

# Analytical Synthesis of Current Mode High-Order OTA-C Filters

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**Abstract:** This paper presents a new general synthesis method for high-order current mode Operational Transconductance Amplifiers and Capacitors, (OTA-C) filters. The method is based on the analytical solution of a single,  $n$ th-order generic filter transfer function and the generation of  $n$ , number of realizable transfer functions implemented using lossless integrators. The method provides systematic synthesis approach to high-order OTA-C filters. Furthermore, the method generates circuits that are optimal in terms of components count, and without trading off any of the VLSI features of OTA-C filters. Simulation results are included validating the synthesis method.

**Index Terms**---Synthesis, Active filters, Operational transconductance amplifiers

## I. INTRODUCTION

An elegant solution to a filter synthesis problem is to decompose analytically the required filter transfer function into a set of equations that can be realized using simple analog processing blocks such as integrators. This analytical approach is demonstrated in [1] in the case of the well-known 2<sup>nd</sup>-order Tow-Thomas active RC biquad. Analytical synthesis has benefits; for example the ability to generate filter circuits in a systematic and structured way, and the ability to obtain filter circuits with different transfer functions simply by suitable choice of equations. Also, analytical synthesis offers the designer explicit relations that provide insight into the filters behaviour.

Despite the importance of analytical synthesis approach, few methods have been reported [2-7, 11]. In [2-6], the synthesis of voltage-mode OTA-C filters was considered, and in [6-7, 11], the structure generation of current-mode OTA-C second-order filters was proposed. Probably, one of the main reasons for the lack of reported work on analytical synthesis of high-order active filters is due to the difficulties involved in developing a coherent approach for solving a single complex equation representing an  $n$ th-order filter transfer function with different responses; under the constraint that the generated set of equations should be realizable using simple circuitry. Also, mechanisms need to be developed for managing efficiently the intermediate solutions generated throughout the synthesis process, and deriving meaningful conclusions from the final solutions. The motivation of this work is to develop an efficient analytical synthesis approach for high-order current mode OTA-C filters. Such filters have become a creditable alternative to switched-capacitor filters, recent examples ([8]–[10] and the references cited therein). The main contributions of this paper are: firstly a new analytical synthesis method for current mode high-order OTA-C filters is proposed. Secondly, generation of OTA-C filter circuits that contain the least number of passive and active elements for a given filter order and function when compared with some recent previous works.

The paper is organized as follows. Section II, introduces the new synthesis method, Sections III and IV, provide design examples and conclusion, respectively.

## II. NEW SYNTHESIS METHOD

The method is basically based on the analytical decomposition of a single,  $n$ th-order generic filter transfer function into  $n$ , number of first order current mode transfer functions produced following the application of a succession of innovative algebra manipulation operations. The simple transfer functions are realized using integrator circuits with suitably selected current injection(s) at their inputs and connected using easy-to-follow rules, to yield the required filter order and function (low-pass, high-pass, band-pass, band-reject, all-pass).

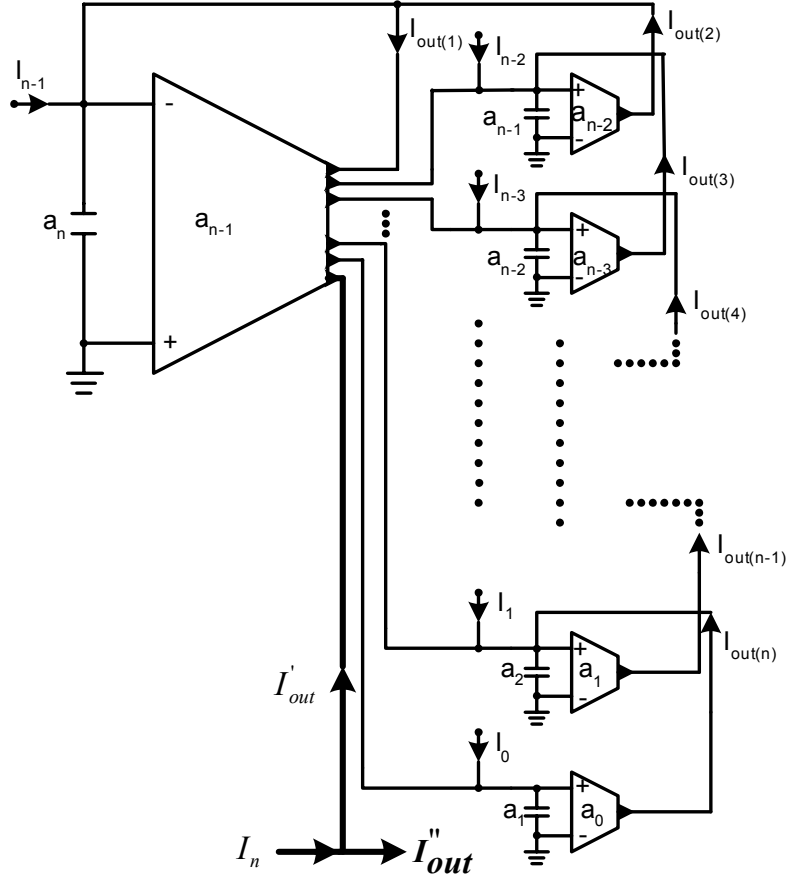


Fig.1. Nth-order OTA-C generic filter structure of the new synthesis method

The  $n$ th-order generic filter structure of the new synthesis method is shown in Fig.1, where  $I_n, I_{n-1}, \dots, I_0$  are the filter input currents whose setting determine the filter function (LP, HP, BP, BR, AP) as shown later, and  $I''_{out}$  is the filter current output. The filter structure of Fig.1 was obtained as follows.

The general transfer function of an  $n$ th-order filter, where the value of the coefficients ( $a_n, a_{n-1}, a_{n-2}, \dots, a_1, a_0$ ) determine the filter function is:

$$I_{out} = \frac{a_n I_n s^n + a_{n-1} I_{n-1} s^{n-1} + a_{n-2} I_{n-2} s^{n-2} + a_{n-3} I_{n-3} s^{n-3} + \dots + a_1 I_1 s + a_0 I_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0} \quad (1)$$

Let,  $I_{out} = I''_{out} + I'_{out}$

where, 
$$I''_{out} = \frac{a_n I_n s^n}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0} \quad (2)$$

$$I'_{out} = \frac{a_{n-1} I_{n-1} s^{n-1} + a_{n-2} I_{n-2} s^{n-2} + a_{n-3} I_{n-3} s^{n-3} + \dots + a_1 I_1 s + a_0 I_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0} \quad (3)$$

Cross multiply Eq. (3), divide by  $a_n s^n$ , and re-arrange, we obtain

$$I'_{out} = \sum_{i=0}^{i=n-1} \left( \frac{a_i}{a_n s^{n-i}} \right) (I_i - I'_{out}) \quad (4)$$

Let  $\frac{a_{n-i}}{a_n s^i} = \prod_{j=0}^{j=i-1} \left( \frac{a_{n-j-1}}{a_{n-j} s} \right)$  for  $i = 1, 2, 3, \dots, n-1, n$ , (5)

the last three terms of Eq. (4) have been re-arranged as

$$\begin{aligned} & \left( \frac{a_{n-1}}{a_n s} \right) (I_{n-1} - I'_{out}) + \left( \frac{a_{n-2}}{a_{n-1} s} \right) \left( \frac{a_{n-1}}{a_n s} \right) (I_{n-2} - I'_{out}) + \left( \frac{a_{n-3}}{a_{n-2} s} \right) \left( \frac{a_{n-2}}{a_{n-1} s} \right) \left( \frac{a_{n-1}}{a_n s} \right) (I_{n-3} - I'_{out}) \\ &= \left[ (I_{n-1} - I'_{out}) + \left[ (I_{n-2} - I'_{out}) + \left( \frac{a_{n-3}}{a_{n-2} s} \right) (I_{n-3} - I'_{out}) \right] \left( \frac{a_{n-2}}{a_{n-1} s} \right) \right] \left( \frac{a_{n-1}}{a_n s} \right) \end{aligned}$$

Generally, Eq. (4) becomes:

$$I'_{out} = \left[ (I_{n-1} - I'_{out}) + \left[ (I_{n-2} - I'_{out}) + \dots + \left[ (I_1 - I'_{out}) + \left( \frac{a_0}{a_1 s} \right) (I_0 - I'_{out}) \right] \left( \frac{a_1}{a_2 s} \right) \dots \right] \left( \frac{a_{n-2}}{a_{n-1} s} \right) \right] \left( \frac{a_{n-1}}{a_n s} \right) \quad (6)$$

If we let,  $I_{out(n)} = \left( \frac{a_0}{a_1 s} \right) (I_0 - I'_{out})$ , (7a)

Then from Eq. (6), we have the following (n-1) individual equations:

$$I_{out(i)} = (I_{n-i} - I'_{out} + I_{out(i+1)}) \left( \frac{a_{n-i}}{a_{n-i+1} s} \right), \text{ for } i = 1, 2, 3, \dots, n-1,$$

where,  $I_{out(1)} \equiv I'_{out}$  (7b)

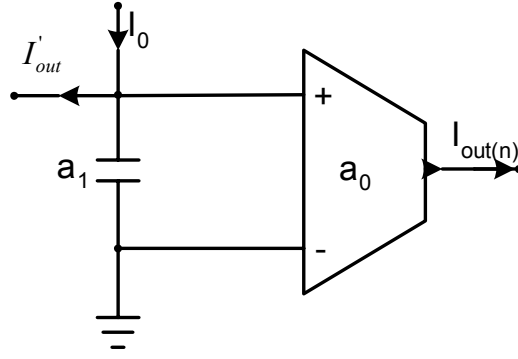


Fig.2. OTA-C realization of the integrator given by Eq. (7a)

Eq.(7a) represents an integrator, and can be realized using the circuit shown in Fig. 2, where  $a_0$  is the transconductance of the OTA, and  $a_1$  is the value of the capacitor. Implementing Eqs. 7(b) using similar circuits to that shown in Fig.2, the combination of the individual circuit yields the generic filter structure shown in Fig.1, a part from the two currents,  $I_n$  and  $I''_{out}$ , marked bold in Fig. 1. Note that all the OTA's are single-input active elements. The multiple output terminals of the OTA with transconductance  $a_{n-1}$  are easily obtained by using current mirrors.

Up to this point, only  $I'_{out}$  of the general transfer function has been implemented. Now,  $I''_{out}$  of Eq. (1) is implemented. From Eq. (2),

$$\frac{I''_{out}}{I_n} = \frac{a_n s^n}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0}$$

This equation can be re-written as

$$\frac{I''_{out}}{I_n} = 1 - \frac{a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0} \quad (8)$$

Let,  $I_{n-1}=I_{n-2}=I_{n-3}=\dots=I_1=I_0=I_n$  in Eq. (3) gives

$$\frac{I'_{out}}{I_n} = \frac{a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0}{a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0} \quad (9)$$

Substituting Eq. (9) into Eq. (8) yields:

$$I''_{out} = I_n - I'_{out} \quad (10)$$

Eq.(10) is easily realized as shown marked bold in the circuit of Fig. 1 but  $I''_{out} = I'_{out}$  under the only condition:  $I_n = I_{n-1} = I_{n-2} = \dots = I_1 = I_0 = I_{in}$ . Note if all the input currents of the structure in Fig. 1 are equal, i.e.,  $I_n = I_{n-1} = I_{n-2} = I_{n-3} = \dots = I_1 = I_0 = I_{in}$ , then the circuit shown in Fig.1 implements a high-pass filter of nth-order. To generate other

filtering functions beside high-pass from the circuit shown in Fig. 1, suitable setting of the filter input currents are needed as follows. Substituting Eq. (3) into  $I''_{out} = I_n - I'_{out}$ , shown in Fig. 1, yields

$$I''_{out} = \left( \frac{1}{\Delta} \right) \left[ a_n I_n s^n + \sum_{i=0}^{i=n-1} a_i s^i (I_n - I_i) \right]$$

$$\text{where } \Delta = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + a_{n-3} s^{n-3} + \dots + a_1 s + a_0 \quad (11)$$

This shows that if an  $n$ th-order inverting low-pass filter is required, setting  $I_0=I_{in}$ , and  $I_1=I_2=I_3=\dots=I_{n-2}=I_{n-1}=I_n=0$  in Eq. (11) will produce a low-pass response.

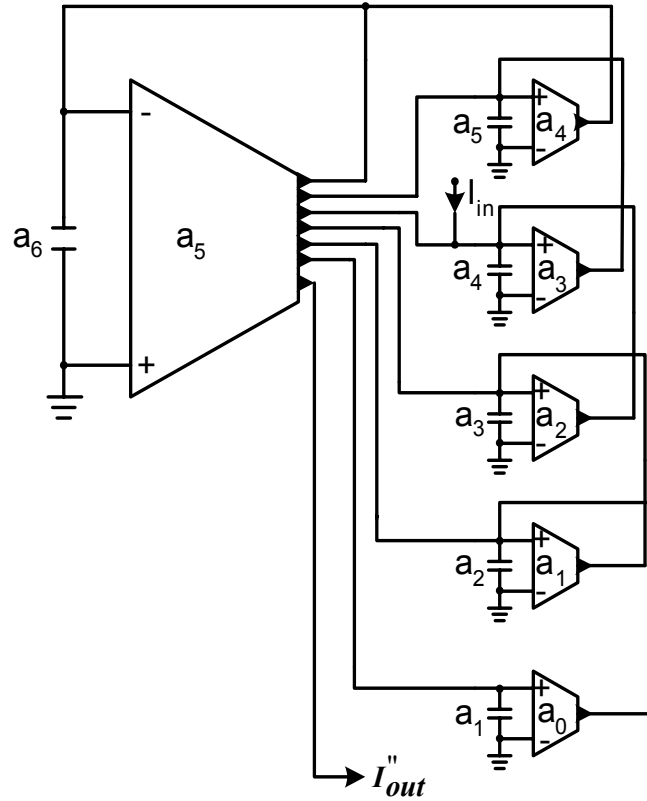


Fig.3. 6<sup>th</sup>-order band-pass OTA-C filter obtained using the new synthesis method

Other filter transfer functions are obtained according to input current conditions:

(i) Band-pass: If  $n$  is even, then  $I_{(n/2)}=I_{in}$ , whilst all the other input currents are zero. If  $n$  is odd, then the input current  $I_{in}$ , is applied to either  $I_{[(n/2)-(1/2)]}$  or  $I_{[(n/2)+(1/2)]}$ . For example, Fig.3 shows a 6<sup>th</sup>-order band-pass filter using the new synthesis technique. The filter output is

$$I''_{out} = \frac{-a_3 s^3 I_{in}}{a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (12)$$

(ii) Band-reject: Set  $I_n=I_{n-1}=I_{n-2}=\dots=I_1=I_{in}$ , and  $I_0=0$ .

(iii) All-pass: If  $n$  is even, then  $I_n=I_{in}$ ,  $I_{n-1}=2I_{in}$ ,  $I_{n-2}=0$ ,  $I_{n-3}=2I_{in}$ ,  $I_{n-4}=0$ , ...,  $I_2=0$ ,  $I_1=2I_{in}$  and

$I_0=0$ . If  $n$  is odd, then  $I_n=I_{in}$ ,  $I_{n-1}=2I_{in}$ ,  $I_{n-2}=0$ ,  $I_{n-3}=2I_{in}$ ,  $I_{n-4}=0$ , ...,  $I_2=2I_{in}$ ,  $I_1=0$  and  $I_0=2I_{in}$ .

If we replace the transconductances  $a_0, a_1, a_2, \dots, a_{n-2}$ , and  $a_{n-1}$  with  $g_1, g_2, g_3, \dots, g_{n-1}$ , and  $g_n$ , and the capacitors  $a_1, a_2, a_3, \dots, a_{n-1}$ , and  $a_n$  with  $C_1, C_2, C_3, \dots, C_{n-1}$ , and  $C_n$  in Fig. 1. Then the transfer function of the circuit in Fig. 1 is shown as follows.

$$I_{out}'' = \frac{\left[ I_n s^n + (I_n - I_{n-1})s^{n-1}b_n + (I_n - I_{n-2})s^{n-2}b_n b_{n-1} + \dots \right. \\ \left. \dots + (I_n - I_1)sb_n b_{n-1} b_{n-2} \dots b_3 b_2 + (I_n - I_0)b_n b_{n-1} b_{n-2} \dots b_3 b_2 b_1 \right]}{s^n + s^{n-1}b_n + s^{n-2}b_n b_{n-1} + \dots + sb_n b_{n-1} b_{n-2} \dots b_3 b_2 + b_n b_{n-1} b_{n-2} \dots b_3 b_2 b_1}$$

$$= \frac{I_n s^n + \sum_{i=0}^{n-1} \left[ (I_n - I_{n-i-1})s^{n-i-1} \prod_{j=n-i}^{j=n} b_j \right]}{s^n + \sum_{i=0}^{n-1} \left[ s^{n-i-1} \prod_{j=n-i}^{j=n} b_j \right]} \text{ where } b_j = \frac{g_j}{C_j} \text{ for } j = 1, 2, 3, \dots, n-1, n. \quad (13)$$

Note that the transfer function shown in Eq. (13) has the same denominator as that proposed in [18].

A comparison in terms components count (passive and active) between filters generated using the new synthesis method and five recently proposed methods is given. In [12] a method for the design of high order OTA based filters using coupled biquads was proposed. It was shown that a 5<sup>th</sup>-order all-pole low-pass filter has seven OTAs and seven grounded capacitors. Using the proposed synthesis method, 5<sup>th</sup>-order low-pass filter has five OTAs and five grounded capacitors. Also, it was shown in [12], that a 6<sup>th</sup>-order band-pass filter has nine OTAs, and six grounded capacitors, compared with six OTAs and six grounded capacitors when the new method is used. Consider the work in [13], it was shown that using the inductor substitution method five OTAs, and three grounded capacitors are required in the case of a 3<sup>rd</sup>-order low-pass filter, and six OTAs and three grounded capacitors in the case a 3<sup>rd</sup>-order high-pass filter. Using the new method, 3<sup>rd</sup>-order low-pass and high-pass filters have three OTAs and three grounded capacitors. Consider the method in [14], it was shown that a 2<sup>nd</sup>-order notch filter or 2<sup>nd</sup>-order high-pass filter has three OTAs, two grounded capacitors and a resistor, compared with two OTAs and two grounded capacitors when the proposed synthesis method is used to design the same filters. In [15], a method for designing current mode OTA-C filters based on simulation of LC ladder was reported. Although, the method produce filters that have  $n$ , single-ended OTAs and  $n$ , grounded capacitors, but they also require current amplifiers and buffers, increasing the complexity of the filter circuit. In [16], a general class of current-mode high-order OTA-C filters by employing a current-mode integrator and a proportional block as basic building units was proposed. It was shown that six OTAs and four grounded capacitors are required to realise a fourth-order filter. However, only four OTAs and four grounded capacitors are required to realise the same filter by using the proposed analytical synthesis method.

### III. DESIGN EXAMPLES

To verify the theoretical analysis of the proposed synthesis method, two filters have been synthesized and simulated using P-Spice. All presented simulation results have been carried out using the multiple output transistor-level OTA circuit given in [17], with level

2 models. The first filter is a 3<sup>rd</sup>-order all-pass having the normalized transfer function,  $(2s^3 - s^2 + s - 2)/(2s^3 + s^2 + s + 2)$ , with a centre frequency of 1 MHz. The filter components are:  $a_0=95 \mu\text{S}$ ,  $a_1=a_2=190 \mu\text{S}$  for transconductances; and  $a_3=15 \text{ pF}$ ,  $a_1=a_2=30 \text{ pF}$  for capacitor values. The all-pass filter circuit and simulated phase frequency response are shown in Figs. 4 and 5 respectively. As can be seen from Fig. 5, there is a close agreement between theory and simulation. It should be noted that the all-pass filter has a magnitude deviation in the pass-band of  $-0.593\text{dB}$  ( $=20\log 0.934\text{dB}$ ), compared to the  $0\text{dB}$  ( $=20\log 1\text{dB}$ ) deviation in the ideal case. The second filter is a 6<sup>th</sup>-order band-pass filter. The filter circuit and simulated magnitude frequency response are shown in Figs. 3 and 6, respectively. The filter component values are:  $a_0=a_1=a_2=a_3=a_4=a_5=95\mu\text{S}$  for transconductances, and  $a_1=a_2=a_3=a_4=a_5=a_6=15\text{pF}$  for capacitor values. Again, as can be seen there is a good agreement between theory and simulation. Note we have used the same labelling ( $a_i$ ) to represent the transconductance of the OTAs and the capacitor values to ensure consistency with the synthesis method described in Section II. The need for using the same symbols for transconductances and capacitors comes from the fact that the presented method is an analysis starting from the  $n$ th-order transfer function, deriving step by step, and then generating  $n$  equations realizable for simple OTA-C circuits. Other filters with different orders and responses including low-pass, and high-pass, have been simulated using the presented synthesis method and found to perform as theory predict.

#### IV. CONCLUSION

It has been shown that it is possible to synthesis high-order current mode OTA filters using an analytical approach. A new synthesis method and its associated filtering structure have been presented. The method produces filters that have the *least* number of passive and active components; indeed they generate canonical structures. A useful feature of the synthesis method is that different filter orders and functions can be obtained from the general filter structure (Fig.1) by suitable selection of current injection(s) at the filter inputs. The presented method should provide valuable addition to the area of analog filter synthesis. Whilst this paper has focused on the analytical synthesis of current-mode OTA-C filters, some of the presented algebra manipulation operations can be applied to voltage-mode filters. However, there are still some challenging problems in developing a complete voltage-mode synthesis approach which is the subject of current research by the authors.

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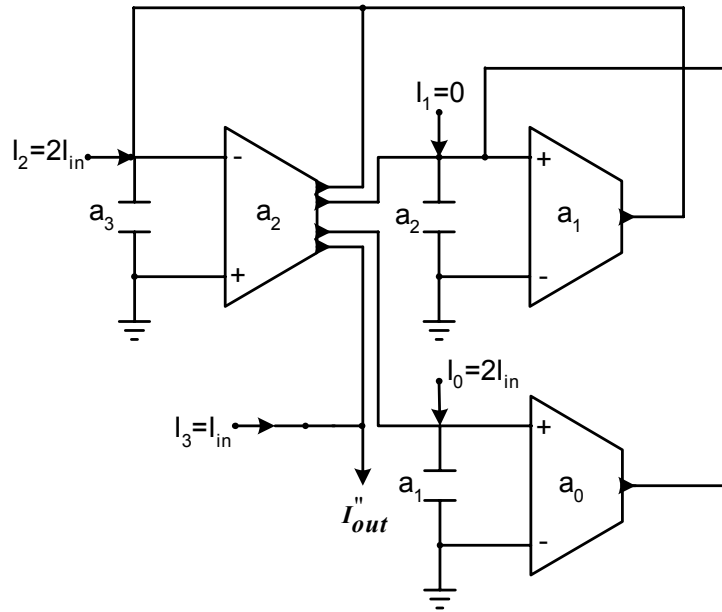


Fig. 4. 3<sup>rd</sup>-order current-mode all-pass OTA-C filter.

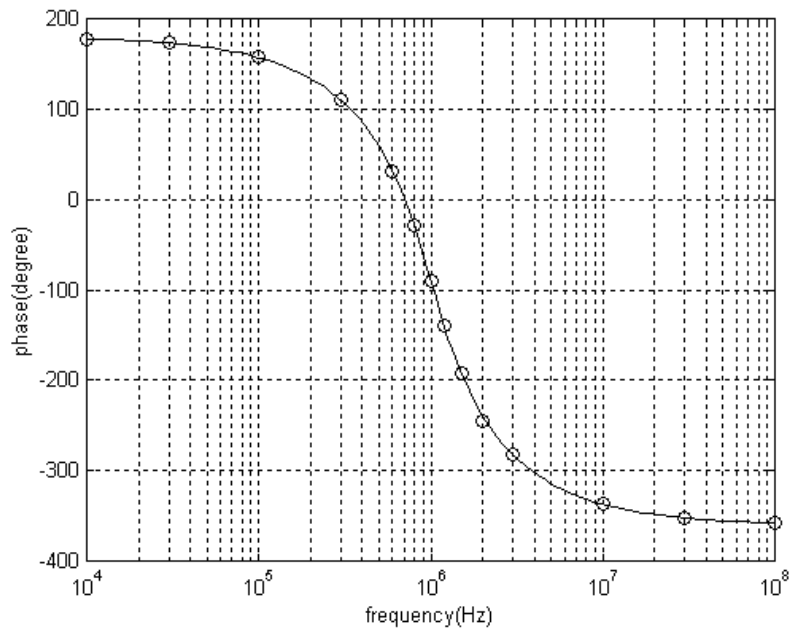


Fig. 5. Phase-frequency response of circuit in Fig. 4 (---, theoretical; ooo, simulation.)

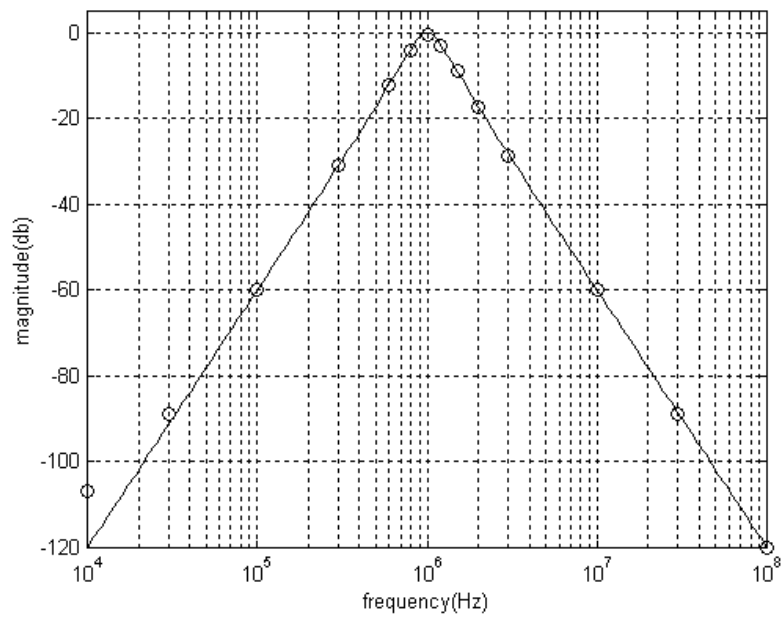


Fig. 6. Frequency response of circuit in Fig. 3 (---, theoretical; ooo, simulation.).