

## Analytical Prediction of Armature-Reaction Field in Disc-Type Permanent Magnet Generators

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**Abstract:** A two dimensional (2D) analytical method to predict the armature-reaction field in the slotted opening airgap/magnet region of a permanent magnet (PM) disc-type generator is presented. The currents in the slot is modeled by a uniform distributed current sheet along an arc at the slot opening. The analysis takes into account the harmonics in the armature current waveforms which are calculated from the voltage equations. The effects of the stator slot openings are being accounted for by the introduction of a 2D relative permeance function which is calculated directly from the nonlinear equation described by conformal transformation. A more refined permeance distribution can then be obtained easily.

**Keywords:** Disc-type Machine, Permanent Magnet, Armature Reaction Field, Analytical Method

### I. INTRODUCTION

The armature-reaction field in the airgap/magnet region of a PM disc-type generator is predicted in this paper. Rigorous field analysis for PM axial-field machines usually necessitates three-dimensional finite element methods. For parametric machine designs it is however reasonable to use simpler methods, such as two-dimensional or magnetic circuit methods, in order to save time and cost [1-4]. The magnetic circuit method is the simplest but it would lead to a significant error and require elaborate test or calibration on a vast number of prototypes. In order to obtain more accurate results both 2D analytical methods and finite element methods have been used to calculate the main fields of slotless disc-type PM machines. Noting that even though the armature reaction field produced by the stator windings in the airgap/magnet region is much smaller than that produced by permanent magnets in a PM disc-type machine, a knowledge of the armature reaction field calculation is important for predicting the axial exciting forces,

the demagnetization withstand capability, as well as the self- and mutual-winding inductances.

Generally the analytical method is more effective for numerical optimal design purpose. When slotted stators are used the axial field computation by the analytical method will be more complex. A two-dimensional relative permeance function has been used in the field computation of cylindrical PM machines [5] in order to account for the effects of the stator slot openings. However only the constant term and the fundamental harmonic are introduced in the simplified calculations because of the complexity of the permeance function. In this paper a 2D analytical method to predict the armature-reaction field in the slotted opening airgap/magnet region of a PM disc-type generator is presented. The armature current in the slot is modeled by a uniform distributed current sheet along an arc at the slot opening. The analysis takes into account the harmonics in the armature current waveforms which are calculated from the voltage equations. The effects of the stator slot openings are being accounted for by the introduction of a 2D relative permeance function which is calculated directly from the nonlinear equation described by the conformal transformation. A more refined permeance distribution can then be obtained. The resultant armature reaction field are calculated from the product of the relative permeance function of the slotted airgap region and the field of the stator windings (when the stator slot openings are neglected).

### II. MAGNETIC FIELD OF ONE WINDING

The disc-type machine being investigated (Fig.1) has 8 poles with 3-phase armature windings accommodated in 6 stator slots [6]. A cylindrical cutting plane is introduced at the mean radius of the stator so as to reduce the 3D problem to 2D boundary value problem. Specifically, the field is symmetric about the horizontal centre line of the magnets. A reduced geometry is shown in Fig.2 alone with a Cartesian reference frame. Here a semi-cylindrical plane is presented due to periodicity considerations. From such simplified geometry, one could develop general expressions for the magnetic field produced by the stator windings in the airgap/magnet region. For simplicity the relative recoil permeability of the permanent magnets is assumed to be unity in the

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calculation of the armature reaction field. Such assumption will not normally introduce any significant error into the calculation of the load field distribution since the armature reaction field is, according to the calculation, only around 2% of the magnitude of the open-circuit field in such machines.

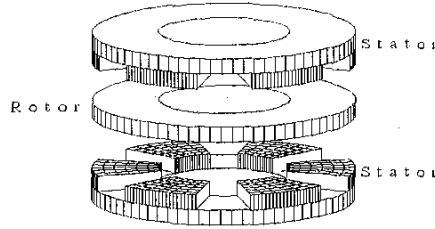


Figure 1 Disc-type PM machine

### A. Current Density Distribution of One Coil

It is assumed that the distributed current sheet of a coil inside the stator slots is uniform along an arc at the slot opening  $b_0$ . For a general case the current density distribution of the  $q$ th coil of one phase winding is shown in Fig.3, where  $\alpha = \theta_q$  corresponds with the axis of the coil;  $\alpha_y = \tau_y/R$  is the span of the coil in radians;  $\tau_y$  is the winding pitch;  $R$  is the mean radius of the stator;  $\alpha_0 = b_0/R$ ;  $N_c i$  is the ampere-turns of the coil. Therefore, as a function of  $\alpha$ , the current density distribution  $J_q$  of the coil can be presented as

$$J_q = \begin{cases} - (N_c i / \alpha_0) & [\theta_q - \alpha_2, \theta_q - \alpha_1] \\ (N_c i / \alpha_0) & [\theta_q + \alpha_1, \theta_q + \alpha_2] \\ 0 & \text{other } \alpha \end{cases} \quad (1)$$

where  $\alpha_1 = (\alpha_y - \alpha_0)/2$ ;  $\alpha_2 = (\alpha_y + \alpha_0)/2$ . The Fourier series expansion then becomes

$$J_q = \sum_{n=1}^{\infty} (a_{nq} \cos n\alpha + b_{nq} \sin n\alpha) \quad (2)$$

where

$$a_{nq} = - (2N_c i / \pi) k_{son} k_{pn} \sin n\theta_q \quad (3)$$

$$b_{nq} = (2N_c i / \pi) k_{son} k_{pn} \cos n\theta_q$$

and

$$k_{pn} = \sin (n\alpha_y/2) \quad (4)$$

is the coil pitch factor, whilst

$$k_{son} = \sin (n\alpha_0/2) / (n\alpha_0/2) \quad (5)$$

is the slot opening factor.

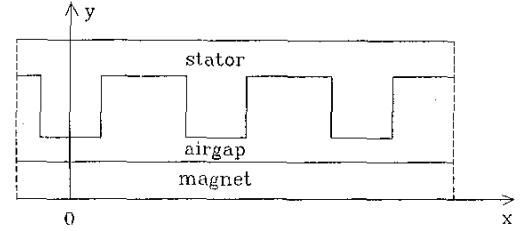


Figure 2 Reduced 2D geometry

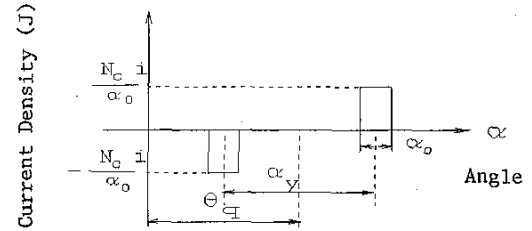


Figure 3 Current density distribution of one coil

### B. Current Density distribution of One Winding

Suppose one winding has  $M$  coils, the overall harmonic coefficients of the current density are given by

$$a_n = \sum_{q=1}^M a_{nq} ; \quad b_n = \sum_{q=1}^M b_{nq} \quad (6)$$

For the disc-type PM machine in this paper there are two coils in each winding. The coils are exactly the same and are displaced from each other by an angle  $\pi$ , in each of the stators. If the centre of the first coil is chosen as the origin ( $\theta_1 = 0$ ), then

$$a_n = 0 \quad (7)$$

$$b_n = (4N_c i / \pi) k_{son} k_{pn} \quad n = 2, 4, 6, \dots$$

The current density distribution of one winding is obtained as follows:

$$J = \sum_{n=2,4,6,\dots}^{\infty} (4N_c i / \pi) k_{son} k_{pn} \sin n\alpha \quad (8)$$

According to this expression the magnetic field produced by one winding can thus be found.

### C. Magnetic Field Produced by One Winding

The governing Laplacian equation in rectangular coordinates is :

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (9)$$

where  $x=R\alpha$ . The general solution for the magnetic field in the airgap and magnet regions, due to the periodicity of the field, has the form of

$$\phi(x,y) = \sum_{n=1}^{\infty} (A_n \cos k_n x + B_n \sin k_n x) * (C_n \cosh k_n y + D_n \sinh k_n y) \quad (10)$$

Assuming the permeability of the stator core is infinite, the boundary conditions are defined by

$$\begin{aligned} H_x |_{y=0} &= - \frac{\partial \phi}{\partial x} |_{y=0} = 0 \\ H_x |_{y=y_1} &= - \frac{\partial \phi}{\partial x} |_{y=y_1} = -J \end{aligned} \quad (11)$$

where  $y_1 = g + h_m/2$ ,  $g$  is the length of the airgap between the rotor and one of the stators, and  $h_m$  is the thickness of the magnet. The complete solution of the scalar magnetic potentials for the airgap and magnet regions can be derived by substituting the boundary conditions and fundamental equations into the general solution

$$\phi(x,y) = - \frac{4N_c i R}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{k_{pn} k_{son}}{n \sinh(ny_1/R)} * \cos(nx/R) \sinh(ny/R) \quad (12)$$

The axial component of the flux density is therefore given as:

$$B_y(x,y) = \frac{4\mu_0 N_c i}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{k_{pn} k_{son}}{\sinh(ny_1/R)} * \cos(nx/R) \cosh(ny/R) \quad (13)$$

The circumferential component of the field is

$$B_x(x,y) = - \frac{4\mu_0 N_c i}{\pi} \sum_{n=2,4,6,\dots}^{\infty} \frac{k_{pn} k_{son}}{\sinh(ny_1/R)} * \sin(nx/R) \sinh(ny/R) \quad (14)$$

With cylindrical coordinates the same results can be also obtained if the radial component of the field is assumed to be zero.

### III. MAGNETIC FIELD OF 3-PHASE WINDING

The magnetic field produced by the 3-phase windings can be

calculated by summing the field produced by each phase winding as long as the current through windings can be obtained.

#### A. The Armature Current Waveform

The armature current can be calculated from the voltage equations of the machine and the loading conditions. In terms of phase coordinates, the stator voltage equation may be written as:

$$v = e - pL_s i - R_s i \quad (15)$$

where  $v = [v_a \ v_b \ v_c]^T$  is the 3-phase terminal voltage vector while  $e = [e_a \ e_b \ e_c]^T$  is the induced voltage vector in the stator windings due to the magnetic field of the permanent magnet;  $i = [i_a \ i_b \ i_c]^T$  is the stator winding current vector; Subscripts a, b, and c represent each of the three phases, respectively; Superscript T denotes the transpose of the matrix;  $p = d/dt$ ;  $R_s = \text{diag}[R_s \ R_s \ R_s]$  is the resistance matrix of the stator windings in which the subscript s refers to the stator.  $L_s$  is the inductance matrix of the stator windings

$$L_s = \begin{bmatrix} L_s & M & M \\ M & L_s & M \\ M & M & L_s \end{bmatrix} \quad (16)$$

where  $L_s$  is the self-inductance of one winding and  $M$  is the mutual-inductance of two windings. Both inductances consists of contributions from three components including the airgap leakage, the slot-leakage and end-winding leakage. The airgap components of the inductances can be calculated from the result of the armature reaction field obtained by the 2D analytical method as outlined in this paper. The slot-leakage and end-winding components are evaluated in the same way as for conventional machines [7]. Since the equivalent airgap length is independent of the rotor position in the disc-type PM machines, the values of the inductances are, therefore, constants. Assuming the disc-type PM generator has a symmetrical star-connected 3-phase winding and is operating under steady-state conditions with a 3-phase symmetrical passive load at a lagging power factor of 0.95, the load voltage equation can be written as:

$$v = pL_L i + R_L i \quad (17)$$

where  $L_L = \text{diag}[L_L \ L_L \ L_L]$  and  $R_L = \text{diag}[R_L \ R_L \ R_L]$  are the inductance and resistance matrix of the load individually, in which subscript L denotes the load. Substituting (17) into (15) gives:

$$e = pLi + Ri \quad (18)$$

where  $L=L_s+L_L$ ;  $R=R_s+R_L$ . Thus if the no load induced voltages  $e$  of the stator windings are obtained, the armature current can be calculated. The typical waveforms of the no load induced voltage, the terminal voltage and the current of one phase winding are shown in Fig.4. Note that the induced voltage has been computed by the 2D analytical method and the terminal voltage and the current are calculated according to the voltage equations. Because of the non-sinusoidal waveform of the induced voltage there are also harmonics in the waveforms of the terminal voltage and the current. Because of symmetry and star-connections of the windings,  $i(\theta)=-i(\theta+2\pi)$ . It is obvious that only odd nontriplen order harmonics exist in the current. The three phase currents can be expressed in Fourier series as:

$$\begin{aligned} i_a &= \sum_k I_k \sin[k(p\omega_r t) - \theta_k] \\ i_b &= \sum_k I_k \sin[k(p\omega_r t - 2\pi/3) - \theta_k] \\ i_c &= \sum_k I_k \sin[k(p\omega_r t - 4\pi/3) - \theta_k] \end{aligned} \quad (19)$$

where  $k=1,5,7,11,13,\dots$ , and  $p$  is the number of pole pairs,  $\omega_r$  is the rotor angular velocity, and  $\theta_k$  is the harmonic phase angle. For the generator being considered the rotor speed is constant.  $p\omega_r = \omega_N$  is the synchronous angular frequency.

#### B. Magnetic Field of 3-phase Winding

The axial component of the instantaneous magnetic field produced by 3-phase windings in the airgap/magnet regions will then be

$$\begin{aligned} B_{wy}(x,y,t) &= B_{ay}(x,y,t) + B_{by}(x,y,t) + B_{cy}(x,y,t) \\ &= i_a \sum B_{yn} \cos(nx/R) + i_b \sum B_{yn} \cos n(x/R - 2\pi/3) \\ &\quad + i_c \sum B_{yn} \cos n(x/R - 4\pi/3) \end{aligned} \quad (20)$$

where

$$B_{yn} = \frac{4\mu_0 N_c i}{\pi} \frac{k_{pn} k_{son}}{\sinh(ny_1/R)} \cosh(ny/R) \quad (21)$$

Hence the magnetic field produced by a 3-phase winding in the disc-type PM machine can be deduced from

$$\begin{aligned} B_{wy}(x,y,t) &= (3/2) \sum_k I_k \sum_n B_{yn} \sin[k\omega_N t - \theta_k \\ &\quad + (nx/R) \mp 3h(2\pi/3)] \end{aligned} \quad (22)$$

$$n = p(3h \mp k), \quad h = 0, \pm 1, \pm 2, \dots,$$

where  $\alpha=0$  corresponds to the axis of the phase A winding.

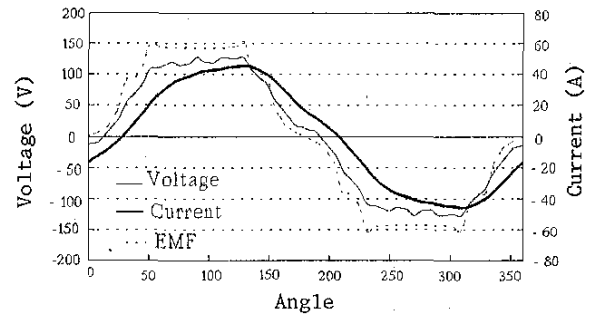


Figure 4 Waveforms of voltage, current and emf

#### IV. STATOR SLOTTING EFFECT

The stator slotting affects both the magnitude and the distribution of the flux density in the airgap/magnet regions. For the sake of simplicity, the effects of the slotted stator are being accounted for by the introduction of a 2D relative permeance function. The relative permeance function of the slotted airgap region are calculated by conformal transformation method with a unit magnetic potential applied between one side of the stator and the rotor horizontal centre surface. Note that the rotor has no magnetic materials. The stators are assumed to have infinite permeability with infinitely deep rectilinear stator slots as shown in Fig.5.

The interior airgap region in the complex plane  $z$  is firstly transformed to the upper half of the complex plane  $w$ , and subsequently to the complex plane  $t$ , by conformal transformation.

The first transformation is given by

$$\begin{aligned} z &= \frac{b_0}{\pi} \left[ \frac{j}{2} \ln \frac{1 - j(w/\sqrt{a^2 - w^2})}{1 + j(w/\sqrt{a^2 - w^2})} + j(2g'/b_0) * \right. \\ &\quad \left. \tan^{-1} (-j2g'w/(b_0\sqrt{a^2 - w^2})) \right] \end{aligned} \quad (23)$$

where  $j$  is the complex operator,  $g' = g + h_m/(2\mu_r)$  is the effective airgap length between the rotor and one of the stators, and

$$a = \sqrt{1 + (2g/b_0)^2} \tag{24}$$

The second transformation is

$$t = (\phi_{m0}/\pi) \ln [(1 + w)/(1 - w)] \tag{25}$$

where  $\phi_{m0}$  is the magnetic potential between the stator and the horizontal centre plane of the magnet.

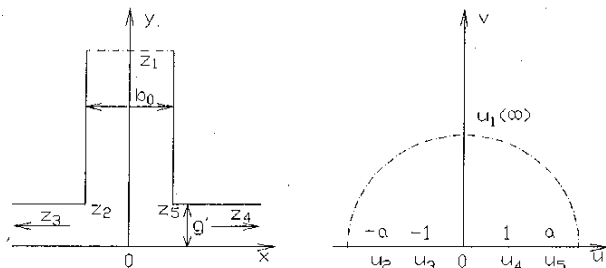


Figure 5 Conformal transformation

In a simplified calculation, an effective slot opening is introduced and the variation of the airgap flux density within the effective slot opening are usually described by a constant term and the fundamental harmonic. The more refined permeance distribution can be derived from the conformal transformation method by calculating the waveform directly from the field distribution described by the first transformation. In this paper the nonlinear equation of the first transformation is solved by numerical methods in which an intermediate complex variable  $s=p+jq$  is introduced, i.e.

$$s = w/(\sqrt{w^2 - a^2}) \tag{26}$$

Given a fixed point  $z=x+jy$  in the  $z$  plane, a corresponding point  $s=p+jq$  is obtained such that another point  $w=u+jv$  in the  $w$  plane can be computed readily.

The permeance  $\lambda(x,y)$  is obtained after the flux density in the airgap/magnet regions is calculated. In this region a unit difference in magnetic potential between the stator and the magnet horizontal centre surfaces is assumed. Hence

$$\lambda = B = \mu_0 H = \mu_0 \frac{dt}{dz} = \mu_0 \frac{dt}{dw} \frac{dw}{dz} = \frac{2\mu_0 \phi_{m0}}{b_0 \sqrt{a^2 - w^2}} \tag{27}$$

Therefore

$$B_{result}(x,y,t) = B_w(x,y,t) \lambda(x,y) \tag{28}$$

is the resultant armature reaction field distribution accounting for the effects of the slotted stator.

#### IV. COMPUTATION RESULT

The resultant field distribution considering the effect of slotted stator is calculated from the product of the field produced by the magnets when the stator slot opening are neglected and the relative permeance function for the slotted airgap region. The variations of the relative permeance waveform with the vertical position in the machine being studied are shown in Fig.6 where the airgap length  $g$  between the rotor and one stator is 1 mm and the half of the magnet thickness ( $h_m/2$ ) is 3.6 mm. The region consisting of half of the stator tooth pitch (in electrical degree) is presented because of periodicity consideration. The calculation predicts an increase in permeance, i.e. in flux density, at the tooth tips, because of the flux concentrating effect which makes the flux stronger near the teeth.

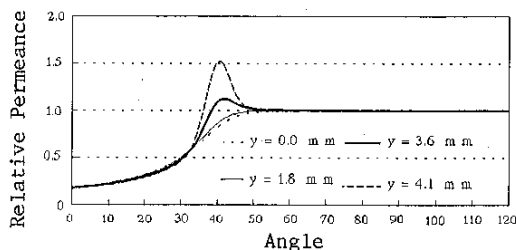


Figure 6 Variation of relative permeance with the  $y$  position

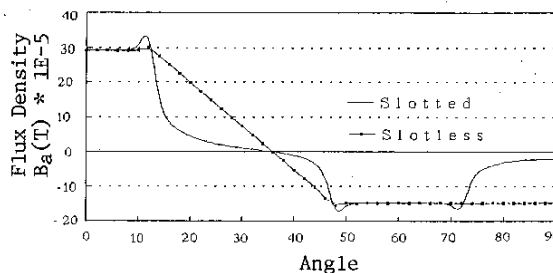


Figure 7 Armature reaction field of one winding

The predictions of the axial components of the flux density distributions in the airgap region produced by one armature winding carrying unit current are shown in Fig.7 for  $y=4.1$  mm. Here only a quarter of the circumferential region is presented because of the periodicity considerations and also because one has to compare the two circumstances, one which is slotless and the other one with slot opening in the figure. It is evident that the armature reaction field will be reduced at the slot opening region. Similarly the circumferential components of the flux density distributions in the airgap region are shown in Fig.8. It can be seen that the circumferential field is almost zero under the stator teeth area. Compared

to the axial components, the circumferential components of the armature reaction field are negligibly. The axial component of the resultant armature reaction field produced by 3-phase stator windings carrying the currents with the waveforms shown in Fig. 4 is shown in Fig. 9 at  $t = 0.0$  s. The prediction of the flux density magnitude in the airgap produced by the permanent magnet of the disc-type PM machine is about 0.8 T. It can be seen that the armature reaction field is much smaller than the main field produced by the magnet such kind of machines. Here the armature reaction field is only about 2 % of the open-circuit field. Therefore it is reasonable that the armature reaction field can be neglected when the voltage equations are solved. This property simplifies the calculation of the machine performances considerably.

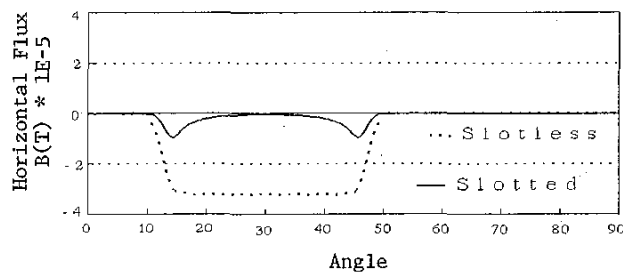


Figure 8 Horizontal field due to one stator winding

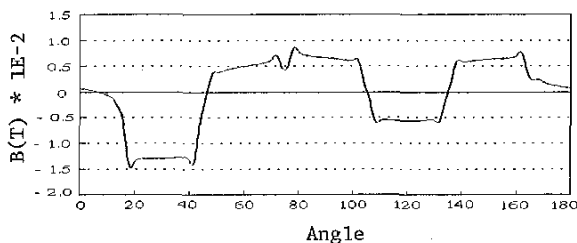


Figure 9 Armature reaction field of the 3-phase windings

## V. CONCLUSIONS

A 2D analytical technique for calculating the armature reaction field distribution produced by 3-phase windings for a permanent magnet disc-type machine has been described. The 3-phase armature current containing higher harmonics can be solved from the voltage equations when the no load induced voltage and the electric circuit parameters have been obtained. The effects of the stator slotting upon the flux density distribution are accounted for by incorporating a "two-dimensional" relative permeance function into the analyses. The more refined permeance distribution is derived by calculating the waveform directly from the field distribution described by the conformal transformation. The analytical method presented in this paper is also appropriate to disc-type PM motor as well, provided that the armature current waveforms are available.

The analytical method as described is an ideal method for rapid parametric calculations and optimal design purpose.

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