ANALYTICAL MODELLING OF INFILLED FRAME STRUCTURES - A GENERAL REVIEW

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ABSTRACT

The analytical modelling of infilled frames is a complex issue because these structures exhibit a highly non-linear inelastic behaviour resulting from the interaction of the masonry infill panel and the surrounding frame. This paper presents a general review of the different procedures used for the analysis of infilled frames, which can be grouped in local or micro-models and simplified or macro-models, depending on the degree of refinement used to represent the structure. The finite element formulation and the equivalent truss mechanism are the typical examples of each group. The advantages and disadvantages of each procedure are pointed out, and practical recommendations for the implementation of the different models are indicated.

INTRODUCTION

Infilled frame structures are used to provide lateral resistance in regions of high seismicity, especially in those places where masonry is still a convenient material, due to economical and traditional reasons. Furthermore, infilled frame buildings designed and constructed before the development of actual seismic codes constitute an important part of the high-risk structures in different countries. The rehabilitation of these buildings to resist seismic actions implies, as a first step, the assessment of the structural behaviour. Consequently, the analytical modelling of this type of structure represents an important issue for engineers and researchers involved in seismic design.

Structural engineers have largely ignored the influence of masonry panels when selecting the structural configuration, assuming that these panels are brittle elements when compared with the frame. The design practice of neglecting the infill during the formulation of the mathematical model leads to substantial inaccuracy in predicting the lateral stiffness, strength and ductility. The reluctance of numerous engineers to consider the contributions of the masonry infills has been due to the inadequate knowledge concerning the composite behaviour of infilled frames, and to the lack of practical methods for predicting the stiffness and strength. It is worth noting that most of the computer programs commonly used by designers are not provided with some rational and specific elements for modelling the behaviour of the masonry infills.

The aim of this paper is to review the approaches used for the analysis of infilled frame structures. The different techniques proposed in the literature for idealizing this structural type can be divided into two groups, namely, local or micro-models and simplified or macro-models. The first group involves the models, in which the structure is divided into numerous elements to take account of the local effects in detail, whereas the second group includes simplified models based on a physical understanding of the behaviour of the infill panel. In the later case, a few elements are used to represent the effect of the masonry infill as a whole. Both types of models will be discussed in the following sections.

It is evident from experimental observations that [1] these structures exhibit a highly non-linear inelastic behaviour. The most important factors contributing to the non-linear behaviour of infilled frames arise from material non-linearity. These factors can be summarized as follows:

- **Infill Panel**: cracking and crushing of the masonry, stiffness and strength degradation.
- **Surrounding Frame**: cracking of the concrete, yielding of the reinforcing bars, local bond slip.
- **Panel-Frame Interfaces**: degradation of the bond-friction mechanism, variation of the contact length.

Geometric non-linear effects can also occur in infilled frames, especially when the structure is able to resist large horizontal displacements. However, these effects do not present any particularity and can be considered in the analysis using the

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same methodologies applied to reinforced concrete or steel structures.

The non-linear effects mentioned above introduce analytical complexities which required sophisticated computational techniques to be properly considered in the modelling. Furthermore, the material properties are difficult to define accurately, especially for masonry. These facts complicate the analysis of infilled frames and represent one of the main reasons to explain why infill panels have been considered as "non-structural elements", despite the strong influence on the global response.

It is worth noting that infilled frame structures cannot be modelled as elasto-plastic systems due to the stiffness and strength degradation occurring under cyclic loading. More realistic models should be used to obtain valid results, particularly in the dynamic analysis of short period structures, where the energy dissipation capacity and shape of the hysteresis loops may have strong influence in the response.

DIAGONAL STRUT MODEL

General Description

Polyakov (as reported by Klinger and Bertero [2] and Mallick and Severn [3]) conducted one of the first analytical studies based on elastic theory. From his study, complemented with tests on masonry walls diagonally loaded in compression, he suggested that the effect of the masonry panels in infilled frames subjected to lateral loads could be equivalent to a diagonal strut (see Figure 1). Later, Holmes [4] took up this idea and proposed that the equivalent diagonal strut should have a width equal to one third of the length of the panel. Stafford Smith [5] refined the approach and started a series of tests to investigate more precisely the width of the equivalent strut. This task was continued by many other researchers. Nowadays, the diagonal strut model is widely accepted as a simple and rational way to describe the influence of the masonry panels on the infilled frame.

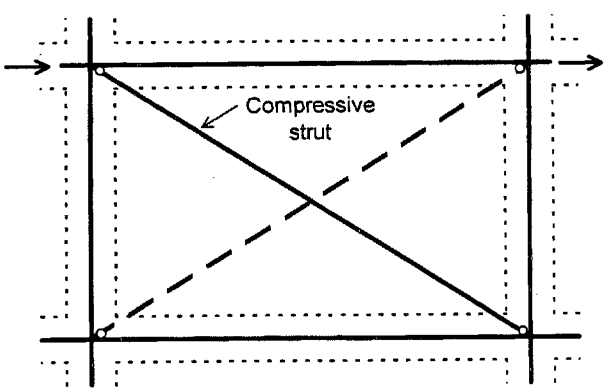

Figure 1: Diagonal strut model for infilled frames.

When the structure is subjected to cyclic or dynamic loading, the use of only one diagonal strut resisting compressive and tensile forces cannot describe properly the internal forces induced in the members of the frame. In this case, at least two struts following the diagonal directions of the panel must be considered to represent approximately the effect of the masonry infill. It is usually assumed that the diagonal struts are active when compressive forces develop in them. However, compression only elements are not available in common elastic computer programs. In this case, Flanagan *et al.* [6] recommend the use of tension-compression truss members with half of the equivalent strut area in each diagonal direction. The use of this simplified model results in significant changes in the internal forces in the surrounding frame, especially the axial forces in the columns (tensile forces decrease, whereas compressive forces increase).

The assumption of a compression only strut is acceptable on the basis that the bond strength at the panel-frame interfaces and the tensile strength of the masonry are very low. Tensile forces, therefore, can be transferred through the interfaces only for small levels of seismic excitation. This consideration may not be valid when either shear connectors are used at the interfaces or the masonry panel is reinforced with horizontal or vertical bars. Refined models, however, can consider the tensile behaviour, which usually does not affect significantly the results.

Modified Diagonal Strut Model

The single diagonal strut model is simple and capable of representing the influence of the masonry panel in a global sense. This model, however, cannot describe the local effects resulting from the interaction between the infill panel and the surrounding frame. As a result, the bending moments and shear forces in the frame members are not realistic and the location of potential plastic hinges cannot be adequately predicted. For these reasons the single diagonal strut model has been modified by different researchers, as illustrated in Figure 2. For simplicity, the struts acting just on one direction have been indicated in this figure.

Zarnic and Tomazevic [7, 8, 9] proposed the model illustrated in Fig. 2 (a) based on their experimental results. In these tests, the damage in the upper zone of the masonry panel occurred off the diagonal, probably due to perturbation introduced by the devices used to apply the lateral and vertical loads in the corners of the frame. Consequently, in the proposed model the upper end of the diagonal strut is not connected to the beam-column joint. This model could be applied in those cases where a shear failure develops at the top of the columns, although it does not represent the mechanism usually observed in laboratory tests.

Figures 2 (b), (c) and (d) show multiple struts models proposed by Schmidt (as reported by König [4]), Chrysostomou [10], and Crisafulli [1], respectively. The main advantage of these models, in spite of the increase of complexity, is the ability to represent the actions in the frame more accurately. Syrmakesis & Vratsanou [11] and San Bartolomé [12] increased the number of struts and used in their analyses a model similar to that illustrated in Fig. 2 (c) with five and nine parallel struts, respectively, in each direction..

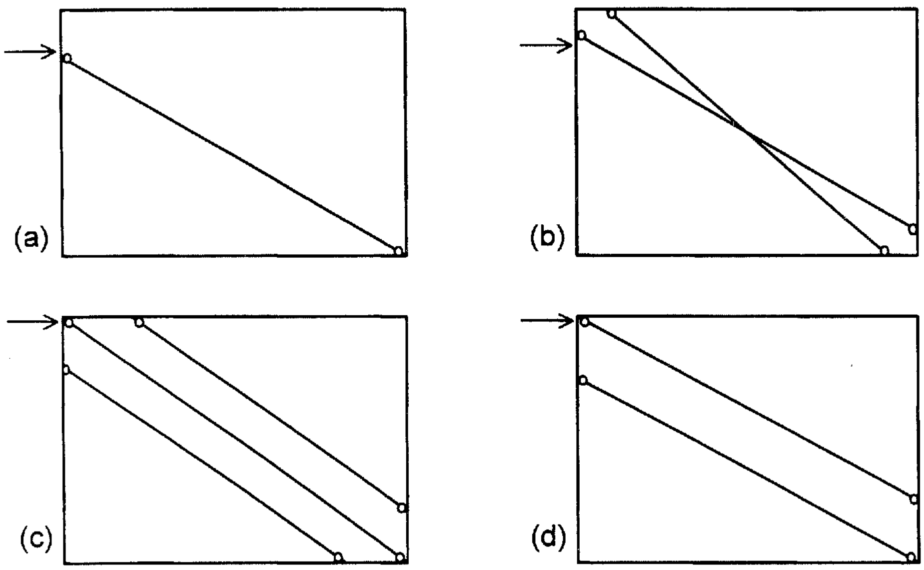

Figure 2: Modification of the diagonal strut model and multiple struts models.

A more complex model was developed by Thiruvengadam [13] for the dynamic analysis of infilled frames. The model consists of a moment resisting frame with a number of pin joined diagonals and vertical struts uniformly distributed in the panel. These diagonals represent the shear and axial stiffness of the masonry infill. In order to take into account the partial separation at the panel-frame interfaces, the contact length is calculated and those ineffective struts are removed. In a similar way, the effect of openings can be considered by removing the struts crossing the opening area. Due to the complexity and refinement involved in this multiple strut model, it may be considered as an intermediate approach between the micro-models and macro-models.

The strut models presented above are not capable of describing the response of the infilled frame system when horizontal shear sliding occurs in the masonry panel. For this case, Fiorato *et al.* [14] proposed a "knee braced frame" to represent the behaviour, and Leuchars & Scrivener [15] suggested the model illustrated in Figure 3. The double strut can depict the large bending moments and shear forces induced in the central zone of the columns. Furthermore, it is possible to consider the friction mechanism developing along the cracks, which mainly controls the strength of the system. According to the author's knowledge, this model was just a suggestion, which was never implemented to verify its accuracy.

Andreaus *et al.* [16] generalized the idea of the diagonal strut and assumed that masonry can be represented using a truss-like system, in order to generate a sort of finite element mesh formed by "cells". Each of these cells represents a four-node element, whose mechanical behaviour is defined by two truss members located along the diagonal directions of the element. This approach can be considered as a micro-model, due to the refinement involved in the representation of the structure. However, it is included here because the formulation of the model was based on the diagonal strut concept. D'Asdia *et al.* [17] applied this approach to model infilled frame structures.

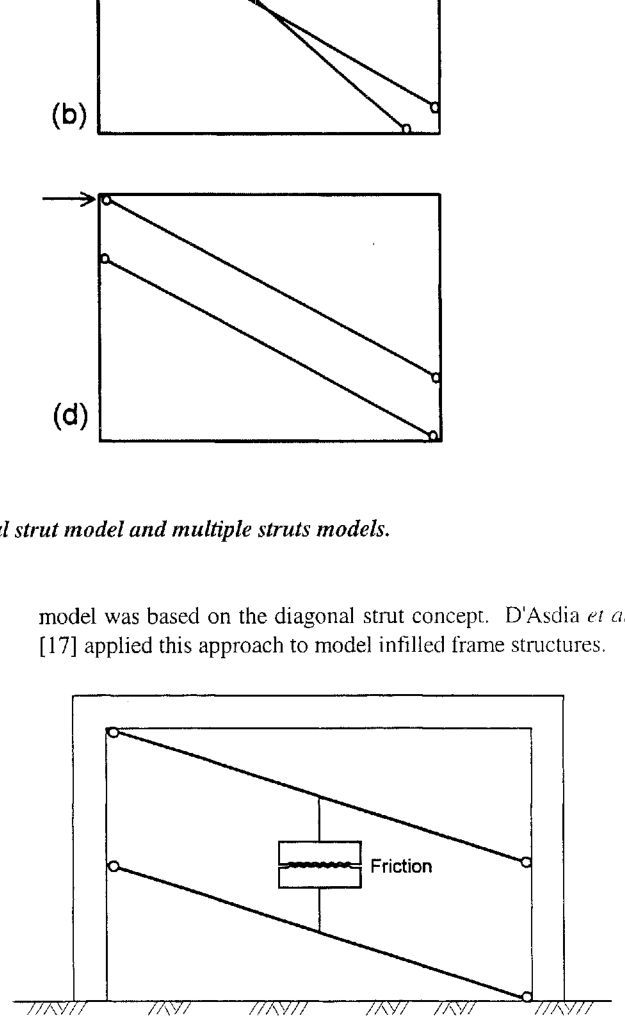

Figure 3:** **Model proposed to represent the response of the infilled frame subsequent to horizontal shear sliding [15].

Properties of the Diagonal Strut

The use of the equivalent strut model is attractive from the practical point of view. Consequently, much experimental research has been directed to define the relationships between the characteristics of the infilled frame system and this simplified model. The properties required for defining the strut model depend on the type of analysis (linear elastic or non-linear) and the type of loading (monotonic, cyclic or dynamic). For linear elastic analysis only the area and length of the strut, and the modulus of elasticity are needed to calculate the elastic stiffness. When non-linear behaviour of the material is considered, the complete axial force-displacement relationship is required. Even more complex is the problem for cyclic or dynamic loading, because the hysteretic behaviour of the material must be established. In this section, only the evaluation of the elastic stiffness is discussed, whereas the hysteretic models are presented in the next section.

It is usually assumed that the ends of the diagonal members coincide with the intersection of the centre lines of the beams and columns of the surrounding frame (see Fig. 1). This implies that the diagonal length in the model is longer than the diagonal length of the masonry panels. The difference, however, is not significant in most cases. The thickness, t, and the elastic modulus, E_m, of the strut are equal to those of the masonry infill. The value of E_m adopted in the analysis obviously depends on the stress level expected in the panels, since the behaviour of masonry is non-linear. Two approaches have been used to calculate the equivalent width, w, of the equivalent strut (see Fig. 4). The first approach is based on measurements from tests of infilled frame structures, whereas in the second procedure analytical results (for example, from finite element analysis) are utilised.

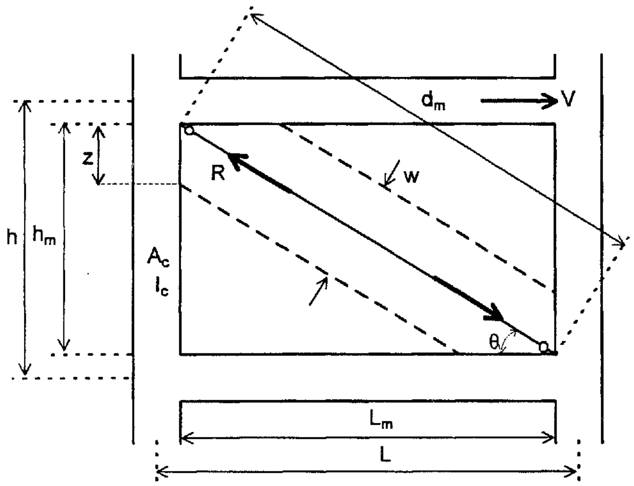

Figure 4: Effective width of the diagonal strut.

The first approximation to calculate the width of the equivalent strut was proposed by Holmes [4], in the lack of experimental data, assuming that:

$$w = \frac{d_m}{3} \quad (1)$$

where d_m is the diagonal length of the masonry panel. Later, Stafford Smith [5, 18, 19, 20] conducted a large series of tests using infilled steel frames and proposed different charts to calculate the equivalent width, w. In the first investigations [5], it was found that the ratio w/d_m varied from 0.10 to 0.25. Additional experimental information [19, 20] allowed a more refined evalution of w, considering the ratio h_m/L_m, and a dimensionless parameter λ_h (which takes account of the relative stiffness of the masonry panel to the frame) defined by:

$$\lambda_h = h \sqrt[4]{\frac{E_m t \sin 2\theta}{4 E_c I_c h_m}} \quad (2)$$

In Equation 2, t and h_m are the thickness and the height of the masonry panel, respectively, θ is the inclination of the diagonal of the panel, E_m and E_c are the modulus of elasticity of the masonry and of the concrete, respectively, and I_c is the moment of inertia of the columns.

Paulay and Priestley [21] pointed out that a high value of w will result in a stiffer structure, and therefore potentially higher seismic response. They suggested a conservative value useful for design proposal, given by:

$$w = 0.25 d_m \quad (3)$$

This equation is recommended for a lateral force level of 50% of the ultimate capacity.

Mainstone [22] and Liauw and Kwan [23] proposed the following equations based on experimental and analytical data, respectively:

$$w = 0.16 \lambda_h^{-0.3} d_m \quad (4)$$

$$w = \frac{0.95 h_w \cos\theta}{\sqrt{\lambda_h}} \quad (5)$$

Figure 5 illustrates the variation of the ratio w/d_m according to the previous expressions. Equations 1 and 3 are independent of the parameter λ_h and they represent just an approximation useful for simplified analysis. Equations 4 and 5 indicate that the ratio w/d_m decreases when the parameter λ_h increases, because the stiffness of the masonry panel is large, when compared with the stiffness of the frame, and the contact length is smaller.

Based on results obtained from framed masonry walls (this is the case in which the masonry wall is built first and then the reinforced concrete frame is cast) tested under lateral forces, Decanini and Fantin [24] proposed two set of equations considering different states of the masonry infill:

Uncracked panel:

$$w = \left(\frac{0.748}{\lambda_h} + 0.085\right) d_m \quad \text{if } \lambda_h \leq 7.85$$

$$w = \left(\frac{0.393}{\lambda_h} + 0.130\right) d_m \quad \text{if } \lambda_h > 7.85 \quad (6a)$$

Cracked panel:

$$w = \left(\frac{0.707}{\lambda_h} + 0.010\right) d_m \qquad \text{if } \lambda_h \le 7.85$$

(6b)

$$w = \left(\frac{0.470}{\lambda_h} + 0.040\right) d_m \qquad \text{if } \lambda_h > 7.85$$

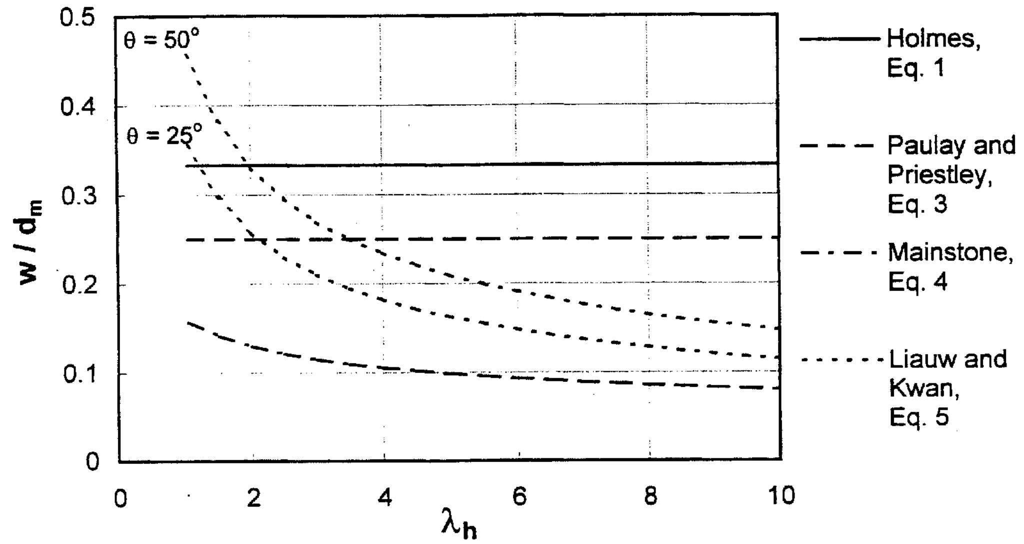

Figure 5: Variation of the ratio w/d_m for infilled frames as a function of the parameter λ_h.

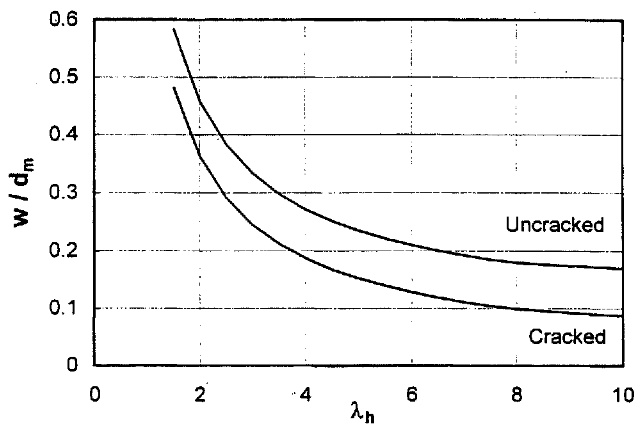

Figure 6: Ratio w/d_m for framed masonry structures according to Decanini and Fantin [24].

The modulus E_m to be used in the calculation of the parameter λ_h is the modulus corresponding to the considered state (uncracked or cracked masonry). These equations are plotted in Figure 6 as a function of the parameter λ_h. The principal advantage of the approach proposed by Decanini and Fantin [24] is the distinction between the uncracked and cracked stages. The comparison of Eqs. 6a and 6b indicates that w reduces significantly after cracking to a value ranging from 50% to 80% of the initial width. The higher reductions occur for large values of the parameter λ_h, because the influence of the infill panel in the response of the system is greater in these cases.

Bazán and Meli [12] proposed also an empirical expression to calculate the equivalent width w for framed masonry:

$$w = (0.35 + 0.22\beta)h \tag{7}$$

where $\beta = (E_c A_c)/(G_m A_m)$ is a dimensionless parameter, A_c is the gross area of the column and $A_m = (L_m t)$ is the area of the masonry panel in the horizontal plane. Figure 7 illustrates the ratio w/d_m according to Eq. 7. It is difficult to compare these results with previous expressions because they are related to two different parameters. Despite this fact, it is observed that Eq. 7 leads to higher ratios w/d_m than Eqs. 6a and 6b in the case of stiffer masonry panels (λ_h and β, in the range of 7 to 10 and 1 to 3, respectively).

It is also important to note that the equivalent width for framed masonry is usually higher than that for non-integral infilled frames, according to the empirical equations presented above. This conclusion is not surprising since framed masonry exhibits better conditions, bond strength and friction, at the panel-frame interfaces.

The simplified expression proposed by Paulay and Priestley [22], Eq. 3, can be considered as an upper limit for the ratio w/d_m. The expressions recommended for framed masonry (Eqs. 5, 6 and 7) lead to higher results only when limit conditions are considered.

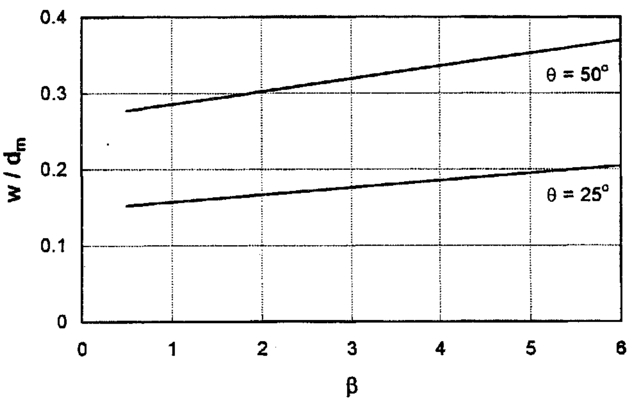

Figure 7: Ratio w/d_m for framed masonry structures (Bazán and Meli [12]).

Stafford Smith [18, 19] pointed out that the length of contact, z, between the frame and the panel (see Fig. 4) can be used as a reference parameter to evaluate either the stiffness or the strength of the infilled frame. They found that the contact length is governed by the relative stiffness parameter, λ_h, and proposed that z can be approximated by the following expression:

$$z = \frac{\pi}{2\lambda_h} h \tag{8}$$

It is worth noting that Eq. 8 was developed from tests conducted on small specimens diagonally loaded in compression. The frames were built with mild steel flat bars of different sizes and the panels were made of mortar. The panel dimension were 150 x 150 x 19 mm. In the author's opinion, the validity of Eq. 8 for infilled frame structures should be verified considering more realistic experimental data.

For the model illustrated in Fig. 2 (a), Zarnic [25] proposed an analytical procedure to calculate the area of the strut. It was assumed that the axial stiffness of the brace is equal to the stiffness of the triangular part of the masonry wall (considering shear and flexural deformations). This triangular part forms in the wall after cracking of the masonry. Therefore, it is possible to obtain the area of the strut as a function of the geometric and mechanical properties of the masonry infill. The equation proposed by Zarnic [25], however, did not consider that both stiffnesses are related to different displacements (axial displacement of the strut and horizontal displacement at the top of the triangular part of the panel). As a result, one of the stiffnesses should be transformed as a function of the inclination of the strut.

Chrysostomou [10] used a different approach to calculate the stiffness of the strut elements of his model, represented in Fig. 2 (c). The compressive force resisted by the masonry panel and their stiffness were calculated as a function of the storey drift, using a modification of the expression proposed by Soroushian *et al.* [26] for masonry shear walls. In order to define the properties of the three struts the following approach was implemented. The behaviour of the central strut was represented by expressions similar to those corresponding to the entire masonry panel. However, it was assumed that the central part of the infill panel deteriorates faster than the other parts. The properties of the off-diagonal struts were evaluated by considering that the forces and stiffnesses of the three struts should be equal to the force and stiffness of the entire masonry wall. The principle of virtual work was used to derive these expressions, assuming one particular displacement field. Chrysostomou's procedure to evaluate the properties of the off-diagonal struts implies that plastic hinges form only at the end of the columns or beams and that the internal work produced in these plastic hinges is negligible. The influence of these hypotheses should be checked to verify the validity of the model.

Comparison of the Response of Different Strut Models

A preliminary study was conducted to investigate the limitations of the single strut model, which is the simplest rational representation used for the analysis of infilled frames. Furthermore, the influence of different multi-strut models on the structural response of the infilled frame was studied, with particular interest in the stiffness of the structure and in the actions induced in the surrounding frame. Numerical results obtained from three strut models were compared with those corresponding to an equivalent finite element model (a detailed description of this model can be found in reference [1]). Figure 8 illustrates the strut models, which are referred as Model A, B and C, respectively. The total area of the equivalent masonry struts, A_{ms}, was the same in all cases. It was assumed in Model C that the sectional area of the central strut was double of that corresponding to the off-diagonal struts. Several series of models were analysed considering a 2.5 m high masonry panel with a length of 3.6 or 5.0 m, and an elastic modulus for the masonry of 2 500 or 10 000 MPa. The dimensions of the frame members were 200 x 200 mm and the elastic modulus of concrete was 25 000 MPa.

According to the objectives of the study, the analyses were conducted under static lateral loading assuming linear elastic behaviour, except for the finite element model in which non-linear effects were considered to represent the separation of the panel-frame interfaces. Results are presented in the following paragraphs in qualitative terms.

The stiffness of the infilled frame was similar in all the cases considered, with smaller values for model B and C. It must be noted that for the multi-strut models, especially Model C, the stiffness may significantly change depending on the distance h_z (see Fig. 8). This distance was evaluated as a fraction of the contact length, z, defined by Eq. 8 . When h_z increases, the stiffness of the infilled frame reduces, being chiefly controlled by the mechanical properties of the columns.

Figure 9 compares the bending moment diagrams obtained from one typical example according to the different models used in this study. Model A underestimates the bending moment because the lateral forces are primarily resisted by a truss mechanism. On the other hand, Model B leads to much larger values than those corresponding to the finite element model. A better approximation is obtained from Model C, although some differences arise at the ends of both columns.

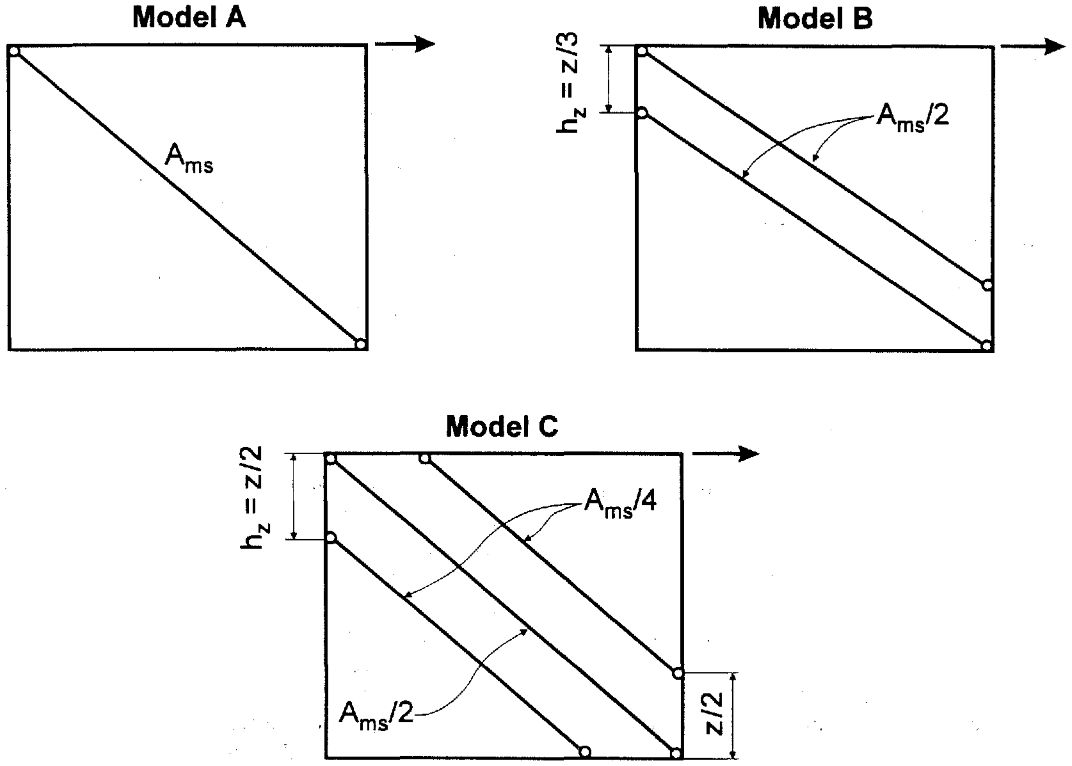

Figure 8: Different strut models considered in the study.

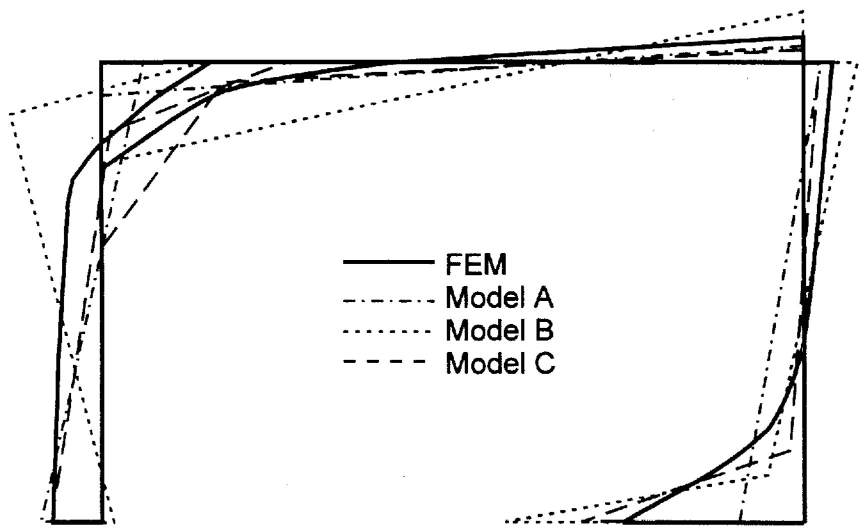

Figure 9: Comparison of the bending moments diagrams corresponding to different strut models.

Similar conclusions can be drawn regarding the shear forces. The maximum axial forces in the frame members are approximately equal in all the models, even though the variation of the axial forces along the columns shows some discrepancy at the top end of the tension column and at the bottom end of the compression column. It can be concluded that the single strut model, despite its simplicity, can provide an adequate estimation of the stiffness of the infilled frame and the axial forces induced in the frame members by lateral forces. However, a more refined model, Model C, is required in order to obtain realistic values of the bending moments and shear forces in the frame. The results obtained here indicate that the single strut model represents an adequate tool when the analysis is focussed on the overall response of the structure.

Hysteretic Behaviour of the Diagonal Struts

In order to conduct non-linear cyclic or dynamic analysis, the force-displacement relationships corresponding to the equivalent strut must be adequately defined. The representation of the hysteretic behaviour increases not only the complexity of the analysis but also the uncertainties of the problem.

Klingner and Bertero [2] developed three different hysteretic models to represent the diagonal strut, each of them involved a slight increase in the complexity. Figure 10 illustrates the characteristics of the third model, in which the envelope was represented by a linear elastic ascending branch followed by an exponential descending curve. Unloading was assumed to be linear with stiffness equal to the initial stiffness, whereas the effect of stiffness degradation was considered for reloading. The comparison of the analytical results with experimental data showed poor agreement, although this model was the first approach to include the non-linear response of infilled frames and represented the basis for further developments. The strength envelope proposed by Klingner and Bertero has been also used for non-linear static analysis in order to represent the effect of strength degradation [27].

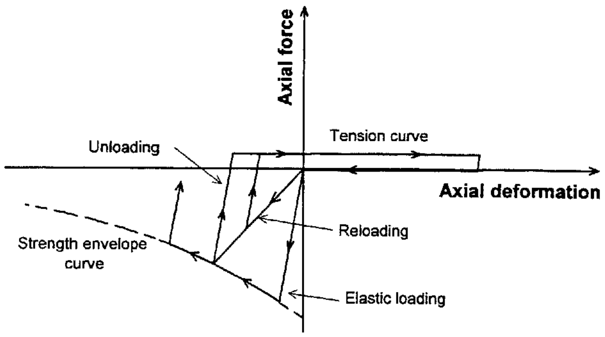

Figure 10: ***Hysteretic behaviour of the strut model proposed by Klingner and Bertero [2].***

The hysteretic model proposed by Doudoumis and Mitsopoulou [28] is shown in Figure 11. This model was developed for non-integral infilled frames, in which a gap normally forms between the masonry panel and the surrounding frame. The envelope curve considered the effect of strength degradation. The hysteresis cycles were described in a very simplistic way assuming that reloading occurs following the elastic branch.

Figure 12 shows the force-displacement relationship adopted by Andreaus *et al.* [16] for representing the mechanical behaviour of the diagonal struts. This model assumes that strength degradation starts immediately after the strength of the strut has been reached. Reloading occurs when the axial deformation is equal to the plastic deformation of the previous loop.

The comparison of the three hysteretic models illustrated in Figures 10, 11 and 12 shows that the envelope curves are similar. The effect of strength degradation appears to be significant and has been considered in all the cases. The representation of the hysteresis loops, however, exhibits important differences. Doudoumis and Mitsopoulou [28] (Fig. 11) assumed that reloading occurs following the initial loading branch. This assumption leads to fat hysteresis loops with high energy dissipation capacity. On the contrary, Andreaus *et al.* [16] (Fig. 12) considered that unloading and subsequent reloading follow the same line, which reduces considerably the area of the loops. The model proposed by Klingner and Bertero [2] (Fig. 10) represents and intermediate situation, which includes also the effect of stiffness degradation.

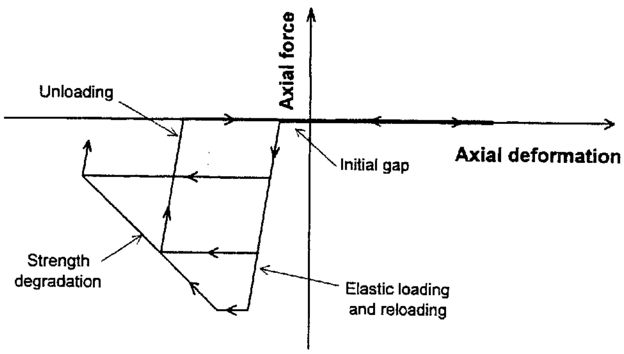

Figure 11: ***Hysteretic model developed by Doudoumis and Mitsopoulou [28].***

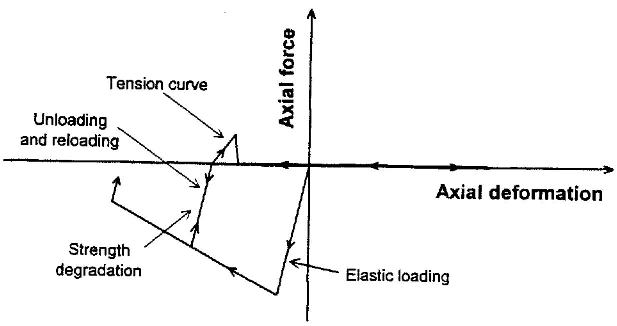

Figure 12: ***Force-displacement relationship assumed by Andreaus et al. [16].***

A different approach was proposed by Soroushian *et al.* [26] for masonry walls, which was later modified by Chrysostomou [10] for representing the behaviour of masonry infills. The hysteretic response is modelled by combining two equations. The first equation (a logarithm exponential function) defines the strength envelope, whereas the second equation (a quartic polynomial function) represents the hysteretic loops, as shown in Figure 13. These expressions, which describe the mechanical behaviour of the infill, were used to derive the force-displacement relationships for the central and off-diagonal struts of the model proposed by Chrysostomou [10] (see Fig. 2 (c)).

Reinhorn *et al.* [29] developed a hysteretic model which combines two mathematical functions to provide a smooth force-displacement relationship. Strength degradation, stiffness decay and pinching of the hysteresis loops can be considered by selecting the proper values of the nine parameters included in the model. Some of these parameters are empirical, whereas the others depend on energy considerations. The implementation of this model is not straightforward and the solution requires the numerical integration of a differential equation.

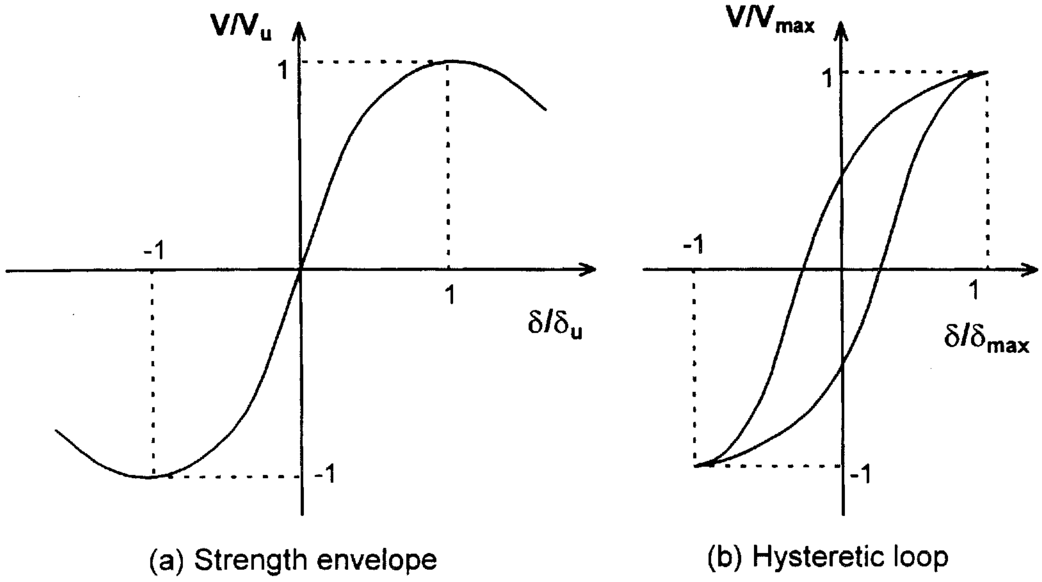

Figure 13: ***Hysteretic model proposed by Soroushian et al. [26] for masonry shear walls and adopted by Chrysostomou [10] for the diagonal strut.***

Crisafulli [1] proposed an analytical formulation to simulate the hysteretic axial response of masonry (stress-strain relationship), and used it to defined the response of the equivalent strut. This model takes into account the nonlinear response of masonry in compression, contact effects in the cracked material, and small cyclic hysteresis, as illustrated in Figure 14. The comparison between experimental and analytical results indicated that a good agreement can be obtained. However, several empirical parameters need to be defined in order to represent adequately the hysteretic response. This model has been implemented in the computer program RUAUMOKO [30] for dynamic inelastic analysis of structures.

It is worth noting that most of the pinching of the hysteresis response, observed in the inelastic models, results from the nonlinear behaviour of the masonry struts, whereas in real structures the influence of sliding shear in the masonry panel and in the columns of the surrounding frame is also important. The representation of this phenomenon with a macro-model is very complex and requires the use of a refined formulation, which is not usually available in most of the existing computer programs. Further research is required to incorporate shear failure mechanisms of reinforced concrete members in the analytical modelling, considering their effect on the structural response can be significant.

STOREY MECHANISM MODEL

The storey mechanism model is a simplified approach developed to investigate the global response of infilled frame structures. According to this approach, the response of a complete storey, or even the entire structure, is represented using a non-linear relationship between the lateral force and the storey drift (or lateral displacement). The model does not consider any distinction between the frame and the masonry panel.

Moroni *et al.* [31] used the storey mechanism model to conduct non-linear static analysis with the objective of comparing the displacement capacity of infilled frames with that required by major earthquakes. Flores and Alcocer [32] proposed a hysteretic rule, which was calibrated from experimental results obtained from infilled frames with and without horizontal reinforcement into the masonry panel. The model was used for non-linear dynamic analyses aimed at investigating the influence of several parameters in the response of infilled frames. Panagiotakos and Fardis [33] proposed a shear force-storey drift relationship which has a multi-linear envelope to represent the most important characteristics of the response (cracking, ultimate strength, post-ultimate falling branch and residual strength). The hysteretic response is controlled by three empirical parameters which define the unloading and reloading branches. These parameters were calibrated to model the pinching effect observed during tests of infilled frames. The general characteristics of the proposed hysteretic model are illustrated in Figure 15.

OTHER MACRO-MODELS

Despite the advantages of the diagonal strut model, other approaches have been used to analyse infilled frame structures. Most of these models were developed to evaluate the stiffness of the structure assuming elastic behaviour, consequently, their applicability is very limited.

The initial uncracked behaviour of infilled frames, providing that there is no gap between the frame and the masonry panel, can be evaluated by assuming that the total system behaves as a

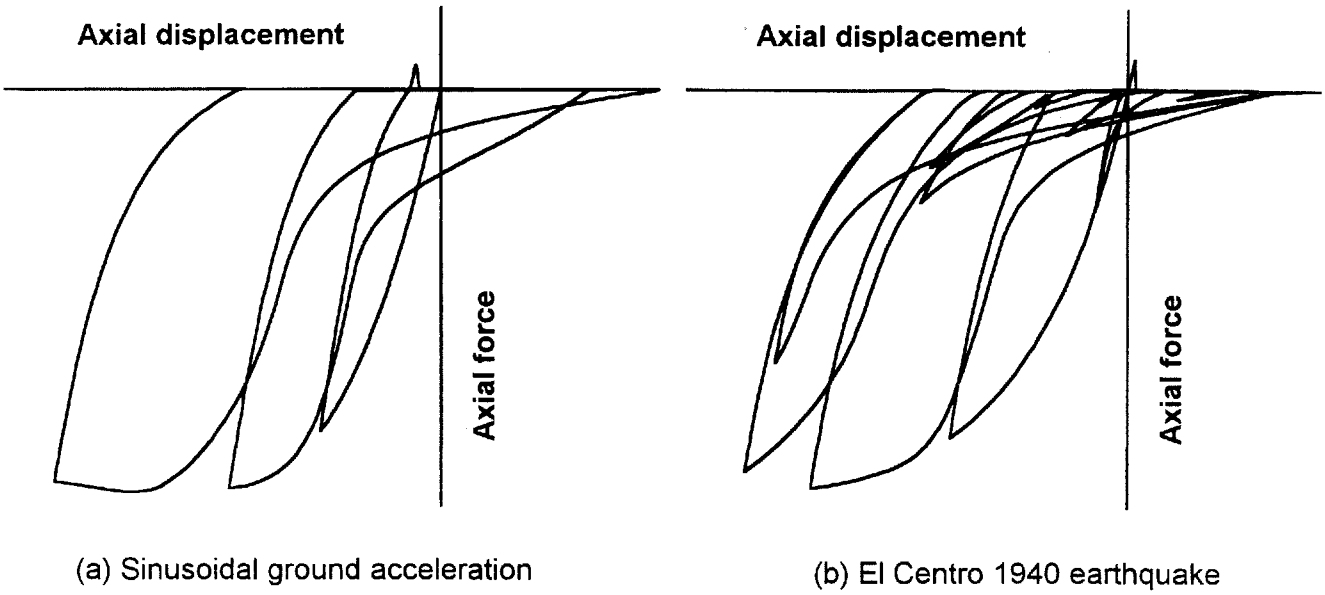

Figure 14: Axial force-displacement relationship for the masonry strut proposed by Crisafulli [1].

single monolithic member [14, 15]. Therefore, the structure can be analysed using standard elastic theory considering the contribution of the flexural and shear deformations of the system to evaluate the horizontal displacements. The validity of this model, usually called "the beam analogy", depends on the bond strength developed at the panel-frame interface. Leuchars & Scrivener [15] reported that the initial uncracked mode can resist forces up to 50% of the ultimate force. Using the same concept of the beam analogy, Thiruvengadam [13] applied a shear-flexure cantilever model to evaluate the natural periods of infilled frames.

Smolira [34] developed an approximate method for analysing infilled frame structures, assuming that the materials obey Hooke's law. The indeterminate variables were the bending moments at the ends of the columns, the diagonal force in the masonry infill and the horizontal displacement. These values were evaluated using a set of equations obtained from conditions of compatibility of deformations and equilibrium of forces.

The equivalent frame method was proposed by Liauw and Lee [35] for infilled frames with openings and shear connectors at the panel-frame interfaces. They assumed that an analogous model can be set up by representing the structure with an equivalent frame. Using the ratio of the elastic modulus of the two materials (masonry and concrete), the actual members are transformed into equivalent sections of infill material. The dimensions of the equivalent frame are obtained from the centroidal axes of the actual infilled frames. The validity of the method depends on the capacity of the shear connectors to sustain the composite action without allowing the separation of the infill.

An approximate substructuring technique, called the constraint approach, was developed by Axley and Bertero [36] to investigate the influence of masonry infills in reinforced concrete frames. Three steps were considered in the formulation of the constraint approach. Firstly, the system is modelled separately, using finite elements for the frame and the panel, and the stiffness matrixes are formed. Then, the stiffness of the masonry infill is reduced to the boundary degrees of freedom by applying static condensation. In the final step, a new transformation is conducted to obtain the stiffness matrix of the panel related to the degrees of freedom of the four corners, which are also the degrees of freedom of the frame. The method was applied to conduct linear elastic analyses and was implemented as a four node element in a computer program for structural analysis. Even though this technique was formulated on the basis of the finite element method, it can be considered as a macro-model from a practical point of view.

Pires *et al.* [37] idealised the infilled frames using a parallel association between frame and the masonry infills (represented by a shear cantilever beam). Both parts of the model were connected with rigid links. Non-linear behaviour was considered for the reinforced concrete frame and masonry infill. This model, however, does not consider that separation between the frame and the panel occurs when the lateral load increases. It is believed that this factor may completely distort the behaviour of the model when compared with the real structure. Furthermore, the complexity of the model increases significantly for large infilled frames.

Valiasis *et al.* [38] developed a phenomenological model in which the relationship between the average shear stress in the panel and the angular deformation was defined. The envelope was represented by two linear ascending branches and one exponential descending branch. The hysteresis rules incorporated the effects of stiffness degradation and pinching using linear branches to approximate the unloading-reloading curves. It was assumed that the contribution of the infill panel is equal to the difference of the response of the infilled and bare frame for each level of lateral displacement. Valiasis *et al.* [38] pointed out that this model includes the influence of the interaction between the masonry panel and the frame and is strongly dependent on the geometric and mechanical characteristics of both the surrounding frame and the infill panel. Later, Michailidis *et al.* [39] implemented this model into a general program for the analysis of plane structures. The entire masonry panel was represented by a four-node isoparametric element with two degrees of freedom at each node.

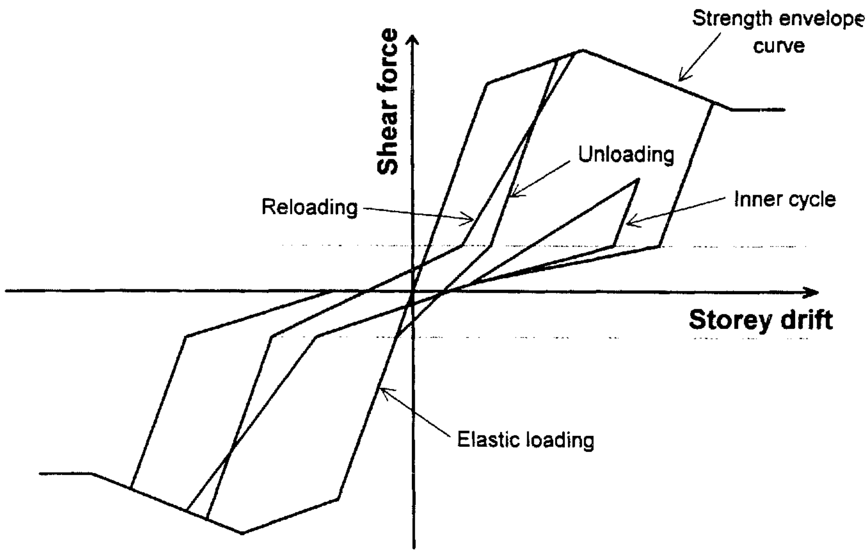

Figure 15: Shear force-storey drift hysteretic model proposed by Panagiotakos and Fardis [33] for infilled frame structures.

FINITE ELEMENT MODELS

Introduction

The finite element method has been extensively used for modelling infilled frame structures, since Mallick and Severn [3] applied this approach in 1967. Due to the composite characteristics of infilled frames, different elements are required in the model: beam or continuum elements for the surrounding frame, continuum elements for the masonry panel and interface elements for representing the interaction between the frame and the panel. Finite element models exhibit obvious advantages for describing the behaviour of infilled frames and the local effects related to cracking, crushing and contact interaction. This implies a greater computational effort and more time in preparing the input data and in analysing the results. For the model to be realistic, the constitutive relationships of the different elements should be properly defined and the non-linear phenomena which occur in the masonry infill and in the panel-frame interfaces must be adequately considered. Otherwise, the validity of the results is jeopardised, despite the great computational effort involved in the analysis.

Even though three-dimensional continuum elements are available for the analysis, it is commonly considered that the use of two-dimensional continuum elements leads to acceptable results. In this case, a state of plane stress is a reasonable assumption for most of the cases of in-plane loading [40].

The modelling of the frame and the masonry panel with finite elements has been amply investigated. Only a brief description of these models is presented here. More attention is given to the modelling of the panel-frame interfaces, which represents a particular characteristic of infilled frame structures.

Modelling of the Masonry Panel

The analytical model used for the masonry panel should reflect the non-linear nature of this material and the influence of the mortar joints. Different approaches have been implemented for representing the masonry panel, which are primarily based on the modelling techniques developed for concrete and rock mechanics [41]. Nevertheless, the behaviour of masonry is more complex due to the planes of weakness introduced by the mortar joints. These approaches can be grouped according to the level of refinement involved in the model [42, 43].

The first approach is the least refined, in which the masonry is represented as a homogeneous material. Consequently, the effect of the mortar joint is considered in an average sense. This approach is suitable for modelling large masonry structures, where a detail stress analysis is not required. The material model should represent the mechanical behaviour of masonry by adequately defining the stress-strain relationship and the failure criterion. Several failure criteria has been specifically developed for masonry structures, although, other criteria have also been used, for example, the Von Mises criterion with tension cut-off [43] or the Drucker-Prager criterion [44].

In the second approach, masonry is represented as a two-phase material. Both masonry units and mortar joints are modelled with continuum elements [40, 45]. The model usually requires a large number of elements and the mechanical behaviour of masonry units and mortar is separately defined. Interface elements should be used to represent the mortar-brick interfaces, where debonding, slip or separation can occur. The model is capable of capturing the different modes of failure, if it is adequately implemented and calibrated. Analyses with such level of refinement require a great computational effort. Consequently, this approach is mainly applied to small structures, usually as a research tool.

The third approach used for modelling masonry panels represents an intermediate situation between the two previous approaches. In this case, masonry units are represented with continuum elements, while the mortar joints are modelled with interface elements [42, 46, 47]. The interface elements not only represent the behaviour of the mortar-brick interfaces, but they also take into account the elastic and plastic deformations occurring in the mortar. In the initial implementation of this approach, conducted by Page [47], the masonry units were assumed to behave elastically. Later developments of the methodology allow the consideration of a more realistic behaviour for the masonry units, including cracking.

Cracking is an important feature that should be also considered in the analysis, independent of the approach used in the discretization of the panel (masonry as homogeneous material or as two-phase material). The smeared crack model is commonly implemented for considering the effect of cracking [4, 44, 46]. This model does not track each individual crack. Instead, the overall cracking within an area is simulated by changing the stress and the material stiffness associated with the integration points. Schnobrich [48] pointed out that there are some doubts about the independence of the solution relative to the grid size used in the analysis (mesh sensitivity). Furthermore, the use of low order finite elements (for example, constant strain triangles) may confuse the cracking situation, due to the inadequate characteristics of these elements to respond to steep stress gradients. It is worth noting that the smeared crack model is a valid tool only for those structures where multiple cracks occur and the response is not sensitive to the precise geometry of cracking. This model should not be used in problems where a few isolated cracks control the behaviour. Furthermore, Shing *et al.* [41] pointed out that the smeared crack approach alone is not able to capture the brittle shear failure of masonry panels and to account for the influence of the mortar joints.

Modelling of the Surrounding Frame

The analytical representation of the frame can be done either with beam elements [3, 49, 50, 51, 52, 53] or with a more refined discretization using continuum elements (two or three-dimensional elements) [4, 44, 46]. The use of these different representations implies increasing levels of complexity in the analysis, resulting in a better accuracy when the model is properly implemented. The main advantage of the beam elements is that they are geometrically simple and have few degrees of freedom. The effect of the steel bars, in reinforced concrete members, is implicitly considered in the definition of the flexural and axial relationships assumed in the analysis. When non-linear analysis is conducted, the effect of slip of the reinforcing bars can be also taken into consideration using rotational springs located at the ends of the member [54]. On the other hand, the use of continuum elements for modelling the frame allows a better description of its behaviour, although many more elements are needed in the discretization. Reinforced concrete members require additional elements to represent the effect of the reinforcing bars. This can be done by using a smeared overlay or discrete bar elements, assuming a hypothesis for the strain compatibility between the steel and the concrete [4, 44, 46].

Modelling of the Interfaces

The structural interfaces between the surrounding frame and the infill panel have been represented in the analytical models by using tie-link or interface elements. The function of these elements is to represent the interaction between deformable structures, along surfaces, where separation and sliding may arise. They allow for geometric discontinuity to occur in the structure. The adequate description of the contact effects developing at the panel-frame interfaces is very important to obtain a realistic response of the model.

The first attempt to take into account the behaviour of the interfaces was developed by Mallick and Severn [3]. They implemented an iterative scheme using a finite element model, in which additional contact forces were introduced in those zones where the panel-frame interfaces were closed. Several researchers [49, 50, 55,] used tie-link elements to connect the boundary nodes of the panel with the surrounding frame. These elements enable two adjacent nodes to be held together or released according to specified conditions. Each node of the element has two translational degrees of freedom. The element is able to transfer compressive and bond forces, but incapable of resisting tensile forces. Large values of the normal and tangential stiffnesses are adopted when the link is active. Conversely, the link is released by setting these values to zero. Figure 16 illustrates schematically the characteristics of the tie-link model.

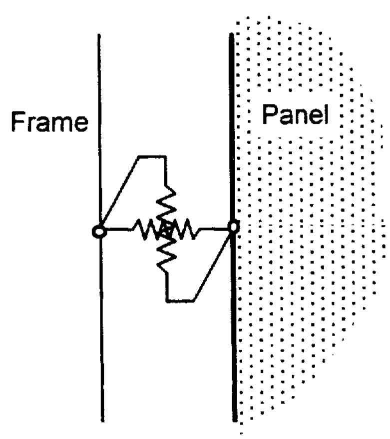

Figure 16:* *Tie-link element used to represent the behaviour of panel-frame interfaces.

A more accurate description of the interaction between the panel and the frame can be achieved by using interface elements [4, 44, 50, 56]. These elements were introduced by Ngo and Scordelis in the area of concrete mechanics and by Goodman et al. in the area of rock mechanics (as reported by Lofti and Shing [42]). Each element requires at least four nodes to represent two adjacent surfaces, as schematically illustrated in Figure 17 (a). The degrees of freedom are related to the normal and shear stresses developed between the surfaces, f_n and v, respectively. The traction transmitted between the surfaces and the relative displacement can be

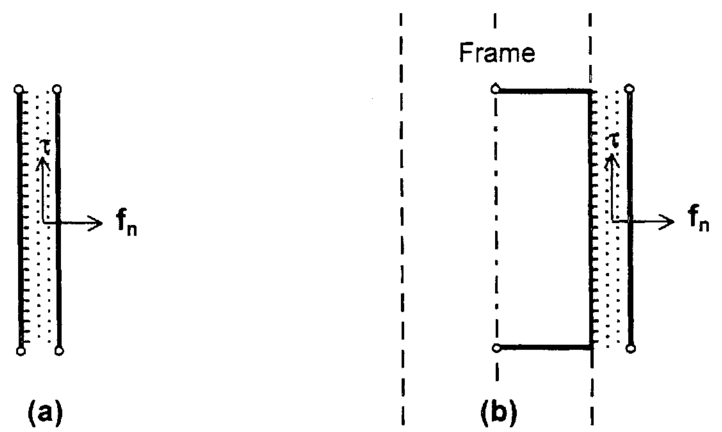

Figure 17: Interface elements: (a) General element, (b) Modified element developed by King and Pandey [15].

represented using different constitutive relationships. However, the friction theory proposed by Coulomb is usually implemented. King and Pandey [50] developed a modified interface element in which one of the surfaces presents two perpendicular, rigid links to represent the depth of the frame member (see Fig. 17 (b)). The nodes related to the rigid links have also a rotational degree of freedom. This modification is useful when the surrounding frame is modelled with beam elements. A similar approach was implemented by Liaum & Kwan [57]. Mosalam *et al.* [44] pointed out that interface elements may be sensitive to the mesh implemented in the analysis. Consequently, the characteristics of the finite element mesh must be carefully selected. A preliminary study with different mesh configurations is recommended.

The constitutive model implemented in the interface element must assure the impenetrability condition, when the surfaces are in contact. The normal stress is usually defined with a linear elastic model. However, the finite value of the normal stiffness violates the impenetrability requirement. By taking a large value of the normal stiffness (in relation to other stiffnesses in the model), this violation is not significant [44].

The friction theory proposed by Coulomb in 1781 is commonly accepted to represent the behaviour of the interfaces. This theory asserts that "relative sliding between two bodies in contact along plane surfaces will occur when the net shear force parallel to the plane reaches a critical value proportional to the net normal force pressing the bodies together. The constant of proportionality is called the coefficient of friction" [58]. Oden and Pires [58] pointed out that the Coulomb's theory is capable of describing only friction effects between rigid bodies. They also considered that is very important to represent adequately the non-local character of the mechanism by which normal stresses are distributed along the surfaces. The stresses are transmitted over junctions formed by asperities on the contact surface. Small tangential displacements occur due to the elastic and inelastic deformation of these junctions. They indicated that, from the mathematical point of view, Coulomb's theory introduced problems in the formulation of the model and formulated a non-local friction law, which includes an additional parameter introduced to consider a small, but non-zero, elastic tangential displacement for shear stresses below the sliding limit.

According to Coulomb's theory, the adequate modelling of the behaviour of structural interfaces requires the consideration of three different stages:

- **Firm contact and no slip**: when the surfaces are in contact, a compressive normal stress, f_n, develops at the interface. It is considered that slip does not occur if the shear stresses, v, satisfies:

$$|v| < \mu f_n \tag{9}$$

  where μ is the coefficient of friction of the interface and f_n is positive when the surfaces are compressed. The condition of no slip is usually approximated by elastic behaviour and the friction theory is implemented with the stiffness method. Consequently, some relative motion (an "elastic slip") can occur between both surfaces. Permitting a relative motion when the surfaces are stuck makes convergence of the solution more rapid, at the expense of solution accuracy. It must be pointed out that no shear deformation should occur when the surfaces are in contact and Eq. 9 holds, since the interface has no thickness. This fact indicates that, in a strict sense, the shear stiffness of the interface should be infinite.

  Different approaches have been implemented to define the shear stiffness of the interface. The non-local friction models assume that impeding motion at a point of contact between deformable bodies will occur when the shear stress at a point reaches a value proportional to a weighted measure of the normal stresses in a neighbourhood of the point [58]. In the model implemented in the program ABAQUS [56], the shear stiffness is chosen in order that the elastic slip (when $v = \mu f_n$) is limited to an allowable value. This value is selected as a small fraction (for example, 0.005) of the length of the interface element. The shear stiffness will change during the analysis because it depends on the normal stress f_n. It can be shown that this implementation represents a non-local friction model.

King and Pandey [50] suggested the use of experimental data to define the shear stress of the interface. They reported values of the shear stiffness, which were obtained from tests of square specimens sliding on steel or concrete surfaces. However, it is not clear what method was used for the evaluation of the shear stiffness. Consequently, the validity of these results cannot be discussed.

Other formulations can be implemented, instead of the stiffness method, to assure that zero relative slip will occur at this stage. For example, it is possible, using by a Lagrange multiplier method, to impose constraints or to prescribe relationships between degrees of freedom in the mathematical formulation of the model [56].

- **Firm contact with slip**: the surfaces remain in contact, although slip occurs because the shear stress is equal to the friction strength of the interface:

$$|v| = \mu f_n \qquad (10)$$

It is usually assumed that the shear stress remains constant (for a constant level of normal stress) as the surfaces slip. However, there is experimental evidence that the coefficient of friction for mortar-brick interfaces decreases after the strength has been achieved [1]. Similar behaviour could be exhibited by other materials as concrete or steel. The decrease of the shear stress after slip occurs could be more significant for those interfaces which exhibit shear bond strength.

- **No contact**: in this stage the surfaces are separated. Neither normal nor shear stresses develop at the interface. It is usually assumed that separation occurs when tensile stresses develop in the normal direction [3, 52]. Other models [56] consider that tensile stresses can be resisted without separation up to some allowable level. In this way, it is possible to represent the effect of tensile bond strength between both surfaces.

It is worth noting that Coulomb's theory does not consider any shear bond between the surfaces in the shear friction mechanism. This effect is not significant in the case of steel infilled frames. However, the effect of the shear bond strength could be important for mortar-concrete or brick-concrete interfaces, especially in framed masonry. Despite this problem, Coulomb's theory has been widely used in the implementation of interface models.

The models described in this section indicate that adequate analytical tools have been developed for representing the behaviour of the interfaces. Unfortunately, this effort has not been accompanied by a similar improvement in the knowledge of the mechanical properties of panel-frame interfaces. In the author's knowledge, there is no data on the tensile and shear bond strength of the interfaces. Only one report has been found with experimental values of the coefficient of friction. These values, obtained by King and Pandey [50], are presented in Table 1. It is believed that some of these results should be revised. For example, the coefficient of friction for concrete on concrete is significantly smaller than that for brick on concrete, which seems to be unrealistic.

The modelling of panel-frame interfaces with shear connectors is more complex. It has been suggested [49, 52] that the tie-link elements can be used to simulate the effect of shear connectors or steel reinforcement connecting the masonry panel and the frame.

Table 1. Coefficient of friction for different materials [50].

Materials	Coefficient of friction, μ
Brick on steel	0.50
Mortar on steel	0.44
Concrete on steel	0.41
Brick on concrete	0.62
Mortar on concrete	0.42
Concrete on concrete	0.44

OTHER MICRO-MODELS

The early analytical investigations related to the behaviour of infilled frames were based on the application of the theory of elasticity. For example, Stafford Smith [5] used a finite difference approximation to solve the stress function for the masonry panel. Polyakov [59] found the stress distribution inside the panel by using variational methods. With the improvement of computer capabilities and the development of the finite element method these types of model have been discarded.

Other researchers [60, 61, 62] have applied the distinct element method, originally developed for fractured rocks, to the analyses of masonry walls. This method allows the study of jointed media subjected to static or dynamic loads. The media is simulated as an assemblage of discrete blocks, which interact through edge contacts. The mathematical model is established by considering two set of equations. The first group of equations is formed by constitutive relationships between the force in the blocks and the relative displacement. The second group represents the kinematic equations which define the motion of the blocks [62]. This numerical method is capable of following the complete process of fracture.

CONCLUSIONS

- Different analytical models have been used to describe the behaviour of infilled frames. These models can be divided into two groups: micro-models and macro-models. The finite element formulation and the equivalent truss mechanism are the typical examples of the first and second group, respectively.
- Macro-models exhibit obvious advantages in terms of computational simplicity and efficiency. Their formulation is based on a physically reasonable representation of the structural behaviour of the infilled frame.
- The single strut model is a simple representation, but it is not able to describe the local effects occurring in the surrounding frame. The use of multi-strut models can overcome this problem without a significant increase in the complexity of the analysis.
- Micro-models can simulate the structural behaviour with great detail, providing that adequate constitutive models are used. However, they are computational intensive and difficult to apply in the analysis of large structures.
- The accurate analysis of infilled frames with finite element models requires the use of at least three types of elements to represent the surrounding frame, the masonry panels and the panel-frame interfaces. In a more refined analysis, the masonry panels can be considered as a two-phase material, in which the masonry units and the mortar joints are modelled separately.
- There is not enough experimental information on the mechanical properties of panel-frame interfaces. More research is required to evaluate these properties, in order to obtain realistic results from finite element models.

ACKNOWLEDGMENTS

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REFERENCES

1. Crisafulli, F. J., *Seismic Behaviour of Reinforced Concrete Structures with Masonry Infills*, PhD Thesis, Department of Civil Engineering, University of Canterbury, 1997, 404 p.
2. Klingner, R. E. and Bertero, V. V., *Infilled Frames in Earthquake-Resistant Construction*, University of California, Berkeley, Report No. EERC 76-32, December, 1976.
3. Mallick, D. V. and Severn, R. T., "The Behaviour of Infilled Frames under Static Loading", *Proceedings of the Institution of Civil Engineering*, Vol. 38, 1967, pp. 639-656.
4. König, G., "The State of the Art in Earthquake Engineering Research", *Experimental and Numerical Methods in Earthquake Engineering*, Edited by J. Donea and P. M. Jones, 1991, pp. 2/1-22.
5. Stafford Smith, B., "Lateral Stiffness of Infilled Frames", *Proceedings of the American Society of Civil Engineering, Journal of Structural Division*, Vol. 88, No. ST6, 1962, pp. 183-199.
6. Flanagan, R. D., Tenbus, M. A. and Bennett, R. M., "Numerical Modelling of Clay Tile Infills", *Proceedings from the NCEER Workshop on Seismic Response of Masonry*, San Francisco, California, February, 1994, pp. 1/63-68.
7. Zarnic, R. and Tomazevic, M., "Study of the Behaviour of Masonry Infilled Reinforced Concrete Frames Subjected to Seismic Loading", *Proceedings of the Seventh International Brick Masonry Conference*, Melbourne, Australia, Vol. 2, 1985, pp. 1315-1325.
8. Zarnic, R. and Tomazevic, M., "An Experimentally Obtained Method for Evaluation of the Behaviour of Masonry Infilled R/C Frames", *Proceedings of the Ninth World Conference on Earthquake Engineering*, Tokyo, Japan, Vol. VI, 1988, pp. 163-168.
9. Zarnic, R. and Tomazevic, M., "The Behaviour of Masonry Infilled Reinforced Concrete Frames Subjected to Cyclic Lateral Loading", *Proceedings of the Ninth World Conference on Earthquake Engineering*, San Francisco, USA, Vol. VI, 1984, pp. 863-870.
10. Chrysostomou, C. Z., *Effects of Degrading Infill Walls on the Nonlinear Seismic Response of Two-Dimensional Steel Frames*, Ph. D. Thesis, Cornell University, 1991.
11. Syrmakezis, C. A. and Vratsanou, V. Y., "Influence of Infill Walls to R.C. Frames Response", *Proceedings of the Eighth European Conference on Earthquake Engineering*, Lisbon, Portugal, Vol. 3, 1986, pp. 6.5/47-53.
12. San Bartolomé, A., *Colección del Ingeniero Civil* (in spanish), Libro No. 4, Colegio de Ingenieros del Peru, 1990, 115p.
13. Thiruvengadam, V., "On the Natural Frequencies of Infilled Frames", *Earthquake Engineering and Structural Dynamics*, Vol. 13, 1985, pp. 401-419.

14. Fiorato, A. E., Sozen, M. A. and Gamble, W. L., *An Investigation of the Interaction of Reinforced Frames with Masonry Filler Walls*, University of Illinois, Urbana, Illinois, Civil Engineering Studies, Structural Research Series No. 370, November, 1970.

15. Leuchars, J. M. and Scrivener, J. C., "Masonry Infill Panels Subjected to Cyclic In-Plane Loading", *Bulletin of the New Zealand National Society for Earthquake Engineering*, Vol. 9, No. 2, 1976, pp. 122-131.

16. Andreaus, U., Cerone, M., D'Asdia, P. and Iannozzi, F., "A Finite Element Model for the Analysis of Masonry Structures under Cyclic Actions", *Proceedings of the Seventh International Brick and Masonry Conference*, Melbourne, Australia, February, Vol. 1, 1985, pp. 479-488.

17. D'Asdia, P., D'Ayala, D. and Palombini, F., "On the Seismic Behaviour of Infilled Frames", *Proceedings of the Ninth European Conference on Earthquake Engineering*, Moscow, Vol. 8, 1990, pp. 162-171.

18. Stafford Smith, B., "Behaviour of Square Infilled Frames", *Proceedings of the American Society of Civil Engineering, Journal of Structural Division*, Vol. 92, No. ST1, 1966, pp. 381-403.

19. Stafford Smith, B. and Carter, C., "A Method of Analysis for Infilled Frames", *Proceedings of the Institution of Civil Engineers*, Vol. 44, 1969, pp. 31-48.

20. Stafford Smith, B., "Methods for Predicting the Lateral Stiffness and Strength of Multi-Storey Infilled Frames", *Building Science*, Vol. 2, 1967, pp. 247-257.

21. Paulay, T. and Priestley, M. J. N., *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley & Sons Inc., 1992, 744p.

22. Mainstone, R. J., "On the Stiffnesses and Strengths of Infilled Frames", *Proceedings of the Institution of Civil Engineers*, 1971, Supplement IV, pp. 57-90.

23. Liauw, T. C. and Kwan, K. H., "Nonlinear Behaviour of Non-Integral Infilled Frames", *Computers & Structures*, Vol. 18, No. 3, 1984, pp. 551-560.

24. Decanini, L. D. and Fantin, G. E., "Modelos simplificados de la mampostería incluida en pórticos. Características de rigidez y resistencia lateral en estado límite" (in Spanish), *Jornadas Argentinas de Ingeniería Estructural*, Buenos Aires, Argentina, 1986, Vol. 2, pp. 817-836.

25. Zarnic, R., "Modelling of Response of Masonry Infilled Frames", *Proceedings of the Tenth European Conference on Earthquake Engineering*, Vienna, Austria, Vol. 3, 1994, pp. 1481-1486.

26. Soroushian, P., Obaseki, K. and Ki-Bong Choi, "Non-linear Modelling and Seismic Analysis of Masonry Shear Walls", *Proceedings of the American Society of Civil Engineers, Journal of Structural Engineering*, Vol. 114, No. 5, 1988, pp. 1106-1119.

27. Chrysostomou, C. Z., Gergely, P. and Abel, J. F., "Preliminary Studies of the Effects of Degrading Infill Walls on the Non-linear Seismic Response of Steel Frames", *Proceedings of the Fourth U.S. National Conference on Earthquake Engineering*, Palm Springs, California, May, 1990, Vol. 2, pp. 229-238.

28. Doudoumis, I. N. and Mitsopoulou, E. N., "Non-linear Analysis of Multistorey Infilled Frames for Unilateral Contact Conditions", *Proceedings of the Eighth European Conference on Earthquake Engineering*, Lisbon, Portugal, 1986, Vol. 3, pp. 6.5/63-70.

29. Reinhorn, A. M., Madam A., Valles, R. E., Reichmann, Y. and Mander, J. B., *Modelling of Masonry Infill Panels for Structural Analysis*, National Centre for Earthquake Engineering Research, Technical Report NCEER-95-0018, December, 1995.

30. Carr, A. J., *RUAUMOKO. Dynamic Nonlinear Analysis*, Department of Civil Engineering, University of Canterbury, Christchurch, New Zealand, 1998.

31. Moroni, M., Astroza, M. and Mesias, P., "Displacement Capacity Required Storey Drift in Confined Masonry Buildings", *Proceedings of the Eleventh World Conference on Earthquake Engineering*, Mexico, 1996, Paper No. 1059.

32. Flores, L. E. and Alcocer, S. M., "Calculated Response of Confined Masonry Structures", *Proceedings of the Eleventh World Conference on Earthquake Engineering*, Mexico, 1996, Paper No. 1830.

33. Panagiotakos, T. B. and Fardis, M. N., "Seismic Response of Infilled RC Frames Structures", *Proceedings of the Eleventh World Conference on Earthquake Engineering*, Mexico, 1996, Paper No. 225.

34. Smolira, M., "Analysis of Infilled Shear Walls", *Proceedings of the Institution of Civil Engineers*, Vol. 55, 1974, pp. 895-912.

35. Liauw, T. C. and Lee, S. W., "On the Behaviour and the Analysis of Multi-Storey Infilled Frames Subject to Lateral Loading", *Proceedings of the Institution of Civil Engineers*, Part 2, Vol. 63, 1977, pp. 641-656.

36. Axley, J. W. and Bertero, V. V., *Infill Panels, Their Influence on Seismic Response of Buildings*, University of California, Berkeley, Report No. UCB/EERC 79-28, September, 1979.

37. Pires, F., Campos-Costa, A. and Raposo, S., "Hysteretic Behaviour of R/C Frames Infilled with Brick Masonry Walls", *Proceedings of the Tenth European Conference on Earthquake Engineering*, Vienna, Austria, 1994, Vol. 3, pp. 1739-1744.

38. Valiasis, T. N., Stylianidis, K. C. and Penelis, G. G., "Hysteresis Model for Weak Brick Masonry Infills in R/C Frames under Lateral Reversals", *European Earthquake Engineering*, Vol. VI, No. 1, 1993, pp. 3-9.

39. Michailidis, C. N., Stylianidis, K. C. and Kappos, A. J., "Analytical Modelling of Masonry Infilled R/C Frames Subjected to Seismic Loading", *Proceedings of the Tenth European Conference on Earthquake Engineering*, Vienna, Austria, 1994, Vol. 2, pp. 1519-1524.

40. Ali, S. S. and Page, A. W., "Finite Element Model for Masonry Subjected to Concentrated Load", *Proceedings of the American Society of Civil Engineers, Journal of Structural Engineering*, Vol. 114, No. 8, 1987, pp. 1761-1784.

41. Shing, P. B., Lotfi, H. R., Barzegarmehrabi, A. and Brunner, J., "Finite Element Analysis of Shear Resistance of Masonry Wall Panels with and without Confining Frames", *Proceedings of the Tenth World Conference on Earthquake Engineering*, Madrid, Spain, 1992, Vol. 5, pp. 2581-2586.

42. Lotfi, H. R. and Shing, P. B., "Interface Model Applied to Fracture of Masonry Structures", *Proceedings of the American Society of Civil Engineers, Journal of Structural Engineering,* Vol. 120, No. 1, 1994, pp. 63-80.

43. Page, A. W., "Modelling the In-Plane Behaviour of Solid Masonry under Static Loading", *Proceedings of the International Workshop on Unreinforced Hollow Clay Tile*, 1992, pp. 2-5.

44. Mosalam, K. M., Gergely, P., White, R. N. and Zawilinski, D., "The Behaviour of Frames with Concrete Block Infill Walls", *Proceedings of the First Egyptian Conference on Earthquake Engineering*, 1993, pp. 283-292.

45. Lafuente, M. and Genatios, C., "Propuestas para el análisis de muros de mampostería confinada" (in Spanish), *Boletín Técnico del Instituto de Materiales y Modelos Estructurales*, Facultad de Ingeniería, Universidad Central de Venezuela, Vol. 32, No. 2, 1994, pp. 43-66.

46. Mehrabi, A. B. and Shing, P. B., "Performance of Masonry-Infilled R/C Frames under In-Plane Lateral Loads: Analytical Modelling", *Proceedings from the NCEER Workshop on Seismic Response of Masonry*, San Francisco, California, February, 1994, pp. 1/45-50.

47. Page, A. W., "Finite Element Model for Masonry", *Proceedings of the American Society of Civil Engineers, Journal of the Structural Division*, Vol. 104, No. ST8, 1978, pp. 1267-1285.

48. Schnobrich, W. C., "The Role of Finite Element Analysis of Reinforced Concrete Structures", *Proceedings of the Seminar on Finite Element Analysis of Reinforced Concrete Structures*, Tokyo, Japan, Published by the American Society of Civil Engineers, 1985, pp. 1-24.

49. Dawe, J. L. and Yong, T. C., "An Investigation of Factors Influencing the Behaviour of Masonry Infill in Steel Frames Subjected to In-Plane Shear", *Proceedings of the Seventh International Brick Masonry Conference*, Melbourne, Australia, February, 1985, Vol. 2, pp. 803-814.

50. King, G. J. W. and Pandey, P. C., "The Analysis of Infilled Frames Using Finite Elements", *Proceedings of the Institution of Civil Engineers*, Part 2, Vol. 65, 1978, pp. 749-760.

51. Mallick, D. V. and Severn, R. T., "Dynamic Characteristics of Infilled Frames", *Proceedings of the Institution of Civil Engineers*, Vol. 39, 1969, pp. 261-287.

52. Moss, P. J. and Carr, A. J., "Aspects of the Analysis of Frame-Panel Interaction", *Bulletin of the New Zealand National Society of Earthquake Engineering*, Vol. 4, No. 1, 1971, pp. 126-144.

53. Mallick, D. V. and Garg, R. P., "Effect of Openings on the Lateral Stiffness of Infilled Frames", *Proceedings of the Institution of Civil Engineering*, Vol. 49, 1971, pp. 193-209.

54. Fathy, M., Abdin, M. and Sobaih, M., "Nonlinear Seismic Analysis of Frames with Reinforced Masonry Infill", *Proceedings of the First Egyptian Conference on Earthquake Engineering*, 1993, pp. 391-400.

55. Franklin, H. A., *Nonlinear Analysis of Reinforced Concrete Frames and Panels*, Department of Civil Engineering, University of California, Berkeley, 1970, Report No. SESM-70-5.

56. *ABAQUS: Theory Manual and User's Manual*, Hibbitt, Karlsson & Sorensen Inc., 1993.

57. Liaum, T. C. and Kwan, K. H., "Non-Linear Analysis of Multistorey Infilled Frames", *Proceedings of the Institution of Civil Engineers*, Part 2, Vol. 73, 1982, pp. 441-454.

58. Oden, J. T. and Pires, E. B., "Nonlocal and Nonlinear Friction Laws and Variational Principles for Contact Problems in Elasticity", *Journal of Applied Mechanics*, Vol. 50, 1983, pp. 67-76.

59. Polyakov, S. V., "Some Investigations of the Problem of the Strength of Elements of Buildings Subjected to Horizontal Loads", *Symposium on Tall Buildings*, University of Southampton, April, 1966, pp. 465-486.

60. Delgado, A., Higashihara, H. and Hakuno, M., "Computer Simulation of Masonry Building Collapse under Earthquake Forces", *Proceedings of the First Egyptian Conference on Earthquake Engineering*, 1993, pp. 381-390.

61. Meguro, K., Iwashita, K. and Hakuno, M., "Fracture Tests of Masonry Concrete Elements by Granular Assembly Simulation", *Proceedings of the Ninth World Conference on Earthquake Engineering*, Tokyo, Japan, 1988, Vol. VI, pp. 181-186.

62. Shabrawi, A. E., Verdel, T. and Piguet, J. P., "The Distinct Element Method: A New Way to Study the Behaviour of Ancient Masonry Structures under Static or Dynamic Loading by the Use of Numerical Modelling", *International Journal of Earthquake Engineering, Egyptian Society for Earthquake Engineering*, Vol. 5., No. 1, 1995, pp 47-59.

APPENDIX A: NOTATION

A_c = area of a column section
A_m = area of a masonry panel in the horizontal plane = t L_m
d_m = diagonal length of the masonry panel
E_c = modulus of elasticity of the concrete
E_m = modulus of elasticity of the masonry
f_n = normal stress at the bed joint
G_m = shear modulus of the masonry
h = storey height
h_m = height of masonry panel
h_z = vertical separation between diagonal struts
I_c = second moment of area (moment of inertia) of a column section
L = span of beam between centre lines of supporting columns
L_m = length of masonry panel between adjacent columns
t = thickness of masonry infill
V = shear force
V_{max} = maximum shear resistance in a given cycle
V_u = ultimate shear resistance
v = shear stress
w = effective width of the equivalent diagonal strut
z = vertical contact length between the masonry panel and the column
β = dimensionless parameter expressing the relative stiffness of the frame to the panel
δ = storey drift
δ_{max} = storey drift corresponding to the shear force V_{max}
δ_u = storey drift corresponding to the ultimate shear resistance, V_u
λ_h = dimensionless parameter expressing the relative stiffness of the infill to the frame
μ = coefficient of friction
θ = inclination of the diagonal of masonry panel respect to the horizontal axis