Analytical study of microchannel and passive microvalve "Application to Micropump simulator"

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ABSTRACT

Microfluidic systems including microchannels and microvalves, fabricated by micromachining technology, are studied with approached models, either analytical or by simulation. The modelling of rectangular channel and passive valves is presented in this paper, which is divided in three parts. At first, the analytical modelling of a channel versus its shape factor and a normalisation of its fluidic comportment is presented. Then a description of the diffuser-nozzle valve is purposed by applying the general Bernoulli equation. The efficiency of this valve is found to be determined by the value of the shape factor and angle of the diffuser element. The third part is dedicated to numerical simulation of a Tesla diode and purpose an optimisation of its efficiency versus the Tesla geometry. Finally, the realisation and characterisation of prototypes are exposed. Characterisation were applied to rectangular channel and showed good agreement with the analytic modelisation. The analytic expressions, that have been found, can be used in simulations of the flow sensors through the construction of an equivalent electric circuit, and subsequently analysed using SPICE or similar tool “Simulink

KEYWORD LIST: Bernoulli equation, circular channel, Rectangular channel, nozzle-diffuser, Tesla valve, micropump, simulator, micro-technology.

1. INTRODUCTION

Recent development in MEMS enables to achieve microstructures for fluidic applications. Two main advantages of micro-fluidic can be highlighted: very small analysis devices (section around 250µm²) reduce the time of analysis and volume of components needed, furthermore a more precise device versus the volume stroke can be obtained. We are developing a general study around this topic. Starting with the realisation of basic component such as micro-channel and micro-diode, a global data base of micro-fluidic structure is under conception. A fabrication is done in order to validate the modelisation. This data base is then implemented in future development around the study of micropumps or bio-devices.

2. STUDY OF MICRO-CHANNEL

For many devices the flow pressure characteristic can be described using a simple analytical formula well known from macroscopic fluid mechanics as for micro-channel [2] and nozzle diffuser [3]. This is the application of the Navier Stokes equation; this approach has been used successfully for channels, valves and flow sensors. But for other devices such as vortex [4] and Tesla valve [5], no analytical solution is known. One solution is to use the ***CFD*** software Computational Fluidic Dynamics *(ANSYS® FLOTRAN)* or ***FlumeCAD*** *(COVENTOR)*. From the numerical simulation results, an analytic formula can be extracted.

2. 1 Flow modelling

In this part the fundamental laws of fluidic element (Bernoulli and Navier-Stokes) are applied to microcomponent in order to develop an analytical database(such as micro-channel "rectangular channel, circular channel", microdiodes "diffuser/nozzle, Tesla diode … …). According to the theoretical models based on the Bernoulli equation, the total pressure drop DP across the channel is the sum of:

- the pressure drop $\left(\frac{1}{2}rU^2(\frac{4L}{D_h})f\right)$ due to laminar friction.
- the pressure drop $\left(\frac{1}{2}rU^2K\right)$ required for the acceleration or deceleration of the liquid [6].

r, L, U *and* D_h are the fluid density, the length channel, mean velocity and the hydraulic diameter, respectively, f is the friction coefficient *and* K the loss coefficient in the singularities. In table 1, the K values are given versus the singularity shape (where β=(s/S), the small cross section/large cross section). We have considered three channels (figure. 1) placed in series "rectangular and diffuser channels"; these channels are traversed by the same flow rate (Similar with electrical current).

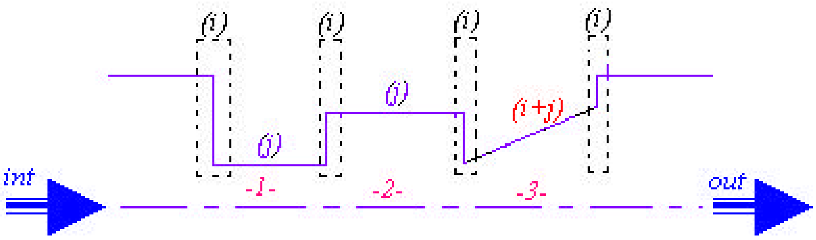

Figure. 1: The schematic of the channels placed in series.

Singularities	K
Widening	$(b-1)^2$
Reduction	$\left(\frac{1}{0.59+0.41\beta^3}-1\right)^2$
Bend circular "90°"	[0.4 to 0.9]
	1.3 to 1.9

Table. 1: the loss coefficient in the singularities[6].

Calling DP_f the sum pressure drop due to laminar friction *(j region)* and DP_s the sum pressure in a singularities *(i region)*. In the gradual cross-section *(channel 3)*, the pressure drops are illustrated with $(DP_f + DP_s)$. The total pressure drop DP_f is found as the sum of the pressure drops of the different regions:

$$DP_{Tot} = \sum_j DP_f + \sum_i DP_s = \sum_j \left(\frac{1}{2} rU^2 \left(\frac{4L}{D_h}\right) f\right)_j + \sum_i \left(\frac{1}{2} rU^2 K\right)_i . \qquad (1)$$

This analytical equation can be used to develop the analytical models and to calculate the static flow rate Φ of the micro-channel and micro-diode(Φ=U*A, where A the cross sectional area). In the section 1, at first the flow-pressure characteristic in a circular channel is presented, then will be studied the fluid characteristic in a rectangular channel and at last the modelisation of the nozzle-diffuser valve *(rectangular)*.

2. 1. 1 Flow regimes

Figure 2 shows the basic structure of an equivalent simple circular micro-channel, defined by its hydraulic diameter $(D_h = D = 4 * channel\ cross-sectionnal\ area / wetted\ perimeter)$ and length L. The volume flow characteristic $F(P)$ can be obtained with a global normalised formula taking into account the laminar flow or turbulent flow.

$$F_N(P) = \frac{F_L(P) \times F_T(P)}{F_L(P) + F_T(P)} \qquad (2)$$

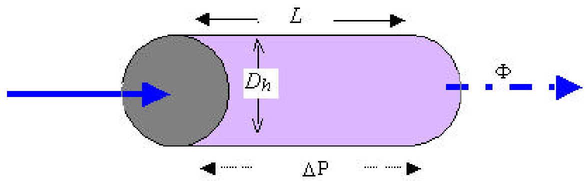

Figure. 2: The schematic of the circular channel.

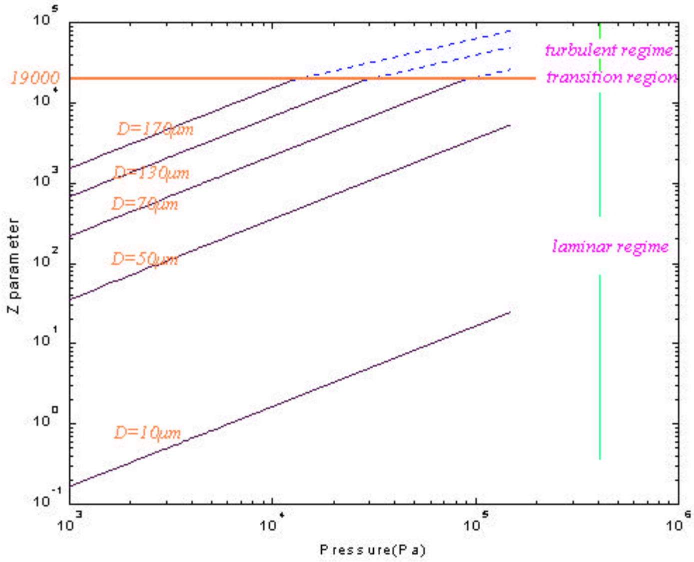

Figure 3. Evolution of the Z parameter at various diameter channel and pressure drop(L=3mm, water liquid).

- $F_L(P)$: the laminar flow characteristic, is given by the first term of the Bernoulli equation " law of Hagen-Poiseuille

$$F_L = \frac{hpD_h}{r}\frac{Z}{64} \qquad \text{with} \qquad f = \frac{16}{R_e}.$$

- $F_T(P)$: The turbulent flow characteristic in the channel, is then calculated from the normalised *BLASIUS* law[9]:

$$F_T = \frac{hpD_h}{r4}\left(\frac{Z}{0.0791}\right)^{4/7} \qquad \text{with} \qquad f = \frac{0.0791}{Re^{1/4}}.$$

Where DP, h and r are the hydrostatic pressure difference, dynamic viscosity and the density of the fluid, respectively, and the Z is the normalised parameter equivalent to the Reynolds number R_e,:

$$Z = \rho\Delta P \cdot D_h^3 / 2L\eta^2, \; R_e = \rho.D_h.V/\eta .$$

The fluid flow through a micro-channel can be laminar or turbulent, this may be verified by computing the Z parameter. Which leads to three different flow regimes:

i. The laminar flow is in the range $(Z \approx 0 \ \ to \ \ 19000)$; here the pressure drop is dominated by viscous losses.
ii. The transition region is defined by $(Z = Z_{Tr} \approx 19000)$.
iii. For $Z \geq 19000$, the flow is turbulent which induce an homogenisation of velocity.

These mathematical expression Φ_N have been used and integrated in a global simulator for a micropump.

Note that all the analytic equations are directly applicable to a rectangular channel by substituting D by D_h. When Dh, case of most of micro-channels obtained by micro-technology, etched on the wafer surface, have a rectangular section. When D_h is around 100µm the flow is Mainly in the laminar regime (see figure. 3), in the second parts we are interested the influence of D_h or the shape factor λ.

2.1.2. Influence of the geometry

The structure of rectangular micro-channel is depicted in figure 4. It's described by its width d, height h and length L. Due to the low Reynolds number; the flow behaviour is supported to be laminar. In this case, two kind of behaviours can be obtained depending on the shape factor λ of the channel, defined as the ratio: $(l = d/h \geq 1)$.

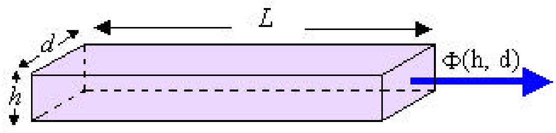

Figure. 4: schematic representation of the rectangular micro-channel

- ***If λ is close to one:***

The rectangular micro-channel is similar to a circular channel with a hydraulic diameter D_h: $D_h = 4\frac{d \cdot h}{2(d+h)}$. The volume flow characteristic $\Phi(P)$ is then calculated from the law of Hagen-Poiseuille "the first term in Bernoulli

$$\Phi_{Ps} = \frac{1}{32}\frac{S.P}{\eta L}D_h^2 \tag{3}$$

Where P, η are the hydrostatic pressure difference and the dynamic viscosity of the fluid, and S is the channel cross section $S = d \cdot h$.

- ***If λ is larger than one:***

In this case, h is very small compared to d; damping effects due to viscosity near the section width are negligible. The volume flow characteristic $\Phi(P, d, h)$ is that given by the Navier–Stokes law:

$$\Phi_{St} = \frac{1}{12}\frac{d \cdot h^3}{L}\frac{P}{\eta} \tag{4}$$

Using the parameters introduced before, the equations (3 and 4) can be rewritten as:

$$\begin{cases} 4\dfrac{\eta L \cdot \Phi_{Ps}}{h^4 P} = \dfrac{1}{2}\dfrac{(d/h)^3}{(1+(d/h))^2} & (5) \\ 4\dfrac{\eta L \cdot \Phi_{Ts}}{h^4 P} = \dfrac{1}{2}(d/h) & (6) \end{cases}$$

These two behaviours can be expressed from λ "the shape factor" with the normalised parameter T. By this way, the relation between the volume flow rate in the channel and geometry parameters *(h and d)* shown in equations 1 and 2 can be expressed as:

Hagen Poiseuille: $$T_{Ps} = \frac{1}{2}\frac{\lambda^3}{(1+\lambda)^2} \tag{7}$$

Navier Stokes: $$T_{St} = \frac{1}{3}\cdot \lambda \tag{8}$$

The use of Hagen Poiseuille relation or Navier Stokes is determined by the value of λ. The limit when the two models give the same behaviour (T_{Ps} equal to T_{St}) is defined by the following value λ=4.45.

2. 1. 3: Fabrication and characterisation of the microchannels

In order to validate the theory, prototypes of rectangular micro-channel have been achieved *(figure. 5)* and characterised *(figure. 6)*. The channels are micromachined and etched in silicon using the *DRIE (Deep Reactive Ion Etching)* technique. All channels were covered with Pyrex glass using anodic bonding.

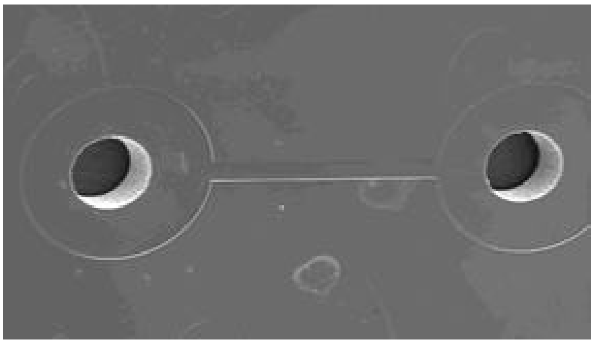

Figure. 5: channel fabricated using micro-machining technology

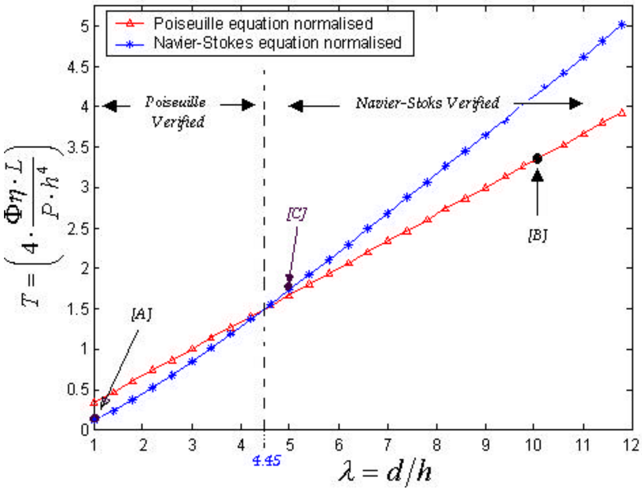

Figure. 6: The relationship between the ratio λ and *T*

Figure. 6 shows the measurement results obtained from micro-channels with different shape factor. These results are compared with the theory models:

- When $\lambda \leq 4{,}5$, Hagen-Poiseuille model is applied*(Point A)*, deduced from Stemme [3].
- When $\lambda \approx 4{,}5$, a similar behaviour is observed*(Point C)*, deduced from M. Anduze [4].
- When $l \geq 4{,}5$, then Navier-Stokes is more convenient, as shown from our own measurements *(*see *Point B)* from the team of Mr Tabeling at ENS Paris (France), using a syringe pump for injected the flow rates and detected the channel pressure-by-pressure sensor.

These mathematical expressions can be used in a global simulation for micro-fluidic system (for example micropumps).

2. 1. 4: Nozzle-diffuser valve

The general task of a diffuser is to decelerate or accelerate a fluid flow by changing its cross section area along the flow axis. When it is used in opposite direction with converging cross-sections it is called a nozzle [7 and 8]; in this paper the diffuser channel used have rectangular cross sections. The structure of the passive microvalve “diffuse- *A(x)* and α are respectively the local variable along the stream axis, the local across sectional area, and angle of the diffuser element.

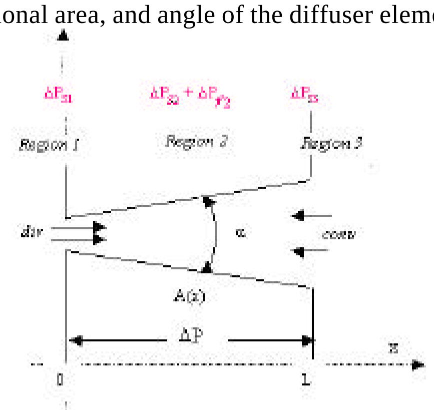

Figure. 7: Definitions of different pressure drops of the different regions.

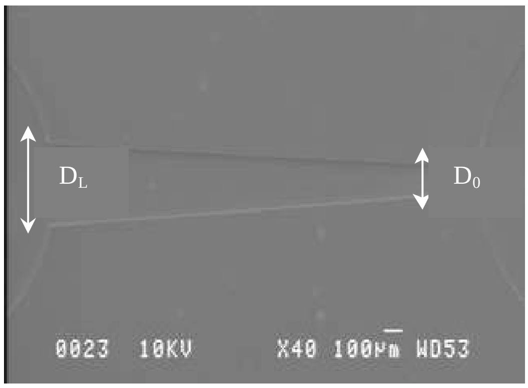

Fig. 8: Nozzle-diffuser fabricated using micro-machining technology (ESIEE - technology *DRIE*).

The geometry of the valve can be separated into three regions. In the region 1 and 3 the friction losses can be neglected and only acceleration losses have to be considered. In the second region the pressure loss is giving by sum of the acceleration (deceleration) and friction losses. The total pressure drop is calculated by the Bernoulli equation:

$$\Delta P_{Total} = \sum_{j=1}^{3} \Delta P_f + \sum_{i=1}^{3} \Delta P_s = \frac{1}{2} \rho K_{Total} \frac{\Phi^2}{A_0^2} + H.\Phi \qquad (9)$$

Where A_0 is the cross sectional area of the flow path $(x=0)$, K_{Total} is the total loss coefficient, in the divergent direction $K_{Total} = K_{div} = K_{d1} + K_{d2} + K_{d3}\left(A_0^2 / A_L^2\right)$ and in the convergent direction it is defined by $K_{Total} = K_{conv} = K_{c1} + \left(K_{c2} + K_{c3}\right)\left(A_0^2 / A_L^2\right)$ [for more details see Ref [8]].

In the above equation, the parameter H is used to simplify the expression, which can be described as:

The Hagen-Poiseuille law:

In this case the shape factor $1 \le \lambda \le 4.5$, the parameter $H = 32\eta \int_0^L \frac{dx}{D_h^2(x) \cdot A(x)}$, the explicit form of this integral is: $H = \frac{8\eta L}{n \cdot h^3}\left[\frac{1}{\left(D_L - D_0\right)} ln\left(\frac{D_L}{D_0}\right) + \frac{2h}{D_L D_0} + \frac{h^2\left(D_L + D_0\right)}{2D_L^2 D_0^2}\right]$

Where *h and n* are the height of channel, and the form factor due to the etching technology, D_0, D_L *are* the width for $x=0$ and $x=L$.

The Navier-Stokes law:

When the shape factor $\lambda \ge 4.5$, the formula: $H = 12\frac{\eta L}{h^3\left(D_L - D_0\right)} ln\left(\frac{D_L}{D_0}\right)$

Solving the Bernoulli equation with the explicit expressions for the H parameter we get for $\Phi(P)$:

$$F_{div} = \frac{-H + \sqrt{H^2 + 2\Delta P_{div} \cdot F_{div}}}{F_{div}} \qquad (10), \qquad F_{conv} = \frac{-H + \sqrt{H^2 + 2\Delta P_{conv} \cdot F_{conv}}}{F_{conv}} \qquad (11).$$

Where $F_{div} = \rho \frac{K_{div}}{A_0^2}$, $\qquad F_{conv} = \rho \frac{K_{conv}}{A_0^2}$

The Reynolds number can be calculated as follows:

$$Re = \frac{\rho D_h}{\eta}\left(\frac{\Phi}{A}\right) \qquad (12)$$

The Reynolds number in Eq. [12], defines the laminar rate flow, in this case the pressure is proportional to the velocity while in the turbulent mode we have a parabolic behaviour. At this stage it is important to notice that the efficiency of the micro valve can be calculated as a function convergent and divergent flow rate as we can see from the following expression: $E = \frac{\left|\Phi_{div} - \Phi_{conv}\right|}{\Phi_{div}}$. (13)

In order to have an important efficiency, λ must be close to one. As shown by the measures experiments, for λ in the range $4 \le \lambda \le 12$, a very low efficiency is obtained(figure. 9). In fact when h increases and approaches d, the inefficiency is maximal. The diffuser angle dependence is also studied by simulation. The optimised value is found to be around 5°(figure. 10).

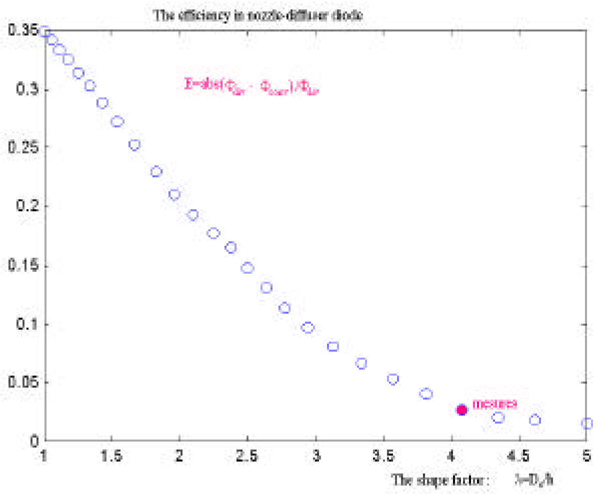

Figure. 9: the efficiency in a nozzle-diffuser valve at various λ, D_0=.100µm, D_L=300µm, L=2.5mm, ΔP=80000Pa(water)

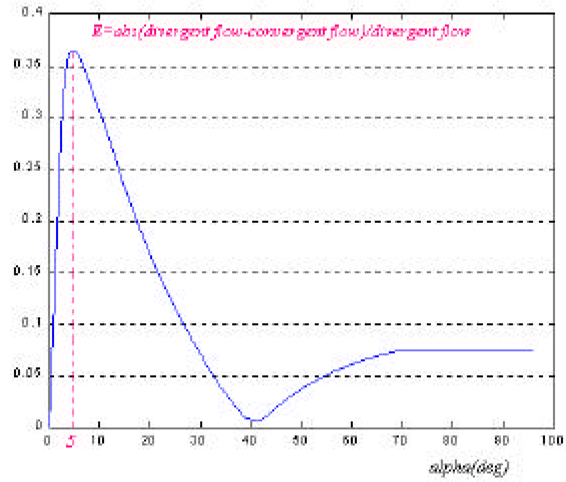

Figure. 10: the efficiency optimised.

In order to have an optimised efficiency value. Nozzle-diffuser may be a ratio factor (λ~1) and a angle 5°.

2.2. Flow simulation "study of tesla valve":

A Tesla diode [5] is similar to a valvular conduit. Its profile is presented in Figure 11. The flow profiles are given by numerical solution (ANSYS®) for a 650µm long, valve and 85µm hydraulic diameter with an internal wall [5]. The valvular conduit allows the fluid (water) to pass in the forward direction, and prevents it from the backward direction by geometric turbulence (Figs. 12 (a, b)):

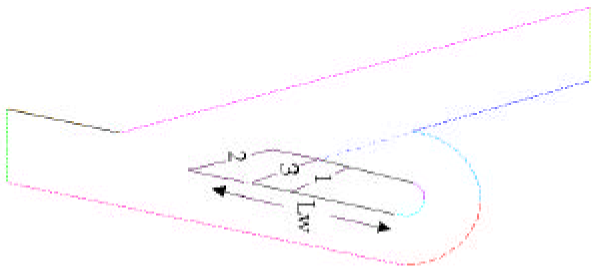

Figure. 11: Tesla channel structures*(1_2_3).*

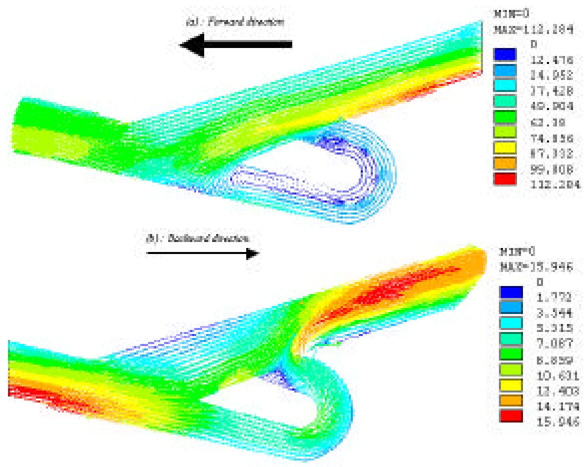

Fig. 12: Velocity distribution of the valve at a pressure of 60000Pa.
(a) : Forward direction.
(b): backward direction.

In order to study the influence of the internal wall length (L_w), simulations of a Tesla channel, for three L_w varied from 1 to 3 (Fig. 11), have been achieved and are shown in Fig. 11. It is interesting to see that forward flow Φ_F and backward flow Φ_B vary differently with L_w in Fig. 13.

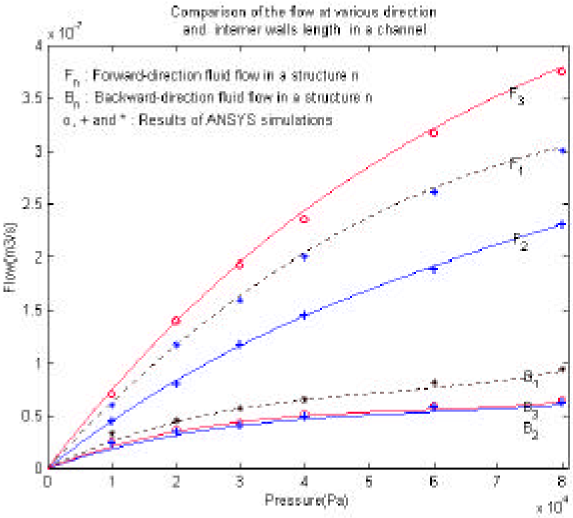

Fig. 13: Flow-pressure characteristic in a Tesla Channel.

Structure 3	
Forward direction	$F = 1.62e-22\ P^3 - 5.31\,e-17\ P^2 + 7.96\ e-12\ P$
Backward direction	$F = 2.24e-22\ P^3 - 3.84\ e-17\ P^2 + 2.42\ e-12\ P$

Table. 2 : Analytical expression of the Tesla diode flow extracted from numeric simulation.

It's important to notice that the optimal structure, denoted by (3), corresponds to a linear prolongation of the entry channel. From numerical results, an analytical expression has been extracted using the least square method (table. 2). For the pressure of 10kPa (corresponding in this case of a Reynolds number of 1000), the efficiency $E = (F_F - F_B)/F_F$ of the Tesla diode is 0,65.

As for the micro-channel, we have achieved some prototypes, which are under test.

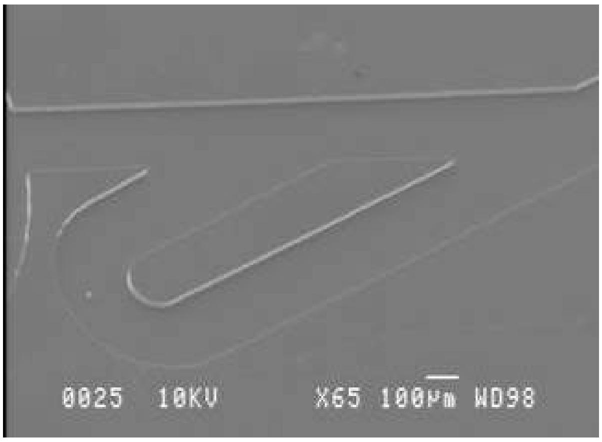

Figure. 14: valvular conduit fabricated using micromachining technology.

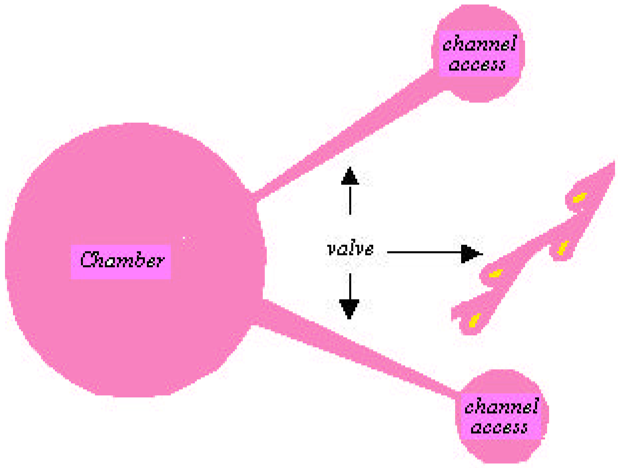

Figure. 15: Micropump structure and valves "nozzle-diffuser or Tesla valve".

Finally, there are two main advantages of using this optimised valve. First, an important efficiency $(E \geq .5)$, second, the valvular conduit contains no moving parts.

2. 3. Fabrication

Fig. 5, 8, 14 and 15 illustrate the photographs of channels fabricated in silicon wafers. These channels were obtained by etching a silicon wafer by Deep Reactive Ion Etching *(DRIE)* technique from 1µm to 200µm. The channels accesses are realised by a complete etching of the bottom side of the silicon wafer. An anodic bonding is done to close the channel with a Pyrex wafer.

The measures were done by forcing the flow in the channel with a syringe pump and measuring the loss pressure between the input and output channel with a differential pressure sensor. The measures were done by controlling the temperature of the global system.

3. CONCLUSION

In this paper, a global modelling of a rectangular micro-channel has been purposed in order to normalise its behaviour. From the value of the micro-channel shape factor, the flow characteristic can be rapidly deduced. Prototypes have been achieved and tested in order to validate the influence of the shape factor on the model. We have demonstrate the relationship between the efficiency of the diffuser-nozzle and its geometric form, in fact for a diffuser angle Alpha =5° and for ratio factor λ close to one we have a maximal efficiency. Then we have extended our study to more complex structure: the Tesla valve. Numerical simulations are presented and enable to optimise the valve efficiency according to its geometry, an analytic formula is extracted from the simulation. In the same way of micro-channel, prototype has been achieved. The direct application of these results is the possibility to integrate the analytic formula in a global simulator of microfluidic device using these structures.

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