Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 273052, 14 pages
http://dx.doi.org/10.1155/2013/273052

Research Article

Analytical Solutions for Steady Heat Transfer in Longitudinal Fins with Temperature-Dependent Properties

Partner L. Ndlovu and Raseelo J. Moitsheki

Center for Differential Equations, Continuum Mechanics and Applications, School of Computational and Applied Mathematics, University of the Witwatersrand, Private Bag 3, Johannesburg 2050, South Africa

Correspondence should be addressed to Raseelo J. Moitsheki; raseelo.moitsheki@wits.ac.za

Received 30 January 2013; Accepted 24 April 2013

Academic Editor: Ireneusz Zbicinski

Explicit analytical expressions for the temperature profile, fin efficiency, and heat flux in a longitudinal fin are derived. Here, thermal conductivity and heat transfer coefficient depend on the temperature. The differential transform method (DTM) is employed to construct the analytical (series) solutions. Thermal conductivity is considered to be given by the power law in one case and by the linear function of temperature in the other, whereas heat transfer coefficient is only given by the power law. The analytical solutions constructed by the DTM agree very well with the exact solutions even when both the thermal conductivity and the heat transfer coefficient are given by the power law. The analytical solutions are obtained for the problems which cannot be solved exactly. The effects of some physical parameters such as the thermogeometric fin parameter and thermal conductivity gradient on temperature distribution are illustrated and explained.

1. Introduction

Fins are surfaces that extend from a hot object (body) to increase the rate of heat transfer to the surrounding fluid. In particular, fins are used extensively in various industrial applications such as the cooling of computer processors, air conditioning, and oil carrying pipe lines. A well-documented review of heat transfer in extended surfaces is presented by Kraus et al. [1]. The problems on heat transfer particularly in fins continue to be of scientific interest. These problems are modeled by highly nonlinear differential equations which are difficult to solve exactly. However, Moitsheki et al. [2–4] have attempted to construct exact solutions for the steady state problems arising in heat flow through fins. A number of techniques, for example, Lie symmetry analysis [2], He's variational iteration method [5], Adomain decomposition methods [6], homotopy perturbation methods [7], homotopy analysis methods [8], methods of successive approximations [9], and other approximation methods [10] have been used to determine solutions of the nonlinear differential equations describing heat transfer in fins.

Recently, the solutions of the nonlinear ordinary differential equations (ODEs) arising in extended surface heat transfer have been constructed using the DTM [11–19]. The DTM is an analytical method based on the Taylor series expansion and was first introduced by Zhou [20] in 1986. The DTM approximates the exact solution by a polynomial, and previous studies have shown that it is an efficient means of solving nonlinear problems or systems with varying parameters [21]. Furthermore, DTM is a computational inexpensive tool for obtaining analytical solution, and it generalizes the Taylor method to problems involving procedures such as fractional derivative (see e.g., [22–24]). Also, this method converges rapidly (see e.g., [25]).

Models arising in heat transfer through fins may contain temperature-dependent properties such as thermal conductivity and heat transfer coefficient. The dependency of thermal conductivity and heat transfer coefficient on temperature renders such problems highly nonlinear and difficult to solve, particularly exactly. Thermal conductivity may be modeled for many engineering applications by the power law and by linear dependency on temperature. On the other hand, heat transfer coefficient can be expressed as a power law for which values of the exponent represent different phenomena (see e.g., [26]).

In this paper, the DTM is employed to determine the analytical solutions to the nonlinear boundary value problem describing heat transfer in longitudinal fins of rectangular, exponential, and convex parabolic profiles. Both thermal conductivity and heat transfer coefficient are temperature dependent. We adopt the terminology exact solutions to refer to solutions given in terms of fundamental expressions such as logarithmic, trigonometric, and exponential. However, analytical solutions will be series solutions and in particular, those constructed using the DTM. The mathematical modelling of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the DTM is be provided in Section 3. The comparison of the exact and analytical solutions constructed by DTM is given in Section 4. In Section 5, we provide analytical solutions for the heat transfer in longitudinal fins of various profiles. Here, heat transfer coefficient is given by the power law, and we consider two cases of the thermal conductivity, namely, the power law and the linear function of temperature. Furthermore, we describe the fin efficiency and the heat flux in Section 6. Some exciting results are discussed in Section 7. Lastly, the concluding remarks are provided in Section 8.

2. Mathematical Models

We consider a longitudinal one dimensional fin of cross-sectional area A_c. The perimeter of the fin is denoted by P and its length by L. The fin is attached to a fixed prime surface of temperature T_b and extends to an ambient fluid of temperature T_a. The fin thickness at the prime surface is given by δ_b and its profile is given by $F(X)$. Based on the one dimensional heat conduction, the energy balance equation is then given by (see e.g., [1])

$$A_c \frac{d}{dX}\left(\frac{\delta_b}{2} F(X) K(T) \frac{dT}{dX}\right) = PH(T)(T - T_a) \qquad 0 \leq X \leq L, \tag{1}$$

where K and H are nonuniform temperature-dependent thermal conductivity and heat transfer coefficients, respectively, T is the temperature distribution, $F(X)$ is the fin profile, and X is the space variable. The length of the fin is measured from the tip to the prime surface as shown in Figure 1. Assuming that the fin tip is adiabatic (insulated) and the base temperature is kept constant, then the boundary conditions are given by

$$T(L) = T_b, \qquad \left.\frac{dT}{dX}\right|_{X=0} = 0. \tag{2}$$

Introducing the following dimensionless variables (see e.g., [1]):

$$x = \frac{X}{L}, \qquad \theta = \frac{T - T_a}{T_b - T_a}, \qquad h = \frac{H}{h_b}, \qquad k = \frac{K}{k_a}, \qquad M^2 = \frac{Ph_b L^2}{A_c k_a}, \qquad f(x) = \frac{\delta_b}{2} F(X), \tag{3}$$

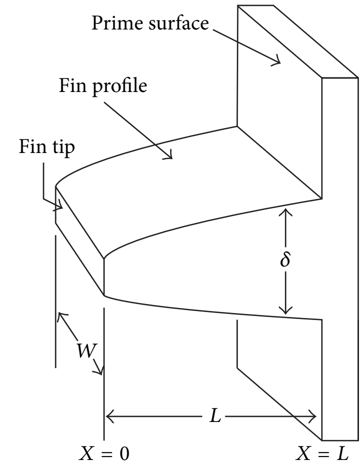


Figure 1: Schematic representation of a longitudinal fin of an unspecified profile.

with k_a being defined as the thermal conductivity at the ambient temperature and h_b as the heat transfer at the prime surface (fin base), reduces (1) to

$$\frac{d}{dx}\left[f(x) k(\theta) \frac{d\theta}{dx}\right] - M^2 \theta h(\theta) = 0, \quad 0 \leq x \leq 1. \tag{4}$$

Here θ is the dimensionless temperature, x is the dimensionless space variable, $f(x)$ is the dimensionless fin profile, k is the dimensionless thermal conductivity, h is the dimensionless heat transfer coefficient, and M is the thermogeometric fin parameter. The dimensionless boundary conditions then become

$$\theta(1) = 1, \quad \text{at the prime surface} \tag{5}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = 0, \quad \text{at the fin tip.} \tag{6}$$

We assume that the heat transfer coefficient is given by the power law

$$H(T) = h_b \left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{7}$$

where the exponent n is a real constant. In fact the values of n may vary as between −6.6 and 5. However, in most practical applications, it lies between −3 and 3 [27]. In dimensionless variables, we have $h(\theta) = \theta^n$. We consider the two distinct cases of the thermal conductivity as follows:

(a) the power law

$$K(T) = k_a \left(\frac{T - T_a}{T_b - T_a}\right)^m, \tag{8}$$

with m being a constant and

(b) the linear function

$$K(T) = k_a \left[1 + \gamma (T - T_a)\right]. \tag{9}$$

The dimensionless thermal conductivity given by the power law and the linear function of temperature is $k(\theta) = \theta^m$ and $k(\theta) = 1 + \beta\theta$, respectively. Here the thermal conductivity gradient is $\beta = \gamma(T_b - T_a)$. Furthermore, we consider a various fin profiles including the longitudinal rectangular $f(x) = 1$, the longitudinal convex parabolic $f(x) = \sqrt{x}$, and the exponential profile $f(x) = e^{ax}$ with a being the constant (see also [16]).

3. Fundamentals of the Differential Transform Method

In this section, the basic idea underlying the DTM is briefly introduced. Let $y(t)$ be an analytic function in a domain $\mathscr{D}$. The Taylor series expansion function of $y(t)$ with the center located at $t = t_j$ is given by [20]

$$y(t) = \sum_{\kappa=0}^{\infty} \frac{(t - t_j)^{\kappa}}{\kappa!} \left[\frac{d^{\kappa} y(t)}{dt^{\kappa}}\right]_{t=t_j}, \quad \forall t \in \mathscr{D}. \tag{10}$$

The particular case of (10) when $t_j = 0$ is referred to as the Maclaurin series expansion of $y(t)$ and is expressed as

$$y(t) = \sum_{\kappa=0}^{\infty} \frac{t^{\kappa}}{\kappa!} \left[\frac{d^{\kappa} y(t)}{dt^{\kappa}}\right]_{t=0}, \quad \forall t \in \mathscr{D}. \tag{11}$$

The differential transform of $y(t)$ is defined as follows:

$$Y(t) = \sum_{\kappa=0}^{\infty} \frac{\mathscr{H}^{\kappa}}{\kappa!} \left[\frac{d^{\kappa} y(t)}{dt^{\kappa}}\right]_{t=0}, \tag{12}$$

where $y(t)$ is the original analytic function and $Y(t)$ is the transformed function. The differential spectrum of $Y(t)$ is confined within the interval $t \in [0, \mathscr{H}]$, where $\mathscr{H}$ is a constant. From (11) and (12), the differential inverse transform of $Y(t)$ is defined as follows:

$$y(t) = \sum_{\kappa=0}^{\infty} \left(\frac{t}{\mathscr{H}}\right)^{\kappa} Y(\kappa), \tag{13}$$

and if $y(t)$ is expressed by a finite series, then

$$y(t) = \sum_{\kappa=0}^{r} \left(\frac{t}{\mathscr{H}}\right)^{\kappa} Y(\kappa). \tag{14}$$

Some of the useful mathematical operations performed by the differential transform method are listed in Table 1.

The delta function $\delta(\kappa - s)$ is given by

$$\delta(\kappa - s) = \begin{cases} 1 & \text{if } \kappa = s, \\ 0 & \text{if } \kappa \neq s. \end{cases} \tag{15}$$

Table 1: Fundamental operations of the differential transform method.

Original function	Transformed function
$y(t) = x(t) \pm z(t)$	$Y(t) = X(t) \pm Z(t)$
$y(t) = \alpha x(t)$	$Y(t) = \alpha X(t)$
$y(t) = \dfrac{dy(t)}{dt}$	$Y(t) = (\kappa + 1)Y(\kappa + 1)$
$y(t) = \dfrac{d^2 y(t)}{dt^2}$	$Y(t) = (\kappa + 1)(\kappa + 2)Y(\kappa + 2)$
$y(t) = \dfrac{d^s y(t)}{dt^s}$	$Y(t) = (\kappa + 1)(\kappa + 2)\cdots(\kappa + s)Y(\kappa + s)$
$y(t) = x(t)z(t)$	$Y(t) = \sum_{i=0}^{\kappa} X(i)Z(\kappa - i)$
$y(t) = 1$	$Y(t) = \delta(\kappa)$
$y(t) = t$	$Y(t) = \delta(\kappa - 1)$
$y(t) = t^s$	$Y(t) = \delta(\kappa - s)$
$y(t) = \exp(\lambda t)$	$Y(t) = \dfrac{\lambda^{\kappa}}{\kappa!}$
$y(t) = (1 + t)^s$	$Y(t) = \dfrac{s(s-1)\cdots(s-\kappa+1)}{\kappa!}$
$y(t) = \sin(\omega t + \alpha)$	$Y(t) = \dfrac{\omega^{\kappa}}{\kappa!} \sin\left(\dfrac{\pi\kappa}{2!} + \alpha\right)$
$y(t) = \cos(\omega t + \alpha)$	$Y(t) = \dfrac{\omega^{\kappa}}{\kappa!} \cos\left(\dfrac{\pi\kappa}{2!} + \alpha\right)$

4. Comparison of Exact and Analytical Solutions

In this section, we consider a model describing temperature distribution in a longitudinal rectangular fin with both thermal conductivity and heat transfer coefficient being functions of temperature given by the power law (see e.g., [2]). The exact solution of (4) when both the power laws are given by the same exponent is given by [2]

$$\theta(x) = \left[\frac{\cosh\left(M\sqrt{n+1}x\right)}{\cosh\left(M\sqrt{n+1}\right)}\right]^{1/(n+1)}. \tag{16}$$

We use this exact solution as a benchmark or validation of the DTM. The effectiveness of the DTM is determined by comparing the exact and the analytical solutions. We compare the results for the cases $n = 1$ and $n = 2$ with fixed values of M.

4.1. Case n=1. Applying the DTM to (4) with the power law thermal conductivity, $f(x) = 1$ (rectangular profile) and given $\mathscr{H} = 1$, one obtains the following recurrence relation:

$$\begin{aligned} \sum_{i=0}^{\kappa} \Big[&\Theta(i)(\kappa - i + 1)(\kappa - i + 2)\Theta(\kappa - i + 2) \\ &+ (i + 1)\Theta(i + 1)(\kappa - i + 1)\Theta(\kappa - i + 1) \\ &- M^2\Theta(i)\Theta(\kappa - i)\Big] = 0. \end{aligned} \tag{17}$$

Exerting the transformation to the boundary condition (6) at a point $x = 0$,

$$\Theta(1) = 0. \tag{18}$$

The other boundary conditions are considered as follows:

$$\Theta(0) = c, \tag{19}$$

where c is a constant. Equation (17) is an iterative formula of constructing the power series solution as follows:

$$\Theta(2) = \frac{M^2 c}{2} \quad (20)$$

$$\Theta(3) = 0 \quad (21)$$

$$\Theta(4) = -\frac{M^4 c}{24} \quad (22)$$

$$\Theta(5) = 0 \quad (23)$$

$$\Theta(6) = \frac{19M^6 c}{720} \quad (24)$$

$$\Theta(7) = 0 \quad (25)$$

$$\Theta(8) = -\frac{559M^8 c}{40320} \quad (26)$$

$$\Theta(9) = 0 \quad (27)$$

$$\Theta(10) = \frac{29161M^{10} c}{3628800} \quad (28)$$

$$\Theta(11) = 0 \quad (29)$$

$$\Theta(12) = -\frac{2368081M^{12} c}{479001600} \quad (30)$$

$$\Theta(13) = 0 \quad (31)$$

$$\Theta(14) = \frac{276580459M^{14} c}{87178291200} \quad (32)$$

$$\Theta(15) = 0$$

$$\vdots \quad (33)$$

These terms may be taken as far as desired. Substituting (18) to (32) into (13), we obtain the following analytical solution:

$$\theta(x) = c + \frac{M^2 c}{2}x^2 - \frac{M^4 c}{24}x^4 + \frac{19M^6 c}{720}x^6 - \frac{559M^8 c}{40320}x^8 + \frac{29161M^{10} c}{3628800}x^{10} - \frac{2368081M^{12} c}{479001600}x^{12} + \frac{276580459M^{14} c}{87178291200}x^{14} + \cdots \quad (34)$$

To obtain the value of c, we substitute the boundary condition (5) into (34) at the point $x = 1$. Thus, we have

$$\theta(1) = c + \frac{M^2 c}{2} - \frac{M^4 c}{24} + \frac{19M^6 c}{720} - \frac{559M^8 c}{40320} + \frac{29161M^{10} c}{3628800} - \frac{2368081M^{12} c}{479001600} + \frac{276580459M^{14} c}{87178291200} + \cdots = 1. \quad (35)$$

Table 2: Results of the DTM and exact solutions for $n = 1$, $M = 0.7$.

x	DTM	Exact	Error
0	0.808093014	0.80809644	0.000003426
0.1	0.810072036	0.81007547	0.000003434
0.2	0.815999549	0.81600301	0.000003459
0.3	0.825847770	0.82585127	0.000003500
0.4	0.839573173	0.83957673	0.000003559
0.5	0.857120287	0.85712392	0.000003633
0.6	0.878426299	0.87843002	0.000003722
0.7	0.903426003	0.90342982	0.000003814
0.8	0.932056648	0.93206047	0.000003820
0.9	0.964262558	0.96426582	0.000003263
1.0	1.000000000	1.00000000	0.000000000

Table 3: Results of the DTM and Exact Solutions for $n = 2$, $M = 0.5$.

x	DTM	Exact	Error
0	0.894109126	0.894109793	0.000000665
0.1	0.895226066	0.895226732	0.000000666
0.2	0.898568581	0.898569249	0.000000668
0.3	0.904112232	0.904112905	0.000000672
0.4	0.911817796	0.911818474	0.000000678
0.5	0.921633352	0.921634038	0.000000685
0.6	0.933496900	0.933497594	0.000000694
0.7	0.947339244	0.947339946	0.000000702
0.8	0.963086933	0.963087627	0.000000694
0.9	0.980665073	0.980665659	0.000000586
1.0	1.000000000	1.000000000	0.000000000

We omit presenting the tedious process of finding c. However, one may obtain the expression for $\theta(x)$ upon substituting the obtained value of c into (34).

4.2. Case n=2. Following a similar approach given in Section 4.1 and given $n = 2$, one obtains the analytical solution

$$\theta(x) = c + \frac{M^2 c}{2}x^2 - \frac{M^4 c}{8}x^4 + \frac{23M^6 c}{240}x^6 - \frac{1069M^8 c}{13440}x^8 + \frac{9643M^{10} c}{134400}x^{10} - \frac{1211729M^{12} c}{17740800}x^{12} + \frac{217994167M^{14} c}{3228825600}x^{14} + \cdots. \quad (36)$$

The constant c may be obtained using the boundary condition at the fin base. The comparison of the DTM and the exact solutions are reflected in Tables 2 and 3 for different values of n. Furthermore, the comparison of the exact and the analytical solutions is depicted in Figures 2(a) and 2(b).

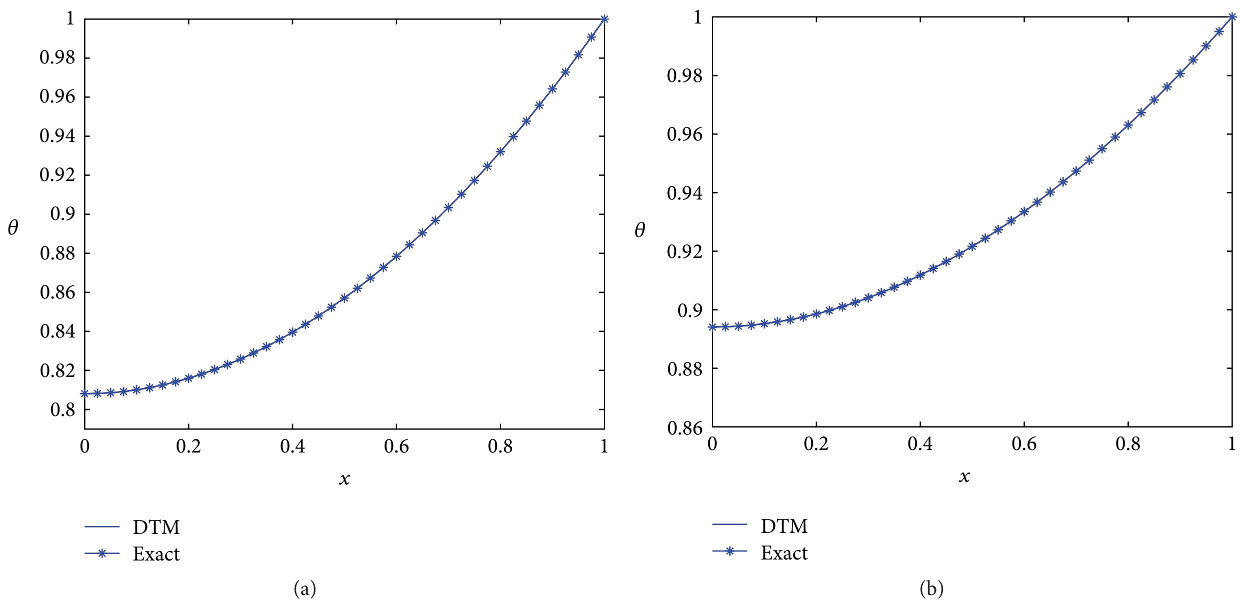


FIGURE 2: Comparison of analytical and exact solutions. In (a) $n = 1$, $M = 0.7$ and (b) $n = 2$, $M = 0.5$.

5. Analytical Solutions

It is well known that exact solutions for ODEs such as (4) exist only when thermal conductivity and the term containing heat transfer coefficient are connected by differentiation (or simply if the ODE such as (4) is linearizable) [4]. In this section we determine the analytical solutions for the nonlinearizable (4), firstly when thermal conductivity is given by the power law and secondly as a linear function of temperature. In both cases and throughout this paper, the heat transfer coefficient is assumed to be a power law function of temperature. These assumptions of the thermal properties are physical realistic. We have noticed that DTM runs into difficulty when the exponent of the power law of the thermal conductivity is given by fractional values and also when the function $f(x)$ is given in terms of fractional power law. One may follow Moradi and Ahmadikia [16] by introducing a new variable to deal with fractional powers of $f(x)$, and on the other hand, it is possible to remove the fractional exponent of the heat transfer coefficient by fundamental laws of exponent and binomial expansion.

Proposition 1. *A nonlinear ODE such as* (4) *may admit the DTM solution if $f(x)$ is a constant or exponential function. However, if $f(x)$ is a power law, then such an equation admits a DTM solution if the product,*

$$f(x) \cdot f'(x) = \alpha, \tag{37}$$

holds. Here α is a real constant.

Proof. Introducing the new variable $\xi = f(x)$, it follows from chain rule that (4) becomes

$$\alpha \frac{d\xi}{dx} \frac{d}{d\xi} \left[k(\theta) \frac{d\theta}{d\xi} \right] - M^2 \theta^{n+1} = 0. \tag{38}$$

The most general solution of condition (37) is

$$f(x) = (2\alpha x + \gamma)^{1/2}, \tag{39}$$

where γ is an integration constant. □

Example 2. (a) If $f(x) = x^{\sigma}$, then $\xi = x^{\sigma}$ transforms (4) into

$$\frac{d}{d\xi} \left[k(\theta) \frac{d\theta}{d\xi} \right] - 4M^2 \xi \theta^{n+1} = 0, \tag{40}$$

only if $\sigma = 1/2$.

This example implies that the DTM may only be applicable to problems describing heat transfer in fins with convex parabolic profile. In the next subsections, we present analytical solutions for (4) with various functions $f(x)$ and $k(\theta)$.

5.1. The Exponential Profile and Power Law Thermal Conductivity.

In this section, we present solutions for equation describing heat transfer in a fin with exponential profile and power law thermal conductivity and heat transfer coefficient. That is, given (4) with $f(x) = e^{\sigma x}$, and both heat transfer coefficient and thermal conductivity being power law functions of temperature, we construct analytical solutions. In our analysis, we consider $n = 2$ and 3 indicating the fin subject to nucleate boiling and radiation into free space at zero absolute temperature, respectively. Firstly, given $n = 3$ and $m = 2$

and applying the DTM, one obtains the following recurrence revelation:

$$\sum_{i=0}^{\kappa}\sum_{l=0}^{\kappa-i}\sum_{p=0}^{\kappa-i-l}\left[\frac{\sigma^p}{p!}\Theta(l)\Theta(i)(\kappa-i-l-p+1)\times(\kappa-i-l-p+2)\Theta(\kappa-i-l-p+2)+2\frac{\sigma^p}{p!}\Theta(l)(i+1)\Theta(i+1)\times(\kappa-i-l-p+1)\Theta(\kappa-i-l-p+1)+2\sigma\frac{\sigma^p}{p!}\Theta(l)\Theta(i)(\kappa-i-l-p+1)\times\Theta(\kappa-i-l-p+1)-M^2\Theta(p)\Theta(l)\Theta(i)\Theta(\kappa-i-l-p)\right]=0. \quad (41)$$

One may recall the transformed prescribed boundary conditions (18) and (19). Equation (41) is an iterative formula of constructing the power series solution as follows:

$$\begin{aligned}\Theta(2)&=\frac{M^2c^2}{2}\\ \Theta(3)&=-\frac{\sigma M^2c^2}{3}\\ \Theta(4)&=\frac{M^2c^2\left(3\sigma^2-2M^2c\right)}{24}\\ \Theta(5)&=-\frac{M^2c^2\left(\sigma^3-4\sigma M^2c\right)}{30}\\ \Theta(6)&=\frac{M^2c^2\left(5\sigma^4-78\sigma^2M^2c+58M^4c^2\right)}{720}\\ &\vdots\end{aligned} \quad (42)$$

The pervious process is continuous and one may consider as many terms as desired (but bearing in mind that DTM converges quite fast). Substituting (18) to (19) and (42) into (13), we obtain the following closed form of the solution:

$$\theta(x)=c+\frac{M^2c^2}{2}x^2-\frac{\sigma M^2c^2}{3}x^3+\frac{M^2c^2\left(3\sigma^2-2M^2c\right)}{24}x^4-\frac{M^2c^2\left(\sigma^3-4\sigma M^2c\right)}{30}x^5+\frac{M^2c^2\left(5\sigma^4-78\sigma^2M^2c+58M^4c^2\right)}{720}x^6+\cdots \quad (43)$$

To obtain the value of c, we substitute the boundary condition (5) into (43) at the point $x=1$. That is,

$$\theta(1)=c+\frac{M^2c^2}{2}-\frac{\sigma M^2c^2}{3}+\frac{M^2c^2\left(3\sigma^2-2M^2c\right)}{24}-\frac{M^2c^2\left(\sigma^3-4\sigma M^2c\right)}{30}+\frac{M^2c^2\left(5\sigma^4-78\sigma^2M^2c+58M^4c^2\right)}{720}+\cdots=1. \quad (44)$$

Substituting this value of c into (43), one finds the expression for $\theta(x)$. On the other hand, given $(n,m)=(2,3)$, one obtains the solution

$$\theta(x)=c+\frac{M^2}{2}x^2-\frac{\sigma M^2}{3}x^3+\frac{M^2\left(\sigma^2c-2M^2\right)}{8c}x^4-\frac{\sigma M^2\left(2\sigma^2c-21M^2\right)}{60c}x^5+\frac{M^2\left(180M^4-186\sigma^2M^2c+5\sigma^4c^2\right)}{720}x^6+\cdots \quad (45)$$

Here, the constant c maybe obtained by evaluating the boundary condition $\theta(1)=1$. The solutions (43) and (45) are depicted in Figures 3(b) and 3(a), respectively.

5.2. *The Rectangular Profile and Power Law Thermal Conductivity.*

In this section, we provide a detailed construction of analytical solutions for the heat transfer in a longitudinal rectangular fin with a power law thermal conductivity; that is, we consider (4) with $f(x)=1$ and $k(\theta)=\theta^m$. The analytical solutions are given in the following expressions.

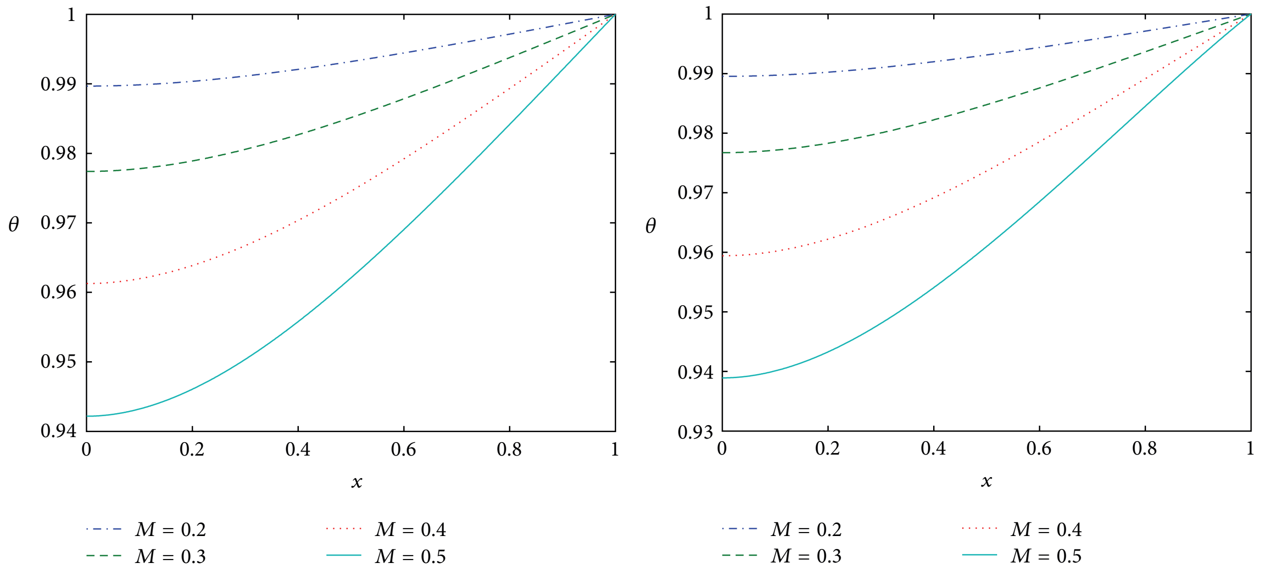
(a) Case $(n,m)=(2,3)$

$$\theta(x)=c+\frac{M^2}{2}x^2-\frac{M^4}{4c}x^4+\frac{M^6}{4c^2}x^6-\frac{33M^8}{122c^3}x^8+\frac{127M^{10}}{336c^4}x^{10}+\cdots. \quad (46)$$

(b) Case $(n,m)=(3,2)$

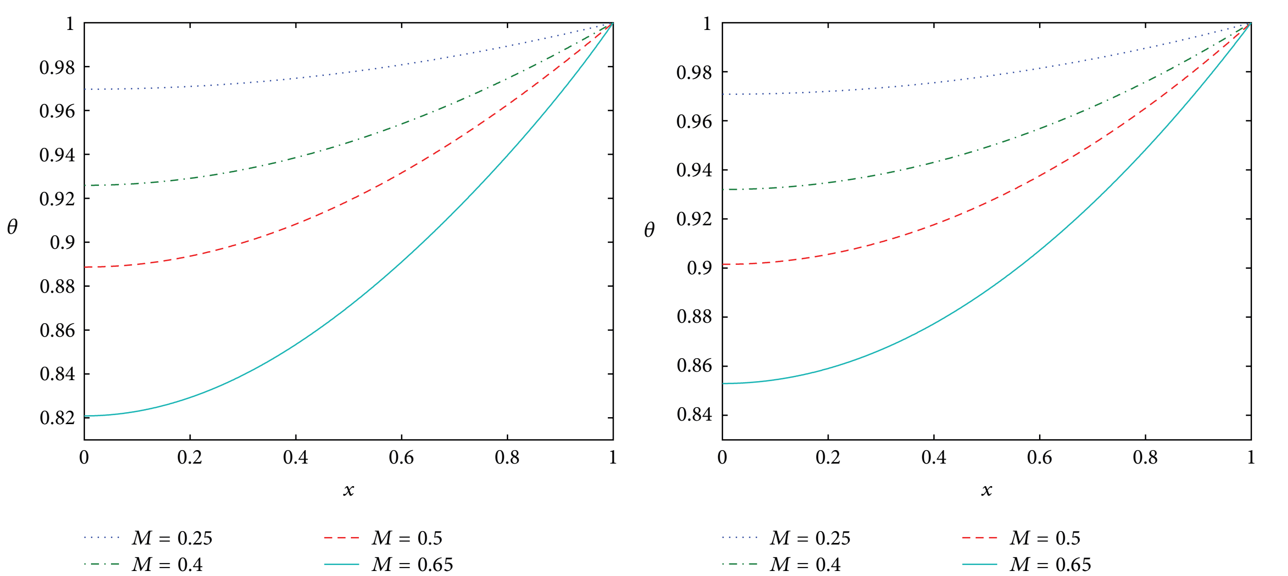
$$\theta(x)=c+\frac{c^2M^2}{2}x^2-\frac{c^3M^4}{12}x^4+\frac{29c^4M^6}{360}x^6-\frac{307c^5M^8}{5040}x^8+\frac{23483c^6M^{10}}{453600}x^{10}+\cdots. \quad (47)$$

The constant c is obtained by solving the appropriate $\theta(x)$ at the fin base boundary condition. The analytical solutions in (46) and (47) are depicted in Figures 4(a) and 4(b), respectively.



(a) (b)

Figure 3: Temperature profile in a longitudinal fin with exponential profile and power law thermal conductivity. In (a) the exponents are $(n, m) = (2, 3)$ and (b) $(n, m) = (3, 2)$.



(a) (b)

Figure 4: Temperature profile in a longitudinal fin with rectangular profile and power law thermal conductivity. In (a) the exponents are $(n, m) = (2, 3)$ and (b) $(n, m) = (3, 2)$.

5.3. The Convex Parabolic Profile and Power Law Thermal Conductivity. In this section, we present solutions for the equation describing the heat transfer in a fin with convex parabolic profile and power law thermal conductivity. Equation (40) is considered. Here we consider the values $\{(n, m) = (2, 3); (3, 2)\}$. The final analytical solution is given by

(a) Case $(n, m) = (2, 3)$

$$\theta(x) = c + \frac{2M^2}{3}x^{3/2} - \frac{2M^4}{5c}x^3 + \frac{23M^6}{45c^2}x^{9/2} - \frac{1909M^8}{2475c^3}x^6 + \frac{329222M^{10}}{259875c^4}x^{15/2} + \cdots . \quad (48)$$

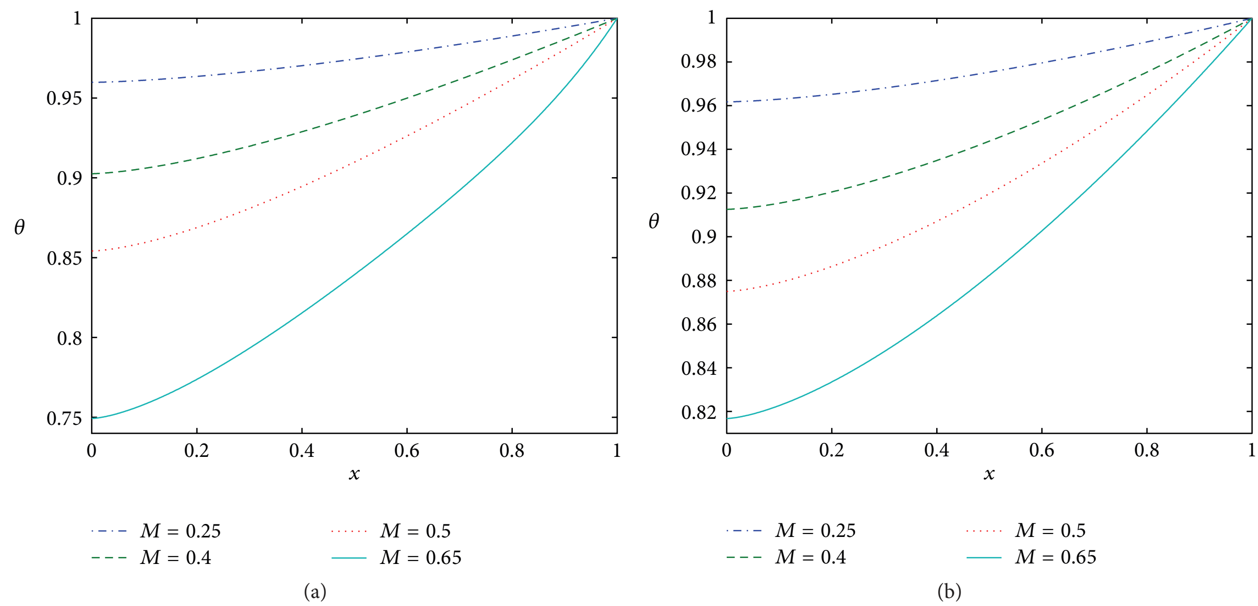


Figure 5: Temperature profile in a longitudinal fin with convex parabolic profile and power law thermal conductivity. In (a) the exponents are $(n, m) = (2, 3)$ and (b) $(n, m) = (3, 2)$.

(b) Case $(n, m) = (3, 2)$

$$\theta(x) = c + \frac{2c^2M^2}{3}x^{3/2} - \frac{4c^3M^4}{45}x^3 + \frac{4c^4M^6}{27}x^{9/2} - \frac{992c^5M^8}{7425}x^6 + \frac{30064c^6M^{10}}{212625}x^{15/2} + \cdots . \quad (49)$$

The constant c is obtained by evaluating the appropriate $\theta(x)$ at the fin base boundary condition. The solutions in (48) and (49) are depicted in Figures 5(a) and 5(b), respectively.

5.4. The Rectangular Profile and Linear Thermal Conductivity. In this section, we present solutions for the equation representing the heat transfer in a fin with rectangular profile and the thermal conductivity depending linearly on temperature. That is, we consider (4) with $f(x) = 1$ and $k(\theta) = 1 + \beta\theta$. The analytical solutions for this problem for different values of n are given by

(a) Case $n = 0$

$$\begin{aligned}\theta(x) &= c + \frac{M^2c}{2(1+\beta c)}x^2 - \frac{M^4c(-1+2\beta c)}{24(1+\beta c)^3}x^4 \\ &+ \frac{M^6c(1-16\beta c+28\beta^2c^2)}{720(1+\beta c)^5}x^6 \\ &- \frac{M^8c(-1+78\beta c-600\beta^2c^2+896\beta^3c^3)}{40320(1+\beta c)^7}x^8 \\ &+ \frac{M^{10}c(1-332\beta c+7812\beta^2c^2-39896\beta^3c^3+51184\beta^4c^4)}{3628800(1+\beta c)^9}x^{10} \\ &+ \cdots . \end{aligned} \quad (50)$$

(b) Case $n = 1$

$$\begin{aligned}\theta(x) &= c + \frac{M^2c^2}{2(1+\beta c)}x^2 - \frac{M^4c^3(-1+2\beta c)}{24(1+\beta c)^3}x^4 \\ &+ \frac{M^6c^4(10-16\beta c+19\beta^2c^2)}{720(1+\beta c)^5}x^6 \\ &- \frac{M^8c^5(-80+342\beta c-594\beta^2c^2+559\beta^3c^3)}{40320(1+\beta c)^7}x^8 \\ &+ \frac{M^{10}c^6(1000-7820\beta c+24336\beta^2c^2-36908\beta^3c^3+29161\beta^4c^4)}{3628800(1+\beta c)^9}x^{10} \\ &+ \cdots . \end{aligned} \quad (51)$$

(c) Case $n = 2$

$$\begin{aligned}\theta(x) &= c + \frac{M^2c^3}{2(1+\beta c)}x^2 + \frac{M^4c^5}{8(1+\beta c)^3}x^4 + \frac{M^6c^7(3+2\beta^2c^2)}{80(1+\beta c)^5}x^6 \\ &+ \frac{M^8c^9(49-20\beta c+66\beta^2c^2-40\beta^3c^3)}{4480(1+\beta c)^7}x^8 \\ &+ \frac{M^{10}c^{11}(427-440\beta c+1116\beta^2c^2-1020\beta^3c^3+672\beta^4c^4)}{134400(1+\beta c)^9}x^{10} \\ &+ \cdots . \end{aligned} \quad (52)$$

(d) Case $n = 3$

$$\begin{aligned}\theta(x) &= c + \frac{M^2c^4}{2(1+\beta c)}x^2 + \frac{M^4c^7(4+\beta c)}{24(1+\beta c)^3}x^4 \\ &+ \frac{M^6c^{10}(52+32\beta c+25\beta^2c^2)}{720(1+\beta c)^5}x^6 \\ &+ \frac{M^8c^{13}(1288+1020\beta c+1212\beta^2c^2-95\beta^3c^3)}{40320(1+\beta c)^7}x^8 \\ &+ \frac{M^{10}c^{16}(52024+45688\beta c+77184\beta^2c^2-680\beta^3c^3+15025\beta^4c^4)}{3628800(1+\beta c)^9}x^{10} \\ &+ \cdots. \end{aligned} \tag{53}$$

The constant c may be obtained from the boundary condition on the appropriate solution. The solutions in (50), (51), (52), and (53) are depicted in Figure 6.

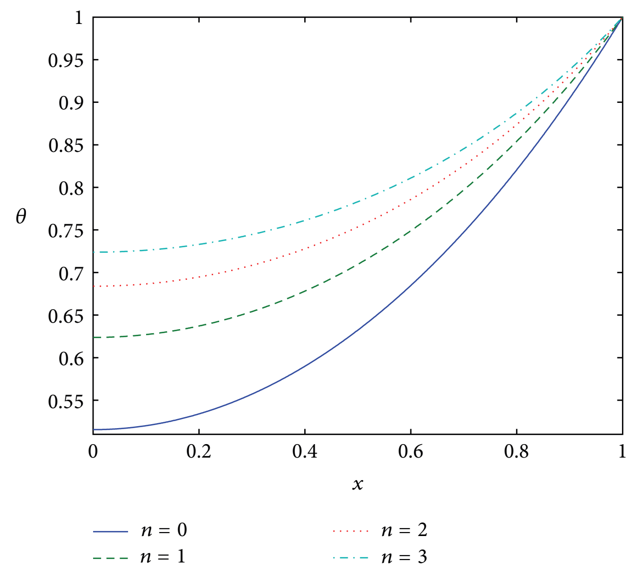


FIGURE 6: Temperature profile in a longitudinal rectangular fin with linear thermal conductivity and varying values of n. Here $\beta = 0.5$ and $M = 1.5$ are fixed.

5.5. The Convex Parabolic Profile and Linear Thermal Conductivity. In this section, we present solutions for the equation describing the heat transfer in a fin with convex parabolic profile and the thermal conductivity depending linearly on temperature. That is, we consider (40) with $k(\theta) = 1 + \beta\theta$. The analytical solution for this problem for different values of n is given by

(a) Case $n = 0$

$$\begin{aligned}\theta(x) &= c + \frac{2M^2c}{3(1+\beta c)}x^{3/2} - \frac{2M^4c(-2+3\beta c)}{45(1+\beta c)^3}x^3 \\ &+ \frac{M^6c(2-25\beta c+33\beta^2c^2)}{405(1+\beta c)^5}x^{9/2} \\ &- \frac{M^8c(-10+599\beta c-3582\beta^2c^2+4059\beta^3c^3)}{66825(1+\beta c)^7}x^6 \\ &+ \cdots. \end{aligned} \tag{54}$$

(b) Case $n = 1$

$$\begin{aligned}\theta(x) &= c + \frac{2M^2c^2}{3(1+\beta c)}x^{3/2} - \frac{2M^4c^3(-4+\beta c)}{45(1+\beta c)^3}x^3 \\ &+ \frac{2M^6c^4(9-11\beta c+10\beta^2c^2)}{405(1+\beta c)^5}x^{9/2} \\ &- \frac{2M^8c^5(-330+1118\beta c-1584\beta^2c^2+1093\beta^3c^3)}{66825(1+\beta c)^7}x^6 \\ &+ \cdots. \end{aligned} \tag{55}$$

(c) Case $n = 2$

$$\begin{aligned}\theta(x) &= c + \frac{2M^2c^3}{3(1+\beta c)}x^{3/2} + \frac{2M^4c^5(6+\beta c)}{45(1+\beta c)^3}x^3 \\ &+ \frac{M^6c^7(16+3\beta c+7\beta^2c^2)}{135(1+\beta c)^5}x^{9/2} \\ &+ \frac{M^8c^9(1160-107\beta c+1116\beta^2c^2-367\beta^3c^3)}{22275(1+\beta c)^7}x^6 \\ &+ \cdots. \end{aligned} \tag{56}$$

(d) Case $n = 3$

$$\begin{aligned}\theta(x) &= c + \frac{2M^2c^4}{3(1+\beta c)}x^{3/2} + \frac{2M^4c^7(8+3\beta c)}{45(1+\beta c)^3}x^3 \\ &+ \frac{4M^6c^{10}(23+17\beta c+9\beta^2c^2)}{405(1+\beta c)^5}x^{9/2} \\ &+ \frac{2M^8c^{13}(5000+4868\beta c+4356\beta^2c^2+363\beta^3c^3)}{66825(1+\beta c)^7}x^6 \\ &+ \cdots. \end{aligned} \tag{57}$$

The constant c may be obtained from the boundary condition on the appropriate $\theta(x)$. The solutions in (54), (55), (56) and (57) are depicted in Figure 7.

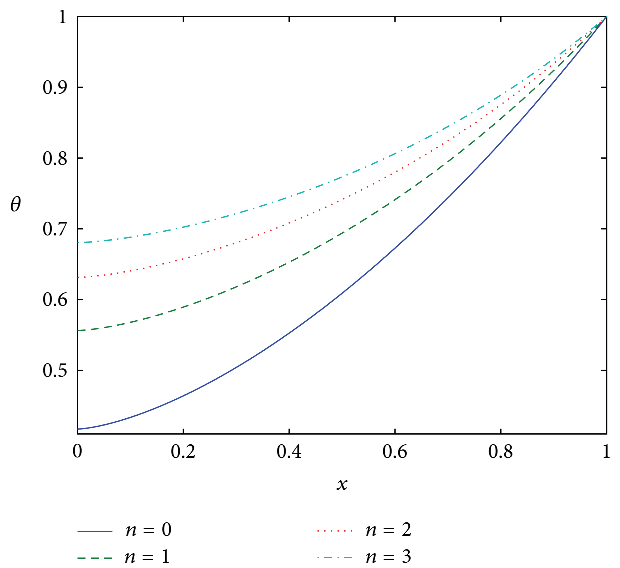


FIGURE 7: Temperature profile in a longitudinal convex parabolic fin with linear thermal conductivity and varying values of n. Here $\beta = 0.5$ and $M = 1.5$ are fixed.

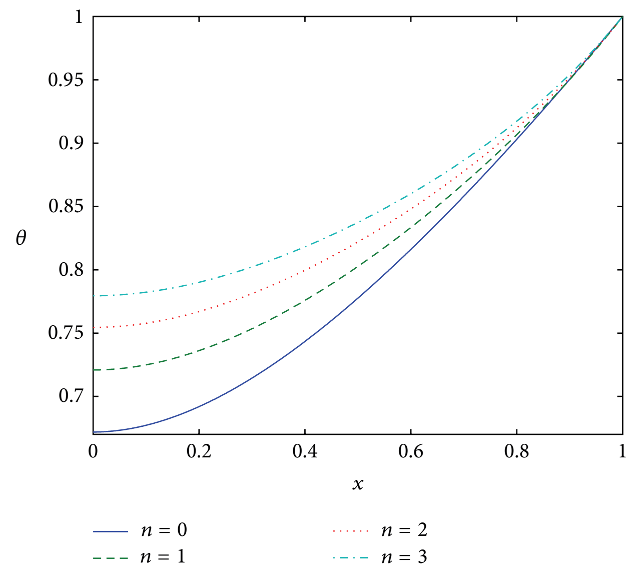


FIGURE 8: Temperature profile in a longitudinal fin of exponential profile with linear thermal conductivity and varying values of n. Here $\beta = 0.5$ and $M = 1.5$ are fixed.

5.6. The Exponential Profile and Linear Thermal Conductivity. In this section, we present solutions for equation heat transfer in a fin with convex parabolic profile and the thermal conductivity depending linearly on temperature. That is, we consider (4) with $k(\theta) = 1 + \beta\theta$ and $f(x) = e^{\sigma x}$. Here σ is a constant. The analytical solutions for this problem for different values of n are given by

(a) Case $n = 0$

$$\theta(x) = c + \frac{M^2 c}{2(1+\beta c)}x^2 - \frac{\sigma M^2 c\left(2+\beta c+\beta^2 c\right)}{6(1+\beta c)^2}x^3 + \frac{M^2 c\left(M^2\left(1-2\beta^2 c\right)+\sigma^2\left(3+3\beta c+\beta^3 c^2+\beta^4 c^2+\beta^2 c(3+c)\right)\right)}{24(1+\beta c)^4}x^4 + \cdots. \quad (58)$$

(b) Case $n = 1$

$$\theta(x) = c + \frac{M^2 c^2}{2(1+\beta c)}x^2 - \frac{\sigma M^2 c^2\left(2+\beta c+\beta^2 c\right)}{6(1+\beta c)^2}x^3 + \frac{M^2 c^2\left(M^2 c\left(2+\beta c-2\beta^2 c\right)+\sigma^2\left(3+3\beta c+\beta^3 c^2+\beta^4 c^2+\beta^2 c(3+c)\right)\right)}{24(1+\beta c)^3}x^4 + \cdots. \quad (59)$$

(c) Case $n = 2$

$$\theta(x) = c + \frac{M^2 c^3}{2(1+\beta c)}x^2 - \frac{\sigma M^2 c^3\left(2+\beta c+\beta^2 c\right)}{6(1+\beta c)^2}x^3 + \frac{M^2 c^3\left(M^2 c^2\left(3+2\beta c-2\beta^2 c\right)+\sigma^2\left(3+3\beta c+\beta^3 c^2+\beta^4 c^2+\beta^2 c(3+c)\right)\right)}{24(1+\beta c)^3} \times x^4 + \cdots. \quad (60)$$

(d) Case $n = 3$

$$\theta(x) = c + \frac{M^2 c^4}{2(1+\beta c)}x^2 - \frac{\sigma M^2 c^4\left(2+\beta c+\beta^2 c\right)}{6(1+\beta c)^2}x^3 + \frac{M^2 c^4\left(M^2 c^3\left(4+3\beta c-2\beta^2 c\right)+\sigma^2\left(3+3\beta c+\beta^3 c^2+\beta^4 c^2+\beta^2 c(3+c)\right)\right)}{24(1+\beta c)^3} \times x^4 + \cdots. \quad (61)$$

The constant c may be obtained from the boundary condition on the appropriate $\theta(x)$. The solutions in (58), (59), (60), and (61) are depicted in Figure 8.

6. Fin Efficiency and Heat Flux

6.1. Fin Efficiency. The heat transfer rate from a fin is given by Newton's second law of cooling:

$$Q = \int_0^L PH(T)\left(T - T_a\right)dx. \quad (62)$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by (see e.g., [1])

$$\eta = \frac{Q}{Q_{\text{ideal}}} = \frac{\int_0^L PH(T)(T - T_a)\,dX}{Ph_b L(T_b - T_a)}. \tag{63}$$

In dimensionless variables, we have

$$\eta = \int_0^1 \theta^{n+1} dx. \tag{64}$$

We consider the solutions (50), (51), (52), and (53) and depict the fin efficiency (63) in Figure 9.

6.2. Heat Flux. The fin base heat flux is given by the Fourier's law

$$q_b = A_c K(T) \frac{dT}{dx}. \tag{65}$$

The total heat flux of the fin is given by [1]

$$q = \frac{q_b}{A_c H(T)(T_b - T_a)}. \tag{66}$$

Introducing the dimensionless variable as described in Section 2 implies

$$q = \frac{1}{Bi} \frac{k(\theta)}{h(\theta)} \frac{d\theta}{dx}, \tag{67}$$

where the dimensionless parameter $Bi = h_b L / k_a$ is the Biot number. We consider a number of cases for the thermal conductivity and the heat transfer coefficient.

6.2.1. Linear Thermal Conductivity and Power Law Heat Transfer Coefficient. In this case (67) becomes

$$q = \frac{1}{Bi}(1 + \beta\theta)\theta^{-n}\frac{d\theta}{dx}. \tag{68}$$

The heat flux in (68) at the base of the fin is plotted in Figure 10.

6.2.2. Power Law Thermal Conductivity and Heat Transfer Coefficient. In this case (67) becomes

$$q = \frac{1}{Bi}\theta^{m-n}\frac{d\theta}{dx}. \tag{69}$$

Not surprisingly, heat flux in one-dimensional fins is higher given values $Bi \ll 1$. The heat flux in (69) is plotted in Figures 11(a), 11(b), and 11(c).

7. Some Discussions

The DTM has resulted in some interesting observations and study. We have observed in Figures 2(a) and 2(b) an excellent agreement between the analytical solutions generated by DTM and the exact solution obtained in [2]. In particular, we considered a fin problem in which both thermal conductivity and heat transfer coefficient are given by the same power law. Furthermore, we notice from Table 2 that an absolute error of approximately 3.5e − 005 is produced by DTM of order $O(15)$. In Table 3, an absolute error of approximately 6.5e − 006 is produced for the same order. This confirms that the DTM converges faster and can provide accurate results with a minimum computation. As such, a tremendous confidence in the DTM in terms of the accuracy and effectiveness was built, and thus we used this method to solve other problems for which exact solutions are harder to construct.

In Figures 3(a), 3(b), 4(a), 4(b), 5(a), and 5(b), we observe that the fin temperature increases with the decreasing

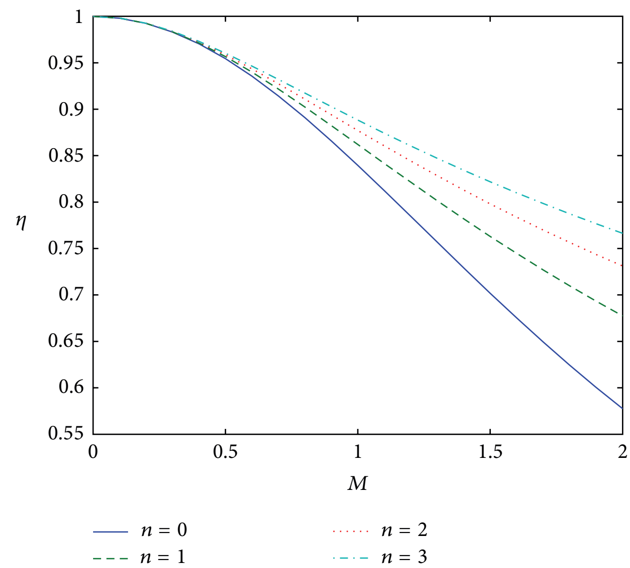


FIGURE 9: Fin efficiency of a longitudinal rectangular fin. Here $\beta = 0.75$.

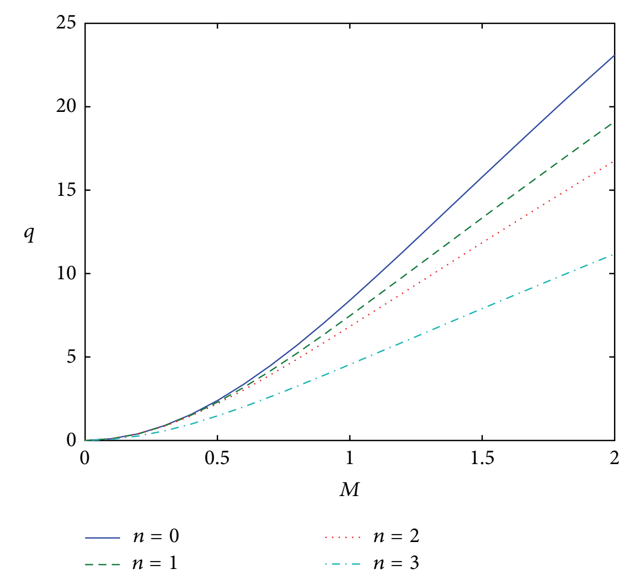


FIGURE 10: Base heat flux in a longitudinal rectangular fin with linear thermal conductivity. Here $\beta = 0.1$.

values of the thermogeometric fin parameter. Here, the values of the exponents are fixed. Also, we observe that fin temperature is higher when $n - m > 0$, that is, when heat transfer coefficient is higher than the thermal conductivity. We observe in Figures 6, 7, and 8 that the fin temperature increases with the increasing values of n. Furthermore, it appears that the fin with exponential profile performs the least in transferring the heat from the base, since the temperature in such a fin is much higher than that of the rectangular and the convex parabolic profiles. In other words, heat dissipation to the fluid surrounding the extended surface is much faster in longitudinal fins of rectangular and convex parabolic profiles. In Figure 9, fin efficiency decreases with increasing thermogeometric fin parameter. Also, fin efficiency increases with increasing values of n. It is easy to show that the thermogeometric fin parameter is directly proportional to the aspect ratio (extension factor) with square root of the Biot number being the proportionality constant. As such, shorter fins are more efficient than longer ones. Else, the increased Biot number results in less efficient fin whenever the space is confined, that is, where the length of the fin cannot be increased. Figure 10 depicts the heat flux at the fin base. The amount of heat energy dissipated from the fin base is of immense interest in engineering [28]. We observe in Figure 10 that the base heat flux increases with the thermogeometric fin parameter for considered values of the exponent n (see also [28]). Figures 11(a), 11(b), and 11(c) display the heat flux across the fin length. We note that the heat flux across the fin length increases with increasing values of the thermogeometric fin parameter.

8. Concluding Remarks

In this study, we have successfully applied the DTM to highly nonlinear problems arising in heat transfer through longitudinal fins of various profiles. Both thermal conductivity and heat transfer coefficient are given as functions of temperature. The DTM agreed well with exact solutions when the thermal conductivity and heat transfer coefficient are given by the same power law. A rapid convergence to the exact solution was observed. Following the confidence in DTM built by the results mentioned, we then solved various exciting problems. The exotic results have been shown in tables and figures listed in this paper.

The results obtained in this paper are significant improvements on the known results. In particular, both the heat transfer coefficient and the thermal conductivity are allowed to be given by the power law functions of temperature, and also we considered a number of fin profiles. We note that exact solutions are difficult if not impossible to construct when the exponents of these properties are distinct.

Perhaps the notable advantage of the DTM is the generalization of the Taylor method to problems involving unusual derivative procedures such as fractional, fuzzy, or q-derivative [22]. Some generalizations have been made by Odibat et al. [24], and they referred to their new method as the Generalized Differential Transform Method (GDTM). This showed great improvement compared to the Fractional Differential

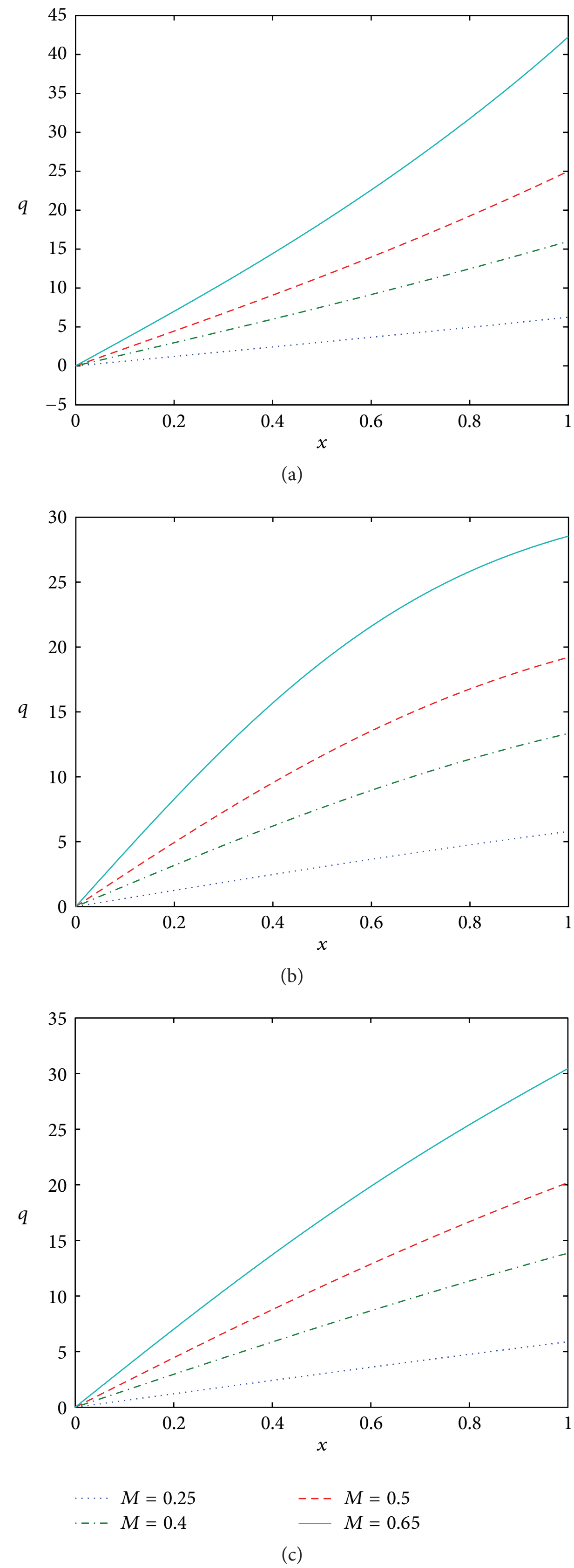


Figure 11: Heat flux across a longitudinal rectangular fin with linear thermal conductivity. (a) $n-m<0$, (b) $n-m>0$, and (c) $n-m=0$. Here $Bi = 0.01$.

Transform Method (FDTM) introduced by Arikoglu and Ozkol [23].

We have shown with the help of an example that DTM may only be applied to fin problems involving heat transfer through fins with convex parabolic profile. Note that given an ODE such as (4) with a power law heat transfer coefficient of a fractional exponent, then one can easily remove the fraction by basic exponential rules and employment of the Binomial expansion. However, using the DTM, one runs into difficulty if the power law thermal conductivity in the same equation is given by the fractional exponent. We do not know whether these observations call for the "modified" DTM to solve problems arising in heat flow through fins with other profiles, such as longitudinal triangular and concave parabolic, and also with fractional power law thermal conductivity.

The main results obtained in this paper give insight into heat transfer in boiling liquids where the heat transfer coefficient is temperature dependent and may be given by a power law. The thermal conductivity of some materials such as gallium nitride (GaN) and Aluminium Nitride (AlN) may be modeled by power law temperature dependency [29, 30]. Thus, the solutions constructed here give a better comparison of heat transfer in terms of material used since in many engineering applications thermal conductivity is given as a linear function of temperature. Furthermore, a good study in terms of performance and efficiency of fin with different profiles is undertaken. These finding could help in the design of fins. It is claimed in [14] that DTM results are more accurate than those constructed by Variational Iteration Methods (VIM) and Homotopy Perturbation Methods (HPM). However, it would be risky to use the DTM approximate solutions as benchmarks for the numerical schemes. Nevertheless, we have also shown that DTM converges rapidly in just fifteen terms to the exact solution.

Acknowledgments

The authors wish to thank the National Research Foundation of South Africa and the University of the Witwatersrand for the generous financial support. The authors are grateful to Professor D. P. Mason and the anonymous reviewers for their invaluable comments which improved this paper significantly.

References

[1] A. D. Kraus, A. Aziz, and J. Welty, *Extended Surface Heat Transfer*, Wiley, New York, NY, USA, 2001.

[2] R. J. Moitsheki, T. Hayat, and M. Y. Malik, "Some exact solutions of the fin problem with a power law temperature-dependent thermal conductivity," *Nonlinear Analysis. Real World Applications*, vol. 11, no. 5, pp. 3287–3294, 2010.

[3] R. J. Moitsheki, "Steady one-dimensional heat flow in a longitudinal triangular and parabolic fin," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 10, pp. 3971–3980, 2011.

[4] R. J. Moitsheki, "Steady heat transfer through a radial fin with rectangular and hyperbolic profiles," *Nonlinear Analysis. Real World Applications*, vol. 12, no. 2, pp. 867–874, 2011.

[5] Mo. Miansari, D. D. Ganji, and Me. Miansari, "Application of He's variational iteration method to nonlinear heat transfer equations," *Physics Letters A*, vol. 372, no. 6, pp. 779–785, 2008.

[6] C. H. Chiu and C. K. Chen, "A decomposition method for solving the convective longitudinal fins with variable thermal conductivity," *International Journal of Heat and Mass Transfer*, vol. 45, no. 10, pp. 2067–2075, 2002.

[7] A. Rajabi, "Homotopy perturbation method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity," *Physics Letters A*, vol. 364, no. 1, pp. 33–37, 2007.

[8] G. Domairry and M. Fazeli, "Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature-dependent thermal conductivity," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 2, pp. 489–499, 2009.

[9] A. Campo and R. J. Spaulding, "Coupling of the methods of successive approximations and undetermined coefficients for the prediction of the thermal behaviour of uniform circumferential fins," *Heat and Mass Transfer*, vol. 34, no. 6, pp. 461–468, 1999.

[10] A. Campo and F. Rodrfguez, "Approximate analytic temperature solution for uniform annular fins by adapting the power series method," *International Communications in Heat and Mass Transfer*, vol. 25, no. 6, pp. 809–818, 1998.

[11] R. Chiba, "Application of differential transform method to thermoelastic problem for annular disks of variable thickness with temperature-dependent parameters," *International Journal of Thermophysics*, vol. 33, pp. 363–380, 2012.

[12] S. Sadri, M. R. Raveshi, and S. Amiri, "Efficiency analysis of straight fin with variable heat transfer coefficient and thermal conductivity," *Journal of Mechanical Science and Technology*, vol. 26, no. 4, pp. 1283–1290, 2012.

[13] M. Torabi, H. Yaghoori, and A. Aziz, "Analytical solution for convective-radiative continously moving fin with temperature dependent thermal conductivity," *International Journal of Thermophysics*, vol. 33, pp. 924–941, 2012.

[14] M. Torabi and H. Yaghoobi, "Two dominant analytical methods for thermal analysis of convective step fin with variable thermal conductivity," *Thermal Science*. In press.

[15] A. Moradi, "Analytical solutions for fin with temperature dependant heat transfer coefficient," *International Journal of Engineering and Applied Sciences*, vol. 3, no. 2, pp. 1–12, 2011.

[16] A. Moradi and H. Ahmadikia, "Analytical solution for different profiles of fin with temperature-dependent thermal conductivity," *Mathematical Problems in Engineering*, vol. 2010, Article ID 568263, 15 pages, 2010.

[17] H. Yaghoobi and M. Torabi, "The application of differential transformation method to nonlinear equations arising in heat transfer," *International Communications in Heat and Mass Transfer*, vol. 38, no. 6, pp. 815–820, 2011.

[18] S. Ghafoori, M. Motevalli, M. G. Nejad, F. Shakeri, D. D. Ganji, and M. Jalaal, "Efficiency of differential transformation method for nonlinear oscillation: Comparison with HPM and VIM," *Current Applied Physics*, vol. 11, no. 4, pp. 965–971, 2011.

[19] A. A. Joneidi, D. D. Ganji, and M. Babaelahi, "Differential transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity," *International Communications in Heat and Mass Transfer*, vol. 36, no. 7, pp. 757–762, 2009.

[20] J. K. Zhou, *Differential Transform Method and Its Applications for Electric circuIts*, Huazhong University Press, Wuhan, China, 1986.

[21] S. Mukherjee, "Reply to comment on 'solutions of the duffing-van der pol oscillator equation by the differential transform method'," *Physica Scripta*, vol. 84, Article ID 037003, 2 pages, 2011.

[22] C. Bervillier, "Status of the differential transformation method," *Applied Mathematics and Computation*, vol. 218, no. 20, pp. 10158–10170, 2012.

[23] A. Arikoglu and I. Ozkol, "Solution of fractional differential equations by using differential transform method," *Chaos, Solitons and Fractals*, vol. 34, no. 5, pp. 1473–1481, 2007.

[24] Z. Odibat, S. Momani, and V. S. Erturk, "Generalized differential transform method: application to differential equations of fractional order," *Applied Mathematics and Computation*, vol. 197, no. 2, pp. 467–477, 2008.

[25] C. W. Bert, "Application of differential transform method to heat conduction in tapered fins," *Journal of Heat Transfer*, vol. 124, no. 1, pp. 208–209, 2002.

[26] F. Khani and A. Aziz, "Thermal analysis of a longitudinal trapezoidal fin with temperature-dependent thermal conductivity and heat transfer coefficient," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 3, pp. 590–601, 2010.

[27] H. C. Ünal, "An anlytical study of boiling heat transfer from a fin," *International Journal of Heat and Mass Transfer*, vol. 31, no. 7, pp. 1483–1496, 1988.

[28] M. H. Chang, "A decomposition solution for fins with temperature dependent surface heat flux," *International Journal of Heat and Mass Transfer*, vol. 48, no. 9, pp. 1819–1824, 2005.

[29] A. Jezowski, B. A. Danilchenko, M. Boćkowski et al., "Thermal conductivity of GaN crystals in 4.2–300 K range," *Solid State Communications*, vol. 128, no. 2-3, pp. 69–73, 2003.

[30] S. Vitanov, V. Palankovski, S. Maroldt, and R. Quay, "High-temperature modeling of AlGaN/GaN HEMTs," *Solid-State Electronics*, vol. 54, no. 10, pp. 1105–1112, 2010.