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Analytical solution of non-Fourier heat conduction problem on a fin under periodic boundary conditions †

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Abstract

Fourier and hyperbolic models of heat transfer on a fin that is subjected to a periodic boundary condition are solved analytically. The differential equation in Fourier and non-Fourier models is solved by the Laplace transform method. The temperature distribution on the fin is obtained using the residual theorem in a complex plan for the inverse Laplace transform method. The thermal shock is generated at the base of the fin, which moves toward the tip of the fin and is reflected from the tip. The current study of various parameters on the thermal shock location shows that relaxation time has a great influence on the temperature distribution on the fin. An unsteady boundary condition in the base fin caused the shock, which is generated continuously from the base and has interacted with the other reflected thermal shocks. Results of the current study show that the hyperbolic heat conduction equation can violate the second thermodynamic law under some unsteady boundary conditions.

Keywords: Analytical solution; Conduction; Fin; Hyperbolic; Periodic

1. Introduction

Classical Fourier heat conduction model is applied to predict the temperature distribution in general engineering problems under regular conditions [1]. The parabolic characteristic of Fourier's laws implies that heat flux is caused simultaneously with the creation of the temperature gradient. This assumption cannot be true because we know that two phenomena that are related to each other cannot exist simultaneously [2]. Such an immediate response leads to a local change in temperature that causes instantaneous temperature perturbations in all regions; thus, heat propagation speed will be infinite [2].

Fourier's law has a nonphysical conclusion for situations that are involved with very high temperature gradients, extremely short times, very low temperatures, and very small structural dimensions, among others. The mathematical description of non-Fourier heat conduction law, which represents the time lag of heat waves, is a hyperbolic type of differential equation. Non-Fourier hyperbolic heat transfer in the fins at short times under periodic boundary conditions has a wide application in micro-devices, such as heating and cooling of microelectronic elements, micro-fabrication technology, and micro-heat exchangers, among others.

Numerous studies are developed to solve the analytical and numerical heat conduction equations in the fins. However, applying analytical methods to solve heat transfer in the fins with complicated boundary conditions, variable physical properties, and thermal discontinuity that are produced in the hyperbolic equations is difficult. Thus, numerical schemes are used in most studies. The major problem of numerical solutions is the presence of oscillations near the thermal discontinuities, whereas analytical methods do not have unreasonable oscillations near the thermal discontinuities. Analytical solutions are used to check the accuracy and convergence of the numerical methods.

Various analytical and numerical methods of the hyperbolic heat conduction equation subjected to periodic boundary conditions were presented in Refs. [3-13] and many others. Most studies solved the fin problems in the Fourier domain by applying the numerical methods. Yen and Wu [14] solved the hyperbolic heat conduction in a finite slab with surface radiation and periodic heat flux using the Laplace transform method. Chang and Juhng [15] analytically solved the hyperbolic heat conduction in a slab under the sinusoidal periodic surface heating process. Aziz and Na [16] adopted a perturbation method to solve the fins with various thermal properties. Convective heat transfer in the fin under a periodic boundary

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condition is analytically solved by Yang [17]. The problem of periodic boundary conditions in the hyperbolic heat conduction was investigated by Tang and Araki [18] and Abdel-Hamid [19]. Periodic boundary conduction in materials with non-Fourier heat conduction model for a one-dimensional slab was examined by Cossali [20] through the "transfer function" method. Abdallah [21] investigate the analytic method of a boundary value problem for a semi-infinite medium with traction-free surface heated by a high-speed laser pulse. He used a Dirac laser pulse boundary that was not periodic.

With regard to the periodic boundary condition, most studies have dealt with conduction heat transfer using the parabolic (Fourier's law) heat equation or numerical schemes, whereas some relied on the hyperbolic heat equation. However, the hyperbolic model of the heat transfer cannot accurately predict the temperature in a medium.

The present work focuses on the analytical scheme in solving the hyperbolic heat conduction in the fin that is subjected to every periodic boundary condition using the Laplace transform method. Unlike other numerical methods, this analytical method is free of oscillations around the thermal discontinuities. The objective of the present work is to investigate the effects of relaxation time by having various boundary conditions on the temperature distribution in the fin, and by assessing the second thermodynamic law in the hyperbolic heat equation model.

2. Physical model and heat transfer in the fin

Phonons propagate at the sound speed depending on the type of solid medium. Thus, a response time with very small order implies a submicron depth penetration, thereby necessitating a simultaneous consideration of the microscopic effect in space. To attain the reliable performance of the microdevices, the effective means for heat removal at short times must be ensured. The response time of the thermal and relaxation time of the energy carriers resulting in high temperature at short times and causing early-time thermal damage before steady state operations can occur.

Microscopic models such as the phonon-electron interaction model [22], phonon scattering model [23], and phonon radiative transfer model [24] resulted from the solutions of the semi-classical Boltzmann transport equation.

The classical Fourier diffusion model describes the correlation between the heat flux and temperature gradient in a macroscale heat transfer. The thermal wave model (CV wave) depicts a temperature disturbance propagating as a wave, with thermal diffusivity acting as a damping effect in heat propagation. The fractal model [25] is employed for describing the conducting path in amorphous material and the scattering of phonons over the correlation length on a small scale. The DPL model [26] includes the delay time effects due to microscale effects on the transient response. In this study, we use a modified heat flux proposed by Vernotte [24] and Cattaneo [27] to solve heat transfer in the fin with time-dependent boundary conditions. The thermal wave model given by Cattaneo and Vernotte is applied for micro-solid materials at very short times. Wang et al. [28] showed that the CV wave model can be used for the thermomass gas. They built a thermomass gas model based on hyperbolic heat conduction theory to describe the fluid-flow-like heat conduction process in a medium. Wang and Guo [29] also presented new governing equations for non-Fourier heat conduction in nanomaterials based on the concept of thermomass.

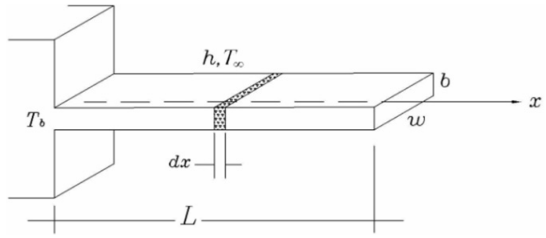

Fig. 1. The fin configuration.

Consider a straight fin with uniform thickness b, width w, and length L, which has an initial temperature T_0 (see Fig. 1). The ratio b/L is a small value, and the fin tip ($x=L$) is adiabatic. At a specific time, a periodic temperature boundary condition is applied to the fin base ($x=0$).

$$T_b(0,t) = T_{b,m} + ACos(\omega t)(T_{b,m} - T_\infty) \tag{1}$$

where T_b, T_∞, and $T_{b,m}$ are periodic base temperature, ambient temperature, and mean base temperature, respectively. A is the input temperature amplitude and ω is the temperature oscillation frequency.

The lateral surfaces of the fin dissipate heat to the environment by convection heat transfer coefficient. The hyperbolic heat conduction equation for the fin is given by Eq. (2):

$$\begin{aligned} &k\frac{\partial^2 T(x,t)}{\partial x^2} - 2\frac{h}{b}(T - T_\infty) = \\ &\tau\rho C\frac{\partial^2 T(x,t)}{\partial t^2} + \rho C\frac{\partial T(x,t)}{\partial t} + 2\tau\frac{h}{b}\frac{\partial}{\partial t}(T - T_\infty) \end{aligned} \tag{2}$$

where $T(x, t)$ represents temperature; k, ρ, and C are the thermal conductivity, density, and specific heat capacity in a medium, respectively. τ is the relaxation time, which means that the free path λ is over phonon velocity and ν (speed of sound in the medium). Relaxation time illustrates that there is a finite lag time for the onset of a thermal current after a temperature gradient is imposed on a medium. In the absence of relaxation time ($\tau = 0$, Eq. (2) is reduced to the classical Fourier's law. Eq. (2) is a heat wave equation that propagates a temperature disturbance in the form of a heat wave; this equation is damped using the diffusivity coefficient α.

The following dimensionless quantities, i.e., dimensionless temperature θ, dimensionless convective heat transfer H, dimensionless time ξ, dimensionless space η, dimensionless frequency of the temperature oscillation Ω, and dimensionless time relaxation β, are introduced as:

$$\eta = \frac{x}{L}\,,\ \xi = \frac{\alpha t}{L^2}\,,\ \beta = \frac{\alpha\tau}{L^2}\,,\ \Omega = \frac{\omega L}{\alpha}\,,\ H = \frac{2hL^2}{bk} \tag{3}$$

$$\theta = \frac{T - T_0}{T_{b,m} - T_0}\,,\ \theta_\infty = \frac{T_\infty - T_0}{T_{b,m} - T_0}\,. \tag{4}$$

Eq. (2) and the relevant boundary conditions are expressed in terms of the above dimensionless variables as:

$$\beta\frac{\partial^2\theta}{\partial\xi^2} + (1+\beta H)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} + H(\theta - \theta_\infty) \tag{5}$$

$$\theta(0,\xi) = 1 + A\cos(\Omega\xi) \tag{6}$$

$$\theta_\eta(1,\xi) = 0 \tag{7}$$

$$\theta(\eta,0) = \theta_\xi(\eta,0) = 0. \tag{8}$$

3. The analytical procedure of temperature periodic boundary condition

The Laplace transform method is used for solving hyperbolic heat transfer in the fin that is subjected to thermal periodic boundary conditions. The main problem of this method is the inverse Laplace transform. In this study, we use the inverse theorem by applying the residue theorem in the complex plan. After taking the Laplace transform of Eq. (5), the following ordinary differential equation is obtained.

$$\frac{d^2\Theta}{d\eta^2} - \left[\beta s^2 + (1+\beta H)s + H\right]\Theta + \frac{H\theta_\infty}{s} = 0 \tag{9}$$

where $\Theta(\eta, s)$ is Laplace transform of $\theta(\eta, \xi)$. By solving Eq. (9) and applying the boundary conditions (6) and (7), we would have

$$\Theta(\eta,s) = \frac{H\theta_\infty}{sm^2} + \left[\frac{1}{s} + \frac{Bs}{(s^2+\Omega^2)} - \frac{H\theta_\infty}{sm^2}\right]\frac{Cosh[m(\eta-1)]}{Cosh(m)}. \tag{10}$$

Eq. (10) is solved using the inverse image functions by calculating residues. Function of $\theta(x, t)$ is the inverse Laplace transform of $\Theta(x, s)$ that is obtained from the complex integral:

$$\theta(x,t) = \frac{1}{2\pi i}\lim_{L\to\infty}\int_{\gamma+iL}^{\gamma+iL} e^{ts}\Theta(x,s)\,ds \tag{11}$$

which is known as the inverse theorem of Laplace transform method [30]. This integral is taken along the infinite line L (line $x=\gamma$) and half circle C_R, where all singular points S_j ($j = 1, 2, \cdots, N$) in circle C_R of radius R enclose the whole integral poles. If $\Theta(x, s)$ is analytic, except for a number of N poles that are all to the left of some line $x=\gamma$, we complete the contour of Eq. (10) by a big contour $L+C_R$ and by enclosing the whole integral poles. If $\Theta(x, s)$ is analytic (except for a number of poles that are all to the left of some line $x=\gamma$) and if it has a branch point at $z=S_j$, then we complete the contour of the inversion integral, including a loop along the cut and around the branch point by introducing a cut along the left side of line $x=\gamma$ (for more details, see Ref. [30]). After applying this theorem to Eq. (12), an accurate temperature distribution in the fin is calculated by the real part of Eq. (12).

$$\begin{aligned}\theta(\eta,\xi) = real\Bigg\{ & m + \theta_\infty\left[(1 - e^{-\xi/\beta}) - \frac{(e^{-H\xi} - e^{-\xi/\beta})}{1-\beta H}\right] \\ & \times(1-m) + B\frac{Cosh\left[(\eta-1)\sqrt{(i\beta\Omega+1)(i\Omega+H)}\right]}{Cosh\left[\sqrt{(i\beta\Omega+1)(i\Omega+H)}\right]}e^{i\Omega\xi} \\ & -\sum_{n=0}^{\infty} a_n\frac{H\theta_\infty}{s_n(s_n+H)}\left[\frac{(e^{s_n\xi} - e^{-\xi/\beta})}{\beta s_n + 1} - \frac{(e^{-H\xi} - e^{-\xi/\beta})}{1-\beta H}\right] \\ & + \sum_{n=0}^{\infty} a_n\left[\left(\frac{1}{s_n} + \frac{Bs_n}{(s_n^2+\Omega^2)}\right)e^{s_n\xi}\right]\Bigg\}\end{aligned} \tag{12}$$

where

$$s_n = \frac{1}{2}\left[-(\frac{1}{\beta}+H) \pm \sqrt{(\frac{1}{\beta}+H)^2 - 4(H + \frac{\lambda_n^2}{\beta})}\right] \tag{13}$$

$$\lambda_n = (2n+1)\frac{\pi}{2} \tag{14}$$

$$m = Cosh\left[(\eta-1)\sqrt{H}\right] / Cosh\left(\sqrt{H}\right) \tag{15}$$

$$a_n = \frac{2Cos\left[(\eta-1)\lambda_n\right]\lambda_n}{\beta(-1)^n(2s_n + \frac{1}{\beta} + H)}\,. \tag{16}$$

4. The arbitrary periodic temperature boundary condition

If the boundary condition at the fin base is periodic with an arbitrary function, we can write it in the Fourier's series form. For example, if the boundary condition is in the step function form shown in Fig. 2, then the Fourier's series is:

$$\theta_b(\xi) = A_0 + \sum_{n=1}^{\infty} A_n\cos\left(\frac{n\pi\xi}{p}\right) + \sum_{n=1}^{\infty} B_n\sin\left(\frac{n\pi\xi}{p}\right) \tag{17}$$

where p is the period of boundary condition and $\Omega=n\pi/p$. We can use the superposition theorem because the governing equation and boundary conditions are linear. Therefore, this problem is divided into three sub-problems with the following boundary conditions:

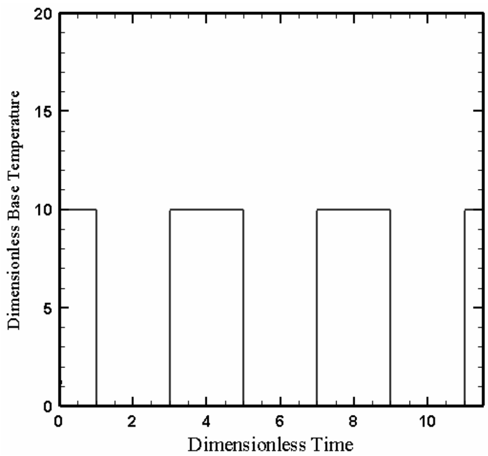

Fig. 2. Step function boundary condition.

$$\begin{aligned}\theta_{b1}(0,\xi) &= A_0\\ \theta_{b2}(0,\xi) &= \cos(\ )\\ \theta_{b3}(0,\xi) &= \sin(n\pi\xi/p).\end{aligned} \tag{18}$$

One of the boundary conditions above (e.g., constant boundary condition) is solved with θ_∞, whereas the other boundary conditions are solved without θ_∞ (or $\theta_\infty=0$) because Eq. (5) is nonhomogenous due to term θ_∞.

When the boundary condition at the fin base has a constant value θ_{b1}, (including θ_∞), the solution can be obtained by applying the Laplace transform and the inverse theorem in complex variables. The solution of this case is expressed as follows:

$$\begin{aligned}\theta_1(\eta,\xi) &= -A_0\cosh\left[(\eta-1)\sqrt{H}\right]/\cosh\left(\sqrt{H}\right)\\ &+\theta_\infty\left[(1-e^{-\frac{\xi}{\beta}})-\frac{(e^{-H\xi}-e^{-\frac{\xi}{\beta}})}{1-\beta H}\right]\times\left[1-\frac{\cosh\left[(\eta-1)\sqrt{H}\right]}{\cosh\left(\sqrt{H}\right)}\right]\\ &+real\left[\sum_{n=0}^{\infty}\frac{2\cos\left[(\eta-1)\lambda_n\right]\lambda_n}{\beta(-1)^n(2s_n+\frac{1}{\beta}+H)}\left(\frac{-A_0}{s_n}e^{s_n\xi}\right)\right.\\ &-\sum_{n=0}^{\infty}\frac{2\cos\left[(\eta-1)\lambda_n\right]\lambda_n}{\beta(-1)^n(2s_n+\frac{1}{\beta}+H)}\left(\frac{H\theta_\infty}{s_n(s_n+H)}\right)\\ &\left.\times\left(\frac{(e^{s_n\xi}-e^{-\frac{\xi}{\beta}})}{\beta s_n+1}-\frac{(e^{-H\xi}-e^{-\frac{\xi}{\beta}})}{1-\beta H}\right)\right]\end{aligned} \tag{19}$$

where s_n, λ_n, and m are defined in Eqs. (13)-(15), respectively. Solving the governing equation with cosine boundary condition θ_{b2} without θ_∞ is similar to solving the equations mentioned in section 3, which is the real part of Eq. (20).

$$\begin{aligned}\theta_2(\eta,\xi) &= real\left[\frac{\cosh\left[(\eta-1)\sqrt{(i\beta\Omega+1)(i\Omega+H)}\right]}{\cosh\left[\sqrt{(i\beta\Omega+1)(i\Omega+H)}\right]}e^{i\Omega\xi}\right.\\ &\left.+\sum_{n=0}^{\infty}\frac{2\cos\left[(\eta-1)\lambda_n\right]\lambda_n}{\beta(-1)^n(2s_n+\frac{1}{\beta}+H)}\left[\frac{s_n}{(s_n{}^2+\Omega^2)}e^{s_n\xi}\right]\right]\end{aligned} \tag{20}$$

If the boundary condition at the fin base is sinusoidal θ_{b3} with $\theta_\infty=0$, the solution named $\theta_3(\eta,\xi)$ will be a real part of Eq. (21).

$$\begin{aligned}\theta_3(\eta,\xi) &= real\left[-\frac{i}{2}\frac{\cosh\left[(\eta-1)\sqrt{(i\beta\Omega+1)(i\Omega+h)}\right]\exp(-i\Omega\xi)}{\cosh\left[\sqrt{(i\beta\Omega+1)(i\Omega+h)}\right]}\right.\\ &+\frac{1}{2}\frac{\cosh\left[(\eta-1)\sqrt{(-i\beta\Omega+1)(-i\Omega+H)}\right]}{\cosh\left[\sqrt{(-i\beta\Omega+1)(-i\Omega+H)}\right]}\exp(i\Omega\xi)\\ &\left.+\sum_{n=0}^{\infty}\frac{2\cos\left[(\eta-1)\lambda_n\right]\lambda_n}{\beta(-1)^n(2s_n+\frac{1}{\beta}+K)}\left[\frac{\Omega}{(s_n{}^2+\Omega^2)}e^{s_n\xi}\right]\right]\end{aligned} \tag{21}$$

Therefore, the solution of the hyperbolic heat transfer in the fin with arbitrary periodic boundary condition is:

$$\theta(\eta,\xi)=\theta_1(\eta,\xi)+\sum_{n=1}^{\infty}A_n\theta_2(\eta,\xi)+\sum_{n=1}^{\infty}B_n\theta_3(\eta,\xi). \tag{22}$$

5. The heating flux periodic boundary condition

In this section, we assume that the heat flux boundary condition at the fin base is:

$$\theta(\eta,\xi)=\theta_1(\eta,\xi)+\sum_{n=1}^{\infty}A_n\theta_2(\eta,\xi)+\sum_{n=1}^{\infty}B_n\theta_3(\eta,\xi). \tag{23}$$

This boundary condition can be a part of the arbitrary periodic boundary condition. θ_e=0 is also assumed. Once the Laplace transform method in Eq. (2) under the boundary condition (23) is applied, then the temperature will be:

$$\begin{aligned}\theta(\eta,\xi) &= real\left[\left(\frac{A}{2}\frac{\beta\Omega i+1}{m|_{s=i\Omega}}\right)\frac{\cosh\left[m|_{s=i\Omega}(\eta-1)\right]}{\sinh\left[m|_{s=i\Omega}\right]}e^{i\Omega\xi}\right.\\ &+\left(\frac{A}{2}\frac{1-\beta\Omega i}{m|_{s=-i\Omega}}\right)\frac{\cosh\left[m|_{s=-i\Omega}(\eta-1)\right]}{\sinh\left[m|_{s=-i\Omega}\right]}e^{-i\Omega\xi}\\ &+\frac{A}{1+(\beta\Omega)^2}\frac{\beta+\beta^3\Omega^2}{1-\beta H}e^{-\frac{\xi}{\beta}}+\frac{A}{H^2+\Omega^2}\frac{H+\beta\Omega^2}{\beta H-1}e^{-H\Omega}\\ &\left.+\sum_{n=1}^{\infty}\frac{2A\left(s_n-\beta\Omega^2\right)}{s_n{}^2+\Omega^2}\frac{\cos\left[\lambda_n(\eta-1)\right]}{2\beta s_n+\beta H+1}(-1)^n e^{s_n\xi}\right]\end{aligned} \tag{24}$$

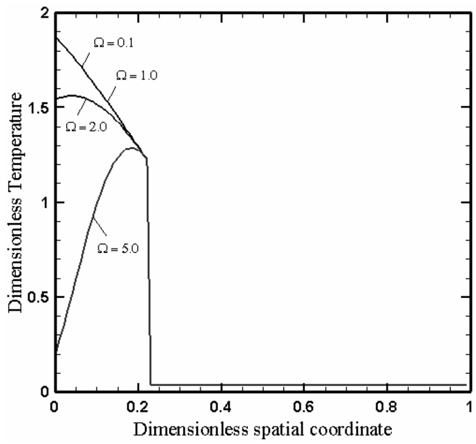

Fig. 3. The temperature distribution on the fin subjected to the boundary condition Eq. (6).

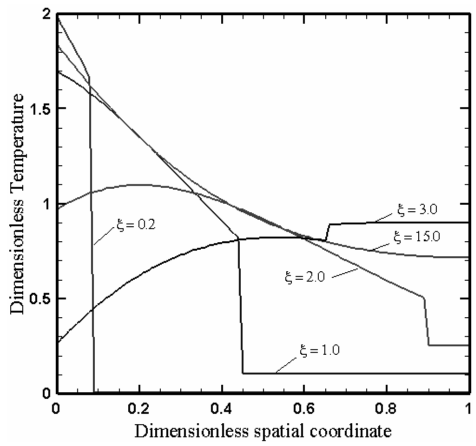

Fig. 4. Temperature distribution on the fin at β=5.

where $m = \sqrt{H}\sinh\left[(\eta-1)\sqrt{H}\right]/\cosh\left[\sqrt{H}\right]$.

6. Discussion of results

6.1 The periodic boundary condition temperature

The fin temperature subjected to the boundary condition $\theta(0,\xi)=1+A\cos(\Omega\xi)$ with dimensionless parameters A=1, H=1.9, and θ_∞=1 is calculated by Eq. (13). The temperature distribution at dimensionless time ξ=0.5 and dimensionless relaxation time β=0.5 for various frequencies of temperature oscillations is shown in Fig. 3. The thermal shock is caused in the temperature because the governing equation is hyperbolic. Fig. 3 shows that the base temperature frequency does not have any influence on the location and span of thermal shock. The temperature before thermal shock influences the base temperature frequency. The temperature after thermal shock is not influenced due to the base temperature after the shock and the finite heat propagation speed. No parameter exerts an influence on the temperature after the shock, except a temperature increase due to convective heat transfer with the environment.

In Fig. 4, the temperature distribution on the fin under the unsteady boundary condition Eq. (6) is shown at various dimensionless times, dimensionless relaxation time β=5, and under conditions A=1, H=1.9, θ_∞=1, and Ω=0.8.

At dimensionless time ξ=0.2, the thermal wave is close to the base of the fin due to its finite velocity. At time ξ=1, the thermal wave moves to the tip of the fin, and the fin temperature is increased after the thermal wave because it has a convective heat transfer with the environment. With an increase in time (at time ξ=3), the thermal shock reaches the tip of the fin where it is reflected. Here, the fin temperature is increased after the thermal shock due to the convective heat transfer.

The thermal shock is reflected back after it reaches the tip of the fin. Some other thermal waves are generated because the base temperature changes rapidly. On one hand, the thermal wave keeps on heating because the fin tip is cooler than that of the environment. Consequently, the fin tip gets warmer than its base and brings about a heat flux toward the fin base. Another heat wave is generated that moves to the fin base because the thermal wave speed is finite (due to relaxation time). The thermal waves then move back and forth until they are damped. At time ξ=15 (a long time), the thermal wave is completely uniform, and that we will not see any thermal waves in the fin.

The dimensionless location of the thermal shock η_s can be obtained by:

$$\eta_s = \xi/\sqrt{\beta}. \tag{25}$$

If η_s is greater than the unit (fin length), we should consider the reflected wave from the fin tip; thus, thermal wave location can be obtained by:

$$\eta_{a,s} = \eta_s - \left[\eta_s\right] \tag{26}$$

where $[\eta_s]$ is the bracket of η_s, and $\eta_{a,s}$ is the actual location of the thermal shock. If $[\eta_s]$ is an even number, then it is evaluated from the fin base; if $[\eta_s]$ is an odd number, then it is evaluated from the fin tip.

The temperature distribution corresponding to the analytical solution Eq. (22) at various non-dimensional relaxation times and dimensionless time ξ=0.5 is presented in Fig. 5. With a decrease in relaxation time, the shock wave location in the fin moves to the right side due to an increase in shock wave velocity and a decrease in relaxation time. At the relaxation time β=0.001, the dimensionless temperature tends to get closer to the temperature obtained from the Fourier's law model; hence, there is no thermal shock in this case.

The temperature distribution under boundary condition Eq. (6), dimensionless time ξ=100 (a long time), and various relaxation times is shown in Fig. 6. The relaxation time has a great influence on temperature distribution. This fact shows that even for long periods, the variation of relaxation time brings about great effects on the temperature distribution in the fin. Thus, applying Fourier heat equation for slightly high relaxation time can lead to significant error.

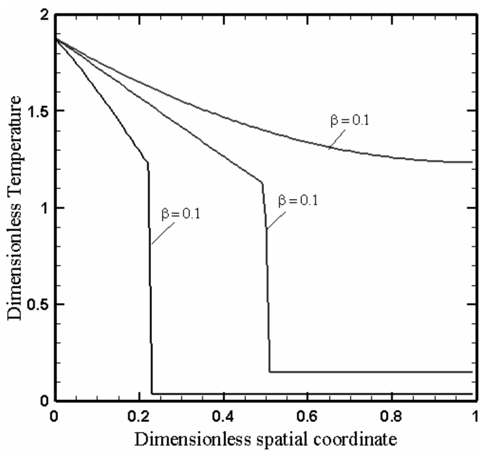

Fig. 5. Temperature distribution on the fin at various relaxation times and ξ=0.5, Ω=1.0, A=1, H=1.9, and θ_∞=1.0.

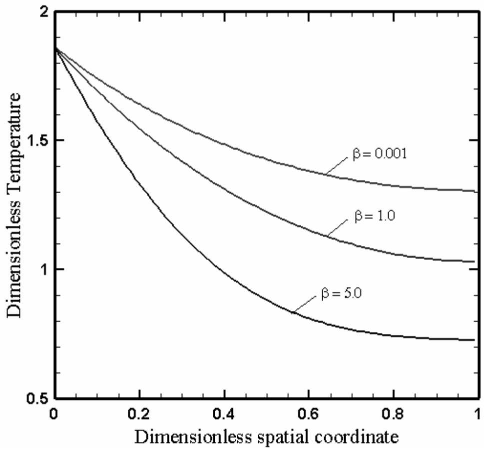

Fig. 6. Temperature distribution on the fin at various relaxation times, and ξ=100, Ω=1.0, A=1, H=1.9, and θ_∞=1.0.

6.2 Studying the accuracy of the hyperbolic heat equation

A problem related to the hyperbolic heat equations is the creation of thermal shocks. Based on the assumption that the speed of thermal wave in the Fourier model is infinite, the thermal shock will not be generated. According to Fig. 5, heat flux is not infinite in this location with regard to the infinite temperature gradient. By checking Eq. (13), this infinite temperature gradient is due to the time derivative of the heat flux in the fin. Thus, creating infinite temperature gradient cannot be a good reason for proving the invalidity of the hyperbolic heat equation model.

To study the accuracy of the thermodynamic laws, consider the following example: the thermal distribution for heat flux periodic boundary condition (23), H=0 (no dissipate heat transfer to the ambient), Ω=0, and ξ=1.75 are shown in Fig. 7. For the periodic boundary condition q=cos($\Omega\xi$) at ξ=0, which is the first quarter of the unit circle, heat is continuously injected into the fin base with a positive value while the temperatures decrease at the initial times as shown in Fig. 7. Thus, for the time interval where cos($\Omega\xi$) has a positive value, both

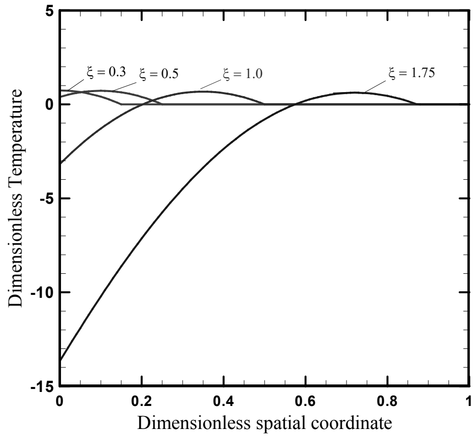

Fig. 7. Dimensionless temperature distribution on the fin in the heat flux boundary condition at ξ=1.75, β=16, Ω=1.0, A=1, and H=0.

the heat flux and temperature gradient have positive values. Therefore, heat flows from a higher temperature to a lower temperature, which is a violation of the second law of thermodynamics. By increasing the relaxation time and frequency of the periodic boundary condition, the violation of the second law of thermodynamics is increased. We can now express a periodic function for the boundary condition, which continues piecewise in terms of both sines and cosines. Therefore, we can find an interval that hyperbolic equations violate the second law of thermodynamics.

In Fig. 7, we observe that the temperature of fin has a negative value (a below ambient temperature). This shows that for a dimensionless time from 0.5 up to 1.75, both the heat flux and temperature gradient at the fin base are positive, which can violate the second law of thermodynamics. According to this viewpoint, temperature decreases while heat is exposed to the fin. Therefore, we conclude that hyperbolic heat equation violates the second thermodynamic law. However, this phenomenon occurs during a very short interval. Moreover, thermodynamic laws attempt to describe equilibrium, whereas non-Fourier conduction seeks to present a correct description of the transient behavior.

7. Conclusion

For the most practical purpose, the effects of non-Fourier conduction are negligible. As the size of the microelectronic devices decreases to tiny portions and the circuit speed increases, Fourier's law cannot be used in heat transfer and temperature prediction. The wave character gives rise to the effects, which do not occur under classical Fourier conduction. In the present study, the non-Fourier hyperbolic heat conduction was solved in the straight small fin that is subjected to thermal and heat flux periodic boundary conditions using analytical solutions. The non-Fourier thermal wave behavior in the small fin for fast phenomenon (high frequency periodic boundary condition) is successfully explained by the results obtained from the hyperbolic heat conduction model. The effects of various parameters on the shock wave show that only the relaxation time has an influence on the location and movement of the shock waves. The frequency and amplitude of the periodic boundary condition and diffusivity coefficient of the fin have a high influence on the strength of the thermal shock waves.

The parabolic (classical diffusion) and hyperbolic equations fail to capture the microscale responses during an unsteady boundary condition. From a physical viewpoint, both models violate the second thermodynamic law in the short-time transient boundary condition. The hyperbolic model is rendering an underestimated temperature in the unsteady heat flux boundary condition. Results show an inductive behavior, discontinuities in the thermal step response, and negative (sub ambient) temperatures during the heating process.

Nomenclature

A : Amplitude of the input temperature
b : Thickness of the fin
C : Specific heat capacity
H : Dimensionless convective heat transfer
h : Convective heat transfer coefficient
K : Thermal conductivity
L : Length of the fin
q : Heat flux
T : Temperature
T_0 : Initial temperature of the fin
T_∞ : Ambient temperature
T_b : Periodic boundary condition
$T_{b,m}$: Mean base temperature
t : Time
w : Width of the fin
x : Spatial coordinate

Greek symbols

α : Diffusivity coefficient
β : Dimensionless relaxation time
η : Dimensionless spatial coordinate
η_s , $\eta_{a,s}$: Dimensionless location of the thermal shock
λ : Means free path between phonons
ν : Sound speed of in the medium
ρ : Density
τ : Relaxation time
ω : Frequency of the temperature oscillation
Ω : Dimensionless frequency of the oscillation
ξ : Dimensionless time
θ : Dimensionless temperature
θ_∞ : Dimensionless ambient temperature

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