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Analytical Solution for Determination of Induction Machine Acceleration Based on Kloss Equation

Nikola Koljčević[1], Željko Fuštić[2], Martin Ćalasan[3]

Abstract: In this paper, the analytical solution for the determination of induction machine acceleration, i.e. acceleration-time characteristic, during no-load direct start-up is proposed. The proposed dependence is derived from the Kloss equation for torque-speed relation for the induction machine. Also, the analytical solution for the maximal value of the acceleration is derived in this paper. The results obtained using developed expressions are compared with corresponding results obtained using the developed Matlab/Simulink model for analyses of the induction machine starting the process. Therefore, very high degree of accuracy is obtained.

Keywords: Induction machine, Kloss equation, Acceleration, Analytical solution, No-load direct start-up.

1 Introduction

An induction machine is the most widely used machine in practice [1]. Applications of these machines can be found in different sectors - industry, electric cars, home applications, etc. These machines are closely related to power electronics, concretely with inverters, which enable their easy speed control.

The induction machine start-up problem is one essential engineering problem [2]. There are different start-up methods used primarily in the industry sector. The basic start-up method is a direct start-up method. The direct start-up method can be realized with or without reducing the voltage [3]. Also, one very popular start-up method is the star-delta method [4]. However, in the last time, the method based on the usage of the soft starter is the most popular. The soft-starter enables a safe and secure machine start-up [5 – 6]. This paper deals with the induction machine direct start-up, as it is an essential start-up method.

The direct start-up is realized by connecting the stator winding to the nominal or reduced voltage at the rated frequency. The problem with this type

[1]Kakaritska gora bb, 81000 Podgorica, Montenegro; E-mail: nkoljcevic@gmail.com
[2]Lička 1, 81000 Podgorica, Montenegro; E-mail: zeljkofustic@gmail.com
[3]University of Montenegro, Faculty of Electrical Engineering, Džordža Vašingtona bb, 81000 Podgorica, Montenegro; E-mail: martinc@ucg.ac.me

of start-up lies in the fact that during direct start-up, at nominal voltage, the phase current can be 5 to 6 times higher than the nominal current value. This high current can damage the stator windings. On the other side, the high value of the stator current can cause a large voltage drop in the local grid where the induction machine is connected. Therefore, a direct start is often realized through a low voltage by using an auto-transformer [3].

The mathematical description of the induction machine speed-time characteristic during the direct start-up is presented in [7]. In the mentioned paper, a Kloss equation [1] was used to obtain the speed-time characteristic. Also, in the mentioned paper an invertible relation between speed and time, during direct start-up, is derived. On the other side, the other invertible relation between speed and time during machine direct start-up is presented in [8]. However, in this paper, the author used the well-known equation for torque-speed relation. Besides these two papers, in the available literature, also other papers that deal with induction machine speed-time curve determination during direct start-up can be found [9 – 14]. However, in [9 – 14] are not presented invertible equation for speed – time representation during direct start-up. Unlike previously published papers [7 – 14], in this paper, we will deal with the mathematical description of acceleration – time characteristic of induction machine during no-load direct-start-up. Furthermore, we will derive an analytical solution for the maximal value of induction machine acceleration during direct start-up. Note, this problem is not analyzed in any of the papers [7 – 14].

This paper is organized as follows. In Section II will be presented literature known mathematical equations for speed-time curve representation during machine direct start-up. Section III will be presented a novel mathematical relation for acceleration-time characteristic during direct start-up, while in Section IV will be presented some numerical results. In Section V the paper review and some of the directions for future work will be presented.

2 Speed-Time Characteristics During Direct Start-up

The speed-time characteristic during no-load direct start-up is presented in [7]. The mentioned relation has the following form:

$$t = A\left(\frac{1-s^2}{2s_{pr}} - s_{pr}\ln(s)\right), \tag{1}$$

where $A = J\omega_s/2M_{pr}$, J represents the machine moment of inertia, s is the slip of the machine, ω_s represent the synchronous machine speed, M_{pr} is the maximum torque, while s_{pr} is the corresponding slip of the machine. The maximal torque, as well as the corresponding slip, can be calculated by using machine data [1]. The mentioned equation is derived taking into account the basic mechanical equation for no-load operation, and ignoring bearings and ventilation losses, as follows:

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = M_{em}. \tag{2}$$

Kloss equation for the torque-speed representation of induction machine has the following expression:

$$M = \frac{2M_{pr}}{\frac{s}{s_{pr}} + \frac{s_{pr}}{s}}. \tag{3}$$

In [7] the invertible equation for the speed-time curve is also presented. It has the following form:

$$\omega = \omega_s - \omega_s s_{pr}\sqrt{W\left(\frac{1}{s^2}\mathrm{e}^{B(\tau-t)}\right)}, \tag{4}$$

where is ω angular speed of induction machine, t is time and W is the solution of Lambert W equation [15 – 19]. Namely, W represents the solution of the equation which form is $x = \gamma\mathrm{e}^{-x}$ and where γ is an integer. In the previous equation

$$\tau = \frac{J\omega_s}{4s_{pr}M_{pr}} \tag{5}$$

and

$$B = \frac{4M_{pr}}{J\omega_s s_{pr}}. \tag{6}$$

3 Analytical Solution for Determination of Induction Machine Acceleration – Time Characteristic

In this section, the analytical calculation of the acceleration of an induction machine is presented. The analytical expression for induction machine acceleration-time characteristic can be derived in three ways (CASE I, II, and III).

3.1 Case 1

Starting from the basic equation for induction machine torque (see (2)), we can define machine acceleration:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}. \tag{7}$$

Therefore, we can write:

$$J\alpha = M \, , \tag{8}$$

as well as

$$\alpha = \frac{M}{J} \, . \tag{9}$$

By including the Kloss equation in the previous equation, we get:

$$\alpha = \frac{2M_{pr}}{J}\frac{s_{pr}s}{s^2 + s_{pr}^2} \, . \tag{10}$$

Finally, as we know that

$$s = s_{pr}\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}$$

we can get the following expression:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{2M_{pr}}{J}\frac{\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right) + 1} \, . \tag{11}$$

Therefore, (11) represents the analytical solution for induction machine acceleration-time characteristic during no-load direct start-up.

3.2 Case 2

The acceleration-time characteristic can be also obtained by using speed-time characteristic during direct start-up (see (4)). The result of the time derivative of (4) is:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \omega_s s_{pr}\frac{1}{2\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}\frac{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)\frac{1}{s_{pr}^2}e^{B(\tau - t)}A}{\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\left(1 + W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)\right)},$$

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\omega_s s_{pr}A}{2}\frac{\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}{1 + W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}. \tag{12}$$

3.3 Case 3

Finally, the acceleration-time characteristic can be obtained by using the time-speed equation for the induction machine during direct start-up (see (1)). Namely, if we make a derivative of (1), we get the following equation:

$$\frac{\mathrm{d}t}{\mathrm{d}s} = -A\left(\frac{s}{s_{pr}} + \frac{s_{pr}}{s}\right). \tag{13}$$

As we know that

$$s = \frac{\omega_s - \omega}{\omega_s}, \tag{14}$$

and

$$\mathrm{d}s = -\frac{1}{\omega_s}\mathrm{d}\omega, \tag{15}$$

we can get

$$\frac{\mathrm{d}t}{\mathrm{d}\omega} = -\frac{A}{\omega_s}\left(\frac{s}{s_{pr}} + \frac{s_{pr}}{s}\right). \tag{16}$$

On the other side, as we know the slip-time relation during direct start-up:

$$s = s_{pr}\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)} \tag{17}$$

we can write the following:

$$\frac{\mathrm{d}t}{\mathrm{d}\omega} = -\frac{B}{\omega_s}\left(\frac{s_{pr}\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}{s_{pr}} + \frac{s_{pr}}{s_{pr}\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}\right) \tag{18}$$

or

$$\frac{\mathrm{d}t}{\mathrm{d}\omega} = \frac{j}{2M_p}\left(\frac{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right) + 1}{\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau - t)}\right)}}\right). \tag{19}$$

As we know that

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{\frac{\mathrm{d}t}{\mathrm{d}\omega}}, \tag{20}$$

we finally get the following

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{2M_{pr}}{J} \frac{\sqrt{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau-t)}\right)}}{W\left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau-t)}\right)+1}. \tag{21}$$

Therefore, in this Section, in three ways, an analytical solution for induction machine acceleration – time characteristic is derived.

4 Analytical Solution for Determination of Induction Machine Maximal Value of Acceleration During Direct Start-up

In order to find the maximal value of the induction machine acceleration during no-load direct start-up, it is necessary to differentiate acceleration per time:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{2M_p}{J} \cdot$$
$$\cdot \frac{-\frac{1}{2\sqrt{W(C)}} \frac{W(C)}{CA(1+W(C))} CA(1+W(C)) + \frac{W(C)}{CA(1+W(C))} CA\sqrt{W(C)}}{(W(C)+1)^2}, \tag{22}$$

where

$$C = \left(\frac{1}{s_{pr}^2}\mathrm{e}^{B(\tau-t)}\right).$$

Obtaining the maximal value of acceleration is possible by solving the following equation:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = 0. \tag{23}$$

When we arrange the expressions (22) and (23), we get:

$$W(C) = 1. \tag{24}$$

From the basic form of the Lambert W function we can found the following time at maximal machine acceleration:

$$t = \tau - \frac{1+2\ln(\mathrm{s}_{sp})}{A}. \tag{25}$$

Therefore, the maximal value of acceleration during direct start-up has the following expression:

$$\alpha = \frac{2M_{pr}}{J}\frac{\sqrt{W(e)}}{W(e)+1}. \tag{26}$$

5 Simulation Results

In this section, for the presentation of simulation results, we observed an induction machine whose data are given in **Table 1**.

Table 1

Induction machines parameters for simulation testing.

Parameters		Machine
P_n [kW]	nominal power	37.3
U_n [V]	nominal voltage	460
f [Hz]	nominal frequency	60
p	number of pair poles	2
R_1 [Ω]	stator resistance	0.087
R_2 [Ω]	rotor resistance	0.228
X_1 [Ω]	stator reactive resistance	0.302
X_2 [Ω]	rotor reactive resistance	0.302
X_m [Ω]	magnetizing reactance	13.08
J_n [kgm^2]*	nominal moment of inertia	1.662

In order to check the analytical solution accuracy, in Matlab/Simulink we have realized a simulation model that consists of a three-phase induction machine, three-phase voltage source, and block for speed measurement (see [7]). In this model, we have used discrete simulation time with the step of 10^{-5}s.

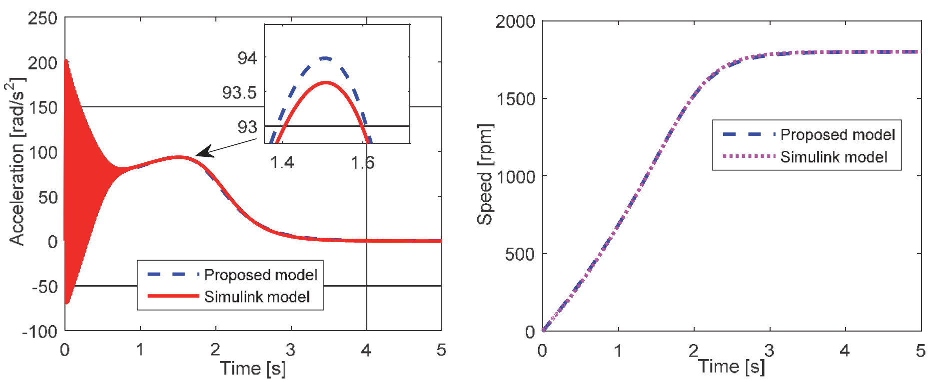

Fig. 1 – *Acceleration-time and speed-time characteristics of the observed machine.*

In Figs. 1 – 2 the acceleration-time and speed-time characteristics of the induction machine during direct start-up are presented ($J = 5J_n$). On this figure the part of characteristics where we can be seen the maximal value of acceleration is also zoomed. On the other side, by using (29) we obtained the maximum value of the acceleration for this case: $\alpha_{max} = 93.98\,\mathrm{rad/s^2}$ and $t_{max} = 1.51\,\mathrm{s}$. Therefore, we see that the analytical solution gives the results in very good agreement with simulation results.

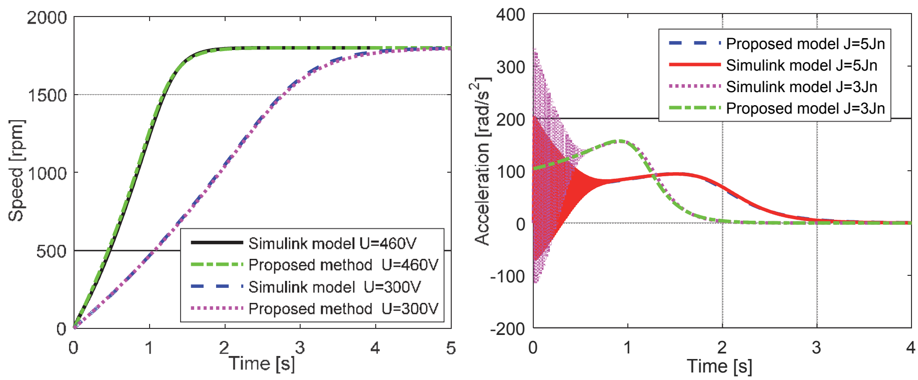

Fig. 2 – *Acceleration-time and speed-time of observed machine for different value of the moment of inertia (U* = 460V*).*

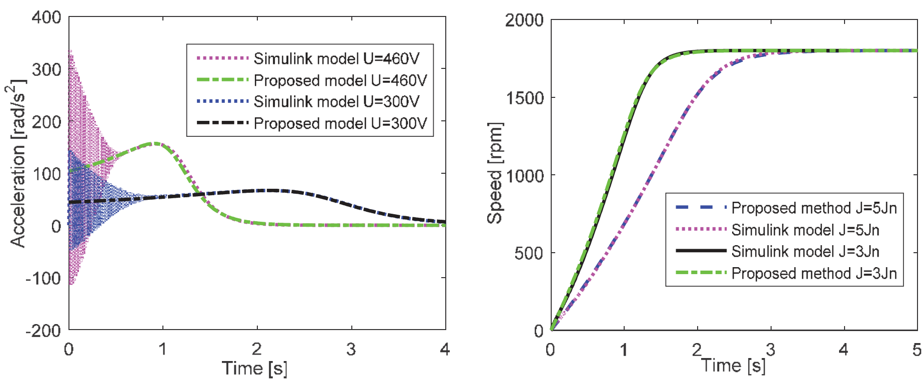

Fig. 3 – *Acceleration-time and speed-time of observed machine for different value of supply voltage ($J = 3J_n$).*

In order to additionally check the accuracy of the proposed analytical solution for acceleration-time characteristics determination, in Figs. 2 – 3 the impact of the moment of inertia and the impact of supply voltage on acceleration-time characteristics are presented, respectively. It is evident from both mentioned figures that the results obtained by using derived equation (24) are very close to the signal obtained by using the Simulink model. Therefore, it can be concluded that the proposed equation is very accurate. Also, in these figures the impact of machine inertia on acceleration can be seen. Namely, the higher value of the machine inertia causes the lower value of the machine maximal acceleration. All these conclusions are in accordance with the derived expression for the induction machine maximum value of acceleration during no-load direct start-up.

6 Conclusion

This paper deals with the determination of acceleration-time expression for induction machine no-load direct start-up. The derived expression is based on the usage of the Kloss equation for torque-speed representation. Also, in this paper, the analytical expression for the maximal value of acceleration during the no-load direct start-up of the induction machine is derived. The results obtained using derived expressions have been compared with corresponding results obtained by using the realized Matlab/Simulink model of the induction machine for starting process analysis. All obtained results confirmed the accuracy and applicability of derived expressions.

In future work, we will deal with the determination of the acceleration-time characteristics based on the induction machine double-cage model.

7 References

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