ANALYZING THE SPECTRAL (A)SYMMETRY OF THE MASSLESS DIRAC OPERATOR ON THE 3–TORUS

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Abstract. We analyze the spectrum of the massless Dirac operator on the 3-torus $\mathbb{T}^3$. It is known that it is possible to calculate this spectrum explicitly, that it is symmetric about zero and that each eigenvalue has even multiplicity. However, for a general oriented closed Riemannian 3-manifold (M, g) there is no reason for the spectrum of the massless Dirac operator to be symmetric. Using perturbation theory, we derive the asymptotic formulae for its eigenvalues and prove that by the perturbation of the Euclidean metric on the 3-torus, it is possible to obtain spectral asymmetry of the massless Dirac operator in the axisymmetric case.

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