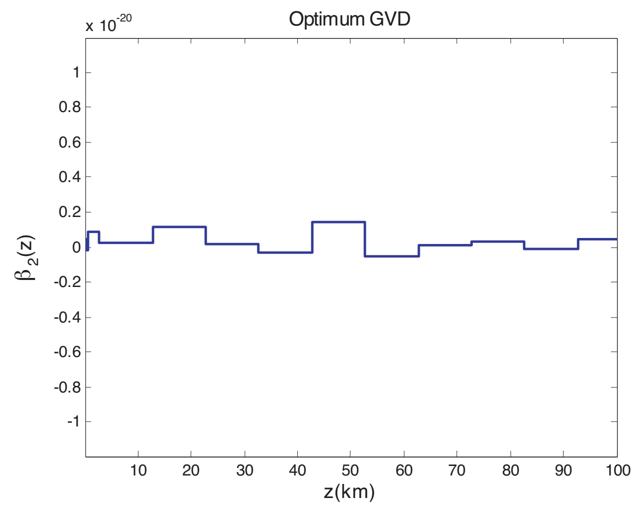

Figure 1. Optimum dispersion profile of optical fiber vs. distance.

For this case the following curve is illustrated to demonstrate the pulse propagation through optimum dispersion compensated media (optimum GVD). It is shown that the input pulse without any perturbation is transmitted through the fiber. The obtained dispersion profile can be realized in practice using selective type and density of doping. As it is shown the output pulse is exactly same as input one.

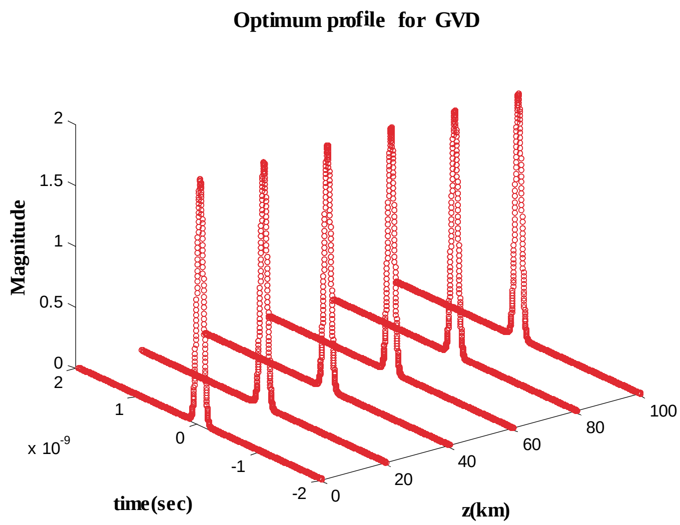

Figure 2. Gaussian pulse propagation through fiber with optimum dispersion of Fig. 1. $n_2 = 6 \times 10^{-13}$, $\alpha = 0.2\,\frac{\mathrm{dB}}{\mathrm{km}}$, $P_0 = 4\,\mathrm{mW}$, $t_0 = 50\,\mathrm{Psec}$.

Also, in previous section our proposed relations show this fact.

In the following figure (Fig. 3) pulse propagation through dispersion compensated by rectangular GVD profile optical fiber is illustrated. It is observed that the propagated pulse is broadened through propagation. On the other hand other profiles for dispersion compensation can't produce output pulse exactly same as input. In the following it is shown.

The evolution of a Gaussian pulse in a dispersion compensated fiber with rectangular GVD profile (not optimum GVD) while nonlinear effect is ignored is demonstrated in Fig. 3. It is shown that the propagating pulse is broadened and the result exactly same as calculated result by derived formula in previous section.

In the following figure (Fig. 4) pulse broadening factor for different values of GVD parameter is illustrated. It is observed that with increasing of the GVD parameter the output pulse width is increased. For description of these results the introduced relations in previous section can be used. The illustrated figure considered only dispersion effect and nonlinear properties are ignored.

Effects of linear and nonlinear phenomena on the pulse broadening factor are considered. It is shown that for given GVD, initial pulse width and light power in the case of linear effects the pulse width is broadened fast compared nonlinear case. This is related to

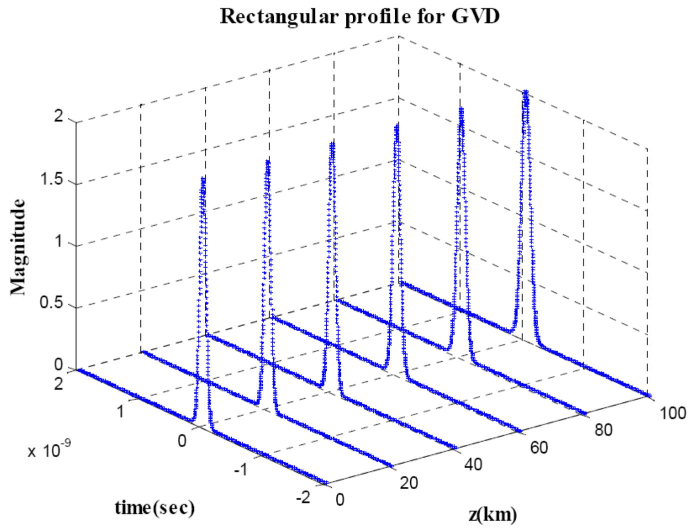

Figure 3. Gaussian pulse propagation through dispersion compensated optical fiber with rectangular GVD profile. $n_2 = 6 \times 10^{-13}$, $\alpha = 0.2\,\frac{\text{dB}}{\text{Km}}$, $P_0 = 4\,\text{mW}$, $t_0 = 50\,\text{Psec}$, $\beta_2 = 25\,\text{P}\sec^2/\text{Km} \cdot \text{nm}$.

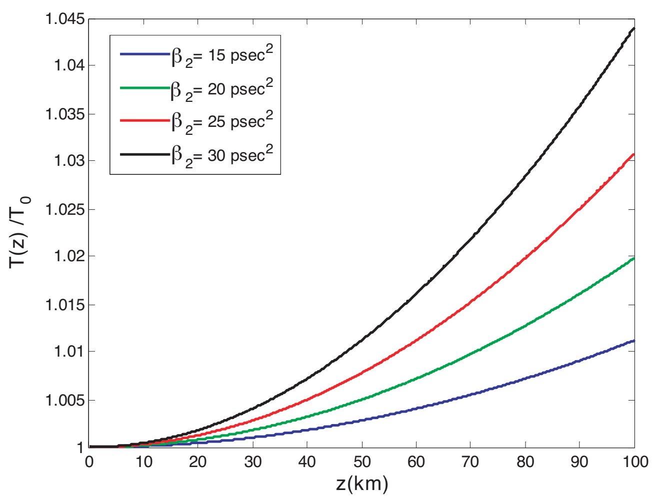

Figure 4. Pulse broadening factor Vs distance for different GVD parameter. $\alpha = 0.2\,\frac{\text{dB}}{\text{Km}}$, $P_0 = 1\,\text{mW}$, $t_0 = 1\,\text{nsec}$, $\beta_2 = 20\,\text{P}\sec^2/\text{Km} \cdot \text{nm}$.

pulse compression properties of nonlinear effects such as self phase modulation (SPM).

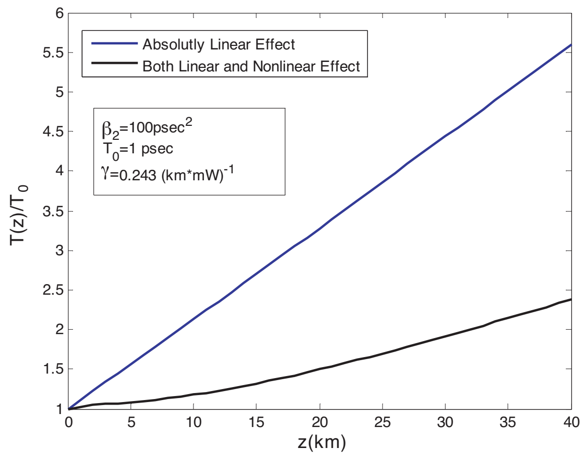

Figure 5. Pulse broadening factor Vs distance for linear and nonlinear effects. $n_2 = 6 \times 10^{-12}$, $\alpha = 0.2\,\frac{\text{dB}}{\text{Km}}$, $P_0 = 1\,\text{mW}$, $t_0 = 1\,\text{Psec}$, $\beta_2 = 100\,\text{P}\sec^2/\text{Km}\cdot\text{nm}$.

In the following simulated curve the pulse broadening due to nonlinear effects only is illustrated. In this curve different parameter is considered. It is observed that with increasing the input power the pulse width is decreased and saturated in small distances. In this case we concentrated on nonlinear behavior of the fiber.

Effect of the input power on the pulse broadening in nonlinear regime is investigated and result illustrated in Fig. 7. It is observed that with increasing the input power the pulse width is broadened faster and in small distance the saturation case occurs.

Effect of chirp on pulse broadening in linear regime is illustrated in Fig. 8. In the case of negative chirp, the pulse broadening is negative first and then going to increase.

Finally, the pulse shape after 100 km propagation through optical fiber of Gaussian input pulse for optimum and rectangular dispersion compensation is illustrated in Fig. 9. It is shown that in the case of optimum dispersion compensation output pulse is exactly the same as input pulse.

In this section some of obtained results in previous section were illustrated graphically. It is shown that in optimum dispersion compensated fibers (Fig. 1), pulse propagates without distortion. Analytical relations for pulse width broadening in general case estimate the nature of pulse propagation in nonlinear optical fibers.

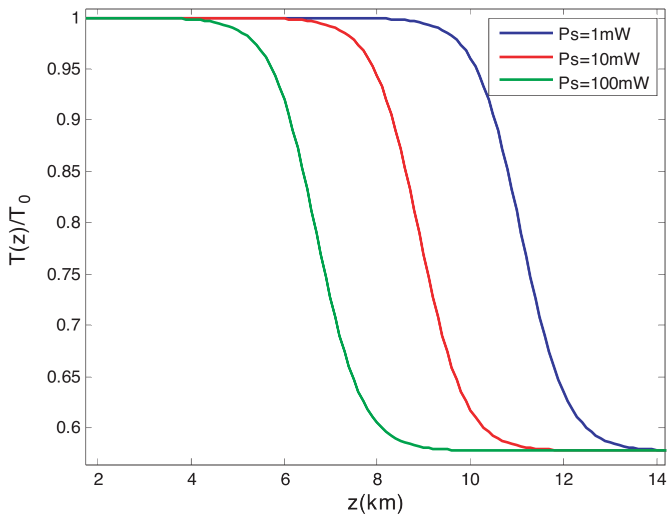

Figure 6. Pulse broadening factor Vs distance (effect of nonlinear behavior). $n_2 = 6 \times 10^{-12}$, $\alpha = 0.2\,\frac{\mathrm{dB}}{\mathrm{Km}}$.

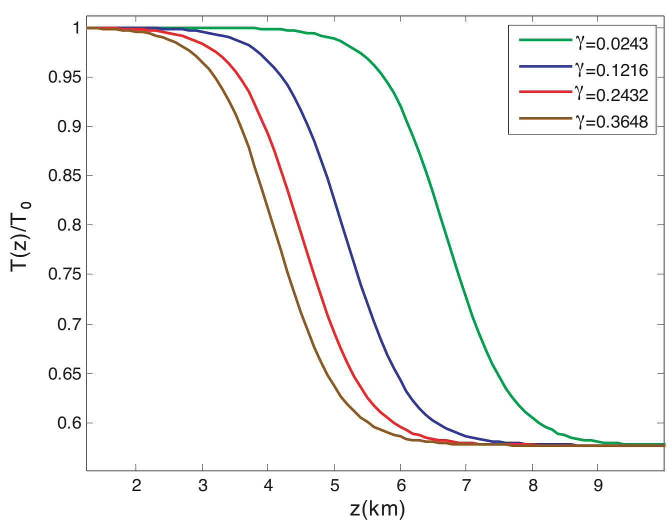

Figure 7. Pulse broadening factor Vs fiber length for different fiber nonlinear factor. $P_0 = 1\,\mathrm{mW}$, $\alpha = 0.2\,\frac{\mathrm{dB}}{\mathrm{Km}}$.

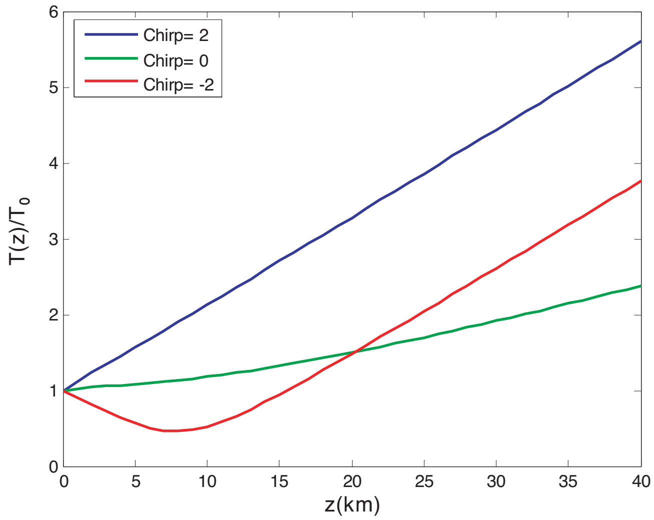

Figure 8. Pulse broadening factor Vs distance (effect of input chirp). $n_2 = 3 \times 10^{-13}$, $\alpha = 0.2\,\frac{\text{dB}}{\text{Km}}$, $P_0 = 4\,\text{mW}$, $\beta_2 = 25\,\text{P}\sec^2/\text{Km}\cdot\text{nm}$.

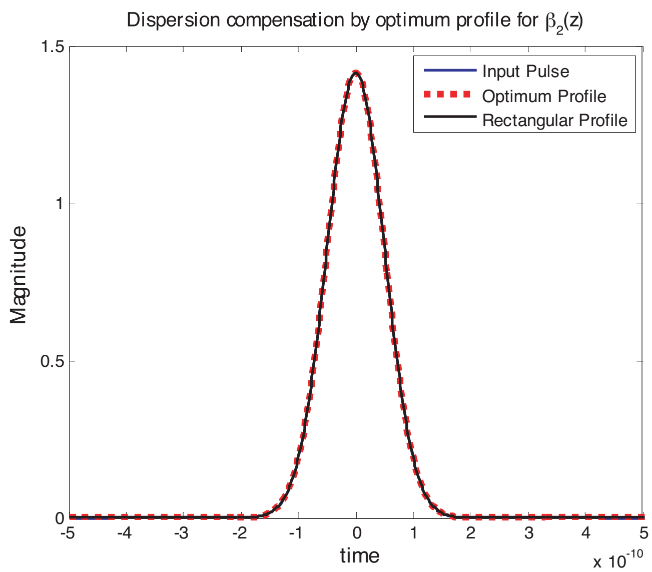

Figure 9. Pulse amplitude Vs time for different profile of dispersion compensation. $\beta_{2\,\text{Rectangular}} = 25\,\text{p}\sec^2/\text{Km}\cdot\text{nm}$, $\alpha = 0.2\,\frac{\text{dB}}{\text{Km}}$.

4. CONCLUSION

In this paper we have obtained analytical relation for pulse broadening factor base on the Volterra series. The proposed relations, for the first time from our point of view, analytically predict the pulse propagation through optical fiber incorporating linear and nonlinear effects. Some simulation results were presented to illustrate ability of derived relations.

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