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Analytical Solution of a Non-isothermal Flow in Cylindrical Geometry

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors contributed to every aspect of the work. Both authors read and approved the final manuscript.

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Abstract

This study proposes analytical solution to the problem of transport in a Newtonian fluid within a cylindrical domain. The flow is assumed to be dominated along the channel axis, and is taken to be axi-symmetric. No-slip boundary condition is considered for velocity while the temperature and concentration have Dirichlet boundary values. The resulting problem is transformed into a set of non-trivial variable coefficient differential equations in a cylindrical geometry. By adopting the series solution method of Frobenius, the closed-form analytical solutions are derived for the flow variables. We conduct an analysis of the derived model, and showed that, indeed, the flow variables are axi-symmetric. We also state and prove another theorem to show that the derived concentration model is positivity preserving – meaning that it yields positive concentration - provided the boundary value is non-negative. Finally, we present graphical results for the flow variables and discuss the effect of the relevant flow parameters. The results showed that (i) an increase in the cooling parameter, λ reduces the fluid velocity, (ii) the temperature decreases as the cooling parameter increases λ and (iii) an increase in the injection parameter, α leads to increase in the concentration.

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Nomenclatures

$u' :=$ Dimensional velocity components
$z' :=$ Dimensional axisymetric flow of the channel
$r' :=$ Dimensional radius
$t' :=$ Dimensional time
$T' :=$ Dimensional fluid temperature
$T'_w :=$ Dimensional wall temperature
$\rho :=$ Fluid density
$M :=$ Magnetic field parameter
$k :=$ Thermal conductivity
$g :=$ Acceleration due to gravity
$\mu :=$ viscosity of the fluid
$\sigma :=$ Electrical conductivity
$C_p :=$ Heat capacity
$\beta :=$ coefficient of volumetric expansion
$q'_r :=$ Radiative heat flux
$\alpha :=$ Concentration injection parameter
$\lambda :=$ Velocity cooling parameter
$\lambda :=$ Temperature cooling parameter
$\theta_a :=$ Temperature boundary parameter
$\varphi_a :=$ Temperature boundary parameter
$Gr :=$ Grashof number
$\Pr :$ Pressure gradient
$u_0(r) :=$ Dimensionless velocity
$u(r) :=$ Velocity profile of the fluid
$\theta_0(r) :=$ Dimensionless temperature
$\theta(r) :=$ Temperature profile
$\varphi_0(r) :=$ Dimensionless concentration
$\varphi(r) :=$ Concentration profile
$Gr :=$ Grashof number
$T_\infty :=$ Free stream temperature
$\tau_w :=$ Shear stress at the wall

1 Introduction

The study of flows in channels has numerous applications in science and technology. Such applications include the human blood vessels and arteries, oil and water flows through reservoirs, chemical engineering for filtration and purification process, movement of natural gas and flows of rivers moving through cylindrical channels. So, understanding channel flows is of great practical importance.

This has attracted numerous studies in the area. For instance, Jahangiri et al. [1] studied the effect of non-Newtonian behavior of blood on wall shear stress in an elastic vessel with simple and consecutive stenosis. They solved their problem numerically and results showed that the power law model is not suitable for simulating the non-Newtonian behavior of blood. In the same vain, Lokendra et al. [2] developed a mathematical model for the blood flow through an overlapping stenosed artery with core region under the effect of magnetic field. The problem were solved analytical and result showed that the velocity of blood and shear stress on the wall of artery due to overlapping stenosis can be controlled using external magnetic field but in their research heat and mass was not considered. Lukendra et al. [3] investigated the pulsatile flow of blood through a porous medium with constant permeability, in an inclined tapered artery with mild stenosis. Their fluid problem was solved analytical and result was summarized as magnetic field, velocity slip, inclination and permeability of the porous medium have significant influence on the flow field, wall shear stress, volumetric flow rate and the effective viscosity. In their work heat and mass was not investigated. More so, Rathod and Shakera [4] considered the pulsatile flow of blood through a porous medium. They solved analytical and concluded that velocity distribution increases with an increase of both body acceleration and permeability of the porous medium, while it decreases as the magnetic parameter increases. They did not consider heat and mass transfer. Nagarani and Sarojamme [5] worked on pulsatile flow of blood through a stenosed artery under the influence of external periodic body acceleration. The problem was solved analytical and result showed the effect of yield stress and stenosis is to reduce flow rate and increase flow resistance. Their result did not take cognizance of heat and mass transfer.

These flows, such as those in human cardiovascular system take place in cylindrical domains; the human blood vessel, for example. Fluid dynamics models in such domains are more challenging than those in Cartesian coordinate systems. The problem gets more complicated when the transport of heat and mass are incorporated into the system.

For instance, [6] studied two finite difference schemes for a channel flow problems. In their results no numerical oscillations were detected for values of a model parameter smaller than the theoretically derived bound. Also in a similar manner, [7] considered the theoretical analysis and applications of a convergent numerical algorithm to the problem of heat transfer in a convective channel flow. They applied numerical scheme and results showed that temperature is enhanced by increasing Brinkman number, while the velocity is enhanced by increasing pressure gradient. In another dimension, [8] presents the flow of an electrically conducting and radiating fluid over a moving heated porous plate in the presence of an induced magnetic field. They applied perturbation method and solved analytically and results showed that suction has more effect on the fluid velocity and magnetic field but less effect on the temperature.

Moreso, [9] considered the flow of a variable- viscosity fluid with heat and mass transfer, taking into account thermal radiation, cross-diffusion and constant-suction. Thus variables are used to transmute the governing partial differential equations (PDEs) into ordinary differential equations later solved using Runge- Kutta methods. In their results they established that the fluid velocity increases with increasing Dufour and Soret parameters. They further buttressed that the flow is only steady, suction is constant, and viscosity variation with temperature is linear.

Jain et al. [10] studied oscillatory flow of blood in a stenosed artery under the influence of transverse magnetic field through porous medium. They solved analytically and Bessel solution was obtained. [11] investigated the flow characterized of the blood flowing through an inclined tapered porous artery with mild stenosis . The problem was solved analytically where Bessel solution was obtained.

Amos et al. [12] worked on boundary layer flow in a rotating MHD fluid. The result shows that the increase in magnetic, Schmidt number, chemical reaction and rotation parameter decreases the velocity of flow in the system while the increase in thermal radiation leads to increase in velocity. Increase in chemical reaction parameter decreases heat transfer while it enhances the mass transfer. [13] considered the MHD peristaltic motion of a third grade fluid in an asymmetric channel under the assumptions of long wavelength and low Reynolds number. Series solutions of axial velocity and pressure gradient were given using perturbation approach when Deborah number is small. In the work of [14] wall stretching and magnetic field with channel flow of copper-kerosene nanofluid was examined. In the cylindrical domain, heat transfer of natural convection with heat sources was highly deliberated by [15]. In another scenario, the coupled system of Maxwell convective heat under Navier- Strokes equation to consider the characteristics of thermal Titania nano-fluids by means of different base in the channel flow. [16] considered the flow of radiating fluid in heated porous media. Results showed that suction is more efficient on the fluid velocity and magnetic field then becomes inefficient on temperature profile.

As mentioned above, the problem of investigating fluid flow in cylindrical geometry is non-trivial, and is compounded by when heat and mass flow are taken account. This difficulty is worsening when analytical solution of the problem is being sought. This is the thrust of this work; to derive analytical solution for the problem of heat and mass flow in a cylindrical channel flow of an incompressible fluid. The challenge in deriving analytical solution is due to the singularity of the resulting fluid models at the origin which is the centre of the channel (tube).This makes it difficult to apply the zero-gradient boundary condition at the origin. This is the problem that could not be resolved in [17] leading to a solution that does not satisfy the no-slip boundary condition at the channel wall. In this work, we resolve this difficulty by imposing the constraint that every flow quantity or variable is finite throughout the channel. This replaces the zero-Newmann boundary condition at the origin, and allows us to set the infinite term, in the general solution of the flow variables, to zero. This way, the desired solution is obtained. This is the novelty of this study.

The paper is presented as follows: In Section 2, we present the physical and mathematical models of the problem, and a detailed analytical solution is derived in Section 3. We present some Theorems with proves, to analyze the derived analytic solution in Section 4. The results are presented and discussed in Section 5. The paper is concluded in Section 6.

This present paper is aimed at extending the work of [11] by including mass and heat transfer and obtaining closed-form analytical solution.

2 Mathematical Formulation

The flow is assumed to be dominated along the channel axis, and is taken to be axi-symmetric. No-slip boundary condition is considered for velocity while the temperature and concentration have non-zero positive constants on the boundary. Let r be the distance from the channel centre and $\vec{u}, T', C'$ are the fluid velocity, temperature and mass concentration as shown in Fig. 1. Let the fluid velocity be $\vec{u} = (0, 0, u')$.

Since the flow is axisymmetric let (r', θ', z') be the cylindrical channel coordinates, where r is the radius of the channel, the directions of flow lies along horizontal axis z and flow is maintained at non-constant temperature (non-isothermal) the directions without flow lies along the vertical axis z see Fig. 1.

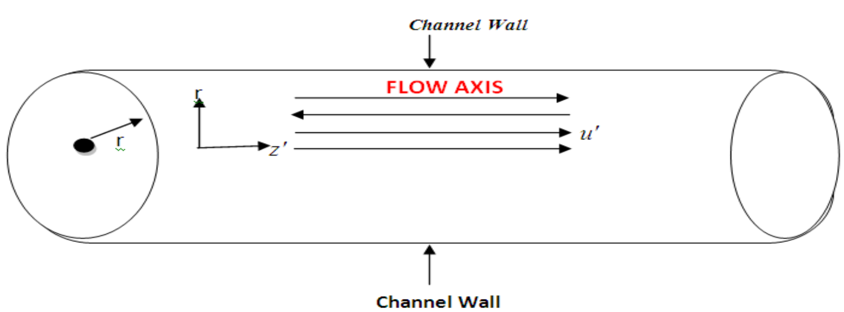

Fig. 1. Geometry of the physical model in horizontal channel

Based on the assumption of Boussinesq approximation of fluid model and taken consideration of induced magnetic field and steady flow; the equations governing the flow are:

$$\frac{\partial u'}{\partial z'} = 0 \tag{1}$$

$$-\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\right) + \rho g \beta_T \left(T' - T_\infty\right) - \alpha B_0'^2 u = 0 \tag{2}$$

$$\frac{k}{\rho C_P}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) - \lambda^2\left(T' - T_\infty\right) = 0 \tag{3}$$

$$\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \underline{\alpha}^2\left(C' - C_\infty\right) = 0 \tag{4}$$

Subject to the following boundary conditions

$$u' = 0\ ,\ \theta' = \theta_a,\ \varphi' = \varphi_a \ \text{ on } r' = a \tag{5}$$

$$u', \theta', \varphi' < \infty \ \text{ for all } r' \tag{6}$$

Where u' w' is the velocity components of fluid, ρ is the fluid density , B_0 is the magnetic field , k is the thermal conductivity, T' is the wall temperature , k is the thermal conductivity, g is the acceleration due to gravity and μ is the viscosity of the fluid, σ is the electrical conductivity , C_p is the heat capacity , q_r is constant magnetic field and β is the coefficient volumetric expansion.

2.1 Non-dimensionalization

From equations (1)-(4) and boundary conditions (5) and (6) respectively the following non- dimensional variables were used:

$$\left.\begin{aligned} & r = \frac{r'}{a}\ , z = \frac{z'}{a}, \theta = \frac{T' - T_\infty}{T_w - T_\infty}, \mathrm{u} = \frac{u'a}{v}, t = \frac{t'v}{a^2}, \rho = \frac{\rho' a^2}{\mu v}, \\ & Pr = \frac{\mu C_\rho}{k} = 1, \quad Gr = \frac{g\beta a^3}{v^2} T_\infty, \lambda^2 = \underline{\lambda}^2\, \mathrm{a}^2, \alpha^2 = \underline{\alpha}^2\, \mathrm{a}^2, \\ & M = \frac{\sigma B^2 a^2}{\mu}, \quad \varphi = \frac{C' - C_0}{C_w - C_0} \end{aligned}\right\} \tag{7}$$

where, Gr is the Grashof number.

Substituting (7) in (2)-(6) gives

$$-\frac{\partial p}{\partial z}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-M^2u+Gr\theta=0 \tag{8}$$

$$\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}-\lambda^2\theta=0 \tag{9}$$

$$\frac{\partial^2 \varphi}{\partial r^2}+\frac{1}{r}\frac{\partial \varphi}{\partial r}+\alpha^2\varphi=0 \tag{10}$$

Subject to

$$u=0,\theta=\theta_a,\varphi=\varphi_a \ 0n \ r=1 \tag{11}$$

$$u,\theta,\varphi<\infty \tag{12}$$

Equation (1) implies that $u \neq u(z)$, hence u is a function of r alone and we arrived at the following system of ordinary differential equations:

$$\frac{d^2 u}{dr^2}+\frac{1}{r}\frac{du}{dr}-M^2u=-p-Gr\theta \tag{13}$$

$$\frac{d^2 \theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-\lambda^2\theta=0 \tag{14}$$

$$\frac{d^2 \varphi}{dr^2}+\frac{1}{r}\frac{d\varphi}{dr}+\alpha^2\varphi=0 \tag{15}$$

Subject to the following boundary conditions

$$u=0, \ \ \theta=\theta_a,\varphi=\varphi_a \ \ on \ \ r=1 \tag{16}$$

$$u,\theta,\varphi<\infty \ \text{ for } 0\le r\le 1 \tag{17}$$

where p is constant term M represents magnetic field parameter, α represents injection term and λ represents cooling term. In the next section, we derive, in detail, an analytical solution of the proposed model.

3 Method of Solution

Equations (13)-(16) are coupled non-linear partial differential equations in the flow variables and .We adopts the method of Ferobenius in solving for $u(r,t),\theta(r,t)$ and $\varphi(r,t)$, Firstly, we solve equation (14), (15) and substituting the result in equation (13) then solving the resulting systems independently to obtain the following flow variables:

3.1 Solution of the temperature model

We assume $\theta(r)=\sum_{n=0}^{\infty}a_n r^{n+c}$, where $a_n, c\in\mathbb{R}$

$$\Rightarrow \theta=\sum_{n=0}^{\infty}a_n r^{n+c}=r^c\sum_{n=0}^{\infty}a_n r^n \tag{18}$$

$$\theta'=\sum_{n=0}^{\infty}a_n\left(n+c\right)r^{n+c-1}=r^{c-1}\sum_{n=0}^{\infty}a_n\left(n+c\right)r^n \tag{19}$$

$$\left.r\theta''=r^{c-2}\sum_{n=0}^{\infty}a_n\left(n+c\right)\left(n+c-1\right)r^n=r^{c-1}\sum_{n=0}^{\infty}a_n\left(n+c\right)\left(n+c-1\right)r^n\right\} \tag{20}$$

Putting (18)-(20) into (14) and performing long algebraic expressions gives

$$a_0c^2r^{c-1}+\sum_{n=0}^{\infty}\left(a_{n+1}\left(n+c+1\right)^2r^{n+c}-a_n\lambda^2r^{n+c+1}\right)=0$$

The indicial equation, therefore gives: $a_0c^2=0, a_0\neq 0, c^2=0, c=0\,(\text{twice})$

$$\theta=r^c\left(a_0+\frac{\lambda^2r^2}{(c+2)^2}+\frac{\lambda^4r^4}{(c+2)^2(c+4)^2}+\frac{\lambda^6r^6}{(c+2)^2(c+4)^2(c+6)^2}+\frac{\lambda^8r^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\ldots\right)$$

That is,

$$\theta=a_0r^c\left(\begin{array}{l}1+\dfrac{\lambda^2r^2}{(c+2)^2}+\dfrac{\lambda^4r^4}{(c+2)^2(c+4)^2}+\dfrac{\lambda^6r^6}{(c+2)^2(c+4)^2(c+6)^2}\\+\dfrac{\lambda^8r^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\ldots\end{array}\right) \tag{21}$$

When $c=c$ (21) becomes

$$\theta=u=A\left(1+\frac{\lambda^2r^2}{2^2}+\frac{\lambda^4r^4}{2^2\times4^2}+\frac{\lambda^6r^6}{2^2\times4^2\times6^2}+\frac{\lambda^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right) \tag{22}$$

Another is given by $v=\dfrac{d\theta}{dc}$

$$\begin{aligned}\frac{d\theta}{dc}=a_0r^c lnr&\left\{1+\frac{\lambda^2r^2}{(c+2)^2}+\frac{\lambda^4r^4}{(c+2)^2(c+4)^2}+\frac{\lambda^6r^6}{(c+2)^2(c+4)^2(c+6)^2}\right.\\&\left.+\frac{\lambda^8r^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\ldots\right\}+a_0r^c\frac{d}{dc}\left\{1+\frac{\lambda^2r^2}{(c+2)^2}+\frac{\lambda^4r^4}{(c+2)^2(c+4)^2}\right.\\&\left.+\frac{\lambda^6r^6}{(c+2)^2(c+4)^2(c+6)^2}+\frac{\lambda^8r^8}{(c+2)^2(c+4)^2(c+6)^2(c+8)^2}+\ldots\right\}\end{aligned}$$

When $c = 0$

$$\theta = v = B\left\{\ln r\left(1+\frac{\lambda^2 r^2}{2^2}+\frac{\lambda^4 r^4}{2^2\times 4^2}+\frac{\lambda^6 r^6}{2^2\times 4^2\times 6^2}+\frac{\lambda^8 r^8}{2^2\times 4^2\times 6^2\times 8^2}+\dots\right)\right.$$
$$\left.+a_0 r^c\left\{-\frac{\lambda^2 r^2}{2^2}-\frac{\lambda^4 r^4}{2^3\times 4^2}-\frac{\lambda^6 r^6}{4^3\times 6^3}+\frac{\lambda^8 r^8}{2^5\times 6^3\times 8^3}-\dots\right\}\right. \tag{23}$$

A linear combination (22) and (23) gives the complete solution

$$\theta = A\left\{1+\frac{\lambda^2 r^2}{2^2}+\frac{\lambda^4 r^4}{2^2\times 4^2}+\frac{\lambda^6 r^6}{2^2\times 4^2\times 6^2}+\frac{\lambda^8 r^8}{2^2\times 4^2\times 6^2\times 8^2}+\dots\right\}$$
$$+B\left\{\ln r\left(1+\frac{\lambda^2 r^2}{2^2}+\frac{\lambda^4 r^4}{2^2\times 4^2}+\frac{\lambda^6 r^6}{2^2\times 4^2\times 6^2}+\frac{\lambda^8 r^8}{2^2\times 4^2\times 6^2\times 8^2}+\dots\right)\right. \tag{24}$$
$$\left.-\frac{\lambda^2 r^2}{2^2}-\frac{\lambda^4 r^4}{2^3\times 4^2}-\frac{\lambda^6 r^6}{4^3\times 6^3}+\frac{\lambda^8 r^8}{2^5\times 6^3\times 8^3}-\dots\right\}$$

Applying the boundary condition (16) then setting $B = 0$ gives

$$A = \frac{\theta_a}{y_1(\text{a})} \tag{25}$$

$$\theta(r) = \frac{\theta_a}{y_1(\text{a})}\left\{1+\frac{\lambda^2 r^2}{2^2}+\frac{\lambda^4 r^4}{2^2\times 4^2}+\frac{\lambda^6 r^6}{2^2\times 4^2\times 6^2}+\frac{\lambda^8 r^8}{2^2\times 4^2\times 6^2\times 8^2}+\cdots\right\} \tag{26}$$

3.2 Solution of concentration model

Similarly from (15) we assume a solution of the form:

$$\varphi(r) = \sum_{m=0}^{\infty} b_m r^{m+k} = r^k \sum_{m=0}^{\infty} b_m r^m \tag{27}$$

$$\varphi' = \sum_{m=0}^{\infty} b_m (m+k) r^{m+k-1} = r^{k-1} \sum_{m=0}^{\infty} b_m (m+k) r^m \tag{28}$$

$$\varphi'' = \sum_{n=0}^{\infty} am(m+k) r^{m+k-2} = r^{k-2} \sum_{n=0}^{\infty} am(m+k)(m+k-1) r^m$$
$$\Rightarrow r\varphi'' = r^{k-1} \sum_{n=0}^{\infty} am(m+k)(n+k-1) r^m \tag{29}$$

Putting equations (27)-(29) into (15) gives

$$r^{k-1}b_0k^2+\sum_{k=0}^{\infty}b_{q+1}\left(q+k+1\right)^2r^{k+1}+\sum_{m=0}^{\infty}b_m\alpha^2r^{m+k+1}=0$$

Equating coefficients of indicial equation, therefore gives: $b_0k^2=0, \mathrm{b}_0\neq 0, \mathrm{k}^2=0, \mathrm{k}=0\,(\text{twice})$

$$\varphi=r^k\left(b_0-\frac{\alpha^4b_0r^4}{(k+2)^2(k+4)^2}+\frac{\alpha^6b_0r^6}{(k+2)^2(k+4)^2(k+6)^2}-\frac{\alpha^8b_0r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}-\ldots\right)$$

That is,

$$\varphi=b_0r^k\left\{1-\frac{\alpha^2r^2}{(k+2)^2}+\frac{\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\} \tag{30}$$

When $k=0$ (30) becomes

$$\varphi=u_1=C_1\left\{1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times4^2}-\frac{\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right\} \tag{31}$$

Another is given by $v_1=\frac{d\varphi}{dk}$

$$\frac{d\varphi}{dk}=b_0r^k lnr\left\{1-\frac{\alpha^2r^2}{(k+2)^2}+\frac{\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\}$$
$$+b_0r^k\frac{d}{dk}\left\{1-\frac{\alpha^2r^2}{(k+2)^2}+\frac{\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\}$$

When $k=0$

$$\varphi=v_2=C_2\left\{\ln r\left(1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times4^2}-\frac{\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)\right.$$
$$\left.+b_0r^k\left\{+\frac{\alpha^2r^2}{2^2}-\frac{\alpha^4r^4}{2^3\times4^2}+\frac{\alpha^6r^6}{4^3\times6^3}+\frac{\alpha^8r^8}{2^5\times6^3\times8^3}-\ldots\right\}\right. \tag{32}$$

A linear combination (31) and (32) gives the complete solution

$$\varphi = C_1\left\{1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times4^2}-\frac{\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right\}$$
$$+C_2\left\{\ln r\left(1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times4^2}-\frac{\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)\right.$$
$$\left.+\frac{\alpha^2r^2}{2^2}-\frac{\alpha^4r^4}{2^3\times4^2}+\frac{\alpha^6r^6}{4^3\times6^3}+\frac{\alpha^8r^8}{2^3\times2^2\times6^3\times8^3}-\ldots\right\} \tag{33}$$

Applying the boundary condition (16) then setting $C_2=0$ in(33) gives

$$C_1=\frac{\varphi_a}{y^*_1(\mathrm{a})} \tag{34}$$

where $y^*_{1(\mathrm{a})}=\left(1-\frac{\alpha^2a^2}{2^2}+\frac{\alpha^4a^4}{2^2\times4^2}-\frac{\alpha^6a^6}{2^2\times4^2\times6^2}+\frac{\alpha^8a^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)$,

$$\varphi=\frac{\varphi_a}{y^*_1(\mathrm{a})}\left\{1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times4^2}-\frac{\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\cdots\right\} \tag{35}$$

3.3 Solution of velocity model

Also we assume a solution of the form:

$$u=\sum_{m=0}^{\infty}b_mr^{m+k}=r^k\sum_{m=0}^{\infty}b_mr^m \tag{36}$$

$$u'=\sum_{m=0}^{\infty}b_m\left(m+k\right)r^{m+k-1}=r^{k-1}\sum_{m=0}^{\infty}b_m\left(m+k\right)r^m \tag{37}$$

$$u''=\sum_{n=0}^{\infty}am\left(m+k\right)r^{m+k-2}=r^{k-2}\sum_{n=0}^{\infty}am\left(m+k\right)\left(m+k-1\right)r^m$$
$$\Rightarrow ru''=r^{k-1}\sum_{n=0}^{\infty}am\left(m+k\right)\left(n+k-1\right)r^m \tag{38}$$

Putting equations (36)-(38) into the left hand side of equation (13) gives

$$r^{k-1}b_0k^2+\sum_{k=0}^{\infty}b_{q+1}\left(q+k+1\right)^2r^{k+1}-\sum_{m=0}^{\infty}b_mM^2r^{m+k+1}=0$$

Equating coefficients of indicial equation, therefore gives: $b_0k^2=0,\mathrm{b}_0\neq0,\mathrm{k}^2=0,\mathrm{k}=0\,(\mathrm{twice})$

$$u(r)=r^k\sum_{m=0}^{\infty}b_m r^m = r^k\left(\begin{array}{l} b_0+\frac{M^2b_0r^2}{(k+2)^2}+\frac{M^4b_0r^4}{(k+2)^2(k+4)^2}+\frac{M^6b_0r^6}{(k+2)^2(k+4)^2(k+6)^2}\\ +\frac{M^8b_0r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\end{array}\right)$$

That is,

$$u(r)=b_0r^k\left\{1+\frac{M^2r^2}{(k+2)^2}+\frac{M^4r^4}{(k+2)^2(k+4)^2}+\frac{M^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{M^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\} \tag{39}$$

When $k=0$ (39) becomes

$$u=u_1=A_1\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right\} \tag{40}$$

Another is given by $v_1=\frac{du}{dk}$

$$\frac{du}{dk}=b_0r^k lnr\left\{1+\frac{M^2r^2}{(k+2)^2}+\frac{M^4r^4}{(k+2)^2(k+4)^2}+\frac{M^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{M^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\}$$
$$+b_0r^k\frac{d}{dk}\left\{1+\frac{M^2r^2}{(k+2)^2}+\frac{M^4r^4}{(k+2)^2(k+4)^2}+\frac{M^6r^6}{(k+2)^2(k+4)^2(k+6)^2}\right.$$
$$\left.+\frac{M^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\ldots\right\}$$

When $k=0$

$$u=v_2=A_2\left\{\ln r\left(1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)\right.$$
$$\left.+b_0r^k\left\{-\frac{M^2r^2}{2^2}-\frac{M^4r^4}{2^3\times4^2}-\frac{M^6r^6}{4^3\times6^3}+\frac{M^8r^8}{2^5\times6^3\times8^3}-\ldots\right\}\right. \tag{41}$$

A linear combination (40) and (41) gives the complete solution

$$u = A_1\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times 4^2}+\frac{M^6r^6}{2^2\times 4^2\times 6^2}+\frac{M^8r^8}{2^2\times 4^2\times 6^2\times 8^2}\right\}$$
$$+A_2\left\{\ln r\left(1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times 4^2}+\frac{M^6r^6}{2^2\times 4^2\times 6^2}+\frac{M^8r^8}{2^2\times 4^2\times 6^2\times 8^2}+\dots\right)\right. \tag{42}$$
$$\left.-\frac{M^2r^2}{2^2}-\frac{M^4r^4}{2^3\times 4^2}-\frac{M^6r^6}{4^3\times 6^3}+\frac{M^8r^8}{2^5\times 6^3\times 8^3}-\dots\right\}$$

Applying the boundary conditions (16) and setting $A_2 = 0$ in (42) gives

$$A_1 = \frac{\theta_a}{y_1(\mathrm{a})} \tag{43}$$

where $y_{1(\mathrm{a})} = \left(1+\frac{M^2a^2}{2^2}+\frac{M^4a^4}{2^2\times 4^2}+\frac{M^6a^6}{2^2\times 4^2\times 6^2}+\frac{M^8a^8}{2^2\times 4^2\times 6^2\times 8^2}+\dots\right)$,

$$u = \frac{\theta_a}{y_1(1)}\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times 4^2}+\frac{M^6r^6}{2^2\times 4^2\times 6^2}+\frac{M^8r^8}{2^2\times 4^2\times 6^2\times 8^2}+\cdots\right\} \tag{44}$$

3.3.1 Non-homogenous part of velocity model

To get the particular solution of (13) which is the non-homogenous part of the problem, we solve using the method of undetermined coefficients.

$$y(r) = A\left\{1+\frac{\lambda^2r^2}{2^2}+\frac{\lambda^4r^4}{2^2\times 4^2}+\frac{\lambda^6r^6}{2^2\times 4^2\times 6^2}+\frac{\lambda^8r^8}{2^2\times 4^2\times 6^2\times 8^2}+\cdots\right\}, A\in\mathbb{R} \tag{45}$$

The complementary solution for (13) is

$$u_c(r) = C_c\left[1+\frac{(Mr)^2}{2^2}+\frac{(Mr)^4}{(2\times 4)^2}+\frac{(Mr)^6}{(2\times 4\times 6)^2}+\frac{(Mr)^8}{(2\times 4\times 6\times 8)^2}+\dots\right] \tag{46}$$

Consider the particular solution:

$$u_p(r) = A_0 + A_1r^2 + A_2r^4 + A_3r^6 + A_4r^8 \tag{47}$$

$$u_p'(r) = 2A_1r + 4A_2r^3 + 6A_3r^5 + 8A_4r^7 \tag{48}$$

$$u_p''(r) = 2A_1 + 12A_2r^2 + 30A_3r^4 + 56A_4r^6 \tag{49}$$

Putting equations(47)-(49) into equation (13) gives

$$\Rightarrow 4A_1 + 16A_2r^2 + 36A_3r^4 + 64A_4r^6 - M^2A_0 - M^2A_1r^2 - M^2A_2r^4 - M^2A_3r^6 - M^2A_4r^8 \equiv$$
$$-p - Gr\theta\left(1 + \frac{(\lambda r)^2}{2^2} + \frac{(\lambda r)^4}{(2\times 4)^2} + \frac{(\lambda r)^6}{(2\times 4\times 6)^2} + \frac{(\lambda r)^8}{(2\times 4\times 6)^2}\right) \quad (50)$$

Combining equations (46) and (47) respectively yields

$$u(r) = \left[c + \frac{cM^2r^2}{4} + \frac{cM^4r^4}{(2\times 4)^2} + \frac{cM^6r^6}{(2\times 4\times 6)^2} + \frac{cM^8r^8}{(2\times 4\times 6\times 8)^2}\right] + A_0 + A_1r^2 + A_2r^4 + A_3r^6 + A_4r^8$$

Taking like terms in the above expressions gives a complete solution of equation (13)

$$u(r) = c + A_0 + \left(\frac{cM^2}{4} + A_1\right)r^2 + \left(\frac{cM^4}{(2\times 4)^2} + A_2\right)r^4 + \left(\frac{cM^6}{(2\times 4\times 6)^2} + A_3\right)r^6$$
$$+\left(\frac{cM^8}{(2\times 4\times 6\times 8)^2} + A_4\right)r^8 \quad (51)$$

In general, we combine the three flow variables to obtain the following results:

Velocity profile:

$$u(r) = c + A_0 + \left(\frac{cM^2}{2^2} + A_1\right)r^2 + \left(\frac{cM^4}{(2\times 4)^2} + A_2\right)r^4 + \left(\frac{cM^6}{(2\times 4\times 6)^2} + A_3\right)r^6$$
$$+\left(\frac{cM^8}{(2\times 4\times 6\times 8)^2} + A_4\right)r^8 \quad (52)$$

Where c, A_0, A_1, A_2, A_3 *and* A_4 are constants stated in appendix.

Temperature profile:

$$\theta(r) = \frac{\theta_a}{y_1(a)}\left\{1 + \frac{\lambda^2r^2}{2^2} + \frac{\lambda^4r^4}{2^2\times 4^2} + \frac{\lambda^6r^6}{2^2\times 4^2\times 6^2} + \frac{\lambda^8r^8}{2^2\times 4^2\times 6^2\times 8^2} + \right\} \quad (53)$$

where $y_{1(a)} = \left(1 + \frac{\lambda^2a^2}{2^2} + \frac{\lambda^4a^4}{2^2\times 4^2} - \frac{\lambda^6a^6}{2^2\times 4^2\times 6^2} + \frac{\lambda^8a^8}{2^2\times 4^2\times 6^2\times 8^2} + ...\right)$

Mass concentration:

$$\varphi(r) = \frac{\varphi_a}{y^*_1(a)}\left\{1 - \frac{\alpha^2r^2}{2^2} + \frac{\alpha^4r^4}{2^2\times 4^2} - \frac{\alpha^6r^6}{2^2\times 4^2\times 6^2} + \frac{\alpha^8r^8}{2^2\times 4^2\times 6^2\times 8^2} + ...\right\} \quad (54)$$

where

$$y_1^*(a) = \left(1 - \frac{\alpha^2 a^2}{2^2} + \frac{\alpha^4 a^4}{2^2 \times 4^2} - \frac{\alpha^6 a^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 a^8}{2^2 \times 4^2 \times 6^2 \times 8^2} - \ldots\right)$$

4 Analysis of the Model

4.1 Axi-symmetric property of the model

Here, the solutions of the three flow variables will be subjected for analysis. Hence, we state the following theorem:

Theorem 4.1(Axi-symmetric Property): The solution (52), (53) and (54) derived for each field variable is symmetrical about the centre of the tube. That is $\frac{du}{dr}|_{r=0} = 0,\ \frac{d\theta}{dr}|_{r=0} = 0,\ \frac{d\varphi}{dr}|_{r=0} = 0$.

Proof

To proof that the three flow variables are axi-symmetric

Recall the solutions in equations (52), (53) and (54)

From (52)

$$u(r) = c + A_0 + \left(\frac{c\beta^2}{2^2} + A_1\right)r^2 + \left(\frac{c\beta^4}{(2\times4)^2} + A_2\right)r^4 + \left(\frac{c\beta^6}{(2\times4\times6)^2} + A_3\right)r^6 + \left(\frac{c\beta^8}{(2\times4\times6\times8)^2} + A_4\right)r^8$$

$$\frac{du}{dr} = 2\left(\frac{c\beta^2}{2^2} + A_1\right)r + 4\left(\frac{c\beta^4}{(2\times4)^2} + A_2\right)r^3 + 6\left(\frac{c\beta^6}{(2\times4\times6)^2} + A_3\right)r^5 + 8\left(\frac{c\beta^8}{(2\times4\times6\times8)^2} + A_4\right)r^7 \qquad (55)$$

$$\frac{du}{dr}|_{r=0} = 0$$

Similarly for temperature (54)

$$\frac{d\theta}{dr} = \left(\frac{2\lambda^2 r}{2^2} + \frac{4\lambda^4 r^3}{2^2\times4^2} + \frac{6\lambda^6 r^5}{2^2\times4^2\times6^2} + \frac{8\lambda^8 r^7}{2^2\times4^2\times6^2\times8^2} + \ldots\right) \qquad (56)$$

$$\frac{d\theta}{dr}|_{r=0} = 0$$

Also for concentration (55)

$$\varphi(r) = \frac{\varphi_a}{y^*_1(a)}\left(1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times 4^2}-\frac{\alpha^6r^6}{2^2\times 4^2\times 6^2}+\frac{\alpha^8r^8}{2^2\times 4^2\times 6^2\times 8^2}-\dots\right)$$

$$\frac{d\varphi}{dr} = -\frac{2\alpha^2 r}{2^2}+\frac{\alpha^4r^4}{2^2\times 4^2}-\frac{\alpha^6r^6}{2^2\times 4^2\times 6^2}+\frac{\alpha^8r^8}{2^2\times 4^2\times 6^2\times 8^2}-\dots \tag{57}$$

$$\frac{d\varphi}{dr}|_{r=0} = 0$$

Hence the claim is true.

4.2 Invariance domain of concentration

Physically concentration should be positive .Hence, if the boundary concentration is positive, then physical reality requires that the model for concentration (15) be positive for all values of $r \in [0,1]$. Below we state and prove theorem to show that our developed model satisfies this physical requirement.

Theorem 4.2: Suppose the boundary concentration φ_a is positive and $0 \le \alpha \le 1$ .Then the model (15) yields a positive concentration for all $r \in [0,1]$.

Proof

$$\text{Let } 0 \le r \le 1, \quad 0 \le \alpha \le 1, \text{and}\, \varphi_a \ge 0.\text{Then}\, \frac{\alpha^2 r^2}{p^2} \le 1 \text{ for } p \ge 1.$$

From the solution of mass Concentration (54), we have the following:

$$\varphi(r) = \frac{\varphi_a}{y^*_1(a)}\left(1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{2^2\times 4^2}-\frac{\alpha^6r^6}{2^2\times 4^2\times 6^2}+\frac{\alpha^8r^8}{2^2\times 4^2\times 6^2\times 8^2}-\dots\right)$$

where

$$y^*_{1(1)} = \left(1-\frac{\alpha^2}{2^2}+\frac{\alpha^4}{2^2\times 4^2}-\frac{\alpha^6}{2^2\times 4^2\times 6^2}+\frac{\alpha^8}{2^2\times 4^2\times 6^2\times 8^2}-\dots\right)$$

$$= \left(\left(1-\frac{\alpha^2}{2^2}\right)+\frac{\alpha^4}{(2\times 4)^2}\left(1-\frac{\alpha^2}{6^2}\right)+\frac{\alpha^8}{(2\times 4\times 6)^2}\left(1-\frac{\alpha^2}{10^2}\right)+\dots\right)$$

$$\geq\left(\left(1-1\right)+\frac{\alpha^4}{(2\times4)^2}\left(1-1\right)+\frac{\alpha^8}{(2\times4\times6)^2}\left(1-1\right)+\ldots\right)$$
$$=\ 0$$
$$\Rightarrow\ y_1^*\left(1\right)\geq0$$

Also define $\underline{\varphi}(r)$ as

$$\underline{\varphi}(r)=\left(1-\frac{\alpha^2r^2}{2^2}+\frac{\alpha^4r^4}{(2\times4)^2}-\frac{\alpha^6r^6}{(2\times4\times6)^2}+\frac{\alpha^8r^8}{(2\times4\times6\times8)^2}+\ldots\right)$$
$$=\left(\left(1-\frac{\alpha^2r^2}{2^2}\right)+\frac{\alpha^4r^4}{(2\times4)^2}\left(1-\frac{\alpha^2r^2}{6^2}\right)+\frac{\alpha^8r^8}{(2\times4\times6\times8)^2}\left(1-\frac{\alpha^2r^2}{10^2}\right)+\ldots\right)$$
$$=\left(\left(1-1\right)+\frac{\alpha^4r^4}{(2\times4)^2}\left(1-1\right)+\frac{\alpha^8r^8}{(2\times4\times6\times8)^2}\left(1-1\right)+\ldots\right)\geq0$$

So $\underline{\varphi}(r):=\frac{\varphi_a}{y_1^*(1)}\underline{\varphi}(r)\geq0$, Since $\varphi_a>0, y_1^*(1)\geq0$ and $\underline{\varphi}(r)>0$

5 Results and Discussion

We have formulated and computed results of non-isothermal flow in a cylindrical channel. The parameter values used from the graphical presentations are shown as follows: However, the following parameter values were explicitly used in the simulation study:

- Vary velocity with magnetic field $\mathrm{Pr}=4,\ Gr=1.0,\ \lambda=180.0$
- Vary velocity with $Gr:\mathrm{M}=1.5,\ \mathrm{p}=1,\ \lambda=1000$
- Vary velocity with $Pr:\mathrm{M}=2.5,\ Gr=0.05,\ \lambda=188$
- Vary velocity with $\lambda:\mathrm{M}=2.5,\ \mathrm{Pr}=6,\ Gr=0.05$
- Vary temperature with $\lambda:\mathrm{M}=\theta_a=1.0$
- Vary temperature with $\theta_a:1.0$
- Vary concentration with $\alpha:\varphi_a=0.1$
- Vary concentration with $\varphi_a:\alpha=0.8$

5.1 Results

Therefore we present our results and discussion:

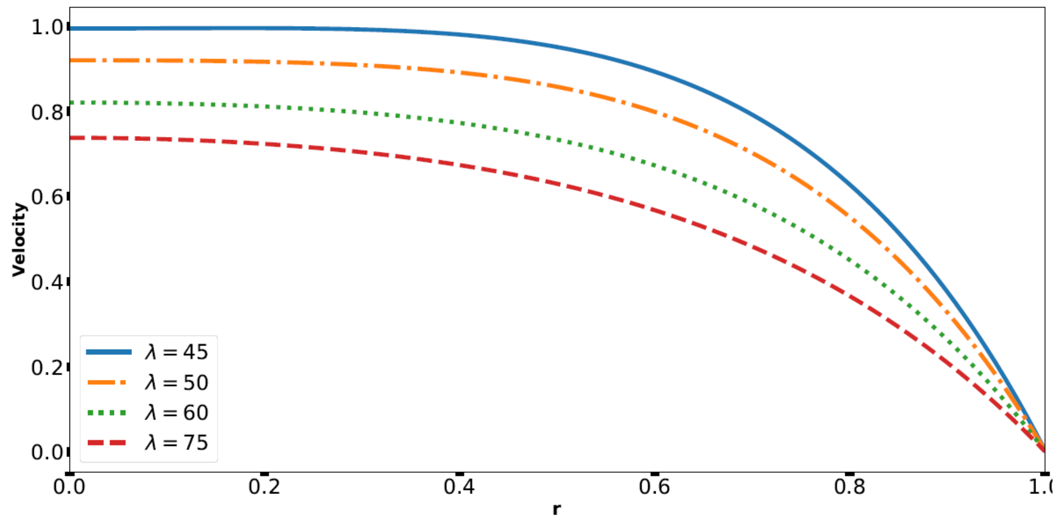

Fig. 2. Velocity profiles for $\Pr = 4,\ Gr = 1.0,\ \lambda = 180.0$ and different values of velocity cooling parameter, λ

Fig. 2 shows the variation of fluid velocity with the cooling parameter. It can be seen that the velocity reduces with increasing cooling parameter. This is physically consistent because an increase in cooling parameter λ reduces fluid temperature which in turn in turn increases the fluid viscosity, hence reduces the fluid velocity. This result is in good agreement with the results of [8].

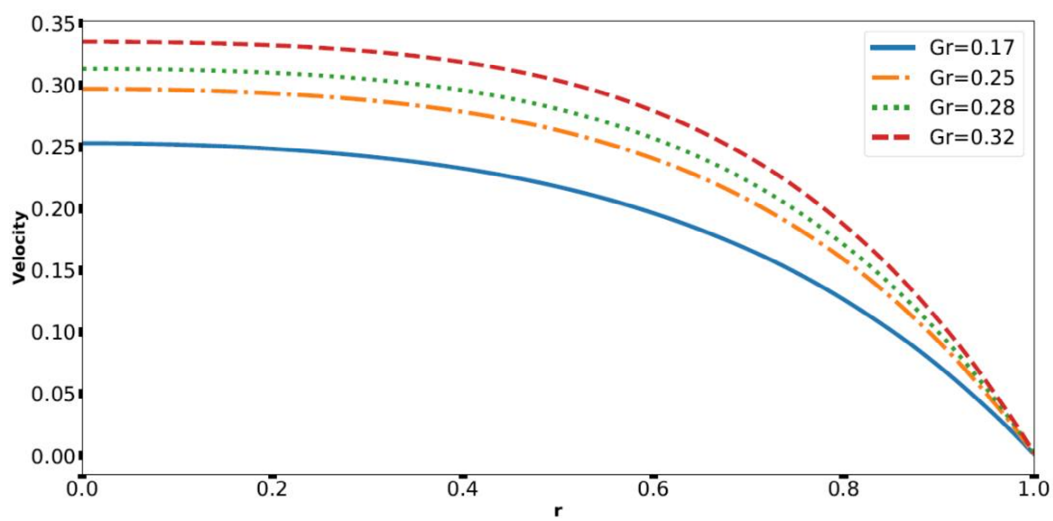

Fig. 3. Velocity profiles for $\mathrm{M} = 1.5,\ \mathrm{p} = 1,\ \lambda = 1000$ and different values of Grashof, number Gr

Fig. 3 shows the variation of fluid velocity with Grashof number. It is clear that an increase in Grashof number leads to an increase in fluid velocity. This agrees with the results of [18] and [19].

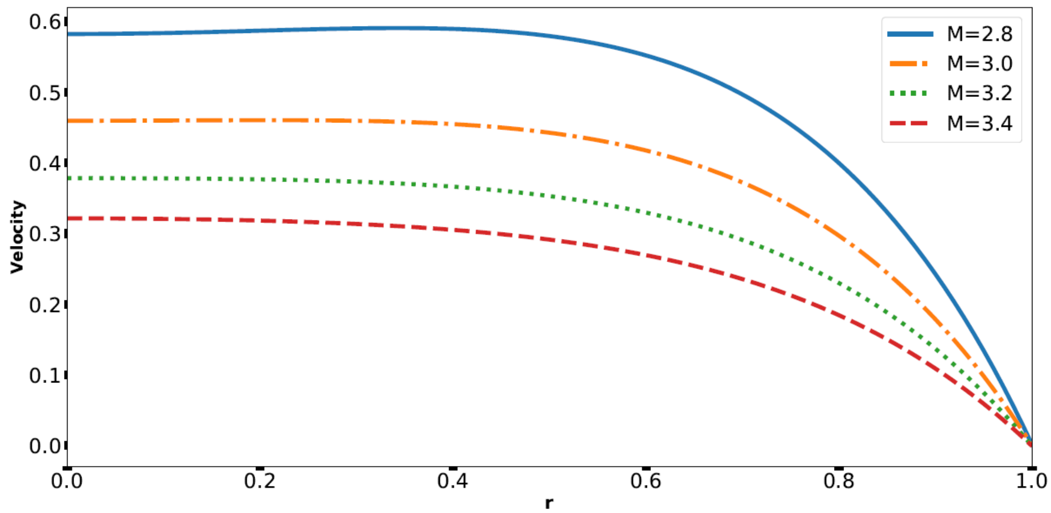

Fig. 4. Velocity profiles for $\mathrm{M} = 2.5,\ Gr = 0.05,\ \lambda = 188$ **and different values of Magnetic field parameter,** M

Fig. 4 shows the variation of fluid velocity with magnetic field parameter. It can be seen that increase in the magnetic field parameter decreases the fluid velocity. This is in accordance with the results of [8,10,16,20,21,22].

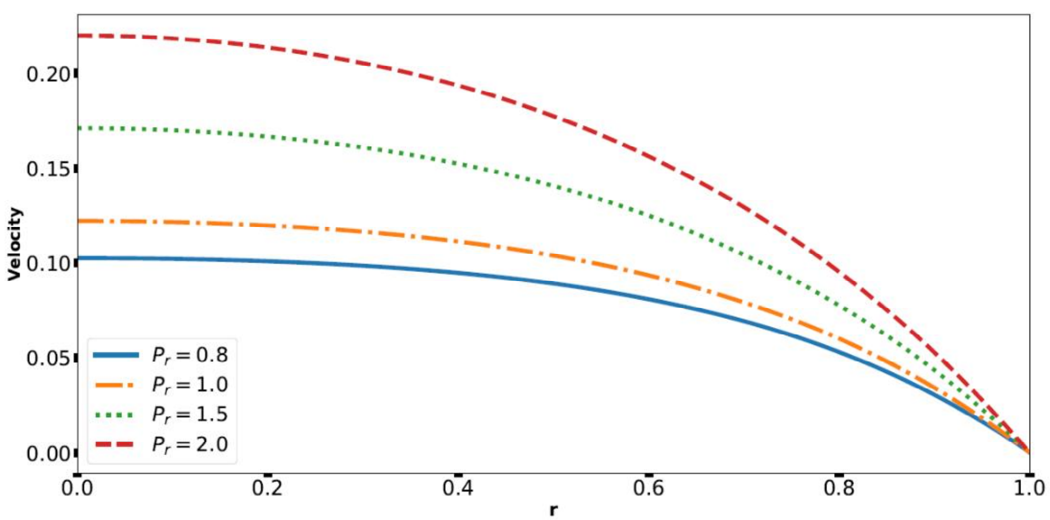

Fig. 5. Velocity profiles for with $\mathrm{M} = 2.5,\ \mathrm{Pr} = 6,\ Gr = 0.05$ **and different values of pressure gradient parameter,** Pr

Fig. 5 shows an increase in pressure gradient leads to an increase in fluid velocity. Therefore, the result is physically obtainable; it is in agreement with those of [10,23].

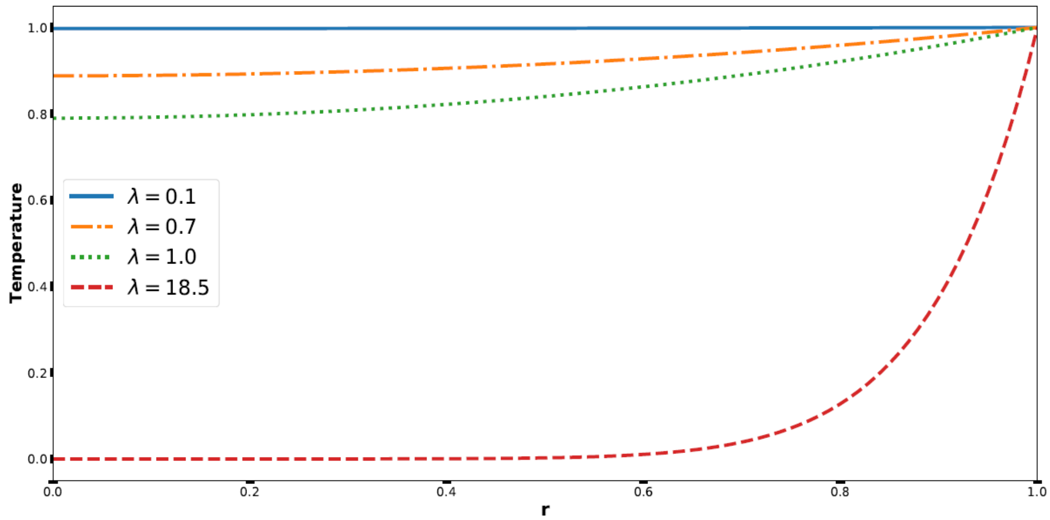

Fig. 6. Temperature profiles for with $\theta_a = 1.0$ and different values of temperature cooling parameter, λ

Fig. 6 shows the variation of fluid temperature with cooling parameter. It is observed that the temperature decreases as the cooling parameter increases. This is quite obvious in our daily lives, if a body is hot and if cooling is applied, the body temperature reduces. This agree with the result of [7].

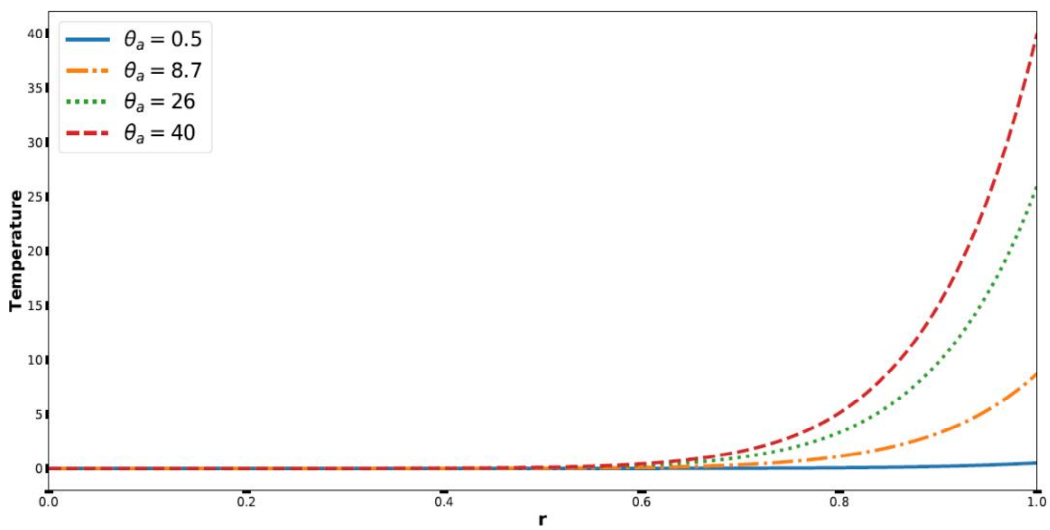

Fig. 7. Temperature profiles for $\lambda = 1.0$ and different values of temperature boundary parameter, θ_a

Fig. 7 shows the variation of temperature with the temperature boundary condition, it can be noticed that an increase in boundary parameter increases fluid temperature.

It is shown in Fig. 8 that an increase in the injection parameter increases concentration. This is physically obvious in the sense that an increasing injection parameter would increase the level by which the pollutant is being added into the fluid; this is also similar with increasing boundary condition parameter in Fig. 9 Therefore, it is realistic. This is in line with result of [7].

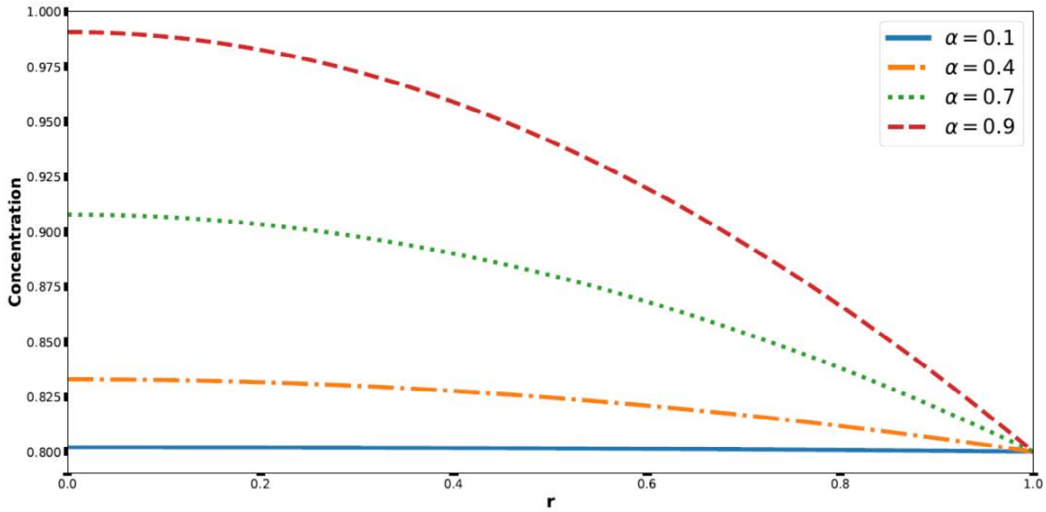

Fig. 8. Concentration profiles for $\varphi_a = 0.1$ and different values of concentration injection parameter, α

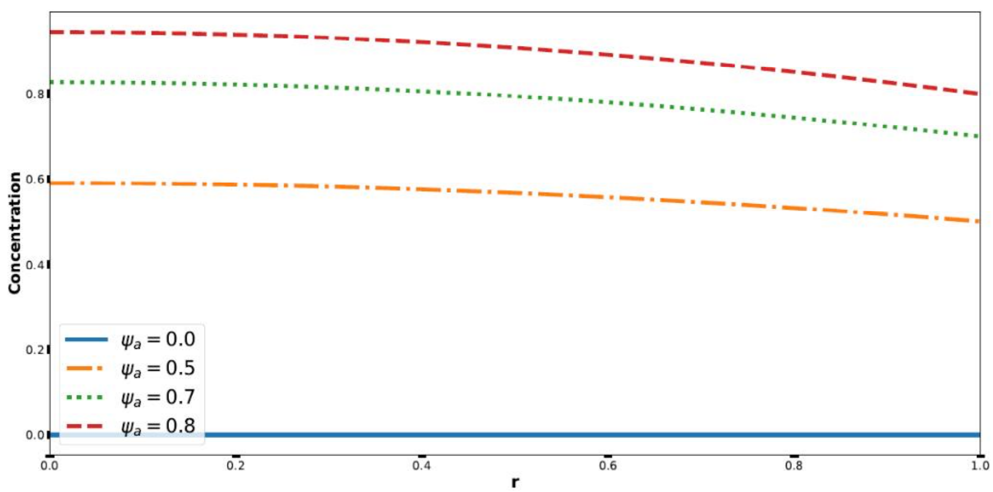

Fig. 9. Concentration profiles for with $\varphi_a = 0.8$ and different values of boundary condition parameter, φ_a

6 Conclusions

This study investigated the problem of heat and mass transfer for simulation of non-isothermal flow in a cylindrical channel. A coupled system of three differential equations is carefully formulated. Detailed analytical solutions are presented; then the positivity of the concentration in was proved theoretically and verified graphically. The graphical results showed (i) an increase in cooling parameter, λ reduces the fluid velocity (ii) temperature decreases as the cooling parameter increases λ (iii) an increase in injection parameter, α leads to increase in fluid concentration.

Competing Interests

Authors have declared that no competing interests exist.

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Appendix

$$\Rightarrow A_4 = -\frac{\lambda^8}{M^2}\frac{Gr\theta}{(2\times4\times6\times8)^2}, \Rightarrow A_3 = -\frac{1}{M^2}\left[64A_4 - \frac{\lambda^6 Gr\theta}{(2\times4\times6)^2}\right], \Rightarrow A_2 = -\frac{1}{M^2}\left[36A_3 - \frac{\lambda^4 Gr\theta}{(2\times4)^2}\right],$$

$$\Rightarrow A_1 = -\frac{1}{M^2}\left[16A_2 - \frac{\lambda^2 Gr\theta}{4}\right], \Rightarrow A_0 = -\frac{1}{M^2}\left[4A_1 + p - Gr\theta\right]$$

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