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ANALYTICAL INVESTIGATION OF THE EFFECT OF COOLING AIR ON TWO-STAGE TURBINE PERFORMANCE

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Cleveland, Ohio

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ABSTRACT

Turbine efficiency is determined as a function of inlet temperature by using the isolated-flow analytical method. The parameters that are varied are coolant total-pressure coefficient, coolant to inlet temperature ratio, and velocity-diagram type. The general results are used to determine the efficiency-temperature trend for two example turbine applications.

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SUMMARY

The effect of cooling air on turbine performance was obtained by using a reference analytical procedure. This effect is depicted as the variation in primary air efficiency as a function of turbine inlet temperature. The range of turbine inlet temperature considered was from 2300° to 3500° R (1278 to 1944 K). Three types of two-stage turbine models were used in this study, each of different aerodynamic severity. The turbines were investigated with coolant to inlet temperature ratios of 0.3, 0.45, and 0.6 and with four values of coolant total-pressure coefficient. Coolant flow schedules were obtained by using relations appropriate to convectively cooled blades.

The trend of efficiency with turbine inlet temperature was shown to be positive, almost zero, or negative depending on coolant to inlet temperature ratio and coolant pressure coefficient. Turbine efficiency increased with increasing values of coolant pressure coefficient because of the increased capacity of the coolant flows to produce useful work output. Turbine efficiency also increased with increasing values of coolant to inlet temperature ratio. The addition of coolant was shown to have a more positive effect on efficiency as the aerodynamic severity of the turbine was increased.

The efficiency variation with turbine inlet temperature was also determined for two example applications, in which the rotative speed, coolant temperature, and uncooled turbine-work output were fixed. The effect on efficiency was shown to be considerably altered from that given by the general-result curves. In these curves the coolant temperature, uncooled turbine-work output, and the square of the blade speed were all varied in proportion to turbine inlet temperature. It was also shown that the results for the example applications could be satisfactorily obtained by interpolation from the general-result curves.

INTRODUCTION

The higher engine-cycle temperatures employed in some advanced aircraft applications necessitate cooling the turbine blading. The cooling method most commonly considered uses air that is bled from the compressor, ducted through the turbine-blade cooling passages, and then discharged into the turbine gas stream. The turbine-work output is affected by the amount of cooling air required by each blade row and the total pressure and total temperature of the coolant in relation to that of the gas stream. The determination of the turbine-work output with coolant flow represents input information that is required to analyze high-temperature engines.

Two analytical methods of determining turbine performance as affected by the addition of coolant flow are described in reference 1, and referred to therein as the mixed-flow and isolated-flow methods. In the mixed-flow analytical method, the coolant flow is assumed to be completely mixed with the gas stream in the injection blade row. For the isolated-flow analytical method, the coolant flow and primary flow are assumed to be entirely uninfluenced by one another. The two methods were applied to two representative types of turbine over a range of arbitrarily assumed coolant flows. The performance results obtained using the two analytical methods were found in reference 1 to be similar in trend and to be reasonably close in efficiency level for small or moderate amounts of coolant flow.

The purpose of the investigation presented in this report was to extend the analysis of reference 1 to include a coolant-flow schedule that was obtained by using convectively cooled blade heat-transfer relations. These coolant rates were estimated for three representative two-stage turbines over a range of turbine inlet temperature and for three values of coolant to inlet temperature ratio. These coolant rates were then applied to the isolated flow performance estimation method to obtain efficiency variation as a function of turbine inlet temperature. The range of turbine inlet temperature investigated is from 2300° to 3500° R (1278 to 1944 K), and the ratios of coolant temperature to turbine inlet temperature considered are 0.3, 0.45, and 0.6. In addition, the effect of coolant total-pressure coefficient was examined by using four values of this parameter. The performance results are obtained as the variation in primary air efficiency caused by coolant addition and are expressed as a fraction of the uncooled turbine efficiency.

ANALYTICAL METHODS

In order to determine the effect of inlet temperature on turbine performance it is first necessary to estimate the coolant rates required by each blade row. These coolant rates can then be applied in the performance estimation procedure. Three types of two-stage turbines were considered and are described in the following section. The coolant rates for the four blade rows of each turbine were estimated by the simplified procedure described in appendix B. (All symbols are defined in appendix A.) The coolant rates were based on convectively cooled blades having a thermal effectiveness of 0.5 with the cooling air being bled from the compressor outlet. A combustor pattern factor of 0.2 was assumed to estimate the elevation of the peak temperature above the average turbine inlet temperature. In addition, the allowable rotor-blade metal temperature was assumed to be 2110° R (1172 K), and that of the stator blading was assumed to be 2210° R (1228 K).

The estimated coolant rates are then used in the performance estimation procedure to determine the effect of turbine inlet temperature on primary air efficiency. As mentioned in the INTRODUCTION, reference 1 describes two such procedures termed mixed-flow and isolated-flow methods. Since the study indicated the methods give comparable results, the isolated-flow method was selected for the study to be described herein. This procedure is used to calculate the change in turbine efficiency, as coolant air is added, from that with no coolant air.

In this procedure it is assumed that there is no exchange of heat or momentum between the primary air and the coolant. It is also assumed that the blading profile losses are unaffected by any changes in blade shape that might be required to accommodate the coolant passages. Therefore the specific work output of the primary air is fixed and any change in efficiency results from the net incremental work output contributed by the coolant flows. In addition, the procedure is assumed to be two-dimensional with flow conditions based on the mean radius velocity diagram. The turbines are assumed to have flexible radial boundaries and the static-pressure levels through the blading are assumed to be the same as for the uncooled turbine regardless of the coolant conditions.

The detailed procedure used is the same as that described in appendix C of reference 1 with the following exceptions:

(1) The total pressure of the coolant introduced in a blade row is determined exclusively herein with a pressure coefficient k_p, and the coolant velocity coefficient mentioned in reference 1 is not used.

(2) The definition of the coolant pressure coefficient is changed slightly from that in reference 1. Herein k_p is defined as the dynamic head of the coolant at the blade outlet related to that of the uncooled turbine. This represents a simplification since it is not necessary to estimate the total-pressure loss across the blade row for the uncooled turbine.

The primary air efficiency, as defined in reference 1, uses the product of the primary air flow and its ideal specific-work output as the total ideal work output. Thus, the change in primary air efficiency directly reflects the change in turbine net work output.

DESCRIPTION OF TURBINE MODELS AND PARAMETRIC VARIABLES

The mean radius velocity diagrams of the three turbine models that were used are shown in figure 1. The diagrams are for the uncooled turbines and are simplified in that the axial velocity is constant and the blade speed is equal to one-half turbine inlet critical velocity. Thus, all of the velocities can be related to the appropriate critical velocity, and the total temperatures at all stations can be expressed in terms of turbine inlet total temperature T_0' (table 1). The turbine models are of different degrees of aerodynamic severity and have stage λ values of 0.55, 0.775, and 1.0. The $\lambda = 1.00$ and $\lambda = 0.775$ turbines have axial leaving velocity. The $\lambda = 0.55$ turbine has a stage outlet angle of 17.3°. This exit tangential velocity was adjusted so that the stage would have impulse hub conditions for a hub-tip radius ratio of 0.7 and free-vortex tangential velocity distribution.

The ratios of coolant temperature to turbine inlet temperature considered herein are 0.3, 0.45, and 0.6. The values of coolant pressure coefficient k_p investigated are 0.25, 0.5, 0.75, and 1.0. A specific-heat ratio γ of 1.3 is used throughout.

TABLE I. - NORMALIZED TOTAL TEMPERATURE VARIATION OF UNCOOLED MODEL TURBINES

Station	Model turbine		
	$\lambda = 0.55$	$\lambda = 0.775$	$\lambda = 1.00$
	Ratio of total temperature to turbine inlet temperature		
0	1.00	1.00	1.00
1 (absolute)	1.00	1.00	1.00
1 (relative)	.92909	.94846	.96739
2 (relative)	.92909	.94846	.96739
2 (absolute)	.88142	.91585	.93478
3 (absolute)	.88142	.91585	.93478
3 (relative)	.81051	.86431	.90217
4 (relative)	.81051	.86431	.90217
4 (absolute)	.76285	.83170	.86956

RESULTS OF ANALYSIS

The results include the cooling requirements of the individual blade rows and the overall turbine performance as affected by the parametric variation of turbine inlet temperature. In addition, the general performance results are used to show the effect of inlet temperature on the performance of two example turbine applications which have fixed values of turbine work, rotative speed, and coolant temperature. Finally, the significance of the stage-work coefficients obtained in the analytical procedure is briefly discussed.

Coolant Rates

Although not a primary purpose of this report, the coolant rates were required (and are estimated in appendix B) to determine the effect of elevated inlet temperature on performance. The coolant rates of the four blade rows of the three turbine models are shown in figure 2. Although there is a minor effect of turbine velocity diagram type, the coolant fractions are affected most by the coolant to inlet temperature ratio. For all three model turbines at a coolant to inlet temperature ratio of 0.6, the coolant fraction of one or more blade rows has reached or exceeded 0.10 at a turbine inlet temperature of 3000° R (1667 K). Therefore, the performance at this coolant to inlet temperature ratio was not determined for higher temperatures.

The coolant rates shown in figure 2 obviously depend on the assumptions employed in the estimation procedure (appendix B). The coolant fractions would be markedly reduced for higher values of allowable blade temperatures and for smaller values of the elevations of the peak gas temperatures. It should also be noted that the trends of coolant required with turbine inlet temperature in the figure are for fixed values of coolant to inlet temperature ratio. Thus, the curves do not show the schedule of coolant required for an actual application where the coolant temperature would be constant.

Effect of Coolant on Overall Performance

The overall effect of coolant on turbine performance is shown in figures 3(a) to (c) for the $\lambda = 0.55$, $\lambda = 0.775$, and $\lambda = 1.0$ model turbines, respectively. The effect is depicted as the change in primary air efficiency from that of the uncooled turbine as a function of turbine inlet temperature. The trend of efficiency with inlet temperature may be positive, almost zero, or negative depending on coolant to inlet temperature ratio T_c/T_0'

TABLE II. - STAGE-WORK COEFFICIENTS FROM ISOLATED FLOW ANALYSIS

Model turbine	Coolant temperature ratio, T_c/T'_0	Coolant fraction	Coolant pressure coefficient, k_p												
			0.25			0.5			0.75			1.00			
			Coolant stage-work coefficient		Overall stage-work coefficient, ΣK	Coolant stage-work coefficient		Overall stage-work coefficient, ΣK	Coolant stage-work coefficient		Overall stage-work coefficient, ΣK	Coolant stage-work coefficient		Overall stage-work coefficient, ΣK
			First stage, K_f	Second stage, K_s		First stage, K_f	Second stage, K_s		First stage, K_f	Second stage, K_s		First stage, K_f	Second stage, K_s	
$\lambda = 0.55$	0.3	A	0.010	0.237	0.247	0.100	0.221	0.321	0.170	0.209	0.379	0.231	0.200	0.431
		B	-.332	.286	-.046	-.250	.281	.031	-.191	.283	.092	-.145	.288	.143
		C	------	.045	.045	------	.148	.148	-----	.224	.224	-----	.289	.289
		D	------	-.314	-.314	------	-.226	-.226	-----	-.164	-.164	-----	-.116	-.116
	0.45	A	0.132	0.429	0.561	0.253	0.412	0.665	0.351	0.401	0.752	0.434	0.394	0.828
		B	-.288	.474	.186	-.188	.476	.288	-.118	.485	.367	-.062	.496	.434
		C	------	.176	.176	------	.316	.316	-----	.421	.421	-----	.508	.508
		D	------	-.266	-.266	------	-.160	-.160	-----	-.085	-.085	-----	-.027	-.027
	0.60	A	0.237	0.602	0.839	0.399	0.586	0.985	0.510	0.579	1.089	0.610	0.575	1.185
		B	-.250	.641	.391	-.136	.649	.513	-.055	.669	.614	.009	.681	.690
		C	------	.290	.290	------	.464	.464	-----	.592	.592	-----	.697	.697
		D	------	-.225	-.225	------	-.104	-.104	-----	-.018	-.018	-----	.048	.048
$\lambda = 0.775$	0.3	A	-0.128	0.178	0.050	-0.019	0.163	0.144	0.068	0.153	0.221	0.142	0.147	0.289
		B	-.534	.203	-.331	-.440	.212	-.228	-.371	.221	-.150	-.316	.231	-.085
		C	------	-.103	-.103	------	.012	.012	-----	.104	.104	-----	.177	.177
		D	------	-.521	-.521	------	-.423	-.423	-----	-.351	-.351	-----	-.294	-.294
	0.45	A	0.013	0.391	0.404	0.158	0.377	0.535	0.274	0.370	0.644	0.371	0.365	0.736
		B	-.484	.412	-.072	-.371	.425	.054	-.288	.439	.151	-.220	.453	.233
		C	------	.048	.048	------	.197	.197	-----	.318	.318	-----	.420	.420
		D	------	-.469	-.469	------	-.351	-.351	-----	-.265	-.265	-----	-.196	-.196
	0.60	A	0.135	0.590	0.725	0.311	0.567	0.878	0.451	0.562	1.013	0.566	0.559	1.125
		B	-.442	.595	.153	-.313	.611	.298	-.218	.629	.411	-.142	.647	.505
		C	------	.172	.172	------	.358	.358	-----	.504	.504	-----	.626	.626
		D	------	-.425	-.425	------	-.289	-.289	-----	-.191	-.191	-----	-.112	-.112
$\lambda = 1.00$	0.3	A	-0.195	0.194	-0.001	-0.074	0.134	0.060	0.021	0.128	0.149	0.100	0.123	0.223
		B	-.693	.151	-.542	-.572	.161	-.411	-.485	.173	-.312	-.414	.185	-.229
		C	------	-.169	-.169	------	-.041	-.041	-----	.057	.057	-----	.138	.138
		D	------	-.681	-.681	------	-.557	-.557	-----	-.466	-.466	-----	-.393	-.393
	0.45	A	-0.024	0.372	0.348	0.129	0.363	0.492	0.246	0.358	0.604	0.344	0.354	0.698
		B	-.630	.370	-.260	-.485	.385	-.100	-.379	.402	-.023	-.294	.418	.124
		C	------	.010	.010	------	.169	.169	-----	.289	.289	-----	.390	.390
		D	------	-.616	-.616	------	-.467	-.467	-----	-.357	-.357	-----	-.269	-.269
	0.6	A	0.123	0.570	0.693	0.301	0.562	0.863	0.436	0.558	0.994	0.550	0.554	1.104
		B	-.576	.560	-.016	-.410	.579	.169	-.289	.599	.310	-.191	.619	.428
		C	------	.163	.163	------	.347	.347	-----	.487	.487	-----	.603	.603
		D	------	-.561	-.561	------	-.390	-.390	-----	-.264	-.264	-----	-.163	-.163

and coolant pressure coefficient k_p. The efficiency level for any given set of parameters (λ, T_0', T_c/T_0') increases with increasing values of k_p, which would be expected because of the increased capacity of the coolant to do useful work. The effect of increasing coolant to inlet temperature ratio is also to increase the efficiency level, as shown in figure 3. The explanation of this effect is complex, however, since it results from changing coolant fractions as well as from changing stage-work coefficients. The effect of λ on overall performance can be adjudged by comparing like parts of figures 3(a) to (c) at a given turbine inlet temperature. The efficiency is seen to increase with a decreasing value of λ. A part of this effect is due to the stage-work output, which varies inversely with λ. Thus, as λ is decreased the negative pumping work expended on the rotor coolant flow becomes a smaller fraction of the stage-work output (table II). Another reason for this effect is that, as λ is decreased, the gas-stream temperatures in the downstream blade rows become lower (table I). Therefore, the temperature of the coolant flows in the second stage more nearly approaches the gas-stream temperature at lower λ values, and the relative capacity of the coolant to produce useful work is increased.

The effect of elevated temperature with coolant additions, as seen in figure 3, is positive over much of the range of conditions. The greatest effect is shown in figure 3 (a-1) to be 0.125. This increase in primary air efficiency, however, results from an expense of total coolant requirement of 0.384, with 0.166 being required for the first-stage rotor blade row. It can be demonstrated from the performance results that if the turbine were rated on the basis of a true thermodynamic efficiency, wherein the turbine is charged with the available energy of coolant flows as well as the primary air, the efficiency would decrease in all cases with the addition of coolant. The primary air efficiency was considered to be a useful and easily understood concept and is, therefore, used herein.

Effect of Inlet Temperature for Example Applications

The general results (fig. 3) are obtained for fixed parameter values. It is therefore of interest to determine the effect of inlet temperature for two example cases where the coolant temperature, uncooled turbine work output, and actual rotative speed are fixed. The base point selected to fix conditions of one example is the $\lambda = 0.55$ turbine at an inlet temperature of 2300° R (1278 K), a coolant supply temperature of 1380° R (767 K), and a blade speed U of $0.5\ V_{cr(2300^\circ\ R)}$. The other example is based on the $\lambda = 0.775$ turbine at an inlet temperature of 2700° R (1500 K), coolant temperature of 945° R (525 K), and a blade speed of $0.5\ V_{cr(2700^\circ\ R)}$. The efficiency effect was determined for temperatures to 3500° R (1944 K) for values of k_p of 0.5 and 1.0. The results are shown in figures 4(a) and (b). Figures 4(a-1) and (b-1) show the total coolant required and the individual blade row requirements for the range of inlet temperature. The total coolant and the individual blade row coolants increase nearly linearly with inlet temperature, whereas in figure 2 the cooling rates increase approximately as inlet temperature squared. This difference occurs primarily because the coolant temperature varies in figure 2 with turbine inlet temperature and is constant in figures 4(a-1) and (b-1).

The efficiency variation is shown for the example applications in figures 4(a-2) and (b-2). The solid curves were obtained by calculating the coolant fractions for each blade row and applying stage-work coefficients interpolated from table II for the correct ratio of coolant to inlet temperature. The dashed curves were obtained by interpolating the efficiency variation from figures 3(a) and (b). The efficiency trend with inlet temperature is considerably altered in figures 4(a-2) and (b-2) from those shown in figures 3(a) and (b). A comparison of the effect of coolant for the two example applications (figs. 4(a-2) and (b-2)) shows that coolant addition has a more positive effect on performance and results in a higher efficiency level for the example application of figure 4(a). This effect occurs because of its lower λ value and the higher coolant temperature. The efficiency trends interpolated from figures 3(a) and (b) are also in good agreement with the calculated trends. This would indicate that the efficiency variation for a given application can adequately be determined by interpolation of the general-result curves.

Stage-Work Coefficients

The stage-work coefficients that are determined as part of the analytical procedure are listed in table II. They show the contribution of the individual coolant flows in the different stages, as well as their overall effect. Herein, efficiency effects were determined for selected values of the coolant pressure coefficient k_p. By using the appropriate stage-work coefficients of table II, the efficiency variation could be determined for cases where the pressure coefficient differed for the individual coolant flows.

SUMMARY OF RESULTS

The effect of coolant air on turbine performance has been obtained analytically for three types of two-stage turbine models. The determination was made over a range of turbine inlet temperature for selected values of coolant to inlet temperature ratio and coolant pressure coefficient. The pertinent results of the study are as follows:

1. The coolant fractions required by the blade rows as turbine inlet temperature increased were affected greatly by the coolant to inlet temperature ratio and only slightly affected by velocity-diagram type.

2. The trend of turbine efficiency with turbine inlet temperature may be positive, almost zero, or negative depending on the coolant to inlet temperature ratio and the coolant pressure coefficient. Increasing either of these two parameters causes the efficiency to increase. Coolant addition also has a more positive effect on efficiency as the λ value is decreased.

3. The procedure was used to determine the efficiency variation over a range of turbine inlet temperature for two example turbine applications for which the rotative speed, coolant temperature, and uncooled turbine-work output were fixed. The trend of efficiency with inlet temperature is considerably altered from that of the general-result curves in which the coolant temperature, uncooled turbine-work output, and the square of the blade speed were all varied in proportion to turbine inlet temperature. It was also shown that the results for the example applications could be obtained satisfactorily by interpolation from the general-result efficiency trends.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 25, 1968,
126-15-02-31-22.

APPENDIX A

SYMBOLS

A	first-stage stator coolant flow expressed as fraction of primary flow
B	first-stage rotor coolant flow expressed as fraction of primary flow
C	second-stage stator coolant flow expressed as fraction of primary flow
$c_{p,c}$	constant-pressure specific heat of coolant, Btu/(lb)($^{\circ}$R); J/(kg)(K)
D	second-stage rotor coolant flow expressed as fraction of primary flow
F_t	factor combining temperature-dependent gas-stream properties defined following eq. (B3)
h_g	blade external heat-transfer coefficient, Btu/(sec)(ft^2)($^{\circ}$R); J/(sec)(m^2)(K)
K	coolant stage-work coefficient, ratio of stage specific-work output of coolant to stage specific work output of uncooled turbine
k	gas conductivity, Btu/(sec)(ft)($^{\circ}$R); J/(sec)(m)(K)
k_p	coolant pressure coefficient, ratio of dynamic pressure of coolant to dynamic pressure of gas stream at injection blade row outlet
l	length of flow path through blade channel, also length of blade mean camber line, ft; m
N	number of blades in a specific blade row
o	average width of flow channel through blading, ft; m
Pr	Prandtl number
S_g	blade surface area, ft^2; m^2
S_p	blade span, ft; m
T_b	allowable blade metal temperature, $^{\circ}$R; K
T_c	temperature of coolant at bleed point, $^{\circ}$R; K
$T_{c,in}$	temperature of coolant supplied to blade row, $^{\circ}$R; K
T_g	effective or peak gas temperature, $^{\circ}$R; K
T_0'	average turbine inlet total temperature, $^{\circ}$R; K
ΔT_{comb}	average temperature rise in combustor, $^{\circ}$R; K
U	blade velocity, ft/sec; m/sec

V	absolute gas velocity, ft/sec; m/sec
V_{cr}	velocity of sound at Mach 1 based on absolute total state, ft/sec; m/sec
W	gas velocity relative to rotor blade, ft/sec; m/sec
W_{cr}	velocity of sound at Mach 1 based on total state relative to moving blade row, ft/sec; m/sec
w_c	coolant flow rate, lb/sec; kg/sec
w_p	primary airflow rate, lb/sec; kg/sec
β	blade angle measured from axial direction
γ	ratio of specific heats
η	turbine efficiency based on total- to static-pressure ratio
η_t	blade thermal effectiveness, $\Delta T_c/(T_b - T_{c,in})$
λ	ratio of blade speed to change of tangential velocity for a turbine stage
μ	primary air viscosity, lb/(ft)(sec); kg/(m)(sec)
ρ	primary air density, lb/ft^3; kg/m^3
σ	solidity, chord/pitch
φ	temperature ratio parameter, eq. (B1)

Subscripts:

f	first stage
s	second stage
un	uncooled turbine
0	station at turbine inlet (see fig. 1)
1	station at first-stage stator outlet
2	station at first-stage rotor outlet
3	station at second-stage stator outlet
4	station at second-stage rotor outlet

Superscripts:

(')	total state absolute
('')	total state relative to moving blade row

APPENDIX B

DETERMINATION OF COOLANT RATES

The effect of coolant on turbine performance depends on the amount of coolant required by the different blade rows, as well as the behavior of the coolant in passing through the turbine. It is therefore necessary to determine coolant rates that vary realistically with coolant temperature and the effective gas temperature of each blade row. The method of estimating the coolant rates used herein is based on convective cooling. The cooling requirements were determined by assuming that the maximum effective gas temperature prevails over the entire blade and that the blade is at a uniform temperature equal to its allowable temperature. By using this assumption, the blade row heat balance equation (eq. (C1) of ref. 2), and the thermal effectiveness relation (eq. (C2) of ref. 2) the following equation was obtained:

$$\varphi = \frac{T_g - T_b}{T_g - T_{c,in}} = \frac{1}{\left(\frac{h_g S_g}{w_c c_{p,c}}\right)\frac{1}{\eta_t} + 1} \tag{B1}$$

which is similar to equation (C3) of reference 2.

The evaluation of the lefthand side of equation (B1) involves the three temperatures. The allowable blade metal temperatures were assumed to be 2210° R (1228 K) for stator blades and 2110° R (1172 K) for rotor blades. The temperature of the coolant entering the blade $T_{c,in}$ is, in the case of stator blade rows, the same as the coolant supply temperature T_c. For rotor blade rows this temperature is assumed to be elevated by the pumping work and is obtained from the equation

$$\frac{T''_{c,in}}{T'_0} = \left[\frac{T_c}{T'_0} + \frac{\gamma - 1}{\gamma + 1}\left(\frac{U}{V_{cr,0}}\right)^2\right] \tag{B2}$$

The maximum gas temperature experienced by the blade rows T_g was determined by estimating the elevation above the average temperature. The average gas temperature is a function of turbine inlet temperature and velocity-diagram type and can be obtained by using table I. The temperature elevation at the turbine inlet was obtained by assuming a pattern factor of 0.2. This factor is defined as

$$0.2 = \frac{T_g - T_0'}{\Delta T_{comb}}$$

where T_g is the maximum gas temperature, T_0' is the average turbine inlet temperature, and ΔT_{comb} is the average temperature rise in the combustor. By using the assumption that the coolant is bled from the compressor outlet, the combustor temperature rise can be related to the turbine inlet temperature and the coolant to inlet temperature ratio. Thus, the temperature elevation $T_g - T_0'$ for the first-stage stator is determinable in terms of the turbine inlet temperature and coolant to inlet temperature ratio. The elevations for the other blade rows are proportioned from that at the inlet according to the following schedule:

Blade row	$\frac{\text{Temperature elevation}}{\text{Temperature elevation at turbine inlet}}$
First-stage stator	1.000
First-stage rotor	.667
Second-stage stator	.688
Second-stage rotor	.542

This schedule was obtained from the design temperature profiles of a high-temperature engine turbine and was applied to the three model turbines used herein. Thus, the temperatures required to evaluate the temperature parameter φ are determined.

The external gas to blade heat-transfer coefficient h_g is approximated by using the flat-plate relation (ref. 3)

$$h_g l = 0.037\, k \left(\frac{\rho V l}{\mu}\right)^{0.8} Pr^{1/3}$$

and the average flow conditions in the blade channel for the Reynolds number, as recommended in reference 2. The ρV term in the Reynolds number can be expressed as

$$\rho V = \frac{w_p}{N S_p o}$$

where o is the average width of the flow channel. The external blade surface area can be expressed as

$$S_g = 2lNS_p$$

By using these substitutions in equation (B1) and rearranging the following equation is obtained:

$$w_c c_{p,c}\left(\frac{1-\varphi}{\varphi}\right)\eta_t = 2NS_p \frac{0.037\,k}{\mu^{0.8}} Pr^{1/3}\left(\frac{w_p l}{NS_p o}\right)^{0.8} \tag{B3}$$

Since Prandtl number Pr, conductivity k, and viscosity μ are functions of the temperature T_g, a term combining these quantities is defined as

$$F_t = \frac{2(0.037)k\,Pr^{1/3}}{\mu^{0.8}}$$

and equation (B3) is divided by w_p and thereby reduced to

$$\frac{w_c}{w_p} c_{p,c}\left(\frac{1-\varphi}{\varphi}\right)\eta_t = F_t \frac{\left(\frac{l}{o}\right)^{0.8}}{\left(\frac{w_p}{NS_p}\right)^{0.2}} \tag{B4}$$

The thermal effectiveness η_t is assumed herein to be 0.5, which is represented as an average, presently accepted value in reference 2. A coolant specific heat $c_{p,c}$ is used that is based on the average coolant temperature in the blade row. This average is determined from the coolant temperature into the blade $T_{c,in}$, thermal effectiveness η_t, and blade metal temperature.

The l/o term in equation (B4) is a function of the blade passage geometry and is independent of pressure level or turbine size. This term is approximated by the equation

$$\frac{l}{o} = 2\sigma \frac{\frac{0.75\pi\Delta\beta}{360} + 0.25 \tan\frac{\Delta\beta}{2}}{\sin\frac{\Delta\beta}{2}\left(\cos\beta_{in} + \cos\beta_{out}\right)} \tag{B5}$$

where $\Delta\beta$ is the turning angle, σ is the solidity, and β_{in} and β_{out} are the blade inlet and outlet angles. The σ value is estimated from the velocity diagrams and the procedure of reference 4.

The remaining term w_p/NS_p does not yield to a general solution since it depends on absolute pressure level and a channel width dimension (such as o) in addition to the quantities determinable from the velocity diagram and assumed inlet temperature. However, the coolant fraction is relatively insensitive to wide variations of this term since it is raised to the 0.2 power. For example, an assumed pressure level of 7 atmospheres predicts the coolant fraction with an accuracy of ±0.15 for a range of pressure level from 3.1 to 14.1 atmospheres. The following conditions were assumed to evaluate the w_p/NS_p term:

(1) Mean diameter, 3.5 feet (1.067 m)

(2) Radius ratio, 0.7

(3) Aspect ratio, 3.0

(4) Total pressure, 7 atmospheres

The coolant fractions were determined for the three turbine types shown in figure 1 for a range of inlet temperature from 2300° to 3500° R (1278 to 1944 K) and for coolant to inlet temperature ratios of 0.3, 0.45, and 0.6. The primary mass flow w_p is assumed to be constant for all blade rows of a given model turbine, and the gas-stream temperature assumed is that of the uncooled turbine. This procedure results in slightly greater coolant fractions for downstream blade rows than that which considers the increased mass flow and decreased gas-stream temperature caused by the addition of coolant in the upstream blade rows. The resulting coolant fractions are shown in figure 2.

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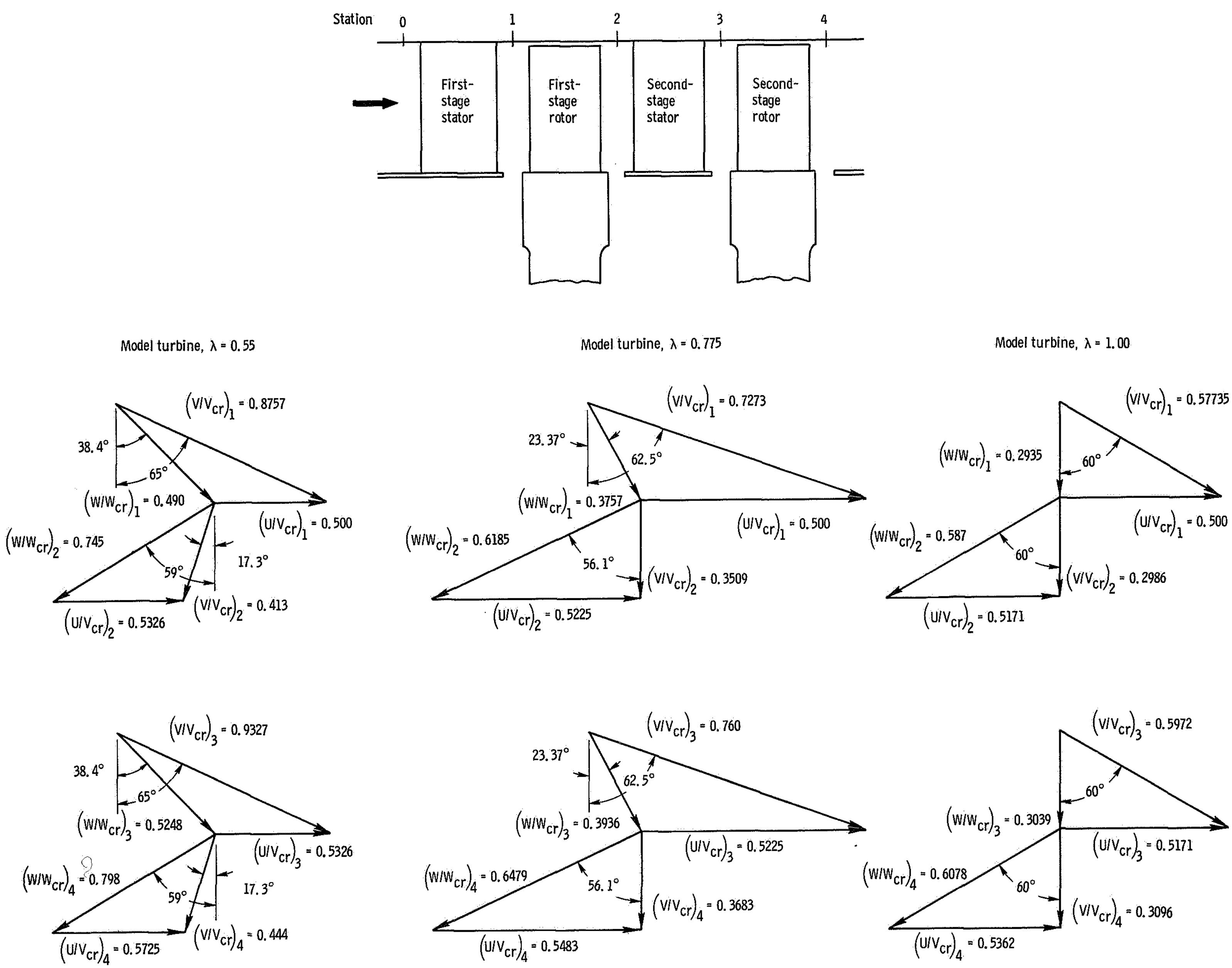
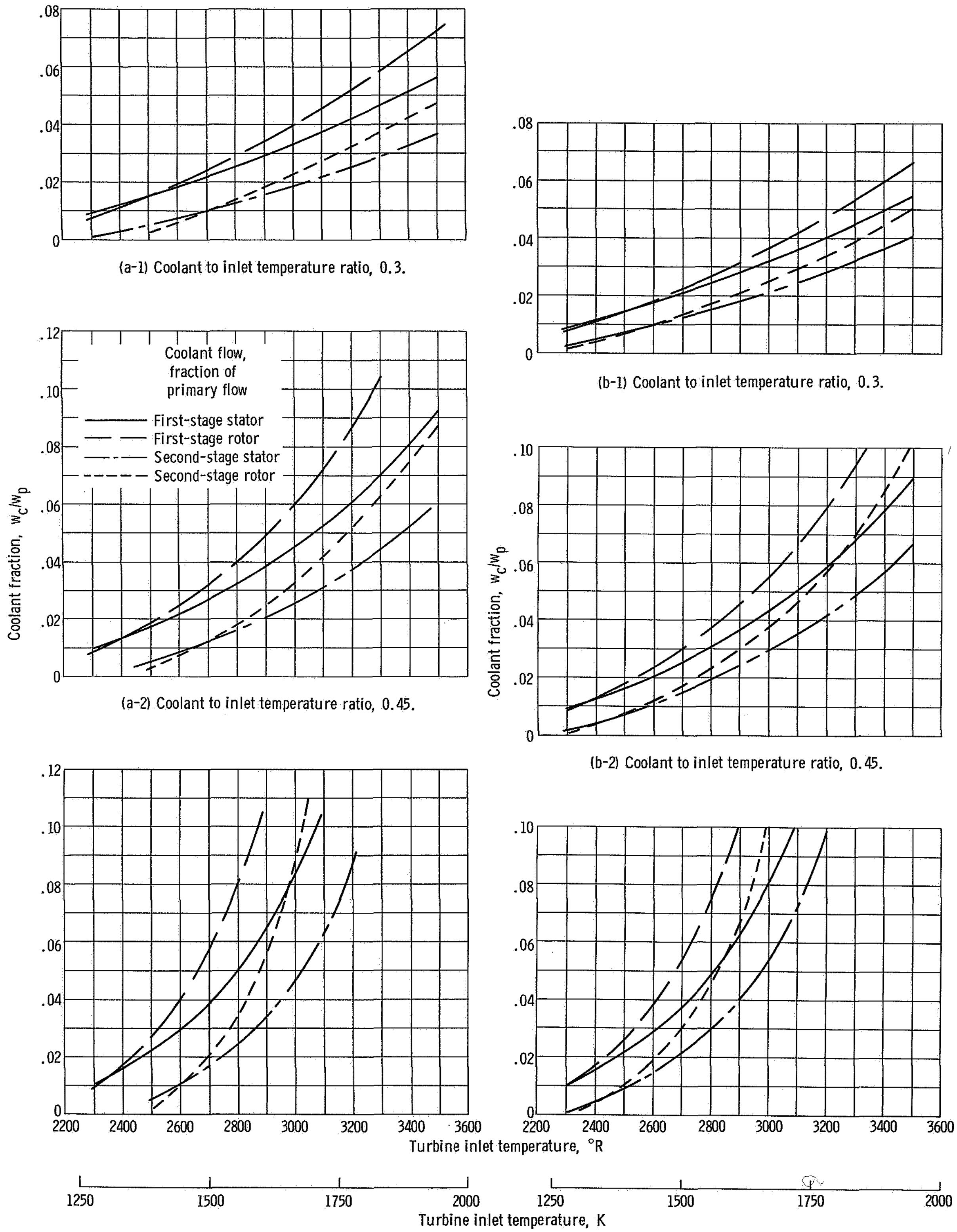


Figure 1. - Velocity diagrams and station locations of model turbines. Diagrams shown are for uncooled turbine.



(a-1) Coolant to inlet temperature ratio, 0.3.

(a-2) Coolant to inlet temperature ratio, 0.45.

(a-3) Coolant to inlet temperature ratio, 0.6.

(a) Model turbine, $\lambda = 0.55$.

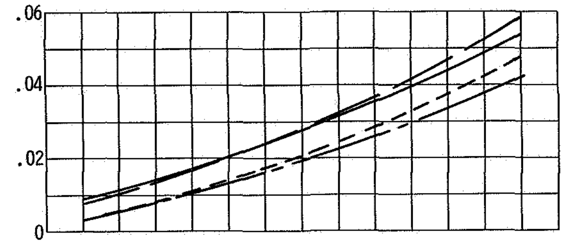
(b-1) Coolant to inlet temperature ratio, 0.3.

(b-2) Coolant to inlet temperature ratio, 0.45.

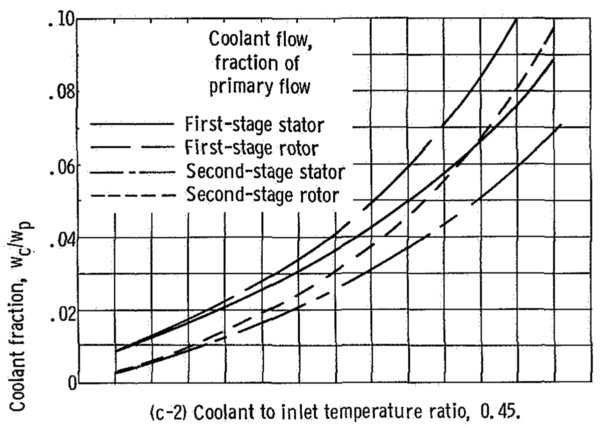
(b-3) Coolant to inlet temperature ratio, 0.6.

(b) Model turbine, $\lambda = 0.775$.

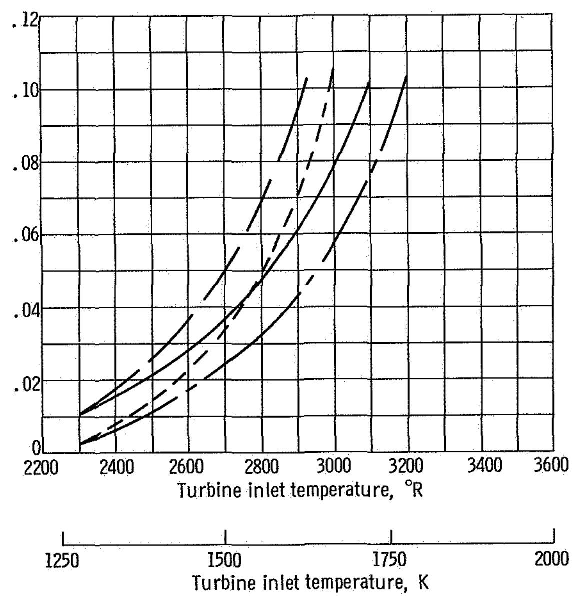
Figure 2. - Estimated coolant fractions for four blade rows of model turbines.



(c-1) Coolant to inlet temperature ratio, 0.3.



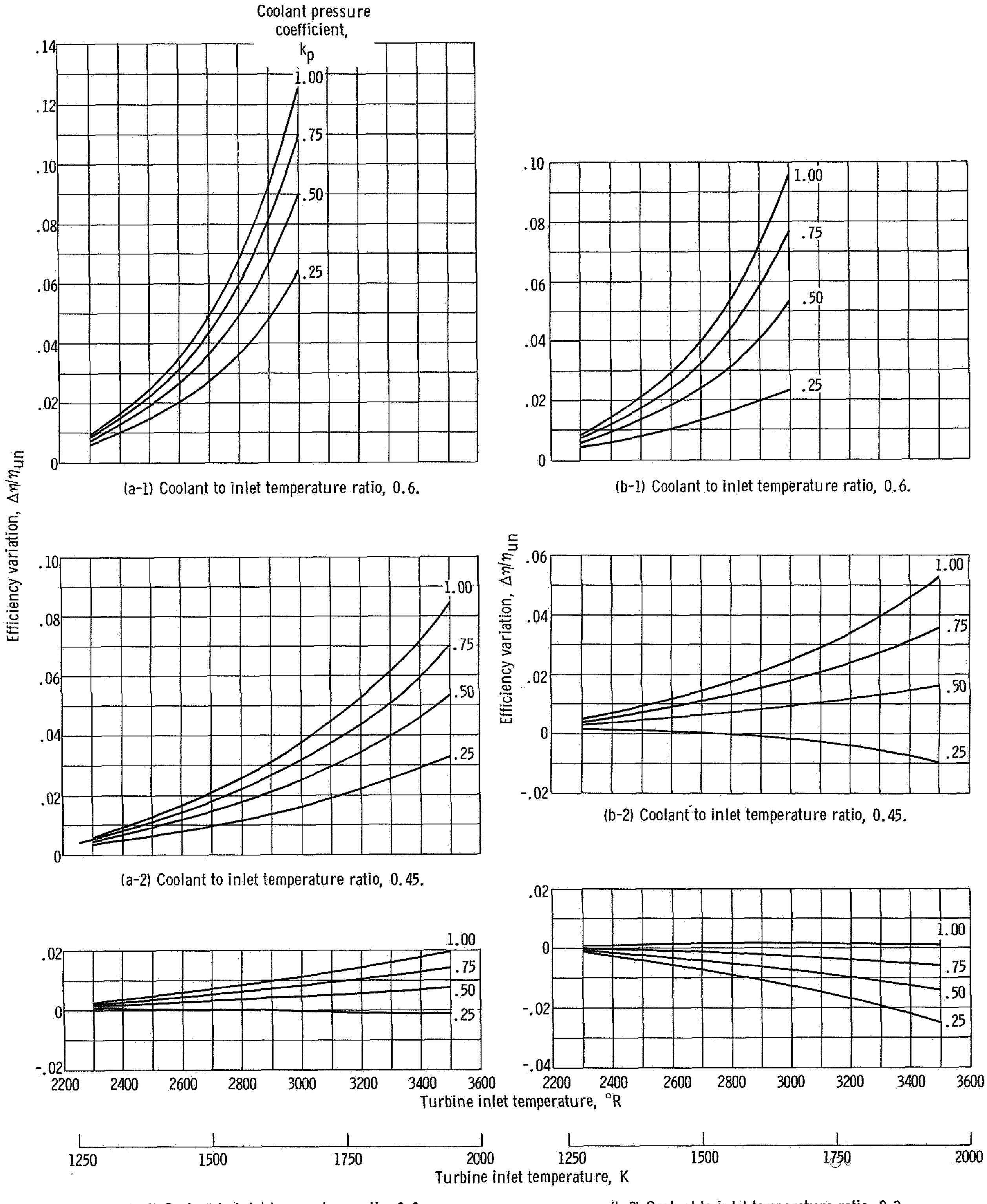
(c-2) Coolant to inlet temperature ratio, 0.45.



(c-3) Coolant to inlet temperature ratio, 0.6.

(c) Model turbine, λ = 1.0.

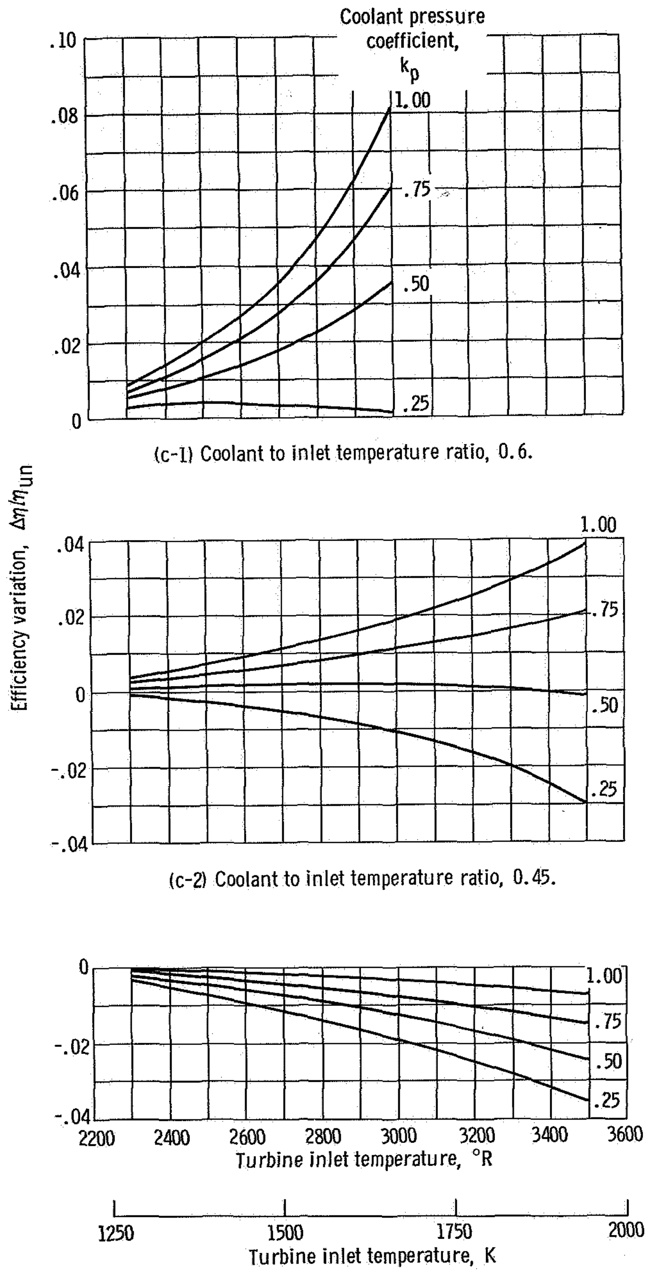
Figure 2. - Concluded.



(a) Model turbine, $\lambda = 0.55$.

(b) Model turbine, $\lambda = 0.775$.

Figure 3. - Effect of turbine inlet temperature on primary air efficiency.



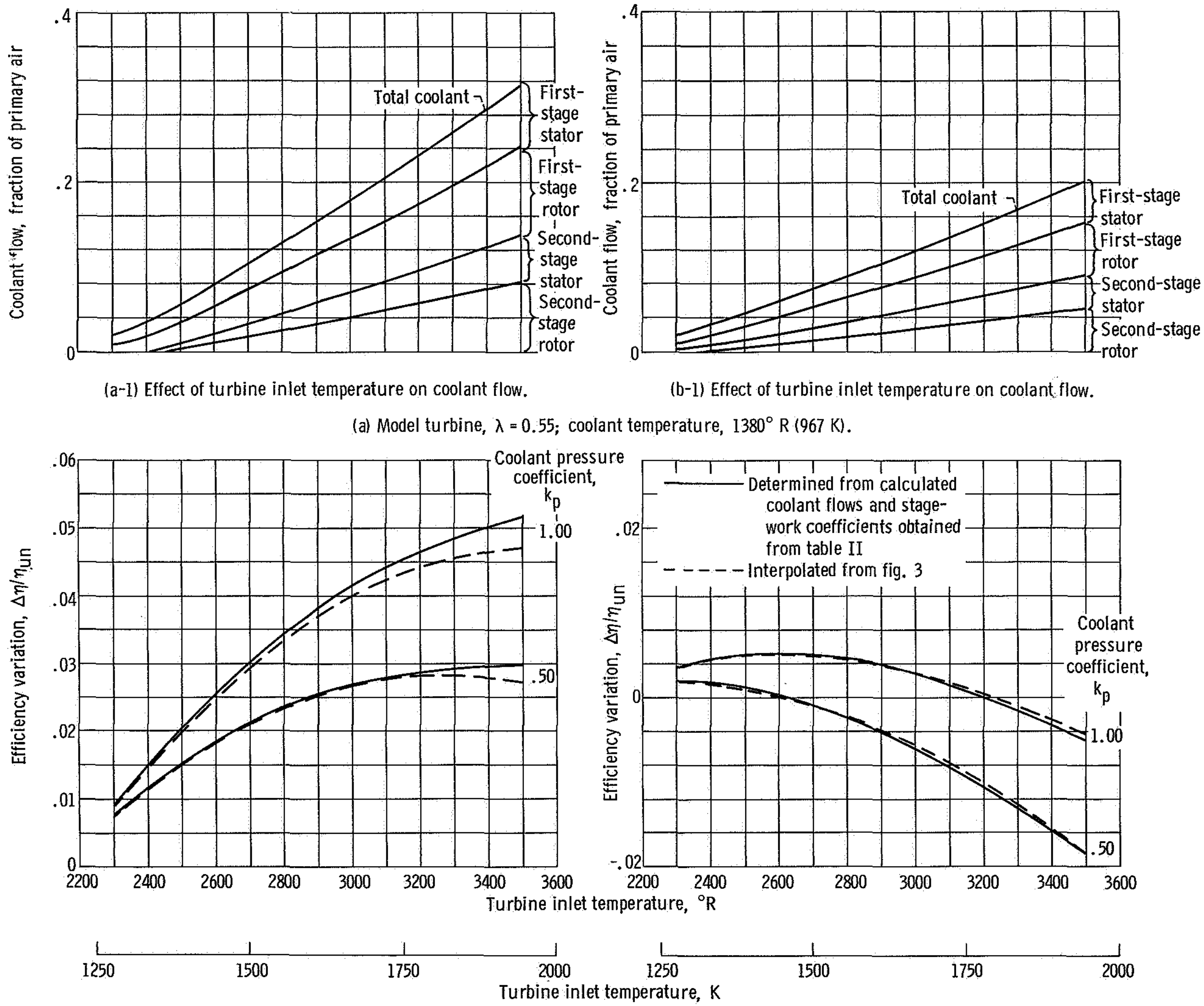
(c-1) Coolant to inlet temperature ratio, 0.6.

(c-2) Coolant to inlet temperature ratio, 0.45.

(c-3) Coolant to inlet temperature ratio, 0.3.

(c) Model turbine, λ = 1.0.

Figure 3. - Concluded.



(a-2) Effect of turbine inlet temperature on efficiency.

(a) Model turbine, λ = 0.55; coolant temperature, 1380° R (767 K).

(b-2) Effect of turbine inlet temperature on efficiency.

(b) Model turbine, λ = 0.775; coolant temperature, 945° R (525 K).

Figure 4. - Effect of turbine inlet temperature on total coolant requirement and efficiency of example turbine application.

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