Ancestor Sampling for Particle Gibbs: Supplemental material

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Abstract

This supplemental material contains the proof of [1, Proposition 1].

1 Proof of Proposition 1

With $M = T - t$ and $w(k) = w_t^k$, the distributions of interest are given by

$$P(k) = \frac{w(k)\prod_{s=1}^{M} h_s(k)}{\sum_l w(l)\prod_{s=1}^{M} h_s(l)} \qquad \text{and} \qquad \widetilde{P}_p(k) = \frac{w(k)\prod_{s=1}^{p} h_s(k)}{\sum_l w(l)\prod_{s=1}^{p} h_s(l)},$$

respectively. Let $\varepsilon_s \triangleq \max_{k,l}\,(h_s(k)/h_s(l) - 1) \leq A\exp(-cs)$ and consider

$$\begin{aligned}\left(\sum_l w(l)\prod_{s=1}^{p} h_s(l)\right)\prod_{s=p+1}^{M} h_s(k) &\leq \sum_l \left(w(l)\prod_{s=1}^{p} h_s(l)\prod_{s=p+1}^{M} h_s(l)(1+\varepsilon_s)\right) \\ &= \left(\sum_l w(l)\prod_{s=1}^{M} h_s(l)\right)\prod_{s=p+1}^{M}(1+\varepsilon_s).\end{aligned}$$

It follows that the KLD is bounded according to,

$$\begin{aligned}D_{\mathsf{KLD}}(P\|\widetilde{P}_p) &= \sum_k P(k)\log\frac{P(k)}{\widetilde{P}_p(k)} \\ &= \sum_k P(k)\log\left(\frac{\prod_{s=p+1}^{M} h_s(k)\left(\sum_l w(l)\prod_{s=1}^{p} h_s(l)\right)}{\sum_l w(l)\prod_{s=1}^{M} h_s(l)}\right) \\ &\leq \sum_k P(k)\sum_{s=p+1}^{M}\log(1+\varepsilon_s) \leq \sum_{s=p+1}^{M}\varepsilon_s \leq A\sum_{s=p+1}^{M}\exp(-cs) \\ &= A\frac{e^{-c(p+1)} - e^{-c(M+1)}}{1-e^{-c}}.\end{aligned}$$

□

References

[1] F. Lindsten, M. I. Jordan, and T. B. Schön, "Ancestor sampling for particle Gibbs," in *Proceedings of the 2012 Conference on Neural Information Processing Systems (NIPS)*, Lake Tahoe, NV, USA, Dec. 2012.