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The cyclical behavior of job creation and job destruction: A sectoral model*

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Summary. Three key features of the employment process in the U.S. economy are that job creation is procyclical, job destruction is countercyclical, and job creation is less volatile than job destruction. These features are also found at the sectoral (goods and services) level. The paper develops, calibrates and simulates a two-sector general equilibrium model that includes both aggregate and sectoral shocks. The behavior of the model economy mimics the job creation and destruction facts. A non-negligible amount of unemployment arises due to the presence of aggregate and sectoral shocks.

1. Introduction

What determines the amount of employment in an economy and its distribution across sectors, or the size of the labor market and its breakdown between those with and without jobs. In U.S. economy job creation is procyclical, job destruction is countercyclical, and job creation is less volatile than job destruction. In a well-known paper, Lilien [10] advanced the hypothesis that variations in sectoral opportunities together with frictions impeding the inter-sector movement of workers play an important role in determining labor market aggregates, and in particular unemployment. The questions raised by these findings are: Can a multi-sector dynamic general equilibrium model replicate the pattern of job creation, and destruction that is observed in the U.S. data? Are sectoral shocks important for determining the average rate of unemployment?

The analysis seeks to explain movements in labor market aggregates as the outcome of the interaction of aggregate and sectoral shocks. The model developed to do this is a multi-sector dynamic competitive general equilibrium framework. The model has three key features. First, each market sector gets hit by both aggregate

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and sectoral shocks. This is similar, in spirit, to the classic Long and Plosser [12] real business cycle model. Second, it takes time to reallocate labor across sectors. Each sector in the market economy can draw new employees from a pool of unemployed workers seeking a job. This pool is made up of agents who entered it in some earlier period, either because they lost their job in a market sector or left the home sector. This feature of the analysis requiring a time cost for job reallocation bears some resemblance to the well-known Lucas and Prescott [14] equilibrium search model. Third, following Hansen [6] and Rogerson [20], it is assumed that labor is indivisible. This assumption ensures that the options of working, searching and staying at home are mutually exclusive.

The model developed reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively well. Workers flow between sectors as jobs are created and destroyed in response to both aggregate and sector-specific shocks. A conclusion of the paper is that aggregate and sectoral shocks contribute a non-negligible amount to the average level of unemployment.[1] Here, approximately one percentage point of the unemployment rate can be accounted for by aggregate and sectoral shocks. The fact that generally some workers are unemployed, but ready to work, allows sectors to expand their output more rapidly in reaction to favorable circumstances in much the same way as inventories of raw materials, parts, etc. do.[2]

The rest of the paper sets out the model in detail and explores its features quantitatively.

2. Model

The multisector dynamic general equilibrium model to be simulated will now be developed.

2.1 Economic environment

A continuum of ex ante identical agents is distributed uniformly over the unit interval. In period t an agent can work in one of two productive sectors, search for a job, or stay at home. To describe this, let $\pi_{i,t}$ represent the fraction of agents who are working in sector i at t, and $\pi_{3,t}$ denote the fraction of agents who are searching. Thus, the fraction of the population currently at home is $1-\sum_{i=1}^{3}\pi_{i,t}$ while $1-\sum_{i=1}^{2}\pi_{i,t}$ is the proportion not working. A description of tastes, technology and the stochastic structure of the model follows.

2.1.1 Tastes Let c_{it} represent an agent's period-t consumption of the commodity produced in sector i. An agent has one unit of non-sleeping time. Labor effort is indivisible with it being assumed that work and search require w and s hours of effort, respectively. Leisure is then given by $1-l_t$, where $l_t \in \{0, s, w\}$. An agent's

[1] Andolfatto [1] studies the equilibrium determination of unemployment within the context of a matching model (that has both aggregate and idiosyncratic shocks).

[2] Clearly technological advances, such as changes in organizational forms, that allow inputs to be allocated more quickly to their end-uses are likely to be desirable.

expected lifetime utility is given by

$$E\left[\sum_{t=0}^{\infty}\beta^{t}\left\{A\ln\left[\sum_{i=1}^{2}\theta_{i}c_{it}^{\rho}(l_{t})\right]^{1/\rho}+(1-A)\ln(1-l_{t})\right\}\right], \tag{1}$$

where $\beta\in(0,1)$, $\rho\in(-\infty,0)\cup(0,1]$, $\theta_i\in(0,1)$, and $\sum_{i=1}^{2}\theta_i=1$.[3]

2.1.2 Production technology Sector i is subject to both aggregate, z_t, and sectoral, $\varepsilon_{i,t}$, disturbances. There is a firm in each sector i that produces output y_{it} according to the production technology:

$$y_{i,t}=z_t\varepsilon_{i,t}(h_{i,t})^{\alpha_i}-I_i(z_t,\varepsilon_{i,t}), \tag{2}$$

where

$$h_{i,t}=w[\pi_{i,t}-\gamma_i(\max[\pi_{i,t}-\pi_{i,t-1},0])^{\lambda_i}]. \tag{3}$$

In (2) $h_{i,t}$ represents the amount of labor input hired by the firm. Hiring new labor is costly. One of the costs is assimilating new workers into the production process, a feature portrayed by (3). This cost is increasing in the number of workers that join the firm. This is equivalent to saying that when new workers are hired in a period they are less productive than experienced workers. The term $I_i(z_t,\varepsilon_{i,t})$ is an output-reducing shock. This function is discussed in more detail below. By substituting (3) into (2) it is easy to see that production is governed by

$$y_{i,t}=z_t\varepsilon_{i,t}w^{\alpha_i}[\pi_{i,t}-\gamma_i(\max[\pi_{i,t}-\pi_{i,t-1},0])^{\lambda_i}]^{\alpha_i}-I_i(z_t,\varepsilon_{i,t}). \tag{4}$$

Firms are owned by households.

2.1.3 Search technology In order to increase its employment a firm must draw new labor from the search pool. Thus, the increase in employment that can occur in a sector is limited by the size of the existing search pool. Specifically,

$$\sum_{i=1}^{2}\max\{0,\pi_{i,t+1}-\pi_{i,t}\}\leq\pi_{3,t}. \tag{5}$$

Note that (5) implies any reallocation of agents between sectors 1 and 2 will involve a one period transition cost.

2.1.4 Stochastic structure The aggregate and sectoral disturbances are independent of one another and follow finite-state first-order Markov processes with supports $Z=\{z^1,z^2,\ldots,z^m\}$ and $E_i=\{\varepsilon_i^1,\ldots,\varepsilon_i^m\}$, respectively. Furthermore, it will be assumed that the shocks in sectors 1 and 2 are inversely related to one another. In particular, let $\varepsilon_1=1/\varepsilon_2\equiv\varepsilon$.

2.2 Planner's problem

Following Rogerson [20] and Hansen [6] the representative household's choice set is extended to include the possibility of a lottery over their consumption and labor allocations. One can think about the lottery mechanism as an employment contract

[3] The case where $\rho=0$ is easily handled by letting the expected value of lifetime ulitity read $E[\sum_{t=0}^{\infty}\beta^t\{A\ln[\Pi_{i=1}^{2}c_{it}(l_t)^{\theta_i}]+(1-A)\ln(1-l_t)\}]$.

specifying for each $l \in \{0, s, w\}$ a (state-contingent) probability $\pi(l)$ that the agent will work l hours, consume $c_i(l)$ units of sector-i output and enjoy $1-l$ units of leisure. Since all agents are alike initially, it follows from using an appropriate law of large numbers that $\pi(0) = 1 - \sum_{i=1}^{3} \pi_i, \pi(s) = \pi_3$, and $\pi(w) = \sum_{i=1}^{2} \pi_i$. The planner's dynamic programming problem that determines the form of this contract is shown below.

$$V(\pi; z, \varepsilon) = \max_{\{c_i(l)\},\{\pi_i'\}} \Bigg\{ \left(1 - \sum_{i=1}^{3} \pi_i'\right)\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2} \theta_i \cdot c_i^{\rho}(0)\right)\right]$$
$$+ \pi_3'\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2} \theta_i \cdot c_i^{\rho}(s)\right) + (1-A)\cdot\ln(1-s)\right]$$
$$+ \left(\sum_{i=1}^{2} \pi_i'\right)\left[\frac{A}{\rho}\ln\left(\sum_{i=1}^{2} \theta_i \cdot c_i^{\rho}(w)\right) + (1-A)\cdot\ln(1-w)\right]$$
$$+ \beta E[V(\pi'; z', \varepsilon') \mid \pi; z, \varepsilon] \Bigg\} \qquad P(1)$$

subject to

$$\left(1 - \sum_{j=1}^{3} \pi_j'\right)c_i(0) + \left(\sum_{j=1}^{2} \pi_j'\right)c_i(w) + \pi_3' c_i(s)$$
$$= z\varepsilon_i w^{\alpha_i}\left[\pi_i' - \gamma_i(\max[\pi_i' - \pi_i, 0])^{\lambda_i}\right]^{\alpha_i} - I_i(z, \varepsilon_i), \quad \text{for } i = 1, 2, \qquad (6)$$

$$\sum_{i=1}^{3} \pi_i' \leq 1, \qquad (7)$$

$$\sum_{i=1}^{2} \max\{0, \pi_i' - \pi_i\} \leq \pi_3, \qquad (8)$$

$$\pi_i' \geq 0, \quad \text{for } i = 1, 2, 3. \qquad (9)$$

The resource constraint for each sector is given by (6). The next constraint limits the aggregate amount of labor that can be used in non-leisure activities. Equation (8) states that the amount of new labor that can be hired by sectors 1 and 2 is restricted by the size of the search pool.

Given the separability of preferences, the planner will select consumption paths that are independent of agents' labor market status.[4] Thus, $P(1)$ can be simplified to

$$V(\pi; z, \varepsilon) = \max_{\{\pi_i'\}} \Bigg\{ \frac{A}{\rho}\ln\left[\sum_{i=1}^{2} \theta_i(z\varepsilon_i w^{\alpha_i}[\pi_i' - \gamma_i(\max[\pi_i' - \pi_i, 0])^{\lambda_i}]^{\alpha_i} - I_i(z, \varepsilon_i))^{\rho}\right]$$
$$+ (1-A)\left[\pi_3' \ln(1-s) + \left(\sum_{i=1}^{2} \pi_i'\right)\ln(1-w)\right]$$
$$+ \beta \cdot E[V(\pi'; z', \varepsilon') \mid \pi; z, \varepsilon] \Bigg\}. \qquad P(2)$$

subject to (7), (8), and (9).

[4] For more detail, see Greenwood and Huffman [5] or Rogerson and Wright [21].

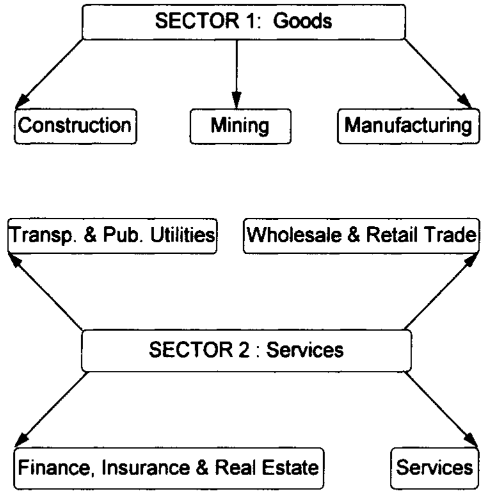

Figure 1.

2.3 Discussion

Multisector frameworks similar to the one presented above have been developed in Rogerson [19] and Hornstein [9]. The planning problem $P(1)$ determines a Pareto-optimal allocation for the economy under study. An interesting question that arises is whether or not this Pareto-optimal allocation can be decentralized as a competitive equilibrium? By extending the analysis of Prescott and Rios-Rull [17], it should be possible to show that this allocation can be supported as a quasi-competitive equilibrium. A key step in doing this is to represent the commodity space as a set of infinite sequences of measures specifying the odds of consuming a given quantity of goods and leisure, contingent upon a particular history of aggregate and sectoral shocks.

3. Calibration

The model is restricted to two sectors, assumed to correspond to the goods and service sectors of the U.S. economy. The industries that make up these sectors are shown in Figure 1; one period is assumed to be one quarter.

3.1 Preference parameters

The quarterly interest rate is taken to be one percent; thus the discount factor, β, is 0.99. Next, data from the Monthly Labor Review shows that, on average, the employed work 39 out of the approximately 100 non-sleeping hours per week available to them; consequently, $w = .39$. According to Barron and Mellow [2], the mean number of hours spent searching per week is approximately 7 which implies $s = .07$. In a similar vein, a value of 0.28 was picked for the coefficient A in the utility function. This results in approximately 25% of aggregate non-sleeping hours being spent at work. In the U.S. data the goods sector is about 58 percent of the size of the service sector, when measured by employment. This occurs in the model's steady state if $\theta_1 = .43$ ($\theta_2 = .57$). Finally, the parameter $\rho \in (-\infty, 1]$ governs the amount of substitution between goods and services in the utility function. Independent evidence on an appropriate value for ρ is hard to come by. In the subsequent analysis, ρ is assigned a value of 0.55.[5]

3.2 Technology parameters

The two production function parameters, α_1 and α_2, are set equal to 0.74 and 0.64 respectively. These numbers are labor's share of income in goods and services sectors for the 1964–1987 period.[6]

3.3 Adjustment costs

The adjustment cost parameters, λ_i and γ_i, are set at 2.0 and 5.5, respectively, for both sectors. These are free parameters that determine the speed of sectoral employment adjustment.[7]

3.4 Shocks

Recall the assumption that $\varepsilon_1 = 1/\varepsilon_2 = \varepsilon$ – this amounts to assuming a single relative sectoral shock. Then, using (2) and data for each sector's output and labor input, the aggregate and sectoral Solow residuals are easy to calculate.[8] By doing this it is found that the aggregate shock has a percentage standard deviation of 0.04 and a serial correlation coefficient of 0.93. The numbers for the sectoral shock are 0.015 and 0.93.

The aggregate and sectoral shocks are two-state Markov processes: $z_t \in Z = \{\exp^{\xi}, \exp^{-\xi}\}$ with $\Pr[z' = z_1 | z = z_1] = \Pr[z' = z_2 | z = z_2]$, and $\varepsilon \in E = \{\exp^{\zeta}, \exp^{-\zeta}\}$ with $\Pr[\varepsilon' = \varepsilon_1 | \varepsilon = \varepsilon_1] = \Pr[\varepsilon' = \varepsilon_2 | \varepsilon = \varepsilon_2]$. The parameters ξ and

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5 The utility function specified in (1) implies that an agent will divide his consumption between goods and services according to the formula $\ln\frac{c_1}{c_2} = \frac{1}{\rho - 1}\ln p$, where p is the relative price of goods in terms of services. Estimation of this equation using instrumental variables yielded a value of .55 for ρ. Unfortunately, this point estimate was insignificant at the 95% level of confidence. Still, on the basis of the time series evidence a value of 0.55 is the best guess for ρ.

6 Labor's share of income for sector i, or α_i, was computed from the formula shown below using data from the National Income and Product Accounts:

$$\alpha_i = \frac{COM_i}{NI_i + CCA_i - PI_i},$$

where COM_i is Compensation of Employees for sector i, NI_i is National Income, CCA_i is the Capital Consumption Allowance, and PI_i is Proprietor's Income.

7 The adjustment costs are quantitatively trivial in magnitude. The loss in labor input due to adjustment costs averages less than 0.0028 percent of total employment in the simulations undertaken.

8 The assumption on the functional form for the sectoral disturbances allows them to be easily identified.

ζ are chosen so that the time series properties for the aggregate and sectoral disturbances in the model inherit the time series behavior of the aggregate and sectoral Solow residuals. This implies setting $\xi = .04$, $\Pr[z' = z_1 | z = z_1] = .965$, $\zeta = .015$ and $\Pr[\varepsilon' = \varepsilon_1 | \varepsilon = \varepsilon_1] = .965$.[9]

3.5 Investment

Finally, in the U.S. economy consumption is relatively smooth, and investment is procyclical and highly volatile. This motivates subtracting a certain amount of output, equal to investment, from the right hand side of the resource constraints.[10] The function $I_i(z, \varepsilon_i)$ is intended to capture this. Let the investment functions, $I_i(z, \varepsilon_i)$, have the form

$$I_i(z, \varepsilon_i) = \begin{cases} e^{\sigma + \sigma_i} I_i^*, & \text{if } z = e^{\xi} \text{ and } \varepsilon_i = e^{\zeta}, \\ e^{\sigma - \sigma_i} I_i^*, & \text{if } z = e^{\xi} \text{ and } \varepsilon_i = e^{-\zeta}, \\ e^{-\sigma + \sigma_i} I_i^*, & \text{if } z = e^{-\xi} \text{ and } \varepsilon_i = e^{\zeta}, \\ e^{-\sigma - \sigma_i} I_i^*, & \text{if } z = e^{-\xi} \text{ and } \varepsilon_i = e^{-\zeta}, \end{cases}$$

where the means and standard deviations of $\ln I_i(z, \varepsilon_i)$ are given by $\ln I_i^*$ and $\sqrt{\sigma^2 + \sigma_i^2}$.

In the U.S., aggregate investment is approximately 20 percent of GNP. This implies that in the model's steady state $I_1 + pI_2 = .2[y_1 + py_2]$, where p is the relative price of good two. Also, the goods producing sector generates two-thirds as much output as the service sector. If it is assumed that investment spending is spread across sectors proportionally, then the model's steady state should display the feature that $I_1/I_2 = y_1/y_2$. Assuming this, along with $I_1 + pI_2 = .2[y_1 + py_2]$, implies $I_1^* = .0378$ and $I_2^* = .0605$. In the U.S. data, investment is four times as volatile as output and the correlation coefficient between aggregate investment and output is 0.95. The percentage standard deviations for the investments were chosen to mimic these observed facts. This involved setting $\sigma = .08$, $\sigma_1 = .06$, and $\sigma_2 = .08$.

4. Findings

The cyclical properties of the above model are developed through simulation. As is now standard, the procedure is to compare a set of stylized facts characterizing the business cycle behavior of the model with a analogous set describing U.S. postwar business cycle behavior over the 1964.1–1987.4 sample period. Appendix A details the computational procedure used to calculate the decision-rules associated with the planner's problem. The procedure used to compute the decision-rules is complicated by the presence of the inequality constraint (8). With these decision-rules in hand,

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[9] It is straightforward to calculate that the percentage standard deviations of the aggregate and sectoral disturbances are given by ξ and ζ. Likewise, the formulae for the autocorrelation coefficients for the shocks are $2\Pr[z' = z_1 | z = z_1] - 1$ and $2\Pr[\varepsilon' = \varepsilon_1 | \varepsilon = \varepsilon_1] - 1$, respectively.

[10] The aggregate disturbance will not affect the solution to the model if there is no investment term in the resource constraint (6). This is immediate from problem $P(2)$. Without the $I_i(z, \varepsilon_i)$ term, it is easy to see that z can be factored out of the first term on the righthand side of $P(2)$. Hence it can't affect the maximization.

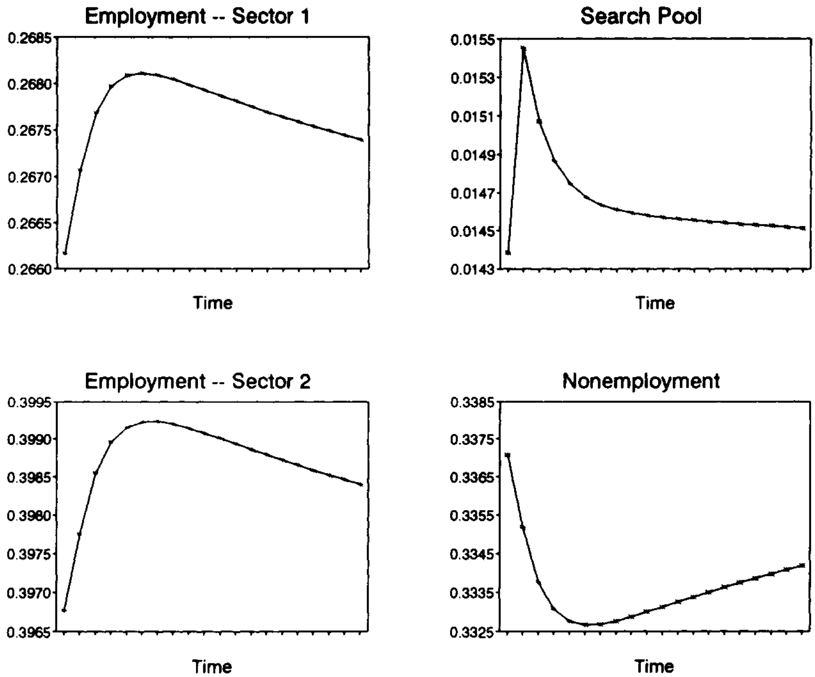

Figure 2.

200 samples of 96 observations (the number of quarters in the U.S. sample period) are simulated. Each simulation run corresponds to a randomly generated sample of 96 realizations of the z and ε processes. The data from the simulations is logged (where applicable) and H-P filtered, as is the data for the U.S. economy, and average moments over the 200 samples are computed for each variable of interest.

4.1 Impulse-response functions

The dynamic effects that aggregate and sectoral disturbances have on sectoral employment and aggregate nonemployment can be represented in terms of impulse-response functions.[11] This is done by fitting a first-order vector autoregression of the form $\pi' = c + b\pi + v$ to the simulated data, where $\pi = [\pi_1, \pi_2, \pi_3]^T$, c and b are 3×1 and 3×3 parameter vectors, and v is a 3×1 vector of approximation errors. Figure 2 plots the impulse response functions associated with an aggregate shock, where the economy is assumed to be in a steady state initially. Employment in both sectors rises, while aggregate nonemployment (or $1 - \pi_1 - \pi_2$) falls. Notice that it takes the economy five periods to move agents out of the searching pool and home sector into work in the two market sectors. This illustrates the influence of adding the search

[11] To be nonemployed is defined here as not working. In the model the number of agents who are nonemployed is $1 - \pi_1 - \pi_2$. This is an exact concept and does not match up precisely with the notion of being unemployed. In the U.S. data an agent is counted as being unemployed if he is not working, but has looked for a job within the last four weeks.

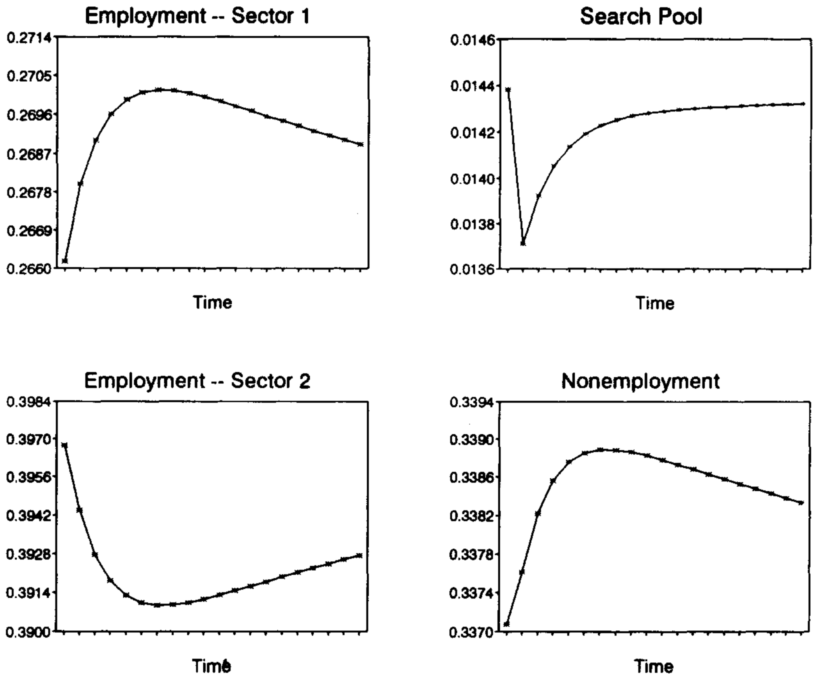

Figure 3.

pool to the model. The results here are consistent with Jovanovic's [11] argument that a positive serially correlated aggregate shock will simultaneously increase sectoral employments and search, and decrease aggregate nonemployment. Similarly, Figure 3 shows the impulse response functions for a sectoral shock. A positive sectoral shock increases the productivity of the goods sector relative to services. Consequently, employment in goods (services) production rises (falls). Again, it takes the economy about five to six periods to go through the adjustment process. Observe that nonemployment rises following the sectoral shock. This transpires since sector two is larger than sector one; more workers are withdrawn from sector two in response to the technology shock than are added to sector one with the difference leaving the labor force.

4.2. Aggregate and sectoral fluctuations

The amount of job creation in sector i during period t is given by $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}$. Thus, the sector-$i$ job creation rate is defined to be $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\pi_{i,t-1}$. Likewise, for sector-$i$ the job destruction rate is $\max\{\pi_{i,t-1} - \pi_{i,t}, 0\}/\pi_{i,t-1}$. The sum of these job creation and destruction rates defines the sector-i job reallocation rate. It follows that the aggregate job creation and destruction rates are $\sum_{i=1}^{2} \max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\sum_{i=1}^{2} \pi_{i,t-1}$ and $\sum_{i=1}^{2} \max\{0, \pi_{i,t-1} - \pi_{i,t}\}/\sum_{i=1}^{2} \pi_{i,t-1}$. The sum of the aggregate job creation and destruction rates defines the aggregate job reallocation rate.

Table 1. Cyclical behavior of U.S. labor market aggregates (Quarterly, 1964.1–1987.4)

Variables	S.D. (%)	Corr/Output	Corr/Employment
Output	2.50	1.00	0.86
Employment	1.66	0.86	1.00
Hours	1.96	0.91	0.98
Nonemployment		−0.93	−0.92
Job creation rate	0.71	**0.46**	**0.12**
Job destruction rate	1.82	**−0.47**	**−0.20**
Job reallocation rate	0.56	**−0.14**	**−0.12**
Productivity	1.06	0.66	**0.20**

Note: The U.S. economy analyzed in this paper consists of two sectors. One of them, Sector 1, is the Goods sector which includes three 1-digit SIC(1987) industries: Mining, Construction, and Manufacturing. The other, Sector 2, is the Service sector which includes the following SIC industries: Transportation and Public Utilities, Wholesale Trade and Retail Trade, Finance-Insurance and Real Estate, and Services. All industry time series are taken from CITIBASE (1989). Since quarterly GNP by industry is not available, National Income is used as substitute. The availability of data on average weekly hours worked by industry determines the sample periods starting from 1964.1 to 1987.4. For productivity, Corr/Employment represents the correlation of productivity and hours. All series are logged (where applicable) and detrended using the Hodrick–Prescott filter.

Table 2. Cyclical behavior of labor market aggregates (Model: 200 simulations with 96 observations each)

Variables	S.D. (%)	Corr/Output	Corr/Employment
Output	1.92	1.00	0.87
Employment	0.29	0.87	1.00
Hours	0.29	0.87	0.99
Nonemployment		−0.87	−1.00
Job creation rate	1.69	**0.26**	**0.03**
Job destruction rate	1.95	**−0.30**	**−0.22**
Job reallocation rate	1.54	**−0.04**	**−0.12**
Productivity	1.67	0.99	**0.81**

Note: All time series are logged (where applicable) and detrended using the H-P filter. The statistics shown in all tables are the average values after 200 simulations of 96 observations each. For productivity, Corr/Employment represents the correlation of productivity and hours.

Descriptive statistics characterizing the cyclical behavior of U.S. labor market aggregates are presented in Table 1. Table 2 presents the same statistics for the model. The model reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively accurately. Specifically,

- In both the model and the data, the job creation rate moves procyclically while the job destruction and reallocation rates are countercyclical. The correla-

tions between these variables, on the one hand, and GNP and employment, on the other, are also close to those found in U.S. data.
- In both the model and the data the job destruction rate is more volatile than either the job creation or reallocation rates. This reflects the importance of the asymmetric nature of the employment process. It is much easier to fire people than to hire them.
- In the data the correlation between hours and productivity is low, as evidenced by the correlation coefficient of 0.20. For the model the number is 0.81, which is too high. On this dimension the model performs more or less the same as the standard model with indivisible labor, but worse than models that include government spending or household production – see Hansen and Wright [8]. Another shortcoming is that hours worked in the model is much less volatile than in the data. Consequently, productivity fluctuates more in the model than in the data. This is due to the presence of adjustment costs for hiring labor.[12]

Next, some stylized facts describing the behavior of U.S. labor market variables at the sector level are given in Table 3. Table 4 presents the same set of facts for the model. The key findings here are:

- In the data, the job creation, destruction and reallocation rates display the same pattern of cyclical behavior at the sectoral level as they do for the economy as a whole. There is, however, one exception: while the job reallocation moves countercyclically in the goods producing sector it moves procyclically in services.[13] The model replicates fairly closely the correlation structure between these variables and output, except for the procyclical movement of the job reallocation rate in the service sector.
- The model and data share the feature that output and employment are more volatile in goods production than in services.
- The model does a much better job matching the hours/productivity correlations observed at the sectoral level.

Finally, Table 5 reports negative correlations between job creation and destruction rates, at both the aggregate and sectoral levels. Similar findings are reported in Mortensen [15]. On this,

- The model yields mixed results here. On the one hand, a positive correlation between aggregate job creation and destruction is displayed by the model. On the other, the model does replicate the negative association between job

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[12] Hours fluctuates more than productivity in the data. Hansen [6] matched this fact by introducing indivisible labor into an otherwise standard stochastic growth model. In the current analysis productivity is more volatile than hours, notwithstanding the use of indivisible labor.

[13] The size of the service sector has increased over time while the volume of goods production has declined. Jobs created in the service sector may accelerate during booms and jobs destroyed in the goods sector may speed up in recessions. This hypothesis is consistent with findings in Loungani and Rogerson [13].

Table 3. Cyclical behavior of sector labor aggregates in U.S. economy (Quarterly, 1964.1–1987.4)

Variables	S.D. (%)		Corr with Output		Corr with Employment	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
Output	4.12	1.50	1.00	1.00	0.87	0.77
Employment	2.94	1.00	0.88	0.77	1.00	1.00
Hours	3.50	1.02	0.93	0.81	0.98	0.98
Job creation rate	1.31	0.56	**0.32**	**0.47**	**0.03**	**0.14**
Job destruction rate	1.68	2.95	**−0.49**	**−0.40**	**−0.24**	**−0.16**
Job reallocation rate	0.96	0.47	**−0.37**	**0.31**	**−0.24**	**0.07**
Productivity	1.54	0.90	0.56	0.75	**0.12**	**0.18**

Note: For productivity, Corr/Employment represents the correlation of productivity and hours.

Table 4. Cyclical behavior of sector labor aggregates (Model: 200 simulations with 96 observations each)

Variables	S.D. (%)		Corr with Output		Corr with Employment	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
Output	2.24	2.16	1.00	1.00	0.76	0.75
Employment	0.68	0.63	0.76	0.75	1.00	1.00
Hours	0.68	0.63	0.76	0.75	1.00	1.00
Job creation rate	2.65	2.15	**0.30**	**0.37**	**0.10**	**0.18**
Job destruction rate	2.97	2.32	**−0.37**	**−0.38**	**−0.22**	**−0.19**
Job reallocation rate	1.85	1.44	**−0.08**	**−0.02**	**−0.10**	**0.00**
Productivity	1.77	1.74	0.95	0.96	**0.57**	**0.57**

Table 5. Relation between job creation and destruction (Correlation)

	Aggregate	Sector 1	Sector 2
U.S. data (Quarterly, 1964.1–1987.4)	−0.50	−0.37	−0.44
Model (200 simulations)	0.40	−0.13	−0.18

creation and destruction observed at the sectoral level, although it understates the size of the correlations.

4.3 The determination of aggregate unemployment

How much of unemployment can be accounted for by aggregate and sectoral shocks? In the absence of technology shocks there would be no steady-state search unemployment in the model. Thus, the average value for π_3 is a measure of the amount of unemployment due to aggregate and sectoral disturbances.[14] On this account 1.20 percent of the labor force is unemployed. To get a rough estimate of how the aggregate and sectoral shocks contribute to unemployment, the aggregate and sectoral shocks can be shut down in turn. When the aggregate shock is shut down (i.e., $\xi = 0$ and $\zeta = .015$) the average value for π_3 falls to 0.97. The average value of π_3 drops to 1.00 when the sectoral shock is turned off (i.e., $\xi = 0.04$ and $\zeta = 0$). Thus, aggregate and sectoral shocks have a similar effect on the average level unemployment. Finally, the procyclical nature of quits in the U.S. economy suggests that job search is procyclical (see Jovanovic [11]). The model predicts that the search is procyclical, that is the correlation between π_3 and output is 0.61.

4.4 Discussion

The job creation and destruction rates computed above represent the lower bounds on the amount of job creation and destruction in the U.S. economy. To ease the burden of the quantitative analysis the economy was dichotomized into two broad sectors, goods and services. If the economy was disaggregated down further into many sectors the amount of job creation and destruction would increase.[15] In fact, the amount of job creation, destruction, and reallocation could be disaggregated down to the level of the plant, as Davis and Haltiwanger [4] do for the manufacturing sector of the U.S. economy. They find that job creation is procyclical, job destruction is countercyclical, and the latter is more volatile than the former. In a model with many sectors, and perhaps many plants within a sector, the amount of steady-state search unemployment due sectoral and plant-specific shocks should increase. On this, in a study of 26 U.S. industries, Loungani and Rogerson [13] find that approximately 5.5 percentage points of unemployment among workers can be accounted for by industry switchers.

5. Concluding remarks

A multisector dynamic general equilibrium model is constructed here to analyze the cyclical pattern of job creation and destruction. The two main ingredients in the model are the Lucas–Prescott [14] idea that it takes time to find employment and the Rogerson [20]/Hansen [6] notion of indivisible labor. It is found that the model can successfully replicate the cyclical patterns of job creation, destruction and reallocation that is observed at both the aggregate and sectoral levels in the U.S. economy. Specifically, job creation rates move procyclically in the model while job destruction rates move countercyclically, as they do in the data. Also, in the model job destruction is more volatile than either job creation or reallocation, a feature

[14] It is being assumed that all agents in the search pool would qualify as being unemployed, as measured in the U.S. data – see footnote 11.

[15] The job creation rate in an N-sector model is given by $\sum_{i=1}^{N} \max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\sum_{i=1}^{N} \pi_{i,t-1}$. Now, consider aggregating the N sectors up into 2 sectors. The rate of job creation for the aggregated 2-sector model would be $\max\{0, \sum_{i=1}^{M} (\pi_{i,t} - \pi_{i,t-1})\}/\sum_{i=1}^{N} \pi_{i,t-1} + \max\{0, \sum_{i=M+1}^{N} (\pi_{i,t} - \pi_{i,t-1})\}/\sum_{i=1}^{N} \pi_{i,t-1}$. Clearly, the latter sum is smaller than the former one.

displayed in the data. Finally, it is found that aggregate and sectoral disturbances contribute non-negligibly to unemployment.

In the model presented here workers were assigned their employment status via a lottery. They were perfectly insured against the possibility of dismissal, in the sense that their consumption in a period was not contingent upon their employment status. One can imagine a world where no such insurance exists. Suppose, instead, that individuals can only insure themselves by saving in the form of a simple asset, such as money or government bonds. Each period those agents currently working in a sector decide whether to stay at work, enter the unemployment pool to search for a new job in another sector, or leave the labor force. Agents in the unemployment pool decide whether to take a job in some sector, remain in the unemployment pool for another period, or leave the labor force. Likewise, those individuals at home must decide whether or not to enter the labor force. Clearly, an individual's decision will be predicated upon both his idiosyncratic circumstance (asset holdings, employment status) and the aggregate situation (the distribution of agents and state of technology in each sector). While computationally more complicated, such an analysis would undoubtedly share many of the features of the above model. But it would permit a much richer analysis along some dimensions. For instance, one could study the effect that government policies, such as unemployment insurance, have on intersector mobility and unemployment.[16] The current analysis can be viewed as a first step toward such a model.

Appendix

A: Computation

Modified discrete state space approach with value function approximation

The neoclassical growth model can be solved using standard discrete state space dynamic programming techniques. In economies with multiple sectors or multiple agents, the standard approach becomes unworkable due to the curse of dimensionality, which limits the practicability of standard discrete state space dynamic programming techniques for large problems.

An alternative treatment of the problem is to store a limited set of coefficients characterizing a parameterized value function and momentary return function.[17] The parameterized objective function can then be maximized using an optimization routine. Two benefits derive from this method: First, computation costs are reduced dramatically; and second, the maximizers are no longer constrained to lie in a discrete subset of the constraint set.

An obvious candidate in the family of simple functions to use to approximate more complicated functions is the polynomial. However, there are two problems associated with polynomial approximation. First, practical concerns prevent using high order polynomials (even given the Weierstrass theorem). Second, the adequacy of polynomial approximations depends on the differentiability properties of the function that is being approximated. Often, for a smooth function a lower degree polynomial can be used.[18]

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[16] This policy experiment could be viewed as embedding the analysis of Hansen and Imrohoroglu [7] into a multisector general equilibrium model of the form presented here.

[17] A discussion of numerical techniques used to solve dynamic equilibrium models can be found in Danthine and Donaldson [3].

[18] Given the assumptions placed on tastes and technology here, the value function will be strictly increasing, and strictly concave (Stokey et al. [22], Chap. 9).

The representative agent's optimization problem, characterized by problem $P(2)$ in Section 2, can be simplified to one with only linear constraints by using the following lemma. For this simplified problem, it is easy to check the convexity of the constraint set.

Lemma 1 *The transition constraint*

$$\sum_{i=1}^{2} \max\{0, \pi_i' - \pi_i\} \le \pi_3 \tag{10}$$

is equivalent to following set of linear inequality constraints:

$$\pi_1' + \pi_2' \le \pi_1 + \pi_2 + \pi_3, \tag{11}$$

$$\pi_1' \le \pi_1 + \pi_3, \tag{12}$$

$$\pi_2' \le \pi_2 + \pi_3. \tag{13}$$

Proof: It is trivial to verify that the set constrained by (10) is same as the one constrained by (11), (12) and (13). If (10) holds, then the following must be true,

$$(\pi_1' - \pi_1) + (\pi_2' - \pi_2) \le \pi_3, \tag{14}$$

$$\pi_1' - \pi_1 \le \pi_3, \tag{15}$$

$$\pi_2' - \pi_2 \le \pi_3. \tag{16}$$

But this is merely (11)–(13). On the other hand, from (14)–(16) it is easy to derive that

$$\max\{0, \pi_1' - \pi_1\} + \max\{0, \pi_2' - \pi_2\} \le \max\{0, \pi_3\}, \tag{17}$$

which is equivalent to the transition constraint (10). □

Let $\mathscr{F}$ represent the space of continuous, bounded functions and consider the mapping $T: \mathscr{F} \to \mathscr{F}$ defined by $P(3)$.

$$\begin{aligned} V^{j+1}(\pi_1, \pi_2, \pi_3; z, \varepsilon_1, \varepsilon_2) = \max\nolimits_{\{\pi_1', \pi_2', \pi_3'\}} \Bigg\{ & \frac{A}{\rho} \ln\left(\sum_{i=1}^{2} \theta_i (z\varepsilon_i w^{\alpha_i} (\pi_i' - \gamma_i (\max[\pi_i' - \pi_i, 0])^{\lambda_i})^{\alpha_i} - I_i(z, \varepsilon_i))^{\rho} \right) \\ & + (1 - A)\left[\pi_3' \ln(1 - s) + \left(\sum_{i=1}^{2} \pi_i' \right) \ln(1 - w) \right] \\ & + \beta E[V^j(\pi_1', \pi_2', \pi_3'; z', \varepsilon_1', \varepsilon_2') | \pi_1, \pi_2, \pi_3; z, \varepsilon_1, \varepsilon_2] \Bigg\}, \end{aligned} \tag{P(3)}$$

subject to the constraints (11)–(13) and

$$\sum_{i=1}^{3} \pi_i' \le 1, \quad \pi_i' \ge 0. \tag{18}$$

The mapping T maps V^j to V^{j+1}. This operator is a contraction mapping that has as its unique fixed point the function V defined by $P(2)$.[19] This last observation motivates the computational procedure used here consisting of the following steps:

1. A grid is defined over the model's state space. Specifically, it is assumed that $\pi_1 \in [.243, .297]$, $\pi_2 \in [.360, .440]$, and $\pi_3 \in [0, .024]$.[20] Three grids of 13 equally spaced points are layered over these intervals. These sets of grid points are denoted by Π_1, Π_2, and Π_3, respectively.
2. An initial guess for the 2nd degree polynomial used to approximate the value function over this grid is made.

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[19] It is trivial to check that $P(3)$ satisfies Blackwell's sufficiency conditions for a contraction mapping – see Stokey et al. (1989).

[20] By simulating the model it was determined that system never left these intervals.

Table 6. Codes of the time series in citibase (Sample period: 1964.1–1987.4)

Variables	Industries						
	1	2	3	4	5	6	7
A: Output							
National income	GYWM	GYWC	GYM	GYWTU	GYNRR + GYNRW	GYFIR	GYS
GNP (82)	GA8G14	GA8G15	GA8GM	GA8GTU	GA8GW + GA8GR	GA8GFE	GA8GS
GNP	GAG14	GAG15	GAGM	GAGTU	GAGW + GAGR	GAGFE	GAGS
B: Labor							
Employment	LPMI	LPCC	LPEM	LPTU	LPT	LPFR	LPS
Weekly hours worked per employee	LWMI6	LWCC	LPHRM	LWTU	LWTWR	LWFR6	LWS
Unemployment rate	LURMI	LURC	LURM	LURTPU	LURWR	LURFS	LURFS
C: Labor share							
Compensation of total employees	GAPMI	GAPCC	GAPM	GAPTPU	GAPW + GAPR	GAPFF	GAPS
Proprietor's income	GAYPMI	GAYPCC	GAYPM	GAYPTU	GAYPTW + GAYPRT	GAYPF	GAYPS
Corporate capital consumption allowance	GACMI	GACCC	GACM	GACTPU	GACW + GACR	GACFF	GACS
D: Population							
Civilian population				PO16			

Note: –The industry numbers are defined in the text of Appendix B.
–All time series, except ones at annual frequency, are seasonally adjusted.
–All time series in groups A and C are nominal, except GNP which is in 1982 dollars.
–All series in groups A and C are annual, except national income which is quarterly, and all series in groups B and D are monthly.

3. Given the guess for the value function, a maximization routine is used to solve the constrained nonlinear optimization problem $P(3)$ for the optimal decision-rules.[21] This is done for each of the 8,788 points in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$.
4. Using the solution obtained for the optimal decision-rules, a revised guess for the value function is computed. This is done by choosing a new 2nd degree polynomial to approximate the value function. In particular, from $P(3)$ a value for V can be computed for each grid point in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$. A 2nd degree polynomial is then fitted to these points via least squares.
5. The decision-rules are checked for convergence.

Once the decision-rules have been obtained, the model can be simulated and various statistics are generated consequently. Note that function values for the decision-rules will have been computed for each point in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$. It is then easy to obtain values for the decision-rules at any point in the space $[.243, .297] \times [.360, .440] \times [0, .024] \times Z \times E$ by using multilinear interpolation – see Press et al. [18]. The adequacy of using a 2nd degree polynomial to approximate the value function can be assessed from a R^2 statistic. The R^2 obtained from using the 2nd degree polynomial was 0.96. Additionally, one could fit a higher order polynomial to the values of V obtained in the grid. Using some appropriate metric, the distance between this polynomial and the 2nd order one can be computed over some desired space. For instance, using the standard Euclidean norm the distance between a 2nd and 3rd degree polynomial was 0.022 when evaluated at some 70,304 points along a mesh spanning the state space.[22] The mesh was constructed by making the original grid twice as fine. While the third degree polynomial fit better (it had an R^2 of 0.99) it involved more computer time without any noticeable change in the results.

B: The data set

As described in Figure 1, the goods-producing and service-producing sectors are made up by seven SIC one-digit industries: Mining (1), Construction (2), Manufacturing (3), Transportation and Public Utilities (4), Wholesale Trade and Retail Trade (5), Finance, Insurance and Real Estate (6) and Services (7). Here the goods-producing sector includes the first three industries while the rest make up the service sector.

All the time series for the postwar U.S. economy are obtained from Citibase. Exceptions are the series for noncorporate capital consumption allowance by industry which came from the National Income and Product Accounts. Output for industry i is measured in 1982 prices. Total hours worked in industry i is the product of employment and the weekly hours worked per employee in that industry. The output, employment, hours and unemployment series are deflated by the civilian population. Citibase codes are contained in Table 6.

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[21] This was done using M.J.D. Powell's GETMIN subroutine developed for solving constrained nonlinear optimization problems.

[22] The average value of $|V|$ over the original grid is about 16.0. Thus, the distance is fairly small in relative terms.

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