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Analyzing Musical Structure and Performance—A Statistical Approach

Jan Beran and Guerino Mazzola

Abstract. Musical performance theory and the theory of musical structure in general is a rapidly developing field of musicology that has wide practical implications. Due to the complex nature of music, statistics is likely to play an important role. In spite of this, up to the present, applications of statistical methods to music have been rare and mostly limited to a formal confirmation of results obtained by other methods. The present paper introduces a statistical approach to the analysis of metric, melodic and harmonic structures of a score and their influence on musical performance. Examples by Schumann, Webern and Bach illustrate the proposed method of numerical encoding and hierarchical decomposition of score information. Application to performance data is exemplified by the analysis of tempo data for Schumann's "Träumerei" op. 15/7. The paper demonstrates why statistics should play a major active part in performance research. The results obtained here are only a starting point and should, hopefully, stimulate a fruitful discussion between statisticians, musicologists, computer scientists and other researchers interested in the area.

Key words and phrases: Bandwidth, cluster analysis, hierarchial smoothing, kernel smoothing, musical analysis, music, musicology, performance theory, regression, tempo.

1. INTRODUCTION

This paper grew out of the attempt to gain better understanding of musical performance of western classical music for which a clearly defined score exists. A second issue that has to be addressed as a prerequisite is whether and how the metric, melodic and harmonic structure of a musical score can be translated into numerical data.

In the last few years, there has been an increasing interest in modelling musical performance. This development is in particular due to dramatic advances in computer technology and the availability of digitized score and performance data. By the very nature of music, "understanding" musical performance is a multidisciplinary task, including in particular aspects of music theory, semiotics, psychology, mathematics, physics, and music history. Nontrivial models of performance must be general enough to incorporate the broad variety of aspects appropriately. In particular, a useful theory of performance has to take into account that there may be a large, or even an infinite, number of adequate performances of the same score. Similarly, experience suggests that, from the point of view of the listener or critic, there is no unique way of "explaining" a performance, that is, of uniquely identifying properties of the score that caused specific features of the physical performance. In view of these general facts, a realistic theory of musical performance cannot be purely causal and deterministic. Statistical and probabilistic methods are therefore likely to play an important role. Nevertheless, up to now, applications of statistical methods to questions in musicology and performance research have been very rare [9, 31, 36, 42, 44]. Apart from a few information theoretical approaches (see, e.g., [44]), most applications of statistics consist of using standard statistical methods as an additional confirmation of results or conjectures that had been "derived" before by psychological or musicologi-

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FIG. 1. *"Träumerei" op. 15/7 by R. Schumann.*

cal reasoning (see, e.g., [9, 36]). In contrast, the aim of our paper is to demonstrate that statistics can play a major *active and more substantial* role in performance research and musicology in general, provided that the special nature of the musical context is taken into account appropriately. The possibly controversial suggestions presented here may stimulate discussions among interested scientists.

The examples discussed here are Schumann's "Träumerei" op. 15/7, Webern's second Variation for Piano op. 28/2, Bach's Canon Cancricans from *Das Musikalische Opfer* BWV 1079 and Schumann's "Kuriose Geschichte" op. 15/2. The scores are dis-

FIG. 2. *Variation für Klavier op. 28/2 by A. Webern.*

played in Figures 1–4. The main focus here will be on investigating the relationship between a score and its performance when score information is coded in a numerical form. The method for extracting numerical data from a score will be discussed only briefly here. For a more detailed account of extracting numerical information from scores see [24]–[22] and [27]. The relationship between score and performance will be illustrated using tempo measurements for Schumann's "Träumerei" op. 15/7. The tempo measurements of 28 performances by various famous pianists were provided to us by

FIG. 3. *Canon Cancricans by J. S. Bach.*

Bruno Repp [32]. The performers are listed in Table 1; see [31] for the complete data.

The reason for restricting attention to the *tempo* of a performance is that only these data were available to us at the present. Clearly, many other aspects of performance will need to be considered in future research. The main question asked with respect to the tempo data is what kind of relationship—if any—there is between the score and the tempo curve of a performance. Note that again this is a simplification, since no other information than that contained in the printed score is considered. A more complete performance analysis would have to include emotional and gestural rationales.

A major difficulty is that it is not clear a priori which information a score contains and how this information can be quantified. Apart from obvious information such as the sequence in which notes are played, a score usually contains explicit but ambiguous information (such as *ritardando*, *accelerando*, *andante*, *p*, *ff*, etc.) as well as hidden structural information. The latter consists, for example, of the harmonic, metric and melodic structure, motivic relationships and so on. This hidden information can be highly complex and ambiguous. Moreover, it is not clear up to which depth such hidden information should be extracted in order to be relevant for the performance. A detailed discussion of this problem involves musicological considerations that would be beyond the scope of this paper. Therefore, only a brief description of the basic principles is given here (in Section 2) that lead to so-called metric, harmonic and melodic weights (or curves).

The structure of the paper is as follows. A brief description of the general background and of a method for encoding score-specific information is given in Section 2.1. The approach yields basic metric, melodic and harmonic weights obtained from an analysis of the score that can be used for the further analysis. The definitions are based on a mathematical music theory first developed in [21] and performance theory, developed in [25]. Software [19] based on this music theory has been used successfully by various composers in the past (e.g., Jan Beran [2, 3, 5], Kurt Dahlke, Wilfried Jentzsch, Guerino Mazzola [20], Dieter Salbert, Karl-Heinz Schöppner, Tamas Ungvary). Some examples of weight curves are discussed in Section 2.2. Section 2.3 discusses the decomposition of these weights according to a natural score-specific hierarchy. In Section 2.4, the results are used to define a matrix Z of explanatory score-variables for Schumann's "Träumerei" that can be related subsequently to the observed tempo y. Section 3 discusses the relationship between Z and y and the consequences for performance theory. Remarks in Section 4 about open statistical problems and the general significance of the proposed method for the analysis

FIG. 4. *"Kuriose Geschichte" op.* 15/2 *by R. Schumann.*

TABLE 1
Repp's list of performances

Abbreviation	Artist	Year of performance
ARG	Martha Argerich	<1983
ARR	Claudio Arrau	1974
ASH	Vladimir Ashkenazy	1987
BRE	Alfred Brendel	<1980
BUN	Stanislav Bunin	1988
CAP	Sylvia Capova	<1987
CO1	Alfred Cortot	1935
CO2	Alfred Cortot	1947
CO3	Alfred Cortot	1953
CUR	Clifford Curzon	1955
DAV	Fanny Davies	1929
DEM	Jörg Demus	1960
ESC	Christoph Eschenbach	<1966
GIA	Reine Gianoli	1974
HO1	Vladimir Horowitz	1947
HO2	Vladimir Horowitz	<1963
HO3	Vladimir Horowitz	1965
KAT	Cyprien Katsaris	1980
KLI	Walter Klien	?
KRU	André Krust	1960
KUB	Antonin Kubalek	1988
MOI	Benno Moisewitsch	1950
NEY	Elly Ney	1935
NOV	Guiomar Novaes	<1954
ORT	Cristina Ortiz	<1988
SCH	Artur Schnabel	1947
SHE	Howard Shelley	<1990
ZAK	Yakov Zak	1960

and synthesis of musical performance conclude the paper.

2. GENERAL BACKGROUND, ENCODING SCORE INFORMATION, EXAMPLES

2.1 Metric, Melodic and Harmonic Weights

In the context of classical western music tradition, performance deals with a complex transformation process that starts from the data of a given score and produces physical sound data. The present status of performance theory is still in its beginning [16]. Recent references to performance theory are, for instance, [25, 38, 39, 40, 41, 17, 11 and 22].

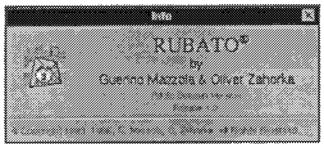
To date, the most investigated subject of performance theory—including corresponding software—is *agogics*, that is, timing microstructure of tempo curves (see [16, 23]). The present work deals with this subject: agogics as an expression of the analysis of harmonic, melodic and rhythmic structures. We investigate *empirically* the basic question *whether, and in which mathematical–statistical conceptualization, agogics may be viewed as being an expression of structural data* obtained from a specific set of musicological analyses.

The numerical encoding of score structures is done by the RUBATO software for musical analysis and performance [24]. In the following, we briefly discuss the approach used by RUBATO. A detailed account of the method can be found in [22, 24, 25, 27 and 46].

A musical score consists of a set of symbols, or "events," with specific (though not always uniquely defined) meanings. In particular, for piano music, a simplified representation of the event "note" in the score is $(t, e) = (t, e_1, e_2, e_3)$, where t is "onset" (the symbolic time in the score at which the note e occurs) and the components of the note $e = (e_1, e_2, e_3)$ are "pitch," "loudness" and "duration," respectively. The notes in a musical score (usually) follow certain structures of compositional grammars. Which relevant structure there is in a score is the subject of (structural) musical analysis. Here we consider three major types of structure, namely, the metric (rhythm), harmonic and melodic structures. In the present approach to performance theory, it is assumed that, depending on the metric, harmonic or melodic structure, each note in the score can be assigned a "weight" that measures its metric, harmonic or melodic "importance," respectively. The two main tasks that have to be carried out are therefore: (1) metric, harmonic and melodic analysis of the score and (2) translation of the analysis into numerical weights of "importance" for each note in the score. In RUBATO, explicit rules, derived from general music theory and practice, are used for analyzing a score and for transforming the results into numerical weights. The user can control the depth and the type of the analysis (by a set of rules), as well as the impact on the resulting weights, by setting various parameters in defined analytical subroutines (so-called Rubettes). Figure 5 shows the flow chart of RUBATO.

The metric analysis used by RUBATO essentially considers all periodically repeating metric structures, so-called local meters. A note in the score is metrically important if it is part of many local meters. The melodic analysis considers similarities between musical motives (configurations of notes) of the score. Essentially, a note is considered melodically important if it is part of motifs that are similar to many other motifs that occur elsewhere in the score. Finally, the harmonic analysis gives higher weights to notes that are harmonically important in the sense of an extended classical theory of harmony (Riemann theory [34]).

As final result, each note event (t, e), at score onset time t is associated with a metric, melodic and



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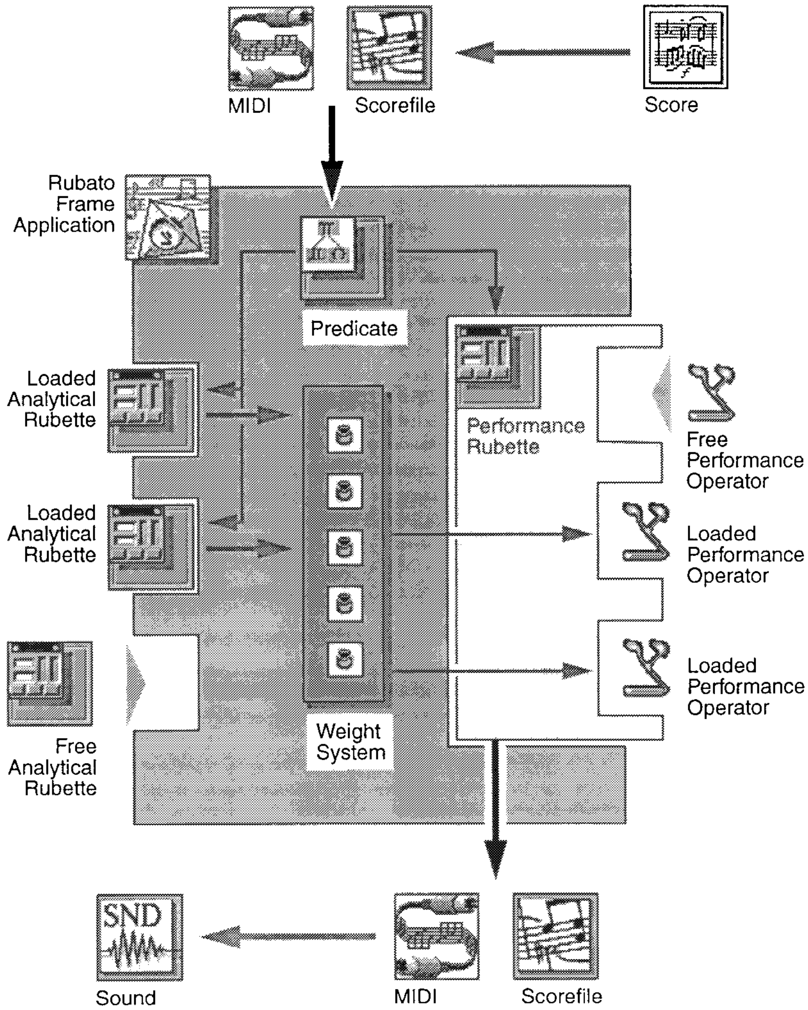

FIG. 5. *Flow chart of the RUBATO platform.*

harmonic weight. [We adopt time units which literally reproduce the score data for duration; i.e., a quarter note has a (symbolic) duration of 0.25, and a 4/4 bar has duration 1.] In principle, at a given score-onset time t, an arbitrary number of note events can occur, for instance, in a chord. Thus, at time t, we may have more than one associated metric, harmonic and melodic weight. To simplify the analysis below, for onset times with multiple note events, the average weight (averaged over all notes occurring at the given onset time) will be used. Thus, for each onset time t with at least one score event, we then have three weights $x_{metric}(t)$, $x_{melod}(t)$ and $x_{harmo}(t)$.

2.2 Four Examples

As previously indicated, the following scores are considered here: Schumann's "Träumerei" op. 15/7 (Kinderszene No. 7), Webern's second Variation for Piano op. 27/II, the Canon Cancricans from Bach's *Musikalisches Opfer* BWV 1079 and Schumann's "Kuriose Geschichte" op. 15/2 (Kinderszene No. 2). Figure 6a–d displays the melodic (dotted, middle), metric (full, lower) and harmonic (dashed, upper)

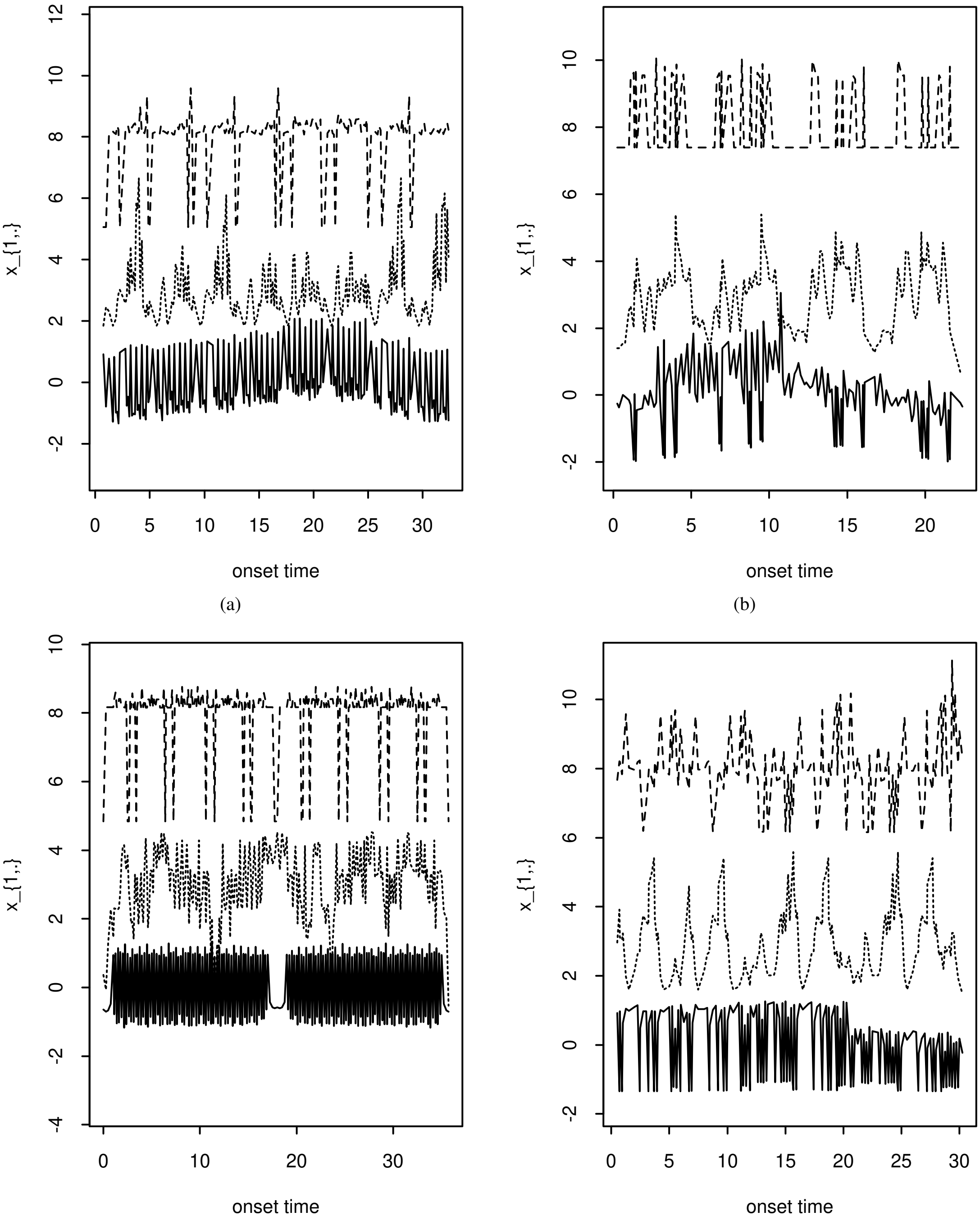

FIG. 6. *Metric, melodic and harmonic weights for* (a) *Schumann's "Träumerei,"* (b) *Webern's Variation op.* 27/2, (c) *Bach's Canon Cancricans and* (d) *Schumann's "Kuriose Geschichte."*

weights for these scores. (The scores themselves appear in Figures 1–4.) For onset times with more than one value of the melodic and harmonic weight, respectively, the average of the values was taken.

It is interesting to look at scatterplots of the three types of weights against each other. Figure 7 shows, for instance, the scatterplots for Bach's Canon Cancricans. For each of the compositions, some simple regular features of the weights can be seen:

- "*Träumerei*"—The score reveals that this composition may be divided into four disjoint parts P_j, $j = 1, 2, 3, 4$, corresponding to the onset intervals $I_1 = [0, 8]$ and $I_j = ((j-1) \cdot 8, j \cdot 8]$, $j = 2, 3, 4$, respectively. These four parts are similar

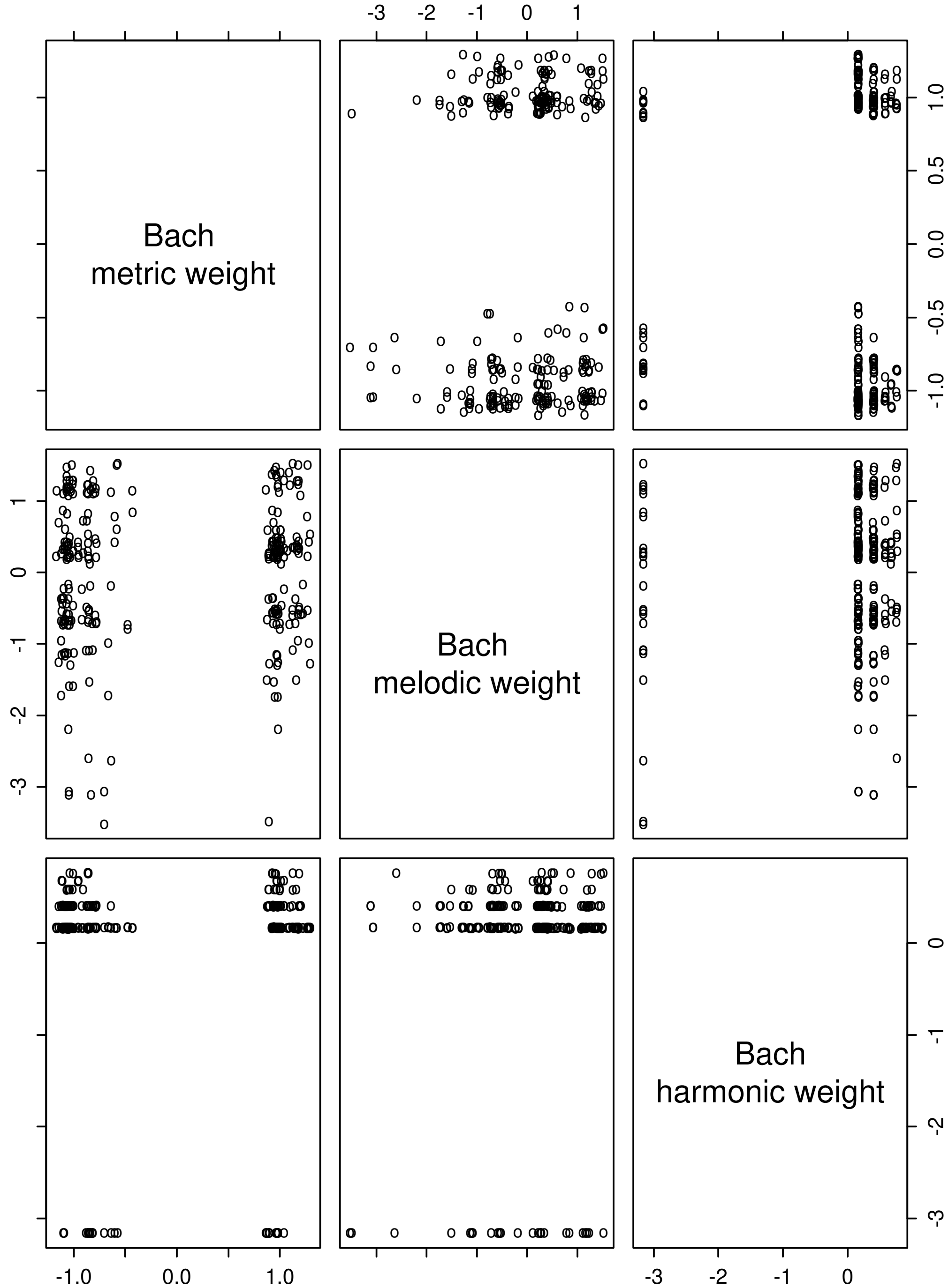

FIG. 7. *Scatterplots of analytic weights for Bach's Canon Cancricans.*

to each other, with P_3 differing most from the other parts. In fact, P_2 is, by definition, an exact replicate of P_1 (except for the slightly different upbeat). In Figure 8a–c, the metric, melodic and harmonic weights for the four parts are plotted on top of each other (i.e., onset time is taken modulo 8). The weights are indeed almost identical to each other. Interestingly, the fact that P_3 differs most from the other parts shows only for the melodic weights. Also, the scatterplots of the weights against each other (not shown) do not indicate any strong relationships between the three weight functions. The sample correlations are all in the range $[-0.01, 0.09]$.

- *Variation op. 27/II*—With respect to the melodic and harmonic weights, and from the score, this

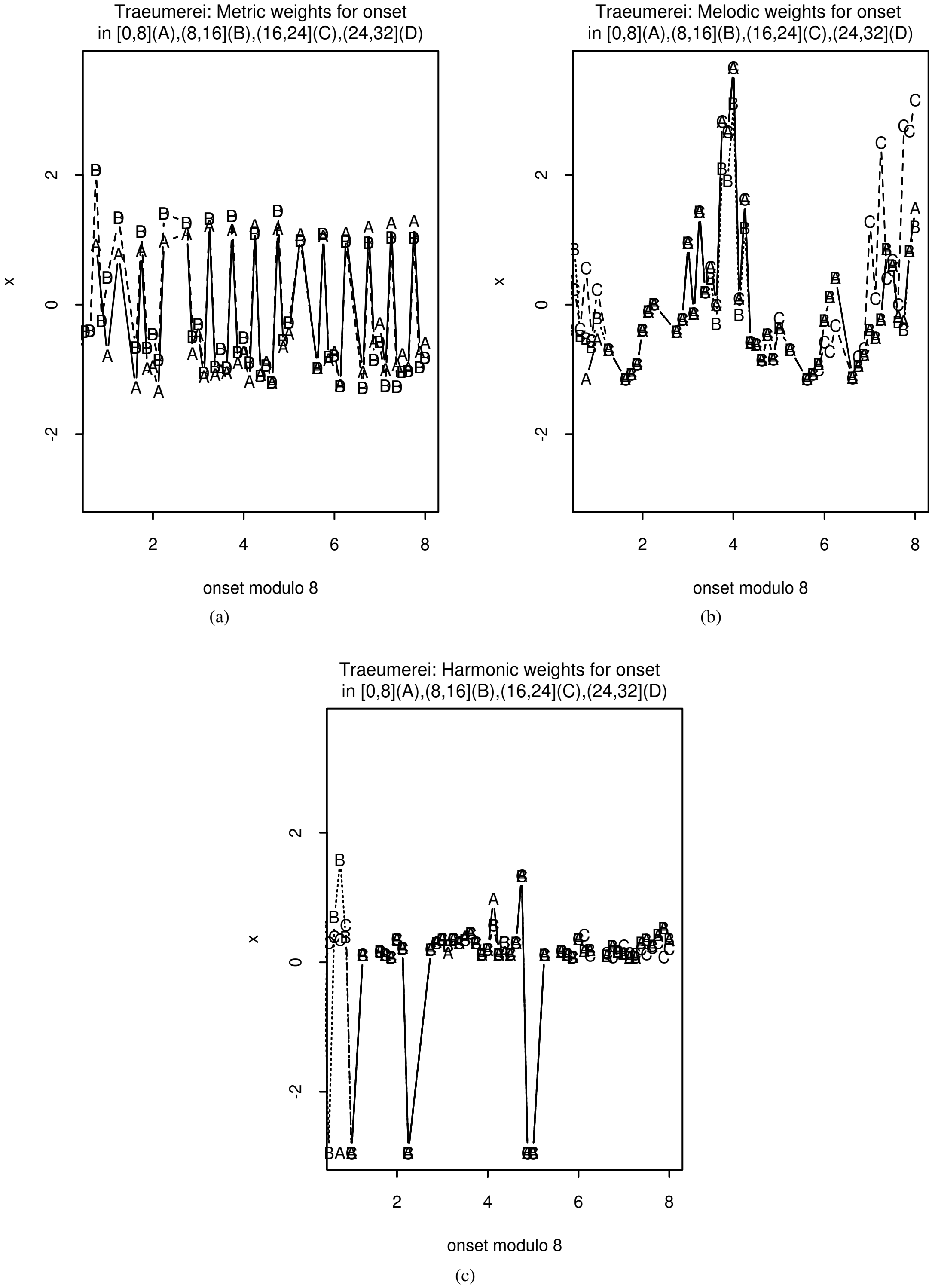

FIG. 8. *Analytic weights for Schumann's "Träumerei" against onset time modulo* 8.

composition again can be seen to be divided into four disjoint parts $P_j,\ j = 1, 2, 3, 4$, corresponding to a division of the onset time into four intervals of equal length. The first two parts are almost identical with respect to the melodic and the harmonic weights. The same is true for the last two parts. For the metric weights, however, P_2 is not a simple replicate of P_1. The same is true for the last two parts. Also, for P_1 and P_2, the maximal values of the metric weights are much higher than for P_3 and P_4. Again, no apparent relationship is shown in the scatterplots between the three weights (not shown). However, the largest correlation (in absolute value) is much larger than in the previous example: namely, -0.31 between metric and harmonic weights.

- *Canon Cancricans*—As expected for a retrograde canon, there is an almost exact time symmetry with respect to the middle of the onset axis. The symmetry is not exact, because the retrograde is not just a reflection of onsets but a transvection in the onset-duration space, parallel to the onset axis (see, e.g., [21]). Also striking is the clustered nature of the weights and the apparently very regular high-frequency oscillation of the metric curve. A high metric weight is almost always succeeded by a low weight and vice versa. Because of the clustered nature of the weights, it is more difficult to detect relationships from scatterplots. The correlations between the weights are again very small, ranging between 0.03 and 0.04.
- "*Kuriose Geschichte*"—Here, the score is again divided into four disjoint parts corresponding to the onset intervals [0,6], (6,12], (12,21], (21,30], with P_1 a repetition of P_2 and P_3 a repetition of P_4. Again, it is difficult to tell whether and how closely the three different curves may be related to each other. Note, however, that the metric weights are much lower for onset times above 21. Thus, for the metric weights, the correspondence between P_3 and P_4 is much weaker. The reason is the breakdown of local meters at bar 21. Similarly to Webern, the strongest correlation between the weights is quite remarkable, namely, -0.33 between melodic and harmonic weights.

It is also interesting to compare the four compositions with each other. The weights of Bach's Canon Cancricans exhibit an extreme high-frequency oscillation that is not observed for the other scores. Ignoring that onset times are not exactly equidistant, this can be seen for instance by comparing the sample autocorrelations of the metric weights (Figure 9).

Another property of interest is the marginal distribution of the weight functions after eliminating global "trends" by taking first differences $x(t_j) - x(t_{j-1})$. Figure 10 shows, for instance, the histograms for metric weights. For the compositions by Schumann and Bach, the first differences of the metric weights can essentially be classified into three clusters (low, medium, high). For Webern's score, the distribution is completely different and in fact rather close to a normal distribution. In contrast, the distributions of the differenced melodic weights turn out to be qualitatively similar for all four scores. Finally, all distributions for the harmonic weights appear to be essentially symmetric. However, while for Schumann's "Träumerei" and the score by Webern there appear to be three clusters, the histograms for Bach and the "Kuriose Geschichte" are essentially unimodal.

In summary, a first look at the weight functions has revealed certain elementary features of the score. In the following it will be demonstrated that a more thorough analysis leads to further new insights about the structure of the scores. In particular, note that the three weight functions were defined by a completely different analytical approach. It may therefore be expected that there is no strong relationship between the curves. The scatterplots of the weights seem to support this conjecture. However, the following analysis will show that certain components of the weight functions are indeed closely related.

2.3 Hierarchical Decomposition of Weight Functions

2.3.1 *General motivation.* As remarked above, the weight functions display certain "obvious" features of the scores. Can additional structural insight be gained by suitable analysis of the analytic weights? The idea of the following method is to find a "natural" decomposition of the weight functions in order to find hidden regularities. In time series terminology, the general problem can be stated as follows: Let $\{x_s(t_i),\ t_i \in R,\ s = 1, \dots, k, i = 1, \dots, n\}$ be a collection of k time series, measured at the time points t_i. The aim is to find a decomposition

$$x_s(t_i) = \sum_{j=1}^{M} x_{j,s}(t_i)$$

such that the components $\{x_{j,s},\ s = 1, \dots, k\}$ reveal a maximal amount of "regular structure." One of the difficulties is to define what is meant by "regular structures" and to define corresponding meaningful measures of the amount of "regular structure." Here, a pragmatic approach is taken, in that the amount of "regular structure" is judged visually. Clearly, more formal definitions could be used.

Before introducing the idea of hierarchical decomposition, a few general remarks should be made:

REMARK 1. Traditionally, one of the main structures of interest for time series is periodicity. In particular, spectral decomposition based on sines and cosines may be used for this purpose (see, e.g., [7, 30]). In our context, this is not applicable, because many compositions are likely to have much

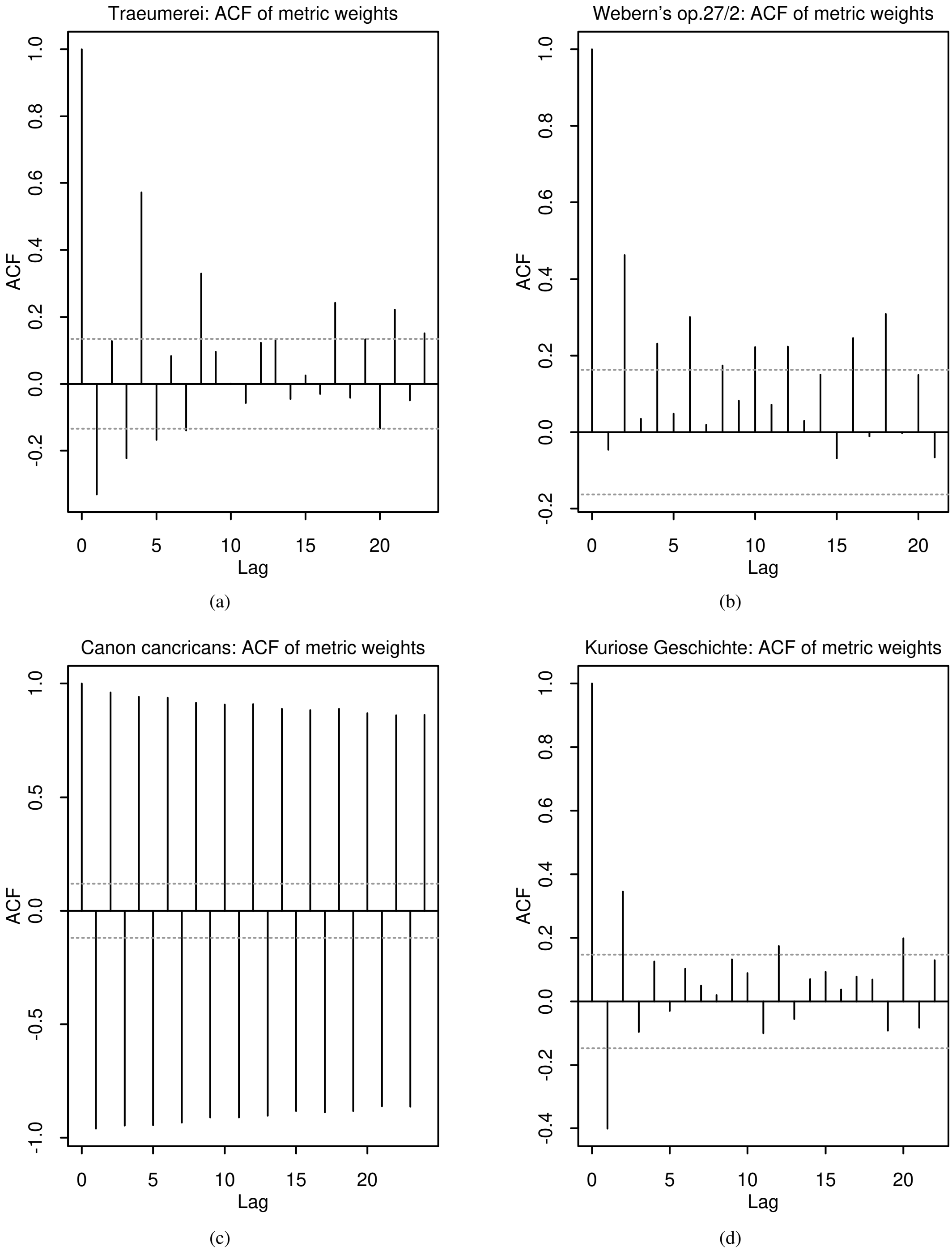

FIG. 9. *Autocorrelograms of metric weights for* (a) *Schumann's "Träumerei,"* (b) *Webern's Variation op.* 27/2, (c) *Bach's Canon Cancricans and* (d) *Schumann's "Kuriose Geschichte."*

more interesting structures than just periodicities. In fact, some scores may not contain any nontrivial periodicities at all. More generally, the problem is that using the same basis of functions, irrespective of the structure of the score, results in focussing on a very limited number of predetermined features that may in fact not be present.

REMARK 2. As a consequence, a nonparametric approach based on kernel smoothing will be pro-

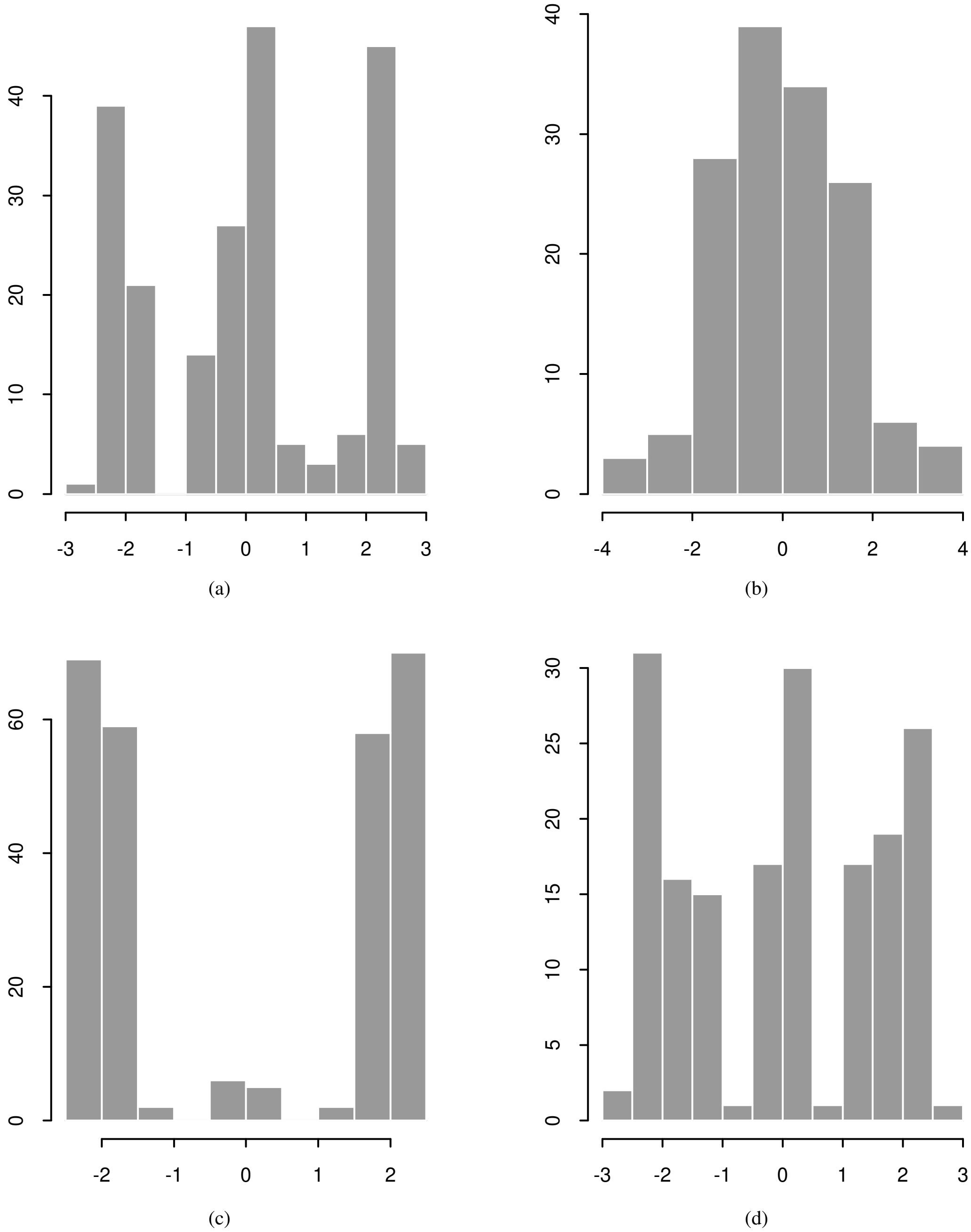

FIG. 10. *Histogram of first difference of metric weights for* (a) *Schumann's "Träumerei,"* (b) *Webern's Variation op.* 27/2, (c) *Bach's Canon Cancricans and* (d) *Schumann's "Kuriose Geschichte."*

posed here. In a traditional setting, the bandwidth b is chosen by minimizing a criterion such as the mean squared error as n tends to infinity. In particular, b tends to zero with increasing sample size. This concept is not directly applicable in our context. The main reasons can be summarized as follows:

1. Based on the definitions given above, the metric, melodic and harmonic aspects of a score are characterized, respectively, by one weight function only. In contrast, a composer is likely to have a hierarchical view. For instance, a piece has on one hand a global harmonic shape that makes the piece coherent as a whole, and on the other hand more local structures. Some composers in fact consciously write a score using a hierarchical approach, first defining a global shape and then refining more and more local structures. Simi-

larly, while rehearsing, a performer is likely to focus first on global features of the score and then sucessively refine more and more local features. This fact was also used in RUBATO to design the process from a prima vista performance (i.e., a performance that only "translates" literarily score information that is written down *explicitly* in the score) to the refined artistic result [27]. In order to obtain a better picture of the structure of a score it is therefore necessary to "extract" the hierarchy that is hidden in the weight functions. For smoothing, this means that there is not just one optimal bandwidth that is of interest. Instead, there is a hierarchy of relevant bandwidths $b_1 > b_2 > \cdots > b_M$. Moreover, the structure of the score, rather than an omnibus statistical criterion (such as the mean squared error), is likely to yield the key information about which sets of bandwidths could be interesting.

2. The weight functions obtained from the analysis above are generally rather complex. In particular, the weights often jump abruptly up and down between very small and very large values (compare Figure 7). This can certainly not be carried over linearly to musical performance. For instance, the tempo of a "musically acceptable" performance is unlikely to change up and down *drastically and repeatedly* within a few seconds. It is therefore reasonable to assume that a performance is not a linear function of the weights but rather a weighted sum of nonlinearly deformed smoothed versions of these functions. Again, there may be a hierarchy of several bandwidths that need to be considered.

These general considerations motivate the idea of *hierarchical smoothing* and *hierarchical decomposition* described below.

2.3.2 *Hierarchical smoothing.* Let $\{x_s(t_i),\ t_i \in R,\ i = 1, \dots, n,\ s = 1, \dots, k\}$ be a k-dimensional time series observed at time points $t_1, \dots, t_n$, and let K_b be a smoothing kernel with bandwidth b and support $[-b, b]$. Applying the smoothing operator

$$K_b x_s(t) = \sum_{i=1}^{n} K_b(t, t_i) x_s(t_i),$$

$t \in R$, for a hierarchy of bandwidths $b_1 > \cdots > b_M$, we obtain a hierarchy of k-dimensional curves

$$\{x_{j,s}(t) = K_{b_j} x_s(t), s = 1, \dots, k\}, \quad j = 1, \dots, M.$$

Here, the Naradaya–Watson kernel

$$K_b(t, t_i) = \frac{K((t - t_i)/b)}{\sum_{j=1}^{n} K((t - t_j)/b)}$$

with a triangular function $K(s) = 1\{|s| \le 1\} \cdot (1 - |s|)$ was used. (Here $1\{A\}$ is the indicator of A.) For $b = 0$, we have $K_b x_s(t) = x_s(t)$. Figures 11 and 12 display hierarchies of smoothed curves for Schumann's "Träumerei" and Bach's Canon Cancricans, resulting from the metric, melodic and harmonic weights. The figures for the other two compositions are omitted to save space. The corresponding bandwidths are given in Section 2.3.3. The figures illustrate that different bandwidths make different features more visible. In particular, for the metric weights, smoothing highlights places where high values occur more frequently. Also, some remarkable similarities between the metric, melodic and harmonic weights become apparent after smoothing.

The statistical technique of using smoothing kernels deserves a comment from the point of view of (inverse) performance theory [22, 25]. Taking into account neighboring values of the analyses by kernel smoothing has a musical meaning: the interpreter is rightly supposed to be conscious of what happened and will happen within a certain time bandwidth b. Conceptually, a related approach was suggested in [25] and [22], where so-called interaction matrices are used to model the influence of different local parts of the score on the performance at a given onset time.

2.3.3 *Hierarchical decomposition.* The approach of hierarchical smoothing suggests a decomposition of the weight function into components of varying smoothness. Thus, let $\{x_s(t_i),\ t_i \in R,\ s = 1, \dots, k,\ i = 1, \dots, n\}$ be a collection of k time series. As discussed above, the aim is to find a decomposition $x_s(t_i) = \sum_{j=1}^{M} x_{j,s}(t_i)$ such that the components $\{x_{j,s}, s = 1, \dots, k\}$ reveal a maximal amount of "regular structure." Structure can be, for instance, the following: symmetry; repeated shapes or periodicities; relationship between different components. Note that, with respect to cross-correlations, a number of methods are known in the literature for testing dependence between stationary time series (see, e.g., [12–14]; also see [30] and references therein). A direct adaptation of these methods is not possible for the following reasons: (1) the series considered here are not stationary in a nontrivial way and can, in particular, not be reduced to white noise by applying a linear filter; (2) the time points are not equidistant; (3) the aim is not only to obtain high cross-correlations but also to highlight regular features of the individual series; (4) not only cross-correlations between "residuals" but between all components are interesting; (5) the musical context suggests that the

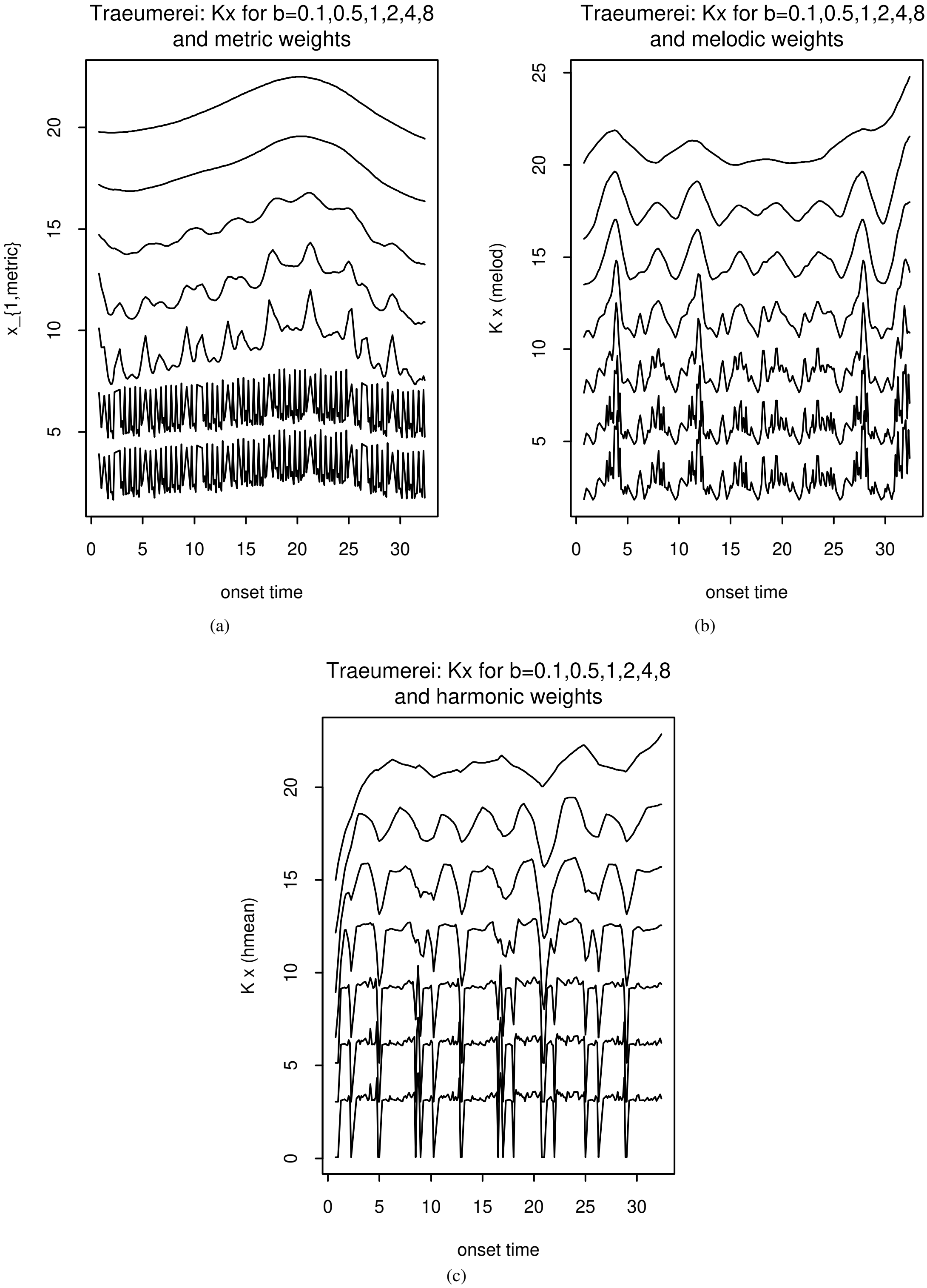

FIG. 11. *Smoothed versions of metric, melodic and harmonic weights for Schumann's "Träumerei."*

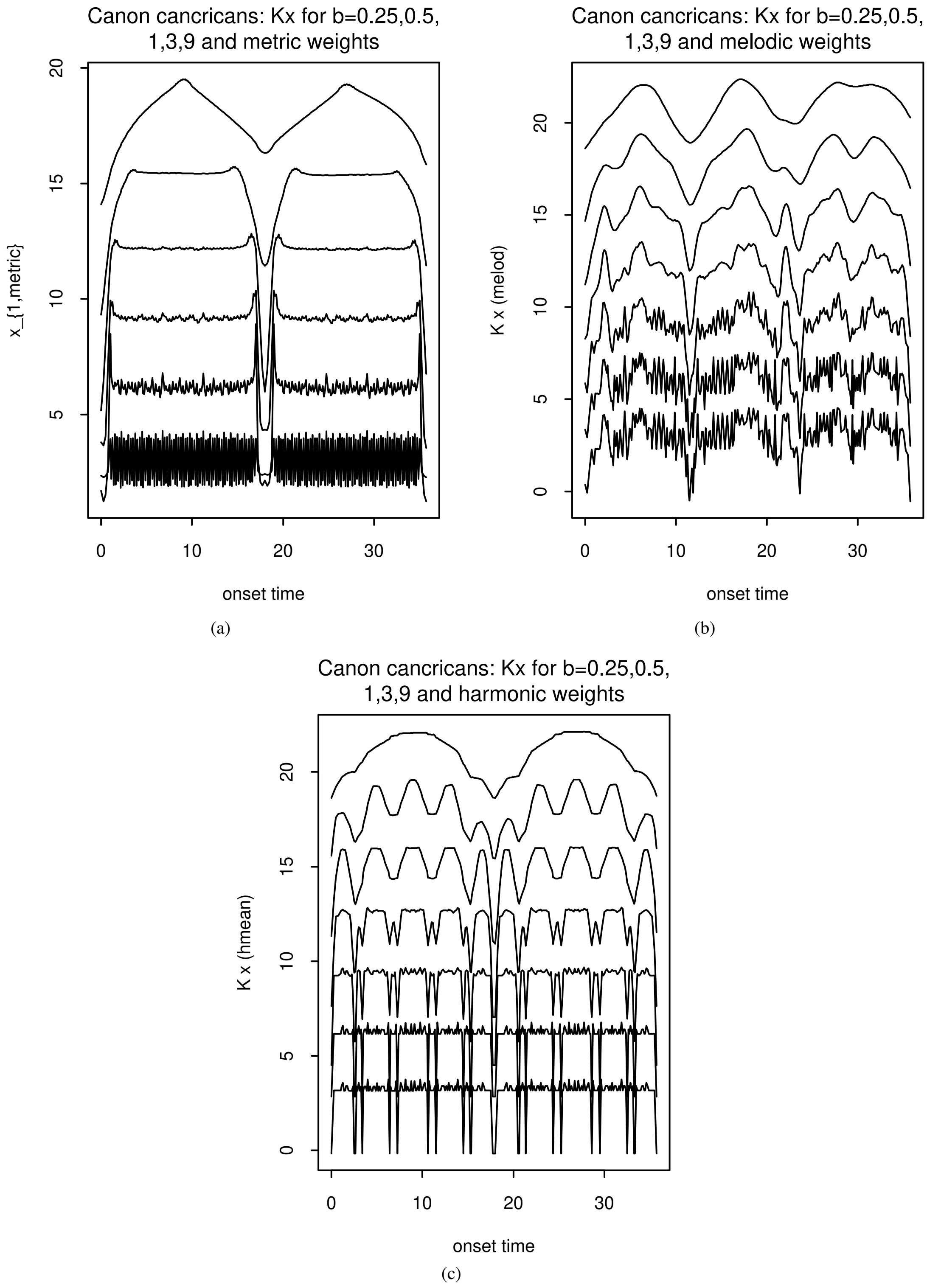

FIG. 12. *Smoothed versions of metric, melodic and harmonic weights for Bach's Canon Cancricans.*

decomposition should be hierarchical in the sense that, with increasing index j, $x_{j,s}$ should contain increasingly local features.

We thus define the following decomposition:

1. Define a hierarchy of bandwidths $b_1 > b_2 > \cdots > b_M = 0$, based on structural information from the score.
2. Define the smoothed function

$$x_{1,s} = K_{b_1} x_s$$

and, for $1 < j \le M$,

$$x_{j,s} = K_{b_j}\left(x_s - \sum_{l=1}^{j-1} x_{l,s}\right).$$

It should be noted that this decomposition is only one of many possible decompositions of x_s. The problem of choosing a meaningful decomposition of a time series is not new. In particular, in the context of the regression analysis in Section 3, it is a special case of the general problem of defining meaningful explanatory variables in regression models. Here, subject-specific considerations provide important guidelines. From a pragmatic point of view, a chosen decomposition can be considered reasonable if the subsequent regression analysis leads to meaningful interpretable results. In our context, the above decomposition appears meaningful, since it decomposes x_s in a simple additive way into components of decreasing smoothness. This translates, in a straightforward way, the generally accepted fact that a musical composition as well as a performance may be considered as a superposition of a hierarchy of local and global "shaping features," obtained by different degrees of "zooming in or out." For a given sequence of bandwidths $b_1 > b_2 > \cdots > b_M = 0$, the first component $x_{1,s}$ represents the most global view of the score (or more specifically of the metric, harmonic or melodic structure, respectively), $x_{2,s}$ represents the next step of refinement by considering, in a more detailed fashion with a smaller bandwidth $b_2 < b_1$, the remaining information (obtained by subtracting the "global information" $x_{1,s}$), and so on.

Specifically, application to the four examples was carried out using $M = 4$. This choice was based on musicological considerations (time signature and bar grouping) as explained in the following. In this sense, the analysis here is exploratory, since no statistical selection criterion was used for choosing M. For possible approaches to choosing M automatically in a related model class see [6]. The following notation will be used here: $x_1 = x_{metric} =$ metric weight, $x_2 = x_{melod} =$ melodic weight, $x_3 = x_{hmean} =$ harmonic (mean) weight, $x_{j,metric} = x_{j,1}$, $x_{j,melod} = x_{j,2}$, $x_{j,hmean} = x_{j,3}$. The choice of the bandwidths was based on the time signature and bar grouping information. Example, Schumann's "Träumerei" is written in 4/4 signature; the grouping is 8+8+8+8. The chosen bandwidths are therefore 4 (4 bars), 2 (2 bars) and 1 (1 bar). The Webern example is written in 2/4 signature; its formal grouping is $1+11+11+11+11$; however, Webern insists on a grouping in 2-bar portions [43], suggesting the bandwidths of 5.5 (11 bars), 1 (2 bars) and 0.5 (1 bar). The Bach example is written in 4/4 signature; the grouping is $9+9+9+9$. The chosen bandwidths are 9 (9 bars), 3 (3 bars) and 1 (1 bar). For the other Schumann example, "Kuriose Geschichte," the time signature is 3/4; the grouping is $8+8+12+12$. The chosen bandwidths are 3 (4 bars), 1.5 (2 bars) and 0.75 (1 bar).

Figures 13 "Träumerei" and 14 (Canon Cancricans), and the corresponding plots for Schumann's "Kuriose Geschichte" and Webern's Variation op. 27/II (not shown here), show remarkable regularities that have not been observed for the original weights. It is in particular remarkable that, for all four compositions, much stronger similarities between the metric, melodic and harmonic components can be observed than for the original weights, especially for $j = 2$ and 3. Moreover, for the first two scores, the same kind of relationship can be observed for $j = 2$ and 3; namely, positive correlation between $x_{j,melod}$ and $x_{j,hmean}$, negative correlation between $x_{j,melod}$ and $x_{j,metric}$ and negative correlation between $x_{j,hmean}$ and $x_{j,metric}$. Particularly surprising is the fact that Webern's score shows the same type of association as Schumann's "Träumerei." This leads to new insights into different approaches to composition and their analyses. In fact, the weight functions are very complex data and deserve a refined "analysis of analysis." Hierarchical smoothing is a possible approach to this problem.

Webern's piece is written in a completely dodecaphonic way, and thus breaks with harmonic and homophonic tradition. *This deserves a special methodologial comment.* The fact that we have nevertheless applied harmonic analysis could be viewed as being in contradiction to Webern's rupture with harmony. Now, we do not claim that this analysis corresponds to Webern's poietic position when composing his "Variationen." Nonetheless, an objective analysis according to the Riemann approach [34] is reasonable for two reasons. (1) Riemann intended to attribute a field of tonalities to *any* possible chord. The fact that he did not succeed in his goal is no reason for refraining from completion of his sketch. The harmonic analysis of RUBATO uses an extended

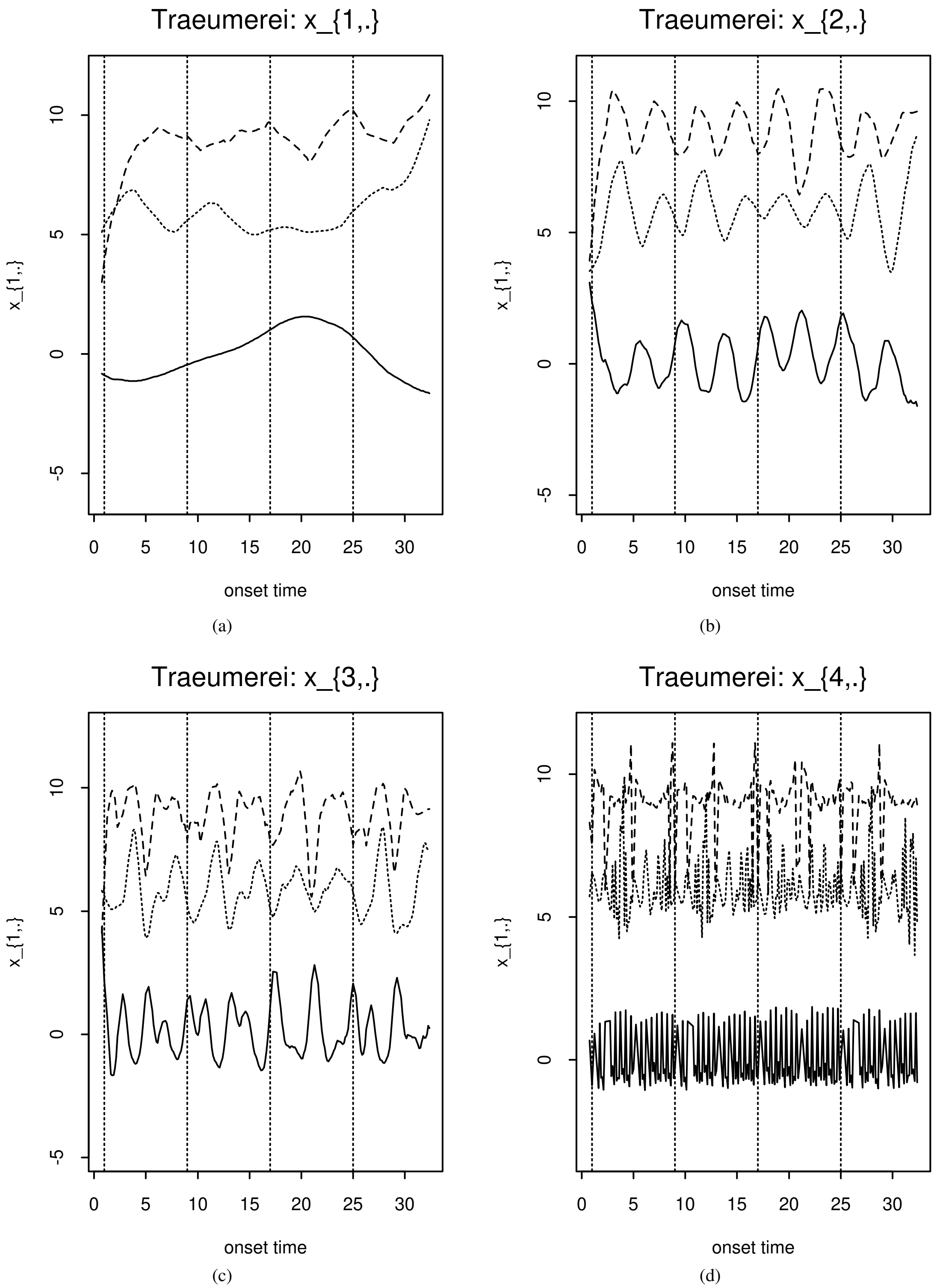

FIG. 13. *Hierarchical components of metric (solid lines), melodic (dotted lines) and harmonic (dashed lines) weights for Schumann's "Träumerei," as defined in Section* 2.3.3: (a) $b = 4$; (b) $b = 2$; (c) $b = 1$; (d) *remaining (residual) series.*

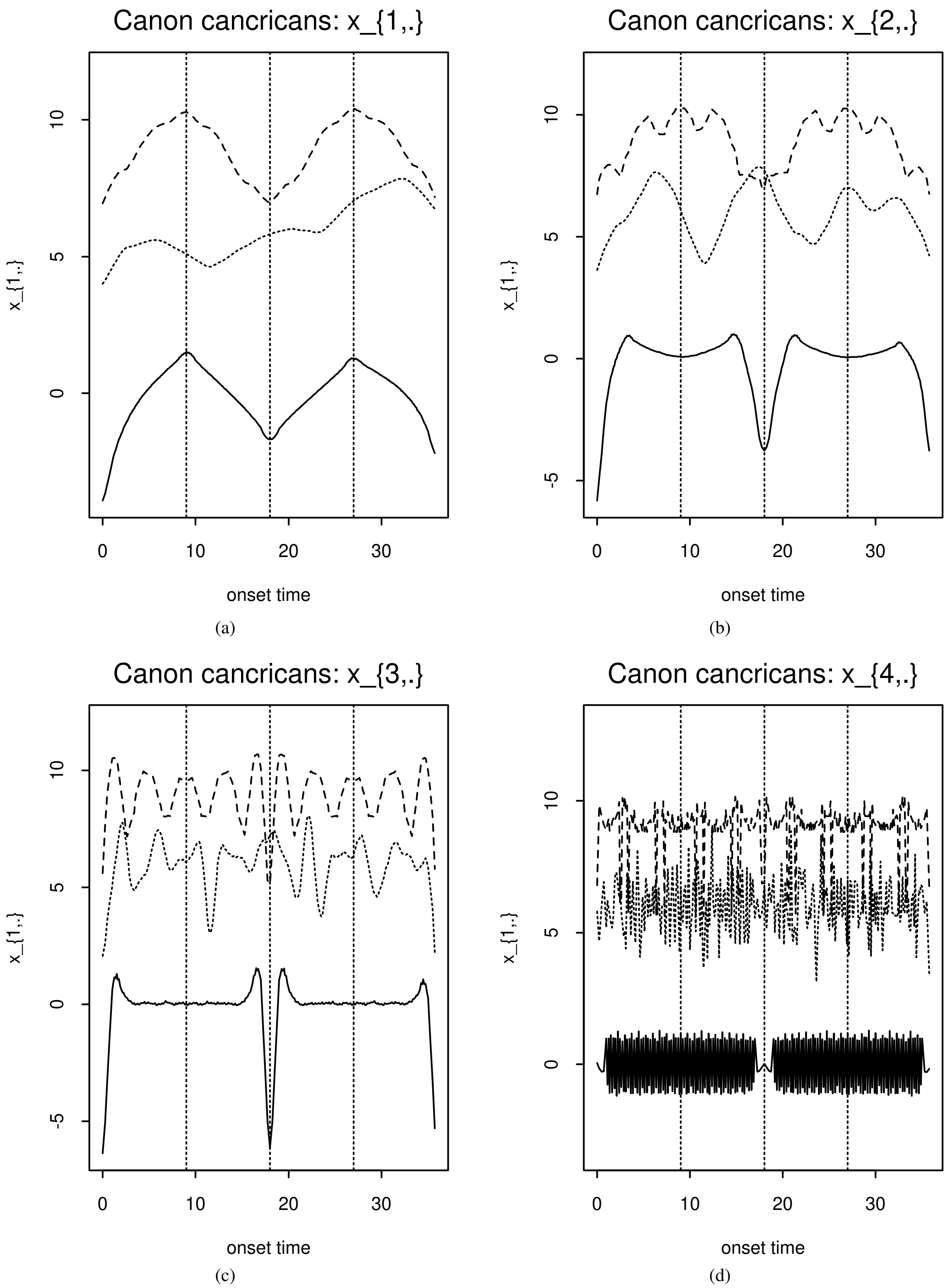

FIG. 14. *Hierarchical components of metric (solid lines), melodic (dotted lines) and harmonic (dashed lines) weights for Bach's Canon Cancricans, as defined in Section* 2.3.3. (a) $b = 9$; (b) $b = 3$; (c) $b = 1$; (d) *remaining (residual) series.*

Riemann theory that indeed allows one to attribute tonality to any possible chord. (2) Applying this approach to apparently atonal compositions is an interesting experiment that is likely to yield a testbed for the universality of Riemann's approach. In view of these considerations the following fact is not completely surprising, although it has not been established explicitly elsewhere in the literature: the correspondence between metric, melodic and harmonic structure in Webern's Variation is very similar to Schumann's "Träumerei." It should be emphasized that this conclusion is obtained by a quantitative analysis of the scores. To our knowledge, this conclusion and in particular its quantitative demonstration is new in the musicological literature.

Schumann's "Kuriose Geschichte" also shows a strong correspondence between the three curves for $j = 1$, 2 and 3. However, this time, the relations are different:

For onset times below 12, we have the following:

1. For $j = 1$, $\mathrm{cor}(x_{1,\,metric}, x_{1,\,melod}) = 0.83$, $\mathrm{cor}(x_{1,\,metric}, x_{1,\,hmean}) = -0.71$, $\mathrm{cor}(x_{1,\,hmean}, x_{1,\,melod}) = -0.63$.
2. For $j = 2$, $\mathrm{cor}(x_{2,\,metric}, x_{2,\,melod}) = 0.00$, $\mathrm{cor}(x_{2,\,metric}, x_{2,\,hmean}) = -0.31$, $\mathrm{cor}(x_{2,\,hmean}, x_{2,\,melod}) = -0.82$.
3. For $j = 3$, $\mathrm{cor}(x_{3,\,metric}, x_{3,\,melod}) = -0.67$, $\mathrm{cor}(x_{3,\,metric}, x_{3,\,hmean}) = -0.20$, $\mathrm{cor}(x_{3,\,hmean}, x_{3,\,melod}) = -0.61$.

It is in particular remarkable that, in contrast to the other scores, melodic and harmonic components are negatively correlated.

After onset time 12, the correlations are as follows:

1. For $j = 1$, $\mathrm{cor}(x_{1,\,metric}, x_{1,\,melod}) = 0.10$, $\mathrm{cor}(x_{1,\,metric}, x_{1,\,hmean}) = -0.38$, $\mathrm{cor}(x_{1,\,hmean}, x_{1,\,melod}) = -0.29$.
2. For $j = 2$, $\mathrm{cor}(x_{2,\,metric}, x_{2,\,melod}) = -0.47$, $\mathrm{cor}(x_{2,\,metric}, x_{2,\,hmean}) = -0.14$, $\mathrm{cor}(x_{2,\,hmean}, x_{2,\,melod}) = -0.11$.
3. For $j = 3$, $\mathrm{cor}(x_{3,\,metric}, x_{3,\,melod}) = -0.75$, $\mathrm{cor}(x_{3,\,metric}, x_{3,\,hmean}) = 0.58$, $\mathrm{cor}(x_{3,\,hmean}, x_{3,\,melod}) = -0.69$.

Finally, for Bach's composition, the only noticeable correlations occur between metric and harmonic weights, namely:

1. For $j = 1$, $\mathrm{cor}(x_{1,\,metric}, x_{1,\,hmean}) = 0.94$.
2. For $j = 2$, $\mathrm{cor}(x_{2,\,metric}, x_{2,\,hmean}) = 0.63$.
3. For $j = 3$, $\mathrm{cor}(x_{3,\,metric}, x_{3,\,hmean}) = 0.61$.

With respect to the shapes of $x_{j,\cdot}$, for $j = 2$ and 3, the two scores by Schumann and the one by Webern are clearly more similar to each other compared with Bach's shapes. From the point of view of music history, this is quite plausible, because Webern's organic composition principle is more related to Schumann's rankly growing romanticism than to Bach's self-disciplined architectural setup (see also the following remarks).

Finally, note that the scatterplots in Figure 7 show that Bach's harmonic weights are highly clustered, and that the smoothed curves in Figure 14a–d are more "edgy" than for the other compositions. In this sense, *Bach's composition exhibits a high degree of organization*. This confirms the general belief that the principle of architectural rather than processual construction plays a dominating role in Bach's music.

Overall, we may conclude that hierarchical decomposition reveals interesting properties, in particular strong similarities between the metric, melodic and harmonic weights, that were not visible in the original series. The results are musically plausible in that the analysis of Bach's score turns out to be the most regular one and the analyses of Webern and Schumann appear to be closer to each other than to Bach's. The results are surprising in that (the analysis of) Webern turns out to be closer to (the analysis of) Schumann than expected. Also, the strong relationship between the three analytic curves could not be expected a priori, because the three weights were calculated using completely different aspects of the score and the scatterplots of the original curves did not show much association.

Based on the results, one may conjecture that appropriate matching of metric, melodic and harmonic structure plays an important role in music, independently of musical style. The tools introduced here provide the possibility of investigating which types of relationships may exist in which musical and historical contexts. An important task for future research will be to investigate such aspects for a larger variety of compositions.

2.4 A more complex design matrix for Schumann's "Träumerei"

The following elaboration of the above method was *applied exclusively to Schumann's "Träumerei"* because, in this case, we dispose of tempo measurements. Of course, it could be applied to the other examples mutatis mutandis.

2.4.1 *Maximal harmonic weights.* Harmonic weights are originally defined for each note. More than one note, and thus harmonic weight, may exist at a given onset time. In the above analysis, the harmonic weight at onset time t_l was defined as the average of all original harmonic weights at t_l. As an alternative, one may consider for instance the maximal harmonic weight. The following notation will be used: $x_{hmean}(t_l)$ is the average harmonic weight at onset time t_l, and $x_{hmax}(t_l)$ is the maximal harmonic weight at onset time t_l. Thus, four different weight functions will be used in the following analysis: x_{metric}, x_{melod}, x_{hmean} and x_{hmax}.

2.4.2 *Derivatives.* In the investigation of the relationship between analytic weight functions and tempo data, "discrete derivatives" (difference quotients) turned out to play an important role. For a given time series $x(t_1), \ldots, x(t_n)$, define

$$dx(t_j) = \frac{x(t_j) - x(t_{j-1})}{t_j - t_{j-1}}$$

and

$$dx^{(2)}(t_{j-1}) = \frac{dx(t_j) - dx(t_{j-1})}{t_j - t_{j-1}}.$$

This definition is applied to x_{metric}, x_{melod}, x_{hmean} and x_{hmax}. Thus, for instance,

$$dx_{metric,\,j}(t_i) = \frac{x_{metric,\,j}(t_i) - x_{metric,\,j}(t_{i-1})}{t_i - t_{i-1}}$$

and

$$d^2x_{metric,\,j}(t_i) = \frac{dx_{metric,\,j}(t_i) - dx_{metric,\,j}(t_{i-1})}{t_i - t_{i-1}}.$$

In a second step, each of the weights and their first and second discrete derivatives is decomposed hierarchically into four components, as decribed above. The bandwidths used for the decomposition are $b_1 = 4$ (weighted averaging over 8 bars), $b_2 = 2$ (4 bars), $b_3 = 1$ (2 bars) and $b_4 = 0$ (residual—no averaging). This gives the following list of a total of 48 functions:

$$\begin{array}{llll}
x_{metric,1} & x_{metric,2} & x_{metric,3} & x_{metric,4} \\
dx_{metric,1} & dx_{metric,2} & dx_{metric,3} & dx_{metric,4} \\
d^2x_{metric,1} & d^2x_{metric,2} & d^2x_{metric,3} & d^2x_{metric,4}
\end{array}$$

$$\begin{array}{llll}
x_{melodic,1} & x_{melodic,2} & x_{melodic,3} & x_{melodic,4} \\
dx_{melodic,1} & dx_{melodic,2} & dx_{melodic,3} & dx_{melodic,4} \\
d^2x_{melodic,1} & d^2x_{melodic,2} & d^2x_{melodic,3} & d^2x_{melodic,4}
\end{array}$$

$$\begin{array}{llll}
x_{hmax,1} & x_{hmax,2} & x_{hmax,3} & x_{hmax,4} \\
dx_{hmax,1} & dx_{hmax,2} & dx_{hmax,3} & dx_{hmax,4} \\
d^2x_{hmax,1} & d^2x_{hmax,2} & d^2x_{hmax,3} & d^2x_{hmax,4}
\end{array}$$

$$\begin{array}{llll}
x_{hmean,1} & x_{hmean,2} & x_{hmean,3} & x_{hmean,4} \\
dx_{hmean,1} & dx_{hmean,2} & dx_{hmean,3} & dx_{hmean,4} \\
d^2x_{hmean,1} & d^2x_{hmean,2} & d^2x_{hmean,3} & d^2x_{hmean,4}
\end{array}$$

2.4.3 *Other essential score information.* Apart from the metric, melodic and harmonic structure, a score typically contains a number of symbolic or verbal performance instructions. Clearly, this information must be included when analyzing performance data. It should be noted, however, that the main aim here is to check whether and in which sense the metric, melodic and harmonic weights "explain" a large part of the tempo. Therefore, the following prima vista functions are defined in the most elementary way:

1. *Ritardandi*—The score shows four onset intervals R_1, R_2, R_3, R_4 for ritardandi, starting at onset times $t_o(R_j)$ $(j = 1, 2, 3, 4)$, respectively. We define the four linear functions

$$x_{rit_j}(t) = 1\{t \in R_j\} \cdot (t - t_o(R_j)), \quad j = 1, 2, 3, 4. \tag{1}$$

2. *Suspensions*—The score shows four onset intervals S_1, S_2, S_3, S_4 for suspensions, starting at onset times $t_o(S_j)$ $(j = 1, 2, 3, 4)$, respectively. We define the four linear functions

$$x_{sus_j}(t) = 1\{t \in S_j\} \cdot (t - t_o(S_j)), \quad j = 1, 2, 3, 4. \tag{2}$$

3. *Fermatas*—The score shows two onset intervals F_1, F_2 for fermatas. We define the two support functions

$$x_{ferm_j}(t) = 1\{t \in F_j\}, \quad j = 1, 2. \tag{3}$$

2.4.4 *Initial design matrix X.* Summarizing, we have a total of $58 = 48 + 4 + 4 + 2$ onset functions of analytical and prima vista types. Call X the *analytical matrix* of these 58 functions. The following notation will be used below: Let A be a $(p \times q_1)$-matrix and B a $(p \times q_2)$-matrix. Then $C = (A, B)$ denotes the $[p \times (q_1 + q_2)]$-matrix obtained by "appending" B on the right-hand side of A. Using the definitions above, we define for $j = 1, 2, 3, 4$ the $(n \times 4)$-matrices

$$X_j(harmo) = \big(x_{hmean,\,j}, x_{hmax,\,j}, x_{metric,\,j}, x_{melod,\,j}\big),$$

$$X_j(metric) = \big(x_{metric,\,j}, x_{hmean,\,j}, x_{hmax,\,j}, x_{melod,\,j}\big),$$

$$X_j(melod) = \big(x_{melod,\,j}, x_{hmean,\,j}, x_{hmax,\,j}, x_{metric,\,j}\big)$$

Furthermore, we define

$$dX_j(harmo) = \big(dx_{hmean,\,j}, dx_{hmax,\,j}, dx_{metric,\,j}, dx_{melod,\,j}\big),$$
$$dX_j(metric) = \big(dx_{metric,\,j}, dx_{hmean,\,j}, dx_{hmax,\,j}, dx_{melod,\,j}\big),$$
$$dX_j(melod) = \big(dx_{melod,\,j}, dx_{hmean,\,j}, dx_{hmax,\,j}, dx_{metric,\,j}\big)$$

$$d^2X_j(harmo) = \big(d^2x_{hmean,\,j}, d^2x_{hmax,\,j}, d^2x_{metric,\,j}, d^2x_{melod,\,j}\big),$$
$$d^2X_j(metric) = \big(d^2x_{metric,\,j}, d^2x_{hmean,\,j}, d^2x_{hmax,\,j}, d^2x_{melod,\,j}\big),$$
$$d^2X_j(melod) = \big(d^2x_{melod,\,j}, d^2x_{hmean,\,j}, d^2x_{hmax,\,j}, d^2x_{metric,\,j}\big)$$

and the $(n \times 10)$-matrix

$$X_{add} = (X_{rit}, X_{sus}, X_{ferm}),$$

where $X_{rit} = (x_{rit_1}, x_{rit_2}, x_{rit_3}, x_{rit_4})$, $X_{sus} = (x_{sus_1}, x_{sus_2}, x_{sus_3}, x_{sus_4})$ and $X_{ferm} = (x_{ferm_1}, x_{ferm_2})$.

Finally, define the $(n \times p)$-matrices (with $p = 58$)

$$\begin{aligned} X(harmo) &= \big(X_1(harmo), X_2(harmo), X_3(harmo), X_4(harmo), dX_1(harmo), dX_2(harmo), \\ &\quad dX_3(harmo), dX_4(harmo), d^2X_1(harmo), d^2X_2(harmo), d^2X_3(harmo), d^2X_4(harmo), X_{add}\big), \end{aligned}$$

$$\begin{aligned} X(metric) &= \big(X_1(metric), X_2(metric), X_3(metric), X_4(metric), dX_1(metric), dX_2(metric), \\ &\quad dX_3(metric), dX_4(metric), d^2X_1(metric), d^2X_2(metric), d^2X_3(metric), d^2X_4(metric), X_{add}\big), \end{aligned}$$

$$\begin{aligned} X(melod) &= \big(X_1(melod), X_2(melod), X_3(melod), X_4(melod), dX_1(melod), dX_2(melod), \\ &\quad dX_3(melod), dX_4(melod), d^2X_1(melod), d^2X_2(melod), d^2X_3(melod), d^2X_4(melod), X_{add}\big). \end{aligned}$$

Each of the matrices $X(metric)$, $X(melod)$ and $X(harmo)$ turned out to be singular, in that the last column can be expressed as a linear combination of the previous columns. Hence, from now on, the last column is omitted. For simplicity of notation, the new $(n \times 57)$-matrices will also be denoted by $X(metric)$, $X(harmo)$ and $X(melod)$.

REMARK 3. In the next section, a regression of tempo curves on the X-space will be performed. In view of the close relationship between the smoothed components of the metric, melodic and harmonic weights for $j = 2$ and 3 (see the discussion in the previous section), it may not be possible to distinguish exactly whether certain characteristics of the tempo curve stem from the metric, the harmonic or the melodic analysis.

REMARK 4. Clearly, the definition of the full design matrix X is based on our specific approach. In particular, based on musicological considerations, the following a priori choices were made: (1) specific definition of melodic, metric and harmonic weights; (2) specific decomposition of the weights by hierarchical smoothing; (3) choice of the bandwidths and of the number of "derivatives." One may ask how the results of the regression analysis below may change, if other choices are made in 1, 2 and 3. It would be beyond the scope of this paper to investigate this question in its full generality. In particular, there is a huge number of possible variations of 1 and 2. Partial answers to 3 are given in [6], where, in a similar but not identical modelling framework, bandwidths and the number of explanatory variables are estimated from the data, instead of being chosen a priori. The general musicological conclusions and, in particular, clusters of performances obtained in [6] are very similar to those obtained here (in Section 3). This indicates that the concrete conclusions are not unduly influenced by our a priori choice of bandwidths and derivatives.

2.4.5 *Orthonormal matrix Z*. Define $Z(harmo)$, $Z(metric)$ and $Z(harmo)$ to be the three $(n \times 57)$-matrices obtained by orthonormalizing successively the columns of $X(harmo)$, $X(metric)$ and $X(harmo)$, respectively. The reason for computing three different matrices is that orthonormalization depends on the initial sequence of the columns. An artificial preference of the variables that are accidentally in the first (or first few) column(s) is avoided by carrying out three separate regression analyses with the respective matrices $Z(harmo)$, $Z(metric)$ and $Z(melod)$ and by comparing the common features of the three results.

By definition, $Z(metric)$ puts first emphasis on metric weights, $Z(melod)$ emphasizes melodic weights and $Z(harmo)$ focusses first on harmonic (mean) weights. Finally, note that, with respect to derivatives, the order is (1) no derivative (all three weights, all hierarchical levels), (2) first derivative (all three weights, all hierarchical levels), (3) second derivative (all three weights, all hierarchical levels). Thus, models with no or low-order derivatives are favored. This appears to be a natural choice, since simple models are preferred and derivatives generally represent more complex features.

3. RELATIONSHIP BETWEEN TEMPO AND SCORE STRUCTURE

3.1 The Tempo Data

The tempo data was provided to us by Bruno Repp. The data consist of tempo measurements (or tempo curves) for $m = 28$ performances. The onset times are on a grid of 1/8th beats. Thus, for instance, grace notes are excluded. From this set of onset times, we consider only onset times where at least one note is actually played. This results in a set T of $n = 212$ nonequidistant onset times t_i, $i = 1, \dots, n$, which are multiples of 1/8. In view of the expectation that a performer may control the tempo in a relative rather than an absolute way, the logarithm of the tempo instead of the original tempo is considered. Moreover, the interest lies in investigating the shape of the tempo curves rather than the absolute tempo values. Therefore, each of 28 tempo curves is standardized to zero sample mean and standard deviation 1. Thus, let $y^*(t_i, j)$ be the (natural) logarithm of the tempo of the jth performance at onset time t_i, $i = 1, \dots, n$, $j = 1, \dots, m$. Then the standardized tempo data are defined by

$$y(t_i, j) = [y^*(t_i, j) - \bar{y}^*(j)]/s^*(j),$$

where $\bar{y}^* = n^{-1}\sum_{i=1}^{n} y^*(t_i, j)$ and $s^*(j) = [(n-1)^{-1}\sum_{i=1}^{n}(y^*(t_i, j) - \bar{y}^*)^2]^{1/2}$. Figure 15 displays the 28 standardized logarithmic tempo curves.

3.2 The Regression Model

Let Z be one of the three matrices $Z(harmo)$, $Z(metric)$ or $Z(melod)$, respectively. The following model for the jth *individual tempo curve* is assumed:

$$y(j) = Z\beta(j) + \varepsilon(j),$$

where $y(j) = [y(t_1, j), y(t_2, j), \dots, y(t_n, j)]^t$, $\beta(j) = (\beta_1(j), \dots, \beta_p(j))^t$, $p = 57$, and $\varepsilon(j) = [\varepsilon(t_1, j), \varepsilon(t_2, j), \dots, \varepsilon(t_n, j)]^t$ is a vector of n identically distributed, but possibly correlated, zero mean random variables $\varepsilon(t_i, j)$, $t_i \in T$, with variance $\mathrm{var}(\varepsilon(t_i, j)) = \sigma^2(j)$. Thus, it is assumed that each performance is essentially characterized by a 57-dimensional parameter vector $\beta(j)$. Due to standardization of y and Z, there is no intercept in the model.

Because $\beta(j)$ is the parameter vector corresponding to the performance number j, it may be assumed to be a random vector, sampled from the space of all "possible" interpretations, with expected value $E[\beta(j)] = \beta$. Thus, we may write

$$\beta(j) = \beta + \eta(j),$$

where $\eta(j)$ is a random vector with $E[\eta(j)] = 0$. The (logarithmic) tempo of the jth performance is then decomposed into an "average performance" $Z\beta$, an individual deviation from the average $Z\eta(j)$ and an unexplained deviation $\varepsilon(t_i, j)$:

$$y(j) = Z\beta(j) + Z\eta(j) + \varepsilon(j).$$

3.3 Results of Regression Analysis

The relationship between analytic weights and the tempo is investigated with respect to (1) existence, (2) type and complexity and (3) comparison of different performances.

3.3.1 *Existence.*

- *Maximal values of* R^2—Disregarding questions of significance and model choice, the values of the unadjusted R^2 for the full model are interesting in order to see how much the regression model can "explain." The result is $0.65 \le R^2 \le 0.85$, depending on the performance.
- R^2 *after variable selection*—A stepwise forward selection with F-to-enter level 0.01 was performed. The remaining coefficients turned out to be significant at the 5% level, even after taking into account possible serial correlations in the residuals. Note that, for variable selection, which of the design matrices $Z(metric)$, $Z(melod)$ or $Z(metric)$ was used plays a role. The results are $0.46 \le R^2 \le 0.79$ for $Z(harmo)$, $0.48 \le R^2 \le 0.78$ for $Z(metric)$ and $0.36 \le R^2 \le 0.77$ for $Z(melod)$. Excluding the performance of Kubalek (with $R^2 = 0.36$), the lower bound for $Z(melod)$ is 0.51. The quality of the fitted models turned out to be very good. A few typical observed and fitted curves [using $Z(melod)$] in Figure 16 illustrate this.

Further evidence that the association found in the regression model is meaningful is provided by the discussion of communalities and diversities below.

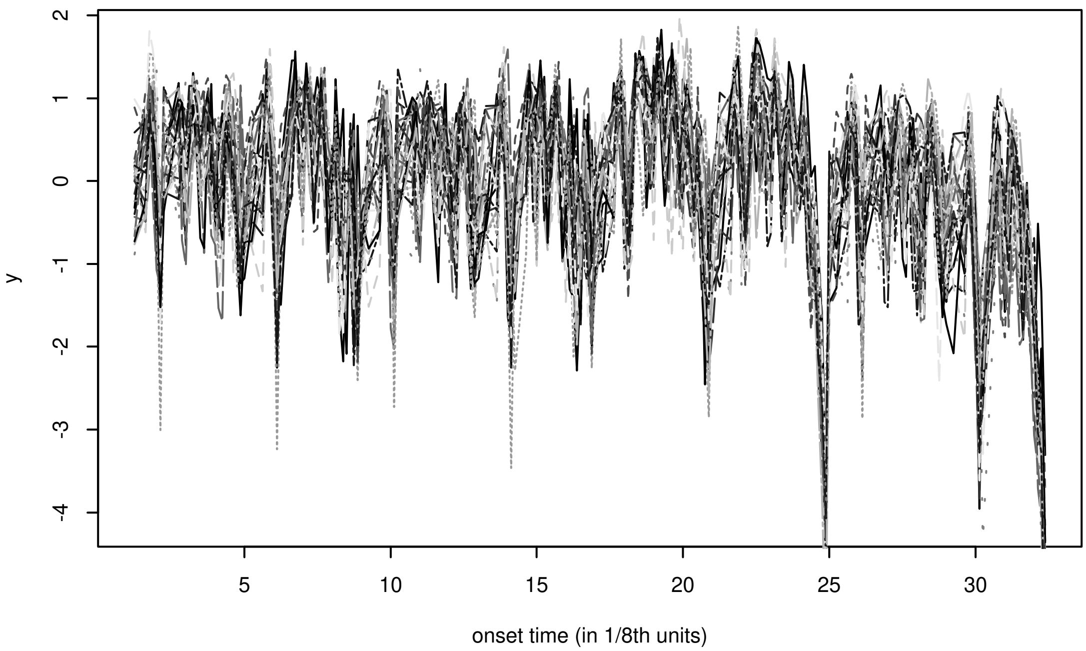

FIG. 15. *Standardized log-tempo curves.*

3.3.2 *Complexity.* Musical performance is considered to be a very complex process. This is confirmed within the given formal setup. For most performances, the selected models turned out to be rather complex. For instance, for the performance by Brendel (Figure 16b) and $Z(melod)$ (with $R^2 = 0.76$), 17 significant variables were selected. The selected variables are: $z_{melod,1}$, $z_{hmean,1}$, $z_{hmean,2}$, $z_{hmax,2}$, $z_{melod,3}$, $z_{hmean,3}$, $z_{hmax,3}$, $z_{melod,4}$, $dz_{melod,1}$, $dz_{melod,2}$, $dz_{hmean,2}$, $dz_{melod,3}$, $dz_{hmean,3}$, $d^2z_{metric,1}$, z_{rit_3}, z_{rit_4} and z_{ferm_1}. In particular, the model contains all four types of weights (metric, melodic, harmonic-mean, harmonic-maximum). Also, all degrees of smoothness, first and second derivatives and additional prima vista variables are included.

3.3.3 *Comparison of different performances.* As noted previously, the smoothed components of metric, melodic and harmonic weights are closely related. Performance is therefore necessarily ambiguous in that a clear decision whether certain features of the tempo are "due to" the harmonic, the metric or the melodic content cannot be reached. This may explain partially the phenomenon that expert opinions about a performance often differ substantially. The following results show, however, that there are also strong similarities between the regression results with $Z(metric)$, $Z(melod)$ and $Z(harmo)$. This indicates that there is at least a core of tempo features that are unambiguously attributable to specific weight functions.

The following aspects are considered here:

- signs of coefficients;
- frequency of selection of a variable;
- relative size of coefficients.

The latter will be used for finding clusters of perfomances.

(a) *Sign of coefficients*. Consider the models obtained after variable selection. Let $p = 57$, and set $\hat{\beta}(j) = [\hat{\beta}_1(j), \ldots, \hat{\beta}_p(j)]^t$, where $\hat{\beta}_k$ is set equal to zero, if the kth variable was not selected. Then for $Z(metric)$, all except 3 coefficients (out of 57) turn out have the same sign for all performances. The same is true for $Z(harmo)$ for all except 2 coefficients, and for $Z(melod)$ for all except 1 coefficient. Thus, the sign of almost all coefficients is common to all performances. The effect of analytic curves has the same direction independently of the performance style. As a general tendency, the results suggest that (1) the tempo decreases as the original (not orthogonalized) harmonic weight increases, (2) the tempo increases as the original (not orthogonalized) metric weight increases and (3) the tempo decreases as the original (not orthogonalized) melodic weight increases. Clearly, this is only an approximate rule under the assumption that all other variables are kept fixed. This is not really the case, because the weights are strongly correlated. The actual relationship between weights and tempo is therefore much more complicated.

(b) *Frequency of inclusion*. Define n_k to be the number of performances for which variable k was

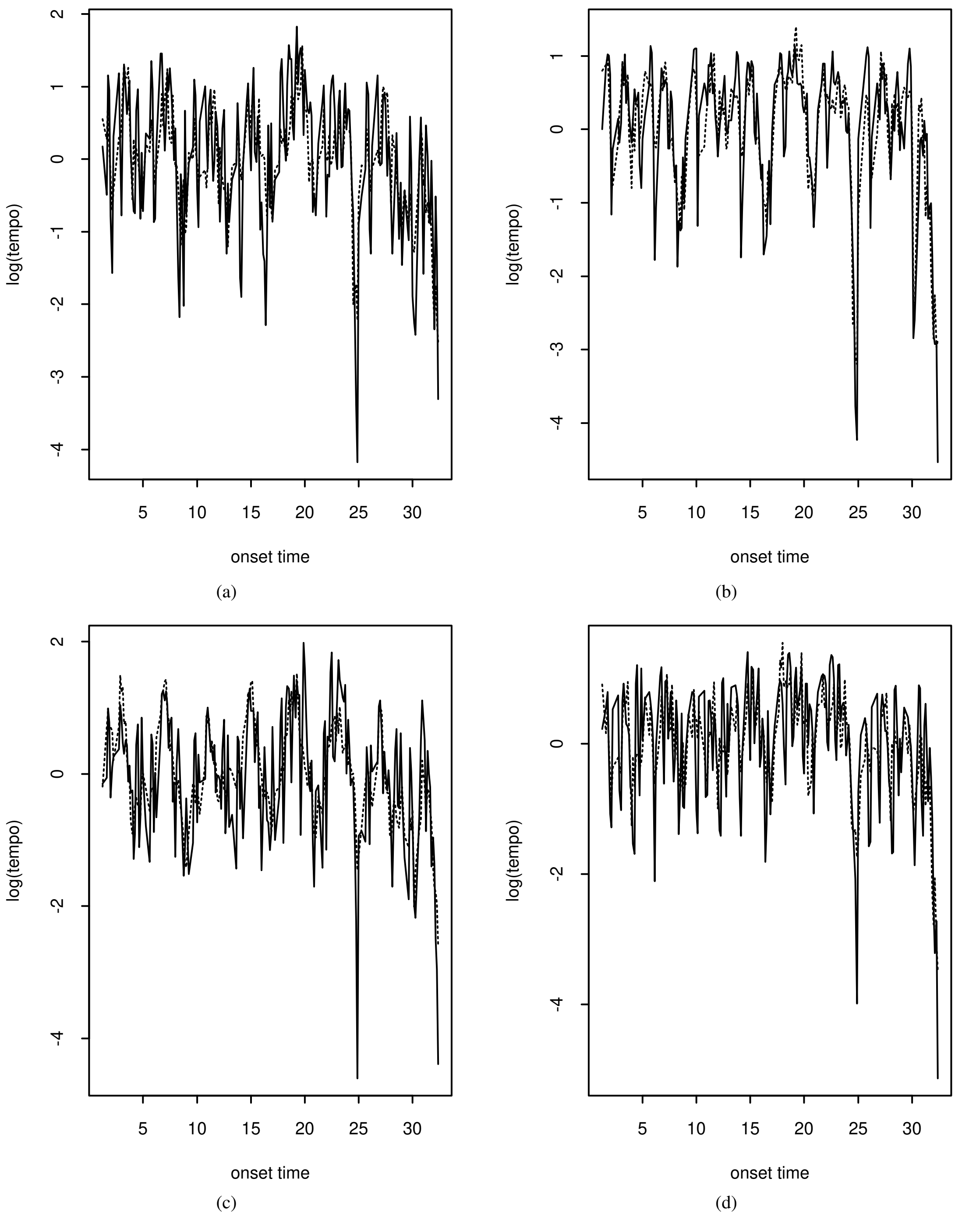

FIG. 16. *Examples of observed and fitted log-tempo curves for* (a) *Argerich*, (b) *Brendel*, (c) *Cortot* (*performance* 3) *and* (d) *Horowitz* (*performance* 1).

included in the final model. Plots of the curves (variables) that were chosen at least 24 times (out of 28) for $Z(harmo)$, $Z(metric)$ and $Z(melod)$, respectively [see Figure 17 for $Z(melod)$] show at least two types of curves that are common to almost all performances, independently of the matrix that is used: (1) very smooth "global" curves, such as $z_{melod,1}$, that shape the overall tendency of the tempo; (2) almost periodic curves, with a period of about 4 measures, corresponding to the approximate periodicity of the harmonic curve $z_{hmean,2}$. It is also remarkable that, independently of the design matrix, $z_{melod,1}$ is chosen for all 28 performances. Thus, the melodic aspect seems to be particularly important. Finally note that, for $Z(melod)$, we have $z_{melod,1} = x_{melod,1}$. For $Z(harmo)$ and $Z(metric)$,

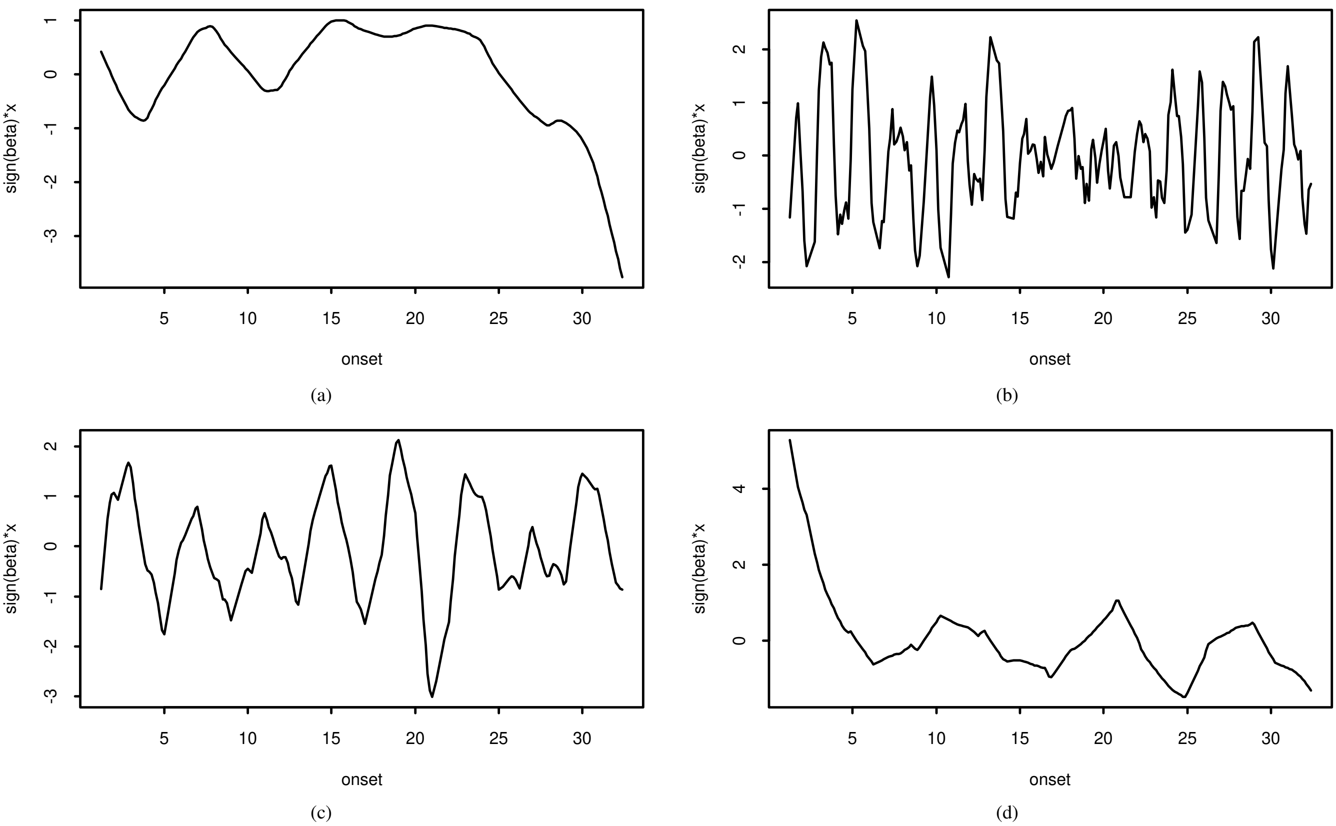

FIG. 17. *Most frequently chosen components of* $X(melod)$. (a) $-z\_\{melod,1\}$ *chosen for* 28 *performances*, (b) $-d\_E\, z\_\{melod,3\}$ *chosen for* 28 *performances*, (c) $-z\_\{hmean,2\}$ *chosen for* 27 *performances*, *and* (d) $-z\_\{hmean,1\}$ *chosen for* 25 *performances.*

$z_{melod,1}$ can be considered as a nonlinear deformation of $x_{melod,1}$. Nonlinear deformations of analytical weights as arguments of refined shaping of performance is also implemented in the performance module of RUBATO.

In summary, the results suggest the existence of a small number of "canonical" analytical weight curves that are relevant for most performances of "Träumerei" and essentially do not depend on the analytical emphasis.

(c) *Relative size of coefficients*. Recall that Z is orthonormal so that coefficients are comparable with respect to their size. For fixed j, let $r_k(j)$ be the rank of $|\hat{\beta}_k(j)|$ in the set $\{|\hat{\beta}_s(j)|, s = 1, \ldots, p\}$. Furthermore, for $1 \le l \le p$, let $f_k(l) = \sum_{j=1}^{m} 1\{r_k(j) > p - l\}$ be the number of performances for which $|\hat{\beta}_k(j)|$ is at least the lth largest.

For $l = 1$, $f_k(1)$ is the number of performances for which the kth variable is most important. For $Z(harmo)$, only four different curves are most important for at least one performance (Figure 18), namely, $z_{hmean,2}$, $z_{hmean,1}$, $z_{metric,1}$, and $z_{melod,4}$. Depending on which of the four curves has the largest coefficient, the individual performances can be classified into four clusters (Table 2a). Performances in the first cluster emphasize the 4-measure periodicity of the harmonic structure. It is in particular interesting that all Cortot performances are included, whereas none of the Horowitz performances is contained. Performances in the second cluster emphasize mainly a globally descending curve. This cluster includes in particular Horowitz1 and Horowitz2. The third cluster, consisting of Bunin and Gianoli, has a global curve with a peak around the 15th measure as dominant feature. The fourth cluster consists solely of the first performance by Horowitz. Apparently, in this performance, a very detailed local structure of the melodic curve $z_{melod,4}$ is dominant. The results for $Z(metric)$ are almost identical and are therefore omitted.

It is remarkable, in particular, that Horowitz's extraordinary first performance from 1947 shows a preference for very detailed local information, from the melodic and from the metrical analysis—contrasting, for instance, with Argerich's highly coherent performance. This observation is confirmed by an investigation of the correlation coefficients in the algebro-geometric analysis of the performance genealogy in the sense of RUBATO's stemma the-

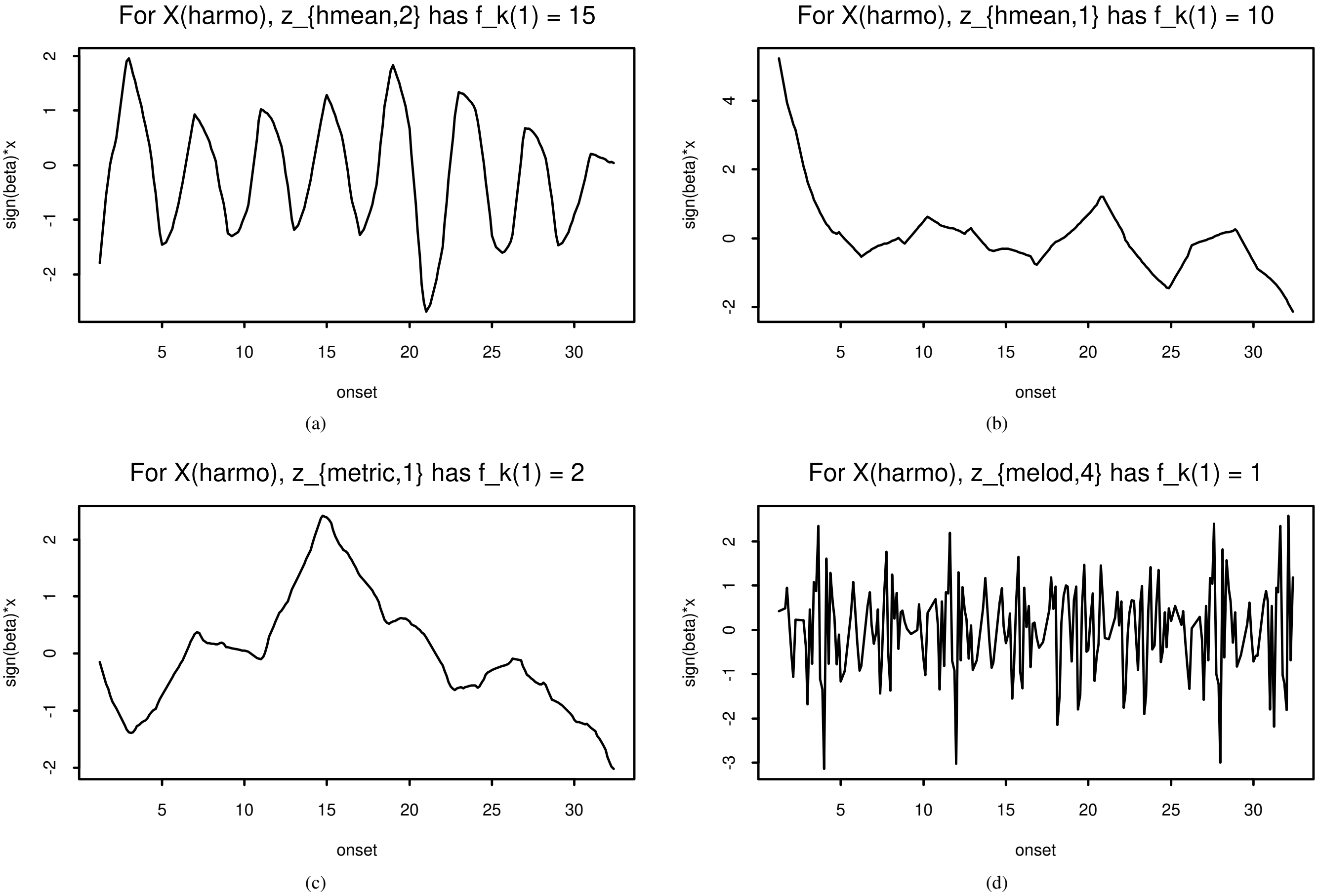

FIG. 18. *Components of $X(harmo)$ with largest coefficient*; $f_k(1)$ *is the number of performances for which the corresponding component was the most important one* (*i.e., had the largest coefficient in absolute value*).

ory (see [22, 25]). When translated into common language, these quantitative results are in perfect coincidence with the judgments of experts on Argerich's and Horowitz's specific differences in performance [1].

For $Z(melod)$ (Figure 19), we obtain the clusters shown in Table 2b. The corresponding variables are $z_{melod,1}$, $z_{hmean,2}$ and $z_{melod,2}$. Here, very simple clusters are obtained (Table 2b). Qualitatively, only two types of curves occur as the most important ones: (1) a globally descending smooth curve or (2) an almost periodic curve (with period 4). (Note, however, that the type-2 curves are not identical.) Only Cortot1, 2 and 3, Krust and Ashkenazy have the second type of curve as dominating feature.

Many more results can be obtained by considering arbitrary values of l. For $l > 1$, $f_k(l)$ is the number of performances for which the kth variable is among the l most important ones. As an example, consider $l = 3$ and $Z(melod)$: Table 3 summarizes the resulting partially overlapping clusters. The variables with $f_k(3) \neq 0$ and the corresponding column labels are $z_{melod,1}$, $z_{hmean,2}$, $z_{hmean,1}$, $dz_{melod,3}$, $z_{melod,2}$, $dz_{melod,2}$, $z_{hmax,2}$, $z_{hmax,3}$, $z_{melod,4}$, $dz_{melod,1}$, $d^2z_{melod,1}$ and z_{sus2}.

The following conclusions can be made. All performers except Ashkenazy put high emphasis on the global shape $z_{melod,1}$. Cortot and "Cortot-type" performances have a high degree of four-measure periodicity (corresponding to the bandwidth $b_2 = 2$). In particular, all Cortot performances have the same three "most important" curves. Apart from the "global" curve $z_{melod,1}$, the two other most important curves are $z_{hmean,2}$ and $dz_{melod,2}$ corresponding to the bandwidth $b_2 = 2$ (and thus a neighborhood of four measures). In contrast, for all three Horowitz performances, the very "local" curve $z_{melod,4}$ corresponding to the bandwidth $b_4 = 0$ is among the three most important explanatory variables. Thus, Cortot and Horowitz have diametrically opposite ways of shaping tempo. The Horowitz-cluster corresponding to the complex local melodic structure of $z_{melod,4}$ suggests that Horowitz puts unusually high emphasis on local structures.

TABLE 2a
Overview of clusters as derived by the criterion in Section 3.3.3(c), *with* $l = 1$ *and* $Z(harmo)$

Artist	$z_{hmean,2}$	$z_{hmean,1}$	$z_{metric,1}$	$z_{melod,4}$
ARG	●			
ARR	●			
ASH	●			
BRE		●		
BUN			●	
CAP	●			
CO1	●			
CO2	●			
CO3	●			
CUR	●			
DAV	●			
DEM	●			
ESC		●		
GIA			●	
HO1				●
HO2		●		
HO3		●		
KAT	●			
KLI		●		
KRU	●			
KUB	●			
MOI		●		
NEY		●		
NOV		●		
ORT		●		
SCH	●			
SHE	●			
ZAK		●		

TABLE 2b
Overview of clusters as derived by the criterion in Section 3.3.3(c), *with* $l = 1$ *and* $Z(melod)$

Artist	$z_{melod,1}$	$z_{hmean,2}$	$z_{melod,2}$
ARG	●		
ARR	●		
ASH			●
BRE	●		
BUN	●		
CAP	●		
CO1		●	
CO2		●	
CO3		●	
CUR	●		
DAV	●		
DEM	●		
ESC	●		
GIA	●		
HO1	●		
HO2	●		
HO3	●		
KAT	●		
KLI	●		
KRU		●	
KUB	●		
MOI	●		
NEY	●		
NOV	●		
ORT	●		
SCH	●		
SHE	●		
ZAK	●		

3.3.4 *Hierarchical decomposition and synthesis of tempo.* Using the size of $|\hat{\beta}_k|$ as criterion for the importance of variable k, we may deduce a natural way of obtaining simplified tempo curves that contain the most important features. For given j and $1 \leq q \leq p$, let the $(p \times 1)$-vector $\gamma_q(j) = [\gamma_{q,1}(j), \gamma_{q,2}(j), \ldots, \gamma_{q,p}(j)]^t$ be defined by $\gamma_{q,k}(j) = \hat{\beta}_k(j) 1\{r_k(j) > p - q\}$. Then $y_q(t_i, j) = Z\gamma_q(j)$ is a simplified tempo curve that corresponds to using the variables (analytic curves) that are among the q most important ones for tempo curve j, importance being measured by $r_k(j)$. Thus, the resulting tempo curve is a simplified curve obtained by superposing the q most important features only. Note that, for $q = p$, this yields the complete curve fitted by stepwise regression. Figure 20a–d displays y_q, $q = 1, \ldots, p$ for $Z(harmo)$ for four typical performances. Using this approach may lead to an objective way of discussing and comparing typical features of performances. On the other hand, the method of successive superposition may also be used for synthesis of tempo curves, an unsolved problem in performance theory.

3.3.5 *Summary.* In summary, the regression model leads to the following conclusions:

- There is a strong association between the metric, melodic and harmonic weights and the tempo.
- The exact relationship is very complex, but a large part of the complexity can be modelled by the proposed method.
- There is no unique way of "explaining" a performance by attributing features of the tempo to exactly one cause (harmonic, metric or melodic analysis). However, there is a small number of essential curves, independent of the chosen priorities (metric, melodic, harmonic), that characterize important commonalities and diversities between tempo curves. It is important to note that these "canonical curves" are score specific.
- Based on the regression results, natural clusters can be defined.
- The proposed method leads to a natural score-specific decomposition of tempo curves into a series of simplified tempo curves containing an increasing number of features.

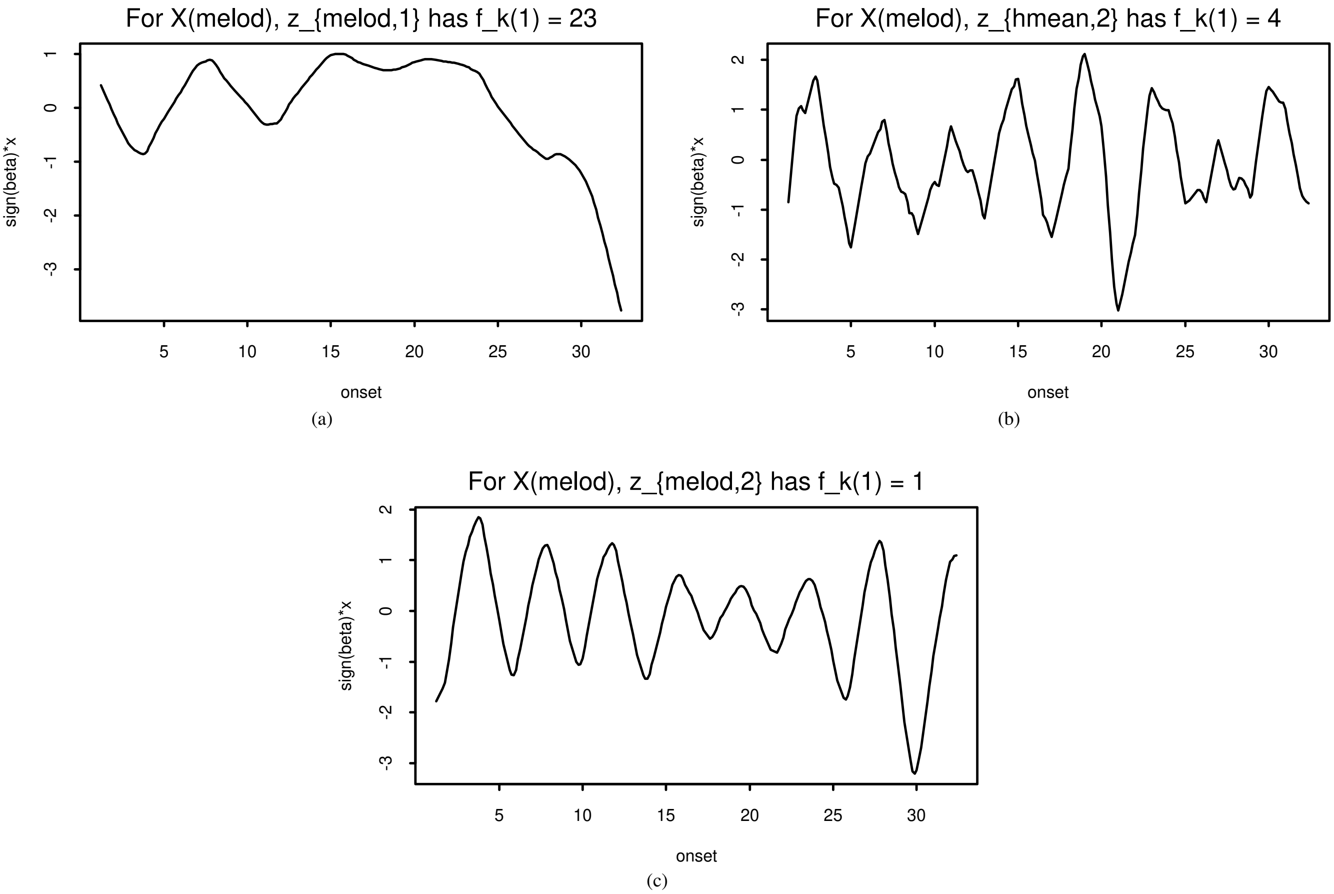

FIG. 19. *Components of $X(melod)$ with largest coefficient*; $f_k(1)$ *is the number of performances for which the corresponding component was the most important one* (*i.e.*, *had the largest coefficient in absolute value*).

4. FINAL REMARKS

In this paper, a method for obtaining numerical results about analytical score structure and its relation to the tempo of a performance was proposed. It was demonstrated that a variety of results can be obtained that are interpretable from the point of view of music theory and performance theory. The method of encoding the score structure essentially consists of two steps: (1) definition of weight functions based on considerations from computational musicology; (2) hierarchical decomposition of the weight functions.

The application to the four scores and to the tempo data shows that the weight functions and the hierarchical approach of decomposing these functions into components of different degree of smoothness seems to be appropriate. Also, hierarchical decomposition is meaningful from the "poietical" point of view of a musician (composer and performer). It is common practice to rehearse first the most global features of a score and then refine the performance successively in greater detail (see [25] for the background of this approach).

In comparison with previous studies about performance structures, our approach is fundamentally different in that the "explanation" of the performance is score specific. It is, however, remarkable that our result that Cortot and Horowitz represent two extreme types of performances confirms a previous result by Repp [31]. Repp came to the same conclusion by applying principal component analysis. In contrast to this standard analysis, our approach yields information about the score-related nature of the commonalities, diversities and clusters. This is due to the fact that the tempo is projected on a space that is defined by the specific score structure, instead of calculating an "omnibus-decomposition" such as principal components or a Fourier decomposition. From the musical point of view this is essential and differs fundamentally from traditional mathematical "omnibus-decompositions."

TABLE 3
Overview of clusters as derived by the criterion in Section 3.3.3(c), *with* $l = 3$

Artist	$z_{melod,1}$	$z_{hmean,2}$	$z_{hmean,1}$	$dz_{melod,3}$	$z_{melod,2}$	$dz_{melod,2}$	$z_{hmax,2}$	$z_{hmax,3}$	$z_{melod,4}$	$dz_{melod,1}$	$d^2z_{melod,1}$	z_{sus2}
ARG	●	●		●								
ARR	●	●		●								
ASH			●		●		●					
BRE	●			●				●				
BUN	●	●									●	
CAP	●	●		●								
CO1	●	●				●						
CO2	●	●				●						
CO3	●	●				●						
CUR	●	●						●				
DAV	●	●			●							
DEM	●			●	●							
ESC	●		●	●								
GIA	●	●				●						
HO1	●			●					●			
HO2	●						●		●			
HO3	●		●						●			
KAT	●	●		●								
KLI	●		●				●					
KRU	●	●			●							
KUB	●	●										●
MOI	●	●	●									
NEY	●		●	●								
NOV	●	●	●									
ORT	●		●							●		
SCH	●	●						●				
SHE	●	●			●							
ZAK	●	●	●									

At this point, the proposed statistical approach is heuristic and exploratory. The definitions used for the analysis were justified intuitively, given background knowledge from musical experience. Much more theoretical and empirical research will be needed to develop a set of statistical tools that are appropriate for data analysis in music. Specific methodological problems within the framework discussed here include the following, for instance:

- *Model extension*—The proposed regression model already "explains" a large part of the observed tempo curves. However, for all performances, an analysis of the residuals, using for instance the approximate maximum likelihood method in [4], revealed clear serial dependencies in the residuals for practically all of the 28 performances. Furthermore, the coherence between the regression residuals and the analytic weights turned out to deviate from zero for certain frequencies. Inclusion of lagged explanatory variables (obtained from Z) does improve the fits and the residuals considerably, but leads to extremely complex models. Clever modelling will be needed to take into account lagged dependencies while keeping the dimension of the explanatory matrix low.
- *Bandwidth choice*—In the decomposition of the score, triangular kernel smoothing was used. Bandwidth choice was done intuitively, based on knowledge about the score. More formal procedures could be defined. In particular, when relating the score to performance data, the various bandwidths could be chosen by optimizing the fit.
- *Hidden structures and other decompositions*—The aim of decomposing the weight functions was to extract more relevant analytical information about the score. The idea was to use a score-specific hierarchy and to avoid predetermined functional shapes. Other types of decompositions may exist that reveal other hidden structures. These may include, for instance, other smoothing procedures, multiplicative instead of additive decomposition, nonlinear transformations and other hierarchies.
- *Semiparametric techniques*—The proposed regression method may be called semiparametric in that it first decomposes the weight functions nonparametrically, and then applies a paramet-

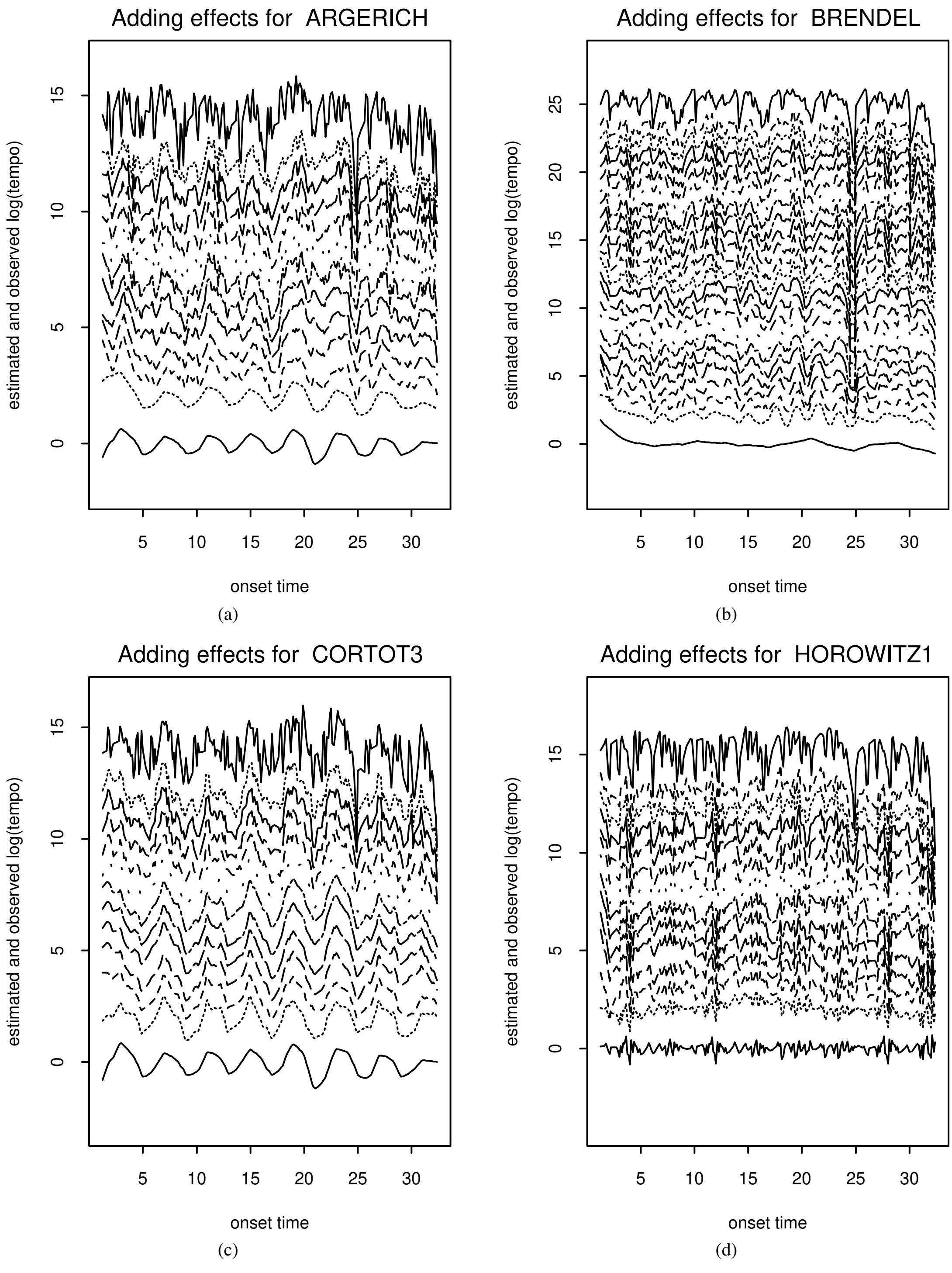

FIG. 20. *Successive approximation of log-tempo curves by* (a) *Argerich*, (b) *Brendel*, (c) *Cortot* (*performance* 3) *and* (d) *Horowitz* (*performance* 1) *by fitted regression curves.*

ric regression model. The success of the model is based on this combination of nonparametric and parametric techniques. Clearly, there may be other models that could be developed in this semiparametric context.

- *Other definitions of analytic weights*—The definitions in Section 3 are based on musicological and mathematical considerations. It is, however, clear that there is no unique way of characterizing metric, melodic and harmonic structures. The given definitions of metric, melodic and harmonic weights, as defined in RUBATO, contain many free "parameters" that need to be set before a score is analyzed. It is an open problem how to define criteria for finding meaningful parameter settings. The basic problem is that meaning is not given a priori, but has to be found without secure guidelines! In our analysis, the choice was based on musicological preferences. Instead, one may for instance try to find parameter settings that correspond best to a performance. How this should be done in detail is unclear because of the large number of free parameters. From the musical point of view, performance-based parameter settings would be meaningful, because different performers may use a different degree of analytical insight. More generally, different definitions of analytic weight functions may be considered, possibly with more precise semantic guidelines.

- *Other aspects of the score*—In the present paper, encoding score information was reduced to the calculation of metric, melodic and harmonic weights. This is only a part of the information contained in the score. For example, contrapuntal structures could also be taken into consideration.
- *Other aspects of performance*—Here, only the tempo of a performance was analyzed. Other aspects of performance will need to be investigated simultaneously. It can be expected that data that encodes most aspects of performances will be high-dimensional and highly structured in a nontrivial way.
- *Other aspects than the score*—Clearly, other aspects than the score may influence a performance. Thus, for instance, psychological, educational or historical circumstances may have to be incorporated. This problem has already been mentioned in the introductory remarks.
- *Computational difficulties*—The definitions of analytic weight functions involve a large number of combinatorial calculations. For example, the motivic calculations exceed any reasonable amount of calculation if handled with ideal boundary conditions. The same is true for harmonic weights. Appropriate sampling and programming techniques may have to be developed to reduce the number of calculations.

Overall, we may conclude that there are a large number of statistical problems that need to be solved in the context of music and performance theory. It can be expected that a vast amount of data will become available that will allow for an empirical development of this scientific discipline. The challenging task for statisticians will be to develop methods for high-dimensional and highly structured data sets, while taking into account the musical context and the large amount of musicological knowledge appropriately. In particular, it must not be ignored that, by the very nature of music, "knowledge" about music is mostly qualitative and partially ambiguous. For many problems the number of good solutions can be infinite. For instance, there is not just one but probably an infinite number of good performances.

The approach presented here provides a possible framework for a more objective discussion about musical structure and performance. It should be emphasized that the aim cannot be to provide a complete "objective" explanation of music. The claim is, however, that certain aspects of music can be discussed in a quantitative semi-objective way. For instance, the decomposition of a tempo curve into a hierarchy of features of decreasing importance leads to a better understanding of the main commonalities and differences between performances. The collaborative effort of statisticians, musicologists and other scientists is likely to bring many exciting insights into music in general.

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REFERENCES

[1] BASTEN, A. (1996). Personal e-mail communication to G. Mazzola, November 22, 1996.
[2] BERAN, J. (1987). *Aniseikonia*. Bison Records.
[3] BERAN, J. (1991). *Cirri*. Centaur Records.
[4] BERAN, J. (1995). Maximum likelihood estimation of the differencing parameter for invertible short and long-memory ARIMA models. *J. Roy. Statist. Soc. Ser. B* **57** 659–672.
[5] BERAN, J. and MAZZOLA, G. (1992). *Immaculate Concept.* SToA 1002.92, Zürich.
[6] BERAN, J. and MAZZOLA, G. (1999). Visualizing the relationship between two time series by hierarchical smoothing models. *J. Comput. Graph. Statist.* To appear.
[7] BROCKWELL, P. J. and DAVIS, R. A. (1987). *Time Series: Theory and Methods.* Springer, New York.
[8] CHATTERJEE, S. and PRICE, B. (1995). *Regression Analysis by Example*, 2nd ed. Wiley, New York.
[9] DE LA MOTTE-HABER, H. (1996). *Handbuch der Musikpsychologie*, 2nd ed. Laaber-Verlag, Laaber.
[10] FRIBERG, A. (1995). *A Quantitative Rule System for Musical Performance.* KTH, Stockholm.
[11] GABRIELSON, A. (1995). Expressive intention and performance. In *Music and the Mind Machine* (R. Steinberg, ed.). Springer, Berlin.

[12] GEWEKE, J. (1981). A comparison of tests of independence of two covariance-stationary time series. *J. Amer. Statist. Assoc.* **76** 363–373.

[13] HAUGH, L. D. (1976). Checking for independence of two covariance stationary time series: a univariate residual cross correlation approach. *J. Amer. Statist. Assoc.* **71** 378–385.

[14] HONG, Y. (1996). Testing for independence between two covariance stationary time series. *Biometrika* **83** 615–626.

[15] JACKENDOFF, R. and LERDAHL, F. (1983). *A Generative Theory of Tonal Music.* MIT Press.

[16] KOPIEZ, R. (1996). Aspekte der Performanceforschung. In *Handbuch der Musikpsychologie* (H. de la Motte, ed.) Laaber-Verlag, Laaber.

[17] KOPIEZ, R. and LANGER, J. (1995). Entwurf einer neuen Methode der Performanceanalyse auf Grundlage einer Theorie oszillierender Systeme. In *Jahrbuch der Deutschen Gesellschaft für Musikpsychologie* (K.-E. Behne and H. de la Motte, eds.) **12**. Deutsche Gessellschaft für Musikpsychologie, Wilhelmshaven.

[18] KOUZES, R. T., MYERS, J. D. and WULF, W. A. (1996). Collaboratories: doing science on the Internet. *Computer* August 1996.

[19] MAZZOLA, G. (1989–1994). *presto.* SToA Music, Zürich.

[20] MAZZOLA, G. (1990). *Synthesis*. SToA 1001.90, Zürich.

[21] MAZZOLA, G. (1990). *Geometrie der Töne.* Birkhäuser, Basel.

[22] MAZZOLA, G. (1995). Inverse performance theory. In *Proceedings of the ICMC 95*. ICMA, Banff.

[23] MAZZOLA, G. and ZAHORKA, O. (1994). Tempo curves revisited: hierarchies of performance fields. *Computer Music Journal* **18**.

[24] MAZZOLA, G. and ZAHORKA, O. (1994). The RUBATO Performance Workstation on NeXTSTEP. In *Proceedings of the ICMC 1994.* ICMA, Aarhus.

[25] MAZZOLA, G. and ZAHORKA, O. (1993-1995). Geometry and logic of musical performance I, II, III. SNSF Research Reports, Univ. Zürich.

[26] MAZZOLA, G. and ZAHORKA, O. (1996). Topologien gestalteter Motive in Kompositionen. In *Festschrift zum 65. Geburtstag J. P. Fricke*. W. Auhagen, Köln. To appear

[27] MAZZOLA, G. and ZAHORKA, O. (1996). RUBATO on the Internet. http://www.ifi.unizh.ch/groups/mml/musicmedia, Univ. Zürich.

[28] MAZZOLA, G., ZAHORKA, O. and STANGE-ELBE, J. (1995). Analysis and Performance of a Dream. In *Proceedings of the 1995 Symposium on Musical Performanc* (J. Sundberg, ed.) KTH, Stockholm.

[29] NOLL, T. (1995). Morphologische Grundlagen der abendländischen Harmonik. Doctoral dissertation, TU Berlin.

[30] PRIESTLEY, M. B. (1981). *Spectral Analysis of Time Series.* Academic Press, New York.

[31] REPP, B. (1992). Diversity and communality in music performance: an analysis of timing microstructure in Schumann's "Träumerei." *Journal of the Acoustic Society of America* **92**.

[32] REPP, B. (1995). e-mail communication of tempo data. Haskins Laboratories, New Haven, June 2, 1995.

[33] RÉTI, R. (1978). *The Thematic Process in Music*, 2nd ed. Westport, Greenwood Press.

[34] RIEMANN, H. (1893). *Vereinfachte Harmonielehre oder die Lehre von den tonalen Funktionen der Akkorde.* London.

[35] *S-Plus.* (1995). MathSoft Inc., Seattle, WA.

[36] STEINBERG, R. ed. (1995). *Music and the Mind Machine.* Springer, New York.

[37] SUNDBERG, J., ed. (1994). *Generative Grammars for Music Performance.* KTH, Stockholm.

[38] SUNDBERG, J. and VERILLO, V. (1980). On the anatomy of the retard: A study of timing in music. *Journal of the Acoustic Society of America* **68** 772–779.

[39] TODD, N. P. MCA. (1985). A model of expressive timing in tonal music. *Music Perception* **3** 33–58.

[40] TODD, N. P. MCA. (1989). Towards a cognitive theory of expression: the performance and perception of Rubato. *Contemporary Music* Review **4**.

[41] TODD, N. P. MCA. (1992). The dynamics of dynamics: A model of musical expression. *Journal of the Acoustic Society America* **91** 3540–3550.

[42] WAUGH, W. A. O'N. (1996). Music, probability, and statistics. In *Encyclopedia of Statistical Sciences* **6** 134–137.

[43] WEBERN, A. (1980). Variationen op.27 mit Interpretationsanweisungen. UE 16845, Wien.

[44] YAGLOM, A. M. and YAGLOM, I. M. (1967). *Wahrscheinlichkeit und Information*. Deutscher Verlag der Wissenschaften, Berlin.

[45] ZAHORKA, O. (1995). PrediBase—controlling semantics of symbolic structures in music. In *Proceedings of the ICMC 95.* ICMA, Banff.

[46] ZAHORKA, O. (1998). From sign to sound—analysis and performance of musical scores on RUBATO. In *Proceedings of Klangart 95* (B. Enders, ed.) Schott, Mainz.