Angular Distribution of *p-p* Elastic Scattering[*)] at High Energies and Radius of the Hard Core

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The problem of the hard core in nuclear force at high energies is studied in connection with the theory of the structure of elementary particles. Reviewed and discussed are various models of the origin of the hard core. Experimental angular distributions for small and large momentum transfers of *p-p* elastic scattering at high energies are analyzed, to deduce the general trend of the variation of the radius of the hard core with incident energy.

§1. Introduction

The Sakata model[1)] of the composite structure of elementary particles proposed in 1956 gave us the way to study various phenomena of elementary particles, on being grasped as the *Erscheinung* of the *Form Grund* — the fundamental particles or urbaryons —, laying in deeper strata than that of the *Form* — various phenomena of elementary particles themselves.[2)] Since then, the composite model of elementary particles has been recognized more and more important, so that one could hardly find out theoretical studies on physics of elementary particles which do not take into consideration more or less the composite structure of elementary particles. At the same time, however, there has been a tendency to treat various kinds of phenomena of elementary particles on single footing, by putting them together.

In reactions of elementary particles in the energy range covered by the high energy accelerators now working, it is expected that certain features of the structure of elementary particles would be revealed, since in those reactions there are possibilities for interacting elementary particles to approach mutually at close distances. For the study of complex phenomena such as reactions of elementary particles at high energies, it is necessary to proceed with caution of the applicability limit of theory. As to the theory of nucleon-nucleon interaction, the Japanese research group of nuclear force studied it step-by-step strategically[3),4)] in close contact with the meson theory due to Yukawa. Experiment on nucleon-nucleon scattering performed with the high energy

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accelerators have enabled us to start studies of the nuclear force at close distances.

As has been stated once by one of the present authors(M.T.),[4] diverse opinions have been presented how the behavior of the hard core would come out in high energy regions, that is known to present in the nuclear force at close distance in low energy regions. Especially, when experiments on nucleon-nucleon scattering in the energy region (10∼20) GeV were per formed, various proposals were presented about interpretation of the results, in connection with models of various kinds for the structure of elementary particles. It should be born in mind, however, that at the present stage of the study of nucleon-nucleon interaction at close distances, we cannot push our way of attacking the problem on relying upon only one of such proposals. It is necessary for us to proceed step-by-step in the study of nuclear force at close distances, by investigating various possibilities strategically, too, in close contact with the Sakata theory of the composite structure of elementary particles.

The aim of the present paper is to discuss the problem of the hard core in nucleon-nucleon interaction along the line mentioned above. In §2, discussions will be given for its relation to the problem of the structure of elementary particles. In §3, we shall analyze the experimental data on the angular distribution of p-p elastic scattering at high energies, in order to see how we can say at present about the problem of the hard core. Section 4 is devoted to the conclusion deduced from our discussion given in this paper.

§2. Hard core and models of elementary particles

2.1 *Hard core in nuclear force at low energies*

As is well known, the existence of meson was predicted by Yukawa as the quantum of the field by the mediation of which nucleons interact mutually. The quantum field theoretical derivation of the nuclear force by means of the meson theory encountered with many difficulties, such as the divergence problem, strong coupling, higher order perturbation, non-static effect, relativistic effect, etc. On confronting with such complexity, there arose tendencies of giving no belief on the meson theory, or, on the contrary, of giving too much belief on mathematical results, such as singularity in nuclear force at the origin, obtained for some special cases.

In 1948∼49 Taketani, Nakamura and Sasaki[5] advanced the method of investigating the nuclear force, deviding the range of nuclear force into parts, in each of which the nuclear force is governed by its respective characteristics. A full account of the method was given in Ref. 6), which is known as T.N.S. (1951), and has paved the way of the studies on the nuclear force by the Japanese research group of nuclear force. As was described in T.N.S. (1951), we proposed to devide the range of action of the nuclear force into the three regions:

Region I: Classical region, static main, quantitative treatment.
Region II: Quantum region, dynamical region, qualitative treatment.
Region III: Phenomenological region.

This division was deduced from the following methodological analyses. The nuclear force at comparatively large distances, that is at $r \gtrsim 1/m_\pi$, where r and m_π are the inter-nucleon distance, and the meson mass, respectively ($\hbar = c = 1$), the one-pion-exchange is dominant. Moreover, for the nuclear force due to one-pion-exchange we have the one and same result as that due to the classical treatment of the meson theory. Thus, for $r \gtrsim 1/m_\pi$, that is in Region I, the nuclear force can be treated quantitatively by means of the meson theory.

At distances smaller than the meson Compton wave length, $1/m_\pi$, contributions from exchange of two or more mesons become effective. In the calculation of contributions of such kind, there appear divergent integrals due to virtual mesons having large momenta, and many dynamical effects like the effect of nucleon recoil, exchange of heavier meson, and so on. The nuclear force at these distances, that is in Region II, should therefore be treated qualitatively, by means of the meson theory, while grasping the meson as the substance of the mediation of the nuclear force. This necessitates the introduction of the cutoff for momentum of meson around the nucleon mass M_N, that is, the cutoff in the coordinate space around the nucleon Compton wave length $1/M_N$. For $r \lesssim 1/M_N$, that is in Region III, we had no reliable theory at that time, and we were led to treat the nuclear force in Region III phenomenologically.

On the basis of the method of investigating the nuclear force outlined above, in 1951 Taketani, Ohnuma and Koide[7] derived that the type of pion should be pseud-scalar, from the analysis of the quadrapole moment of deuteron, which is governed mainly by the nuclear force at large distances, by using the meson theoretical potentials for the nuclear force at large distances, while assuming a hard core in the nuclear force at small distances. Although a hard core was also introduced by Jastrow (1950)[8] to describe *p-p* scattering at 300 MeV, it was not possible to get such a clue to the meson theory, since he used a phenomenological potential for the nuclear force at large distances, too. Many theoretical analyses performed since then in Japan[9] along the directions given in T.N.S. (1951) established the existence of the hard core of radius of about $2/M_N$ in the nuclear force at low energies, which gives an extension with a radius of about $1/M_N$ for each nucleon.

At the same time, through comprehensive strategical studies of the nuclear force, we determined in Region I the coupling constant of meson-nucleon interaction, and clarified in Region II the dynamical effects such as the two pion exchange, nucleon recoil, relativistic effect, etc., and also the effect of exchange of heavy meson, its relation with multiple meson exchange, and so on, as is summarized in Refs. 3) and 4). It seems worth while to note here our success in establishing the one-pion exchange potential did prepare the starting point of the modified phase shift analysis and of the pology of the dispersion relation, as has been mentioned in Ref. 4).

In order to attack the nuclear force in Region III, we have to solve problems of the nucleon-nucleon scattering in presence of the inelastic effect, because high energy should be given to incident nucleon in order to realize its close approach with target nucleon. Hoshizaki and Machida[10] made their analyses of p-p scattering at 660 MeV, assuming that the pion production occurs exclusively through the 3–3 resonance channel. Later on, Hoshizaki and his coworkers[11] extended their approach to experimental data up to a few GeV. According to their analyses, they concluded that the hard core of the nuclear force exists in these energy regions.

2.2 *Hard core and the Sakata model*

The Sakata model casted new lights on the problem of the hard core in the nuclear force. In 1949, Taketani and Nakamura[12] discussed the problem of the elementarity and compositeness of particles, and made the suggestion which states that the question whether a particle under consideration is elementary or composite can be answered with reference to the Compton wave length of its constituent particles, as long as we accept the fundamental aspects of the relativistic quantum mechanics. This is because by relativistic quantum mechanics we know that a particle with a mass μ cannot be confined to the region of dimension smaller than its Compton wave length $1/\mu$, and we should lay emphasis definitely on the meaning of the Compton wave length in the theory of elementary particles. Though we cannot say anything about the validity of the statement given above in a future theory, we believe, in accordance with Ref. 12), that the Compton wave length has an important implication for the transit from the present theory to a future theory. Taketani and Nakamura[12] proposed, furthermore, that the criterion whether the particle concerned behaves as an elementary particle with the quantum mechanical character or not will be found in processes of its pair creation. The Sakata model has, as one of its bases, the understanding of the unification of the elementarity and compositeness in elementary particles.[13]

According to the original form of the Sakata model, the pion is a composite system of a nucleon and anti-nucleon pair. Combined with the criterion according to Taketani and Nakamura,[12] the pion could not have a dimension smaller than $1/M_N$. As was pointed out by Taketani and Fujimoto (1965),[14] a large part of the mass of the nucleon pair is lost by the binding energy because the pion mass m_π is far smaller than the nucleon mass M_N, and this statement on the meson structure is meaningful when the bound state of the pair is within the limit of validity of the relativistic quantum mechanics. On the other hand, in view of the successes we have had with the meson theory for the nuclear force in Region I and Region II, of which $1/m_\pi$ gives the order of magnitude of the characteristic distances, the linear dimension over which the pion structure extended should be small compared with $1/m_\pi$. Thus, one has now

$$r_\pi \sim 1/M_N, \tag{1}$$

where r_π is the size of a pion, or more generally

$$r_\pi \sim \lambda\!\!\!^{-}_B = 1/M_B, \tag{2}$$

in terms of the baryon Compton wave length $\lambda\!\!\!^{-}_B$ or baryon mass M_B.

These discussions given above, therefore, suggest strongly that the hard core in the nuclear force at low energies originates from the pion structure. One may argue that a quantum mechanical effect of exchange of some heavy meson — Vector meson, for example — would give rise to a strong repulsion in the nuclear force at short distances. One should remember, however, that heavy mesons also are composite systems of a baryon and an anti-baryon pairs, according to the Sakata model. They have, as well, structures with linear dimension equal to more or less the baryon Compton wave length $\lambda\!\!\!^{-}_B$. The system we have in relation to the nuclear force under consideration is a complicated mixture of $(2N)$, $(2N+\pi)=(3N+\bar{N})$, $(2N+\rho)=(4N+2\bar{N})$, and so on.[14] Even more complicated configurations would become effective when the inter-nucleon distance is closer. On account of the fundamental interactions acting between the pair of $(N, \bar{N})$ and between that of (N, N) or $(\bar{N}, \bar{N})$ which are considered to be of short range, there would be various effects which make the energy levels of such complicated configuration higher than simpler ones, giving rise to a strong repulsion like a hard core. It would be probable therefore that a large part of the hard core is due to internal structure of mesons, though the possibility that it is in part due to a quantum mechanical effect of exchange of heavy mesons could not be ruled out.

As to the nuclear force due to exchange of heavy mesons, we should mention one thing in addition. Ogawa, Yonezawa and their co-workers[15] proposed in 1961 the One-Boson-Exchange Model (OBEM) for nuclear force, that is the model to study nucleon-nucleon scattering by including potentials due to exchange of a boson of various kinds in the nuclear force. It was based on the success of the one-pion-exchange potential mentioned above, and on the Sakata model which makes it possible to consider heavy mesons on the same footing as the pion. It was also related to the problem of π-π correlation resulting from effective interaction between pions. In the sense of π-π correlation, heavy bosons such as ρ and ω mesons are considered to be clusters of pions. In 1953 Taketani,[16] on noting that the expansion taken in the theory of strong coupling in the inverse power of the coupling constant $g^2/\hbar c$ is effectively an expansion in the power of $\hbar$, and is, in this sense, nothing but to make quantum-mechanical correction to the classical, pointed out that as an ideal such an approximation method should be established, in which the approximation involving any arbitary number of pions, or the one not being characterized by the number of pions, is obtained in its first approximation.[*] Regarding to π-π correlation itself, Japanese research group of nuclear force established[17] that the nuclear forces due to two and three pion exchanges are well described as the exchanges of a ρ meson and of a ω meson, respectively. OBEM thus fulfilled the requirement given by Taketani[16] mentioned just above, owing to the Sakata model.[4]

The arguments given above for the pion structure rest implicitly on the assumption that the linear dimension of a baryon r_B is small compared with $\bar{\lambda}_B$, that is to say with r_π. If r_B is comparable or larger than $\bar{\lambda}_B$, a baryon should be a particle with a character much different from that of an elementary particle.[14] We shall return to the problem of the baryon structure in the next subsection. Our discussion given in the present subsection may be

Table I. Inter-relation between the regions of nuclear force and the structures of hadrons.

Distance or Size[a]	Region of Nuclear Force	Structure of Mesons	Structure of Baryons
0			Composite model (beyond quantum mechanics?)
r_B			
	Region III	Composite model	
$\bar{\lambda}_B$			
	Region II	Point model (with correlation)	Point model
$\bar{\lambda}_\pi$			
∞	Region I	Point model	

a) r_B is the size of a baryon, and $\bar{\lambda}_B$ and $\bar{\lambda}_\pi$ are the Compton wave lengths of a baryon and a pion, respectively.

*) In a recent review article on multiple production of hadrons, Feinberg writes that Feynman has made in 1969 an argument in which it is noted that $g^2/\hbar c$ with large g^2 must produce quasiclassical effects [c.f., E.L. Feinberg[43] and R. Feynman[44]]. This was noted, as is mentioned in the text above, already in 1953 by Taketani.[16] In Feynman's report, however, this is used in a way different from that in Taketani.[16] Feynman has used it to make the so-called Bremsstrahlung analogy for multiple production of mesons. One is referred to other papers in the present issue of Prog. Theor. Phys. Suppl. for discussion on the Bremsstrahlung analogy. In Taketani,[16] it has been applied as mentioned in the text above to the theory of nuclear force, with the results of OBEM.

summarized as is shown in Table I, in accordance with Taketani and Fujimoto (1965).[14)]

2.3) *Hard core and difficulties in non-relativistic quark model*

In 1959, Ogawa, Ohnuki and Ikeda[18b)] succeeded in introducing group theoretical treatment of the meson octet, applying $U(3)$ for the Ogawa symmetry[18a)] that treats symmetrically p, n and Λ which are supposed as fundamental particles in the original form of the Sakata model. Later, Gell-Mann[19a)] and Zweig[19b)] in 1964 showed that the baryon octet can be treated also by such a group theoretical method as in Ogawa, Ohnuki and Ikeda.[18b)] Gell-Mann called those fundamental particles in his model as quarks that have fractional charges. They are a kind of urbaryons in the sense of the Sakata model. Their speciality lies on their fractionality of charges that is assumed in order to treat both baryon and meson octets on the same footing.

Groups of various kinds have been applied by many authors to systematize baryon resonances and/or mesons observed in experiments, on classifying a number of them together. $SU(6)$ is a such example, and it includes quark's spin into treatments of the symmetry. Because it is done in a non-relativistic manner, there arouse the so-called non-relativistic quark model in which baryons and mesons are thought to be composed of quarks through the non-relativistic mechanics. Together with it, there appeared some reasonings to justify the use of non-relativistic mechanics for the composition of baryons and mesons out of quarks.

One example is that of Murpurgo[20a)] and Nambu[20b)] which said that kinetic energies of quarks are small compared with their rest mass M_q, when they move in a potential well of range a_q which is large compared with the quark Compton wave length $1/M_q$. According to this reasoning, a large amount of the rest masses of quarks ($2\,M_q$ for a meson and $3\,M_q$ for a baryon) is canceled out by a potential well with a finite depth of about $2\,M_q$ or $3\,M_q$. It was shown, however, by one of the present authors (M.N.),[21)] that this is not the case for Dirac particles interacting through a potential. Strength of such a potential should be large compared with the rest mass of Dirac particle, in order to realize a bound state with an energy small compared with the rest mass of Dirac particle. Roughly speaking, one should remember that (negative) kinetic energy of a quark outside the potential well assumed in such a reasoning is not small in its absolute value compared with the rest mass itself in such a state.

Another example is that of Van Hove[20c)] which said that if the inter-quark interaction is mediated by a meson of mass m, the range of the inter-quark interaction is of the order of $1/m$ [as in the case of the nuclear force discussed in §2.1)], while the momentum interchanged by a quark with that meson is of the order of m which is small compared with M_q when $1/m \gg 1/M_q$. This reasoning forgets that in order to realize a bound state of nearly zero energy, the coupling constant of the interaction should be strong enough,[22] so that effects of multiple meson exchange and others should be taken into consideration, for which the recoil momentum is not small, as one has experienced for a long time in the theory of nuclear force described in §2.1).

Horwitz[23] got a rather small coupling constant of 1 GeV in his calculation of the masses of ρ and ω mesons with the use of the Bethe-Salpeter equation together with a separable interaction for a pair of a quark and an anti-quark of a mass as large as 10^3 GeV. As pointed out in §2.2), however, in the case of a separable interaction, the wave function of the bound state representing a meson behaves asymptotically, when the inter-quark distance r is large, as $e^{-\mu r}/r$ for $K \gg \mu$, where $K^2 = M_q^2 - (m/2)^2$ in terms of the quark and meson masses, M_q and m, respectively, and μ is the parameter in the separable interaction in momentum space given through $V(\boldsymbol{p}, \boldsymbol{p}') = v(\boldsymbol{p}) v(\boldsymbol{p}')$ with $v(\boldsymbol{p}) = g/(p^2 + \mu^2)$. Since Horwitz assumed, on the line of the non-relativistic quark model, that $1/M_q \ll 1/\mu$, the wave function decays with the factor $e^{-\mu r}$ outside the range of the interaction, for a meson with $m \ll M_q$, in a sharp contrast to the factor e^{-Kr} demanded by the relativistic relation between the energy and momentum.

Now, in such non-relativistic models the size of a baryon as well as that of a meson becomes of the order of the baryon Compton wave length $1/M_B$, if one assumes as usually done that the range of the inter-quark interaction is about one order of magnitude larger than the quark Compton wave length $1/M_q$, while M_q is about one order of magnitude larger than M_B. That is, for both baryons and mesons one has more or less same sizes as the hard core in the nuclear force. In the non-relativistic models, the hard core should therefore be related to the structure of baryon itself. Thus, Namiki and Machida[24] considered that the hard core in the nuclear force at low energies originates from the Pauli exclusion principle acting for urbaryons of quark type in two nucleons, on the analogy of the strong repulsive force at short distances between two α-particles due to the Pauli principle acting for nucleons in α-particles. Same analogy was stressed also by Otsuki, Tamagaki and Yasuno.[25]

With such a view of the origin of the hard core in the nuclear force at low energies, Otsuki, Tamagaki and Wada,[26] assuming a "soft" core instead of the hard core for the nuclear force at low energies, conjectured that the strong repulsion at low energies tends to the strong absorption at tens GeV, that is, they considered that inner degree of freedom of the baryon structure frozen at low energies are excited at high energies. The use of a soft core instead of the hard core is related to Machida's conjecture[27] that the fundamental force between a baryon and an anti-baryon in the original form of the Sakata model would reverse into a strong repulsion for a system of two baryons. One should be aware of the difference in which the fundamental forces concern with bare baryons without meson clouds, while the hard core in the nuclear force concerns with a physical nucleons with meson clouds.[14]

The non-relativistic quark model has, beside the difficulties with it mentioned above, following difficulties in relation to the problem of the baryon structure. As is mentioned in the preceding subsection, if the size of baryon r_B is comparable with the baryon Compton wave length $1/M_B$, a baryon should be a particle with character much different from an elementary particle. In this connection, it should be remembered that $1/M_N$ is fairly large as about 0.2 yukawa (yukawa being 10^{-13} cm). And, if the quantum mechanics is valid for the composition of a baryon out of quarks, r_B cannot be smaller than $1/M_q$. On the other hand, since r_B should be small compared with $1/M_B$ as is mentioned just above, one will have the estimation $r_B \sim 1/M_q$ in contrast with the assumption that $r_B \sim 1/M_B$ taken in the non-relativistic quark model, under the assumption that the quantum mechanics is valid for quarks. In this connection, one has to take in mind the possibility that urbaryons would be beyond the validity of the quantum mechanics.[14] If this is the case, the estimation that $r_B \sim 1/M_{\mathrm{urb}}$ would not be much reliable. In that case, however, current calculations in detail with quark models could not stand in any way.

In other words, if one takes the estimation that $r_B \sim 1/M_q$ in contrary to the non-relativistic quark model, one will have no meaning for the inner degrees of freedom of baryons assumed in it to give rise to the hard core at low energies. Moreover, under the assumption of quark model to compose both baryons and bosons out of quarks in a straight forward way, the use of this estimation implies that $r_\pi \sim 1/M_q$, too. This is not compatible with calculations in detail with quark models of hadron reactions at several GeV. To avoid this, if one assume that $r_B \sim 1/M_q$ and $r_\pi \sim 1/M_B$ by supposing in the frame of the quantum mechanics different inter-quark interactions for the compositions of baryons and mesons, the assumption of quark model to treat both baryons and mesons on the same footing loses its stand point. Then, one is not concerned with quarks, but with urbaryons in general in the sense of the Sakata model.

From our discussions given both in §§2.2 and 2.3, one sees that the stratum or level revealing the internal structure of baryons is likely to be different from that concerned with the composite character of mesons. The former level seems to lie at one step deeper place than the latter in the structure of Nature. In this sense, mesons will be like molecules if baryons are considered on the analogy of atoms, as was pointed out by one of the present authors (M.T.)[28] in 1964. To this idea, Sakata[29] gave the names: the compositenesses of "atom-type" and "molecule-type".

§3. Hard core and absorption at high energies

3.1) *Absorption at high energies*

As has been discussed in the preceding section, in the non-relativistic quark model one considers that the hard core in the nuclear force at low energies as being due to the internal structure of baryons. On the analogy of the repulsive force at short distances in α-α interaction at low energies, in the non-relativistic quark model one has a "soft" core, instead of the hard core, in the nuclear force at low energies, which tends into strong absorption at high energies, where many inelastic channels become open on account of the breakdown of "frozen" internal structure of baryons at low energies.

In making such a conjecture, Otsuki, Tamagaki and Wada[26] used, as a support for their conjecture, Auerbach and Brown's conclusion[30] that the real part of the potential to describe nucleon-nucleon elastic scattering in GeV region of incident laboratory energy should be very small if a fit to experimental data was to be achieved within Serber's model.[31] Serber's model was an optical model in which use was made of a purely imaginary potential and Klein-Gordon wave equation to describe absorptions in p-p elastic scattering at several GeV/c of incident laboratory momentum. Serber used for the shape of the absorptive potential V_{abs}, the Yukawa potential

$$V_{\mathrm{abs}}=i\eta e^{-r/r_0}/r, \tag{3}$$

where η and r_0 are the strength and range of the absorptive potential, respectively. In Serber's calculation η was fixed to be one, in order to avoid a diffraction zero which appears for $\eta>1$. Auerbach and Brown's calculation used Serber's absorptive potential with η fixed to be one together with the real potential mentioned above.

As was pointed out by Nagasaki, Hirasawa and Taketani (1967),[32] however, occurrence of diffraction mini-maxima should not be excluded a priori in a model in which a restrictive method of analysis is used, though experimental data on the angular distribution of p-p elastic scattering at that time did not show any diffraction mini-maxima. In fact,[33] the value of $\eta=1$ is a particular one in Serber's model, since with $\eta=1$ one has an angular distribution proportional to $|t|^{-6}$ at large angles, where t is the four-momentum transfer squared, while with $\eta=1+2\,\delta$ one has that proportional to $|t|^{-4(1+\delta)}$. Moreover, with $\delta>0$ one has a diffraction zero, while with $\delta\leq 0$ there is no diffraction zero. Auerbach and Brown, however, described this particular situation as "the carefully arranged phase cancellation produced by $\eta=1$." As was shown also in Ref. 33), if one uses, instead of that given in Eq. (3), a potential Vabs of the form:

$$V_{\mathrm{abs}}=i\eta(e^{-r/r_0}-e^{-r/r_0'})/r \tag{4}$$

with $r_0'<r_0$, which is cut off the singularity at the origin of V_{abs} in Serber's model, one has the angular distribution at high momentum transfers proportional to $|t|^{-4}$ both for $\delta<0$ and $\delta>0$. There is no particularity in the angular distribution such as one has in Serber's model with η around one. With V_{abs} given by Eq. (4), one has diffraction mini-maxima for a great enough values of η, the value of $\eta=1$ not being particular as in Serber's model.

Auerbach and Brown's result could not therefore be a support for Otsuki, Tamagaki and Wada's conjecture that the hard core in nuclear force at low energies tends to strong absorption at high energies. The occurrence of diffraction mini-maxima does not simply imply the complete absence of the hard core, since there could be a hard core on whose surface a strong absorptive process takes place, as was pointed by Nagasaki, Hirasawa and Taketani (1967).[32] It was also pointed out in Ref. 32) that experimental check with good angular resolution of diffraction mini-maxima would be interesting, any way. In 1968, Durand and Lipes[34] and Chou and Yang[35] made calculations of the so-called "asymptotic" p-p differential cross section,[*] in which results diffraction mini-maxima were shown. Frautschi and Margolis (1968)[36] made calculations of the p-p differential cross section at small and moderate momentum transfers up to the incident laboratory energy of about 2 TeV, in which results diffraction mini-maxima were also shown.

Experimentally, Allaby et al. (1968)[37] made measurements of p-p elastic angular distributions in the range 7 to 12 GeV/c over the range of center-of-mass angle $40°\sim90°$, and found indications of a break at a value of $|t|\approx1.5$ (GeV/c)^2. This break was confirmed also at 19.2 and 21.1 GeV/c of incident laboratory momentum by measurements[38] done at CERN using a high-resolution and single-arm spectrometer. Nowadays, this break is known to locate at $|t|\simeq1.3$ (GeV/c)^2 for the p-p elastic differential cross section in the range 10 to 24 GeV/c of the incident laboratory momentum.[39] Moreover, recent experiment[40] done with CERN ISR on the p-p elastic scattering at the center-of-mass energy 53 GeV has shown that there is a minimum at $|t|\simeq1.3$ (GeV/c)^2. The present authors[41a),41b)] have shown that this break or minimum can be explained as a diffraction minimum with a simple spatial distribution of absorption. We shall return to this problem later in the next subsection.

This may be compared with Otsuki (1968),[42] in which it was written under a headline of "No structure of diffraction peak" that "it is well known that $d\sigma/d|t|$ of p-p elastic scattering has no structure like dip or shoulder in the diffraction region," and "on the contrary, $d\sigma/d|t|$ of $\bar{p}$-p, π^+-p and π^--p elastic scattering have characteristic dips at $t\simeq-(0.6\sim0.8)$ GeV^2 at intermediate energies."

[*] For discussion of the idea of the so-called "asymptotia", one should refer to other papers in the present issue.

Now, about the problem of the hard core at high energies, Kanada, Otsuki, Sakai and Yasuno[43] wrote in 1967 that it was quite difficult to guess a modification which just happened to mask the hard core effect without giving rise to large diffraction pattern. In their calculation of p-p elastic differential cross section in the region (6–22) GeV/c of incident laboratory momentum P_{Lab}, by means of the partial wave expansion of the scattering amplitude $f(\theta)$ with the neglect of nucleon spins,

$$f(\theta)=(2ip_{\mathrm{cm}})^{-1}\sum_{l=0}^{\infty}(2l+1)(e^{2i\delta_l}-1)P_l(\cos\theta), \tag{5}$$

where p_{cm} is the center-of-mass momentum, the imaginary part $i\chi_l$ of the phase shift δ_l of the l-th partial wave was chosen at first to be such that gives the angular distribution given by

$$d\sigma/d|t|=Ae^{-\gamma|t|}. \tag{6}$$

For the real part δ_l^R of δ_l, they chose that for a hard core of radius a, or that given by the T-matrix in the Born approximation in the case of a soft core of the Gaussian shape ($\delta_l^R=\tan^{-1}\ T_l^{\mathrm{Born}}$).

After showing that the calculated differential cross sections for $a=1$ and 2 (GeV/c)$^{-1}$ are large compared at large angles with experimental data, they made their modification of χ_l's from those given from Eq. (6). That is, in their modification they assumed that the smallest partial waves up to $l=2$, 3 or 4 are completely absorbed, or assumed that the reflexion coefficients $e^{-2\chi_l}$'s are all equal to 3/4 up to $l=4$ or are equal to arbitarally chosen values 1, 1, 0.9, 0.8 and 0.65 for $l=0$, 1, 2, 3 and 4, respectively, and so on. These stepwise modifications of $e^{-2\chi_l}$'s for the smallest partial waves naturally gives rise to large diffraction mini-maxima in the elastic differential cross section, which has according to Otsuki[42] no structure of diffraction peak. This was the reason why they wrote as mentioned above that it was quite difficult to guess a modification which just happened to mask the hard core effect without giving rise to large diffraction pattern. Moreover, a wide range of the radius of hard core a should be taken into consideration, as an example given in Nagasaki, Hirasawa and Taketani,[32] since we do not know a priori how the hard core displays at high energies the structure of elementary particles. In the non-relativistic quark model, on which Otsuki et al.[42),43] standed, the hard core should disappear at high energies tending into absorption, however, as has been discussed in the preceding section.

3.2) *Hard core surrounded by absorptive medium*

Let us now investigate further the elastic scattering due to a hard core of radius a surrounded by an absorptive medium. Neglecting nucleon spin, and supposing that the absorptive medium gives us the imaginary phase shifts $i\chi_l$'s, we have for δ_l in Eq. (6)

$$\delta_l = i\chi_l + \delta_l^{\mathrm{hc}}, \tag{7}$$

where δ_l^{hc} is the phase shift of the l-th partial wave due to the hard core. With this δ_l, in our present case, the scattering amplitude given by Eq. (5) can be rewritten as

$$f(\theta) = f_{\mathrm{ab}}(\theta) + f'_{\mathrm{hc}}(\theta), \tag{8}$$

with

$$f_{\mathrm{ab}}(\theta) = i(2p_{\mathrm{cm}})^{-1}\sum_{l=0}^{\infty}(2l+1)(1-e^{-2\chi_l})P_l(\cos\theta) \tag{9}$$

and

$$f'_{\mathrm{hc}}(\theta) = (2ip_{\mathrm{cm}})^{-1}\sum_{l=0}^{\infty}(2l+1)e^{-2\chi_l}(e^{2i\delta_l^{\mathrm{hc}}}-1)P_l(\cos\theta). \tag{10}$$

f_{ab} represents the scattering due to the absorptive medium only, and f'_{hc} represents the scattering due to the hard core reduced by the absorption of incoming and outgoing waves in the absorptive medium surrounding the hard core. In fact, if χ_l does not vary much for partial waves which contribute effectively to δ_l^{hc}'s, we have

$$f'_{\mathrm{ch}}(\theta) \simeq e^{-2\langle\chi_l\rangle} f_{\mathrm{hc}}(\theta), \tag{11}$$

where $f_{\mathrm{hc}}(\theta)$ is the scattering amplitude for the hard core only, and $\langle\chi_l\rangle$ is a mean value of χ_l's for such partial waves.[32]

Introducing the impact parameter approximation, $f_{\mathrm{ab}}(\theta)$ can be rewritten as

$$f_{\mathrm{ab}}(\theta) = ip_{\mathrm{cm}}\int_0^{+\infty}(1-e^{-2\chi(b)})J_0(b\sqrt{|t|})b\,db, \tag{12}$$

where b is the impact parameter,

$$b = \sqrt{l(l+1)}/p_{\mathrm{cm}}, \tag{13}$$

$\chi(b)$ denotes χ_l as a function of b, and $J_0(\chi)$ is the Bessel function. The impact parameter approximation is suitable one for a short incident wave for which the separation into the transverse and longitudinal directions is meaningful.

For the distribution of the absorption due to the absorptive medium surrounding the hard core, we assume that the absorption coefficient $(1-e^{-2\chi})$ is given by [41a)]

$$1-e^{-2\chi(b)} = (e^{-b^2/2\gamma} - \beta' e^{-b^2/2\beta\gamma})/(1-\beta'), \tag{14}$$

where γ, β and β' are parameters to be fixed. Inserting Eq. (14) into Eq.

(12), and using a well-known integral formula for the Bessel function, we have f_{ab} as a function of t as

$$f_{ab}(t)=ip_{cm}\gamma(e^{-\gamma|t|/2}-\beta\beta' e^{-\beta\gamma|t|/2})/(1-\beta'). \tag{15}$$

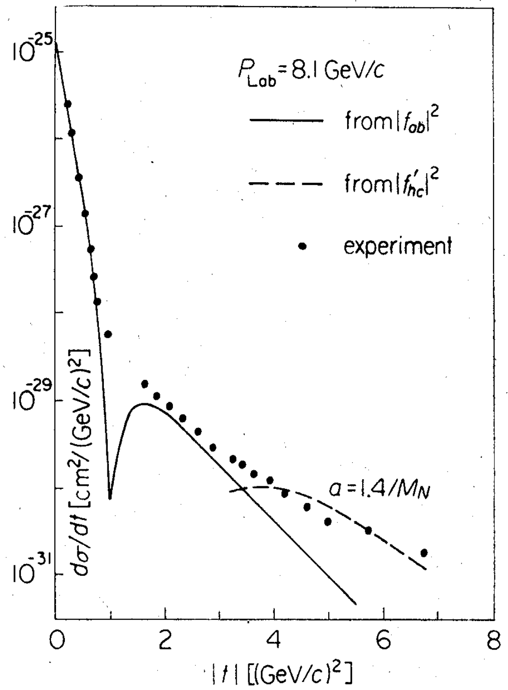

Fig. 1. The angular distribution of p-p elastic scattering at P_{Lab}=8.1 GeV/c.

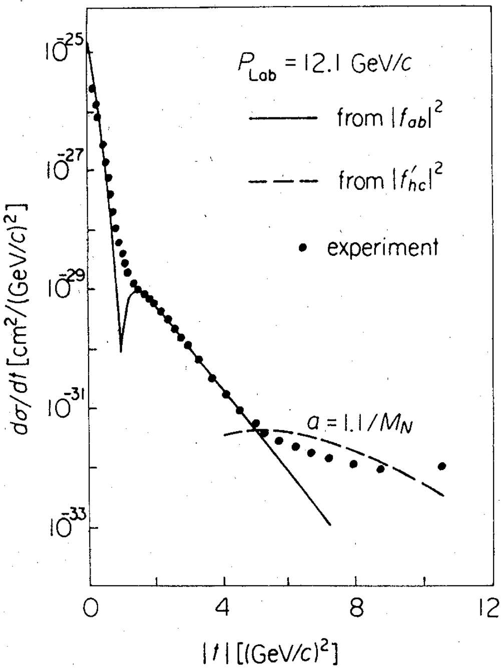

Fig. 2. The angular distribution of p-p elastic scattering at P_{Lab}=12.1 GeV/c.

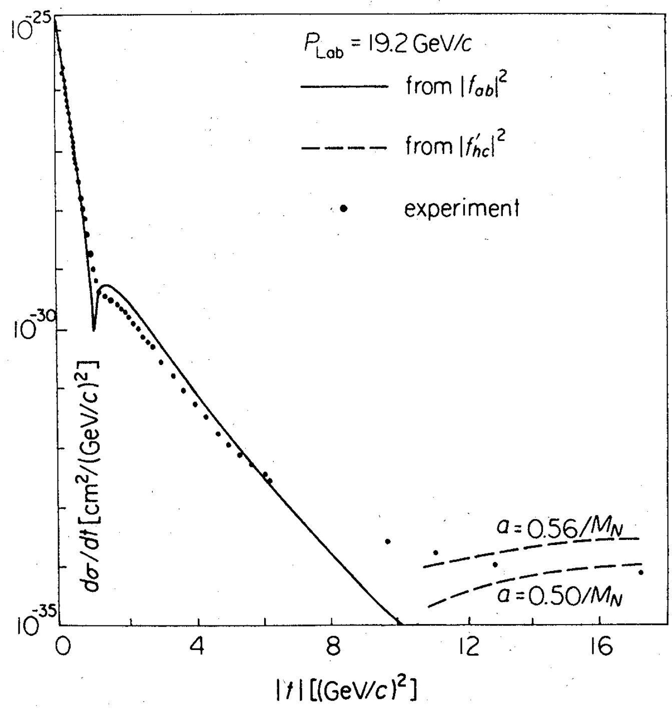

Fig. 3. The angular distribution of p-p elastic scattering at P_{Lab}=19.2 GeV/c.

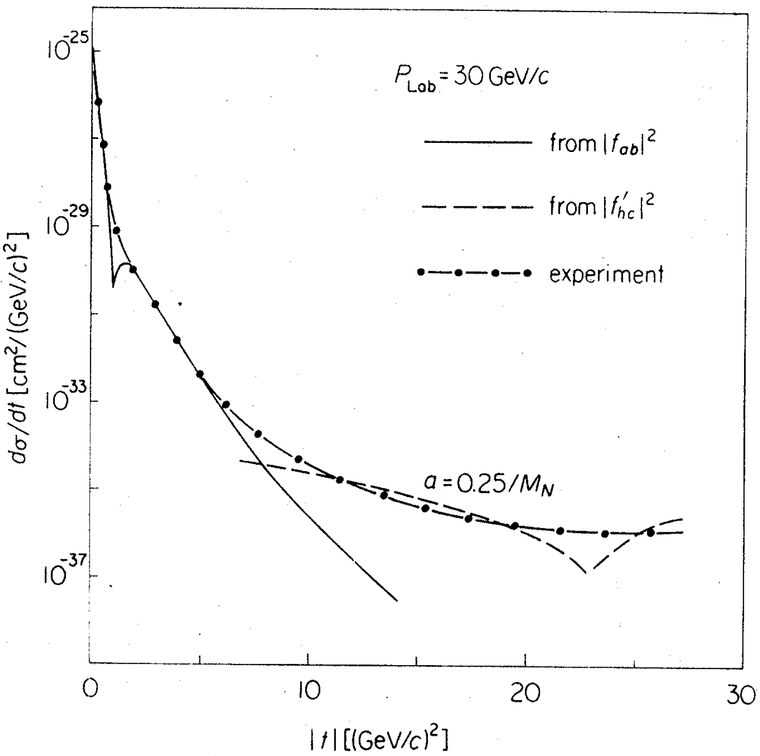

Fig. 4. The angular distribution of p-p elastic scattering at $P_{\text{Lab}}=30$ GeV/c.

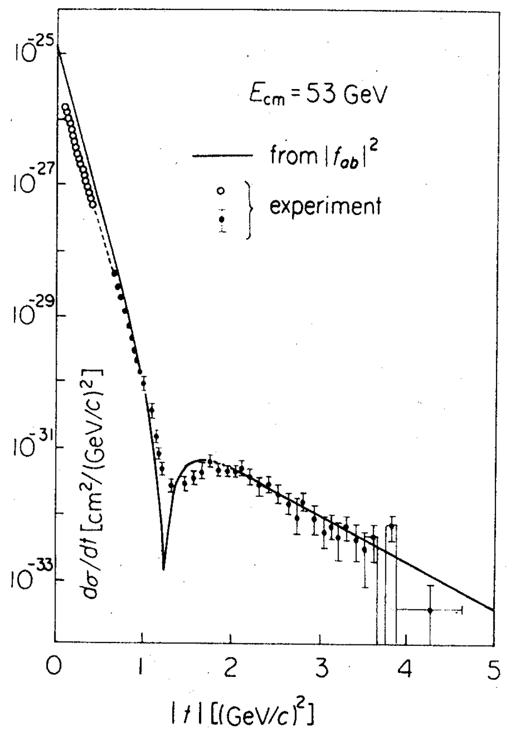

Fig. 5. The angular distribution of p-p elastic scattering at $E_{\text{cm}}=53$ GeV.

$f_{ab}(t)$ given by Eq. (15) has a diffraction minimum. The values of the parameters γ, β and β' can be fixed by supposing that $f_{ab}(t)$ represents the p-p elastic differential cross section at small and moderate $|t|$. In Figs. 1~4 the angular distributions of p-p elastic scattering at $P_{Lab}=8.1$, 12.1, 19.2 and 30 GeV/c due to $|f_{ab}(t)|^2$ are shown by full lines. The sets of the values (γ, β, β'), γ being in $(\mathrm{GeV}/c)^{-2}$, used for these figures are (8.0, 0.178, 0.178), (9.0, 0.178, 0.178), (9.0, 0.200, 0.141) and (9.5, 0.189, 0.111), for $P_{Lab}=$ 8.1, 12.1, 19.2 and 30 GeV/c, respectively.[41a)] Figure 5 shows similar one for the case of $E_{cm}=53$ GeV, for which we have (11, 0.150, 0.021) for the set of the values (γ, β, β').[41a)] In the cases of $P_{Lab}=19.2$ GeV/c and 30 GeV/c, we have used instead of Eq. (14) the following one:

$$1-e^{-2\chi(b)}=(e^{-b^2/2\gamma}-\beta' e^{-b^2/2\beta\gamma}-\nu' e^{-b^2/2\nu\gamma})/(1-\beta'-\nu'), \tag{16}$$

with which one has

$$f_{ab}(t)=ip_{cm}\gamma(e^{-\gamma|t|/2}-\beta\beta' e^{-\beta\gamma|t|/2}-\nu\nu' e^{-\nu\gamma|t|/2})/(1-\beta'-\nu'). \tag{17}$$

The values of ν and ν' used in Fig. 3 are 0.110 and 0.0150, respectively, and those used in Fig. 4 are 0.0842 and 0.00749, respectively. This term proportional to ν' in Eq. (16) is added in order to give a better fit to experimental data for $|t|$ around several $(\mathrm{GeV}/c)^2$.

The absorption coefficient $(1-e^{-2\chi})$ given by Eq. (14), or more generally that given by Eq. (16), behaves for small b as

$$1-e^{-2\chi(b)}=1-c_1(b^2/2\gamma)-c_2(b^2/2\gamma)^2\cdots\cdots, \tag{18}$$

with the expansion coefficients c_1 and c_2 given by

$$c_1=(1-\beta'/\beta-\nu'/\nu)/(1-\beta'-\nu'), \tag{19}$$

and

$$c_2=(\beta'/\beta^2+\nu'/\nu^2-1)/2(1-\beta'-\nu'), \tag{20}$$

respectively. The values of the parameters used for these figures are chosen to make c_1 as small as possible, so that $(1-e^{-2\chi})$ decreases more slowly at small b than $e^{-b^2/2\gamma}$ usually taken.

The angular distribution due to $|f_{ab}(t)|^2$ shown in Figs. 1, 2 and 3 for $P_{Lab}=8.1$, 12.1 and 19.2 GeV/c, respectively, has a diffraction minimum at $|t|\simeq 1.3$ $(\mathrm{GeV}/c)^2$, while one has a break there in the experimental data. We consider that effects of nuclear force other than the hard core under consideration mask the diffraction minimum. Such a force is also necessary in order to reproduce the experimental value of $\mathrm{Re}f(0)/\mathrm{Im}f(0)$. Nuclear force in Region II will give non-negligible contribution for such effects.

With these values given above of the parameters to represent the absorption, $f'_{hc}(\theta)$ can be calculated by means of Eqs. (10), (13) and (14) or

(16), for given values of a. The broken lines in Figs. 1, 2, 3 and 4 are the angular distributions resulting from $|f'_{hc}(\theta)|^2$ for $a=1.52$, 1.22, 0.53 and 0.27 $(\mathrm{GeV}/c)^{-1}$, respectively. In the units of the nucleon Compton wave length, they are 1.43, 1.14, 0.50 and 0.25 M_N^{-1}, respectively.

As has been noted in 41a) and 41b), the total cross section σ_{tot} for the cases shown in Figs. 1∼5, calculated from $\mathrm{Im} f_{ab}(0)$ with the values of the parameters given above, are (18∼24) % larger than the experimental optical theorem values. This shows that the absorption coefficient $(1-e^{-2\chi})$ should decrease for large b a little differently from the first term in Eq. (14) or Eq. (16). This is because the absorption at large b contributes non-negligibly to the total cross section.

In order to reduce the absorption given by Eq. (14) or Eq. (16) at large b, we now add the following correction term $\Delta e^{-2\chi}$ to the reflection coefficient $e^{-2\chi}$:

$$\Delta e^{-2\chi(b)}=\{\mu'(b^2/2\mu_1)e^{-b^2/2\mu_1}-\alpha'(b^2/2\alpha)e^{-b^2/2\alpha}\}(1-e^{-b^2/2\mu_2}), \tag{21}$$

where μ_1, μ_2, μ', α and α' are parameters, with the condition that $\mu_1<\alpha$ and $\mu'>\alpha'$. The term proportional to μ' in Eq. (21) is to reduce the absorption at large b, and that proportional to α' is to keep the corrected absorption coefficient non-negative always or in effect up to a large enough b beyond which the corrected absorption coefficient is sufficiently small in its order of magnitude. The impact parameter approximation gives then the correction Δf_{ab} to f_{ab}:

$$\begin{aligned} i\Delta f_{ab}(t)/p_{cm}&=\mu_1\mu' e^{-\mu_1|t|/2}(1-\mu_1|t|/2)-\alpha\alpha' e^{-\alpha|t|/2}(1-\alpha|t|/2)\\ &-(\tilde{\mu}^2\mu'/\mu_1)e^{-\tilde{\mu}|t|/2}(1-\tilde{\mu}|t|/2)+(\tilde{\alpha}^2\alpha'/\alpha)e^{-\tilde{\alpha}|t|/2}(1-\tilde{\alpha}|t|/2), \end{aligned} \tag{22}$$

where $\tilde{\mu}=\mu_1\mu_2/(\mu_1+\mu_2)$ and $\tilde{\alpha}=\alpha\mu_2/(\alpha+\mu_2)$. The total cross section calculated from $\mathrm{Im} f_{ab}(0)$ through the optical theorem is corrected by the amount

$$\Delta\sigma_{tot}=-4\pi\{\mu'(\mu_1-\tilde{\mu}^2/\mu_1)-\alpha'(\alpha-\tilde{\alpha}^2/\alpha)\}. \tag{23}$$

μ_1 cannot take too large a value in order that the corrected absorption coefficient is kept non-negative in the way mentioned above, since $\mu_1\mu'$ is restricted through Eq. (5) by a given value of $\Delta\sigma_{tot}$. Too small a value of μ_1, on the other hand, gives too large an effect upon the angular distribution around the forward peak. This relation can be used to determine the value of μ_1 within a narrow range. It has been found that a value of μ_1 equal to or a little larger than that of γ is suitable for our purpose. In order to have $e^{-2\chi}$ almost unaffected by Δ_e^{-2x} at smallest b's, μ_2 is chosen as large as possible provided that the angular distribution around the break or minimum is not much affected by Δf_{ab}.

Figure 6 shows the variation of the absorption coefficient versus b for a typical case of $E_{cm}=53$ GeV. The values of the parameters γ, β and β' are readjusted to be 11.0 $(\mathrm{GeV}/c)^{-2}$, 0.21 and 0.037, respectively. The values of μ_1, μ_2, and ν are 11.0, 0, and 16.0 in $(\mathrm{GeV}/c)^{-2}$, respectively, and those of μ' and ν' are 0.364 and 0.0625, respectively. The full line in Fig. 6 shows the corrected absorption coefficient. It varies approximately as $\mathrm{Exp}\{-b^2/2\times13\ (\mathrm{GeV}/c)^{-2}\}$ for $b\lesssim1\ M_N^{-1}$, and as $0.6\ \mathrm{Exp}\{-b^2/2\times9\ (\mathrm{GeV}/c)^{-2}\}$ for $5\ M_N^{-1}\lesssim b\lesssim9\ M_N^{-1}$. The broken line in Fig. 6 shows the uncorrected absorption coefficient. In comparison, usual Gaussian absorption coefficient is shown by the dotted line in Fig. 6. Figure 7 shows the angular distribution resulting from $(f_{ab}+\Delta f_{ab})$ for the same case. There is a slight break at $|t|\simeq0.1$ $(\mathrm{GeV}/c)^2$ in the resulting angular distribution. The corrected value of σ_{tot} is 39 mb with these values of the parameters given above.

Recent experiments performed with CERN ISR have reported that in ISR region of energy σ_{tot} is about 10% or more larger than its average value in $10\sim10^2$ GeV region. In that case, the correction to the absorption coefficient becomes less than that made here, and the resulting angular distribution becomes more or less similar to the average of those shown in Figs. 5 and 7.

In Fig. 8, the angular distribution resulting from the correction to the absorption coefficient at large b's given in Eq. (21) is shown for the case of $P_{Lab}=19.2$ GeV/c. The full lines in Fig. 8 at the largest $|t|$'s and at smaller

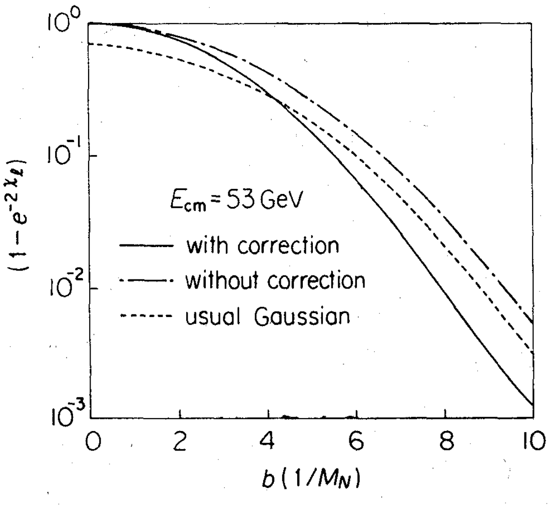

Fig. 6. The absorption coefficient $(1-e^{-2\chi})$ versus the impact parameter b for the case of $E_{cm}=53$ GeV. (M_N^{-1}= the nucleon Compton wave length.)

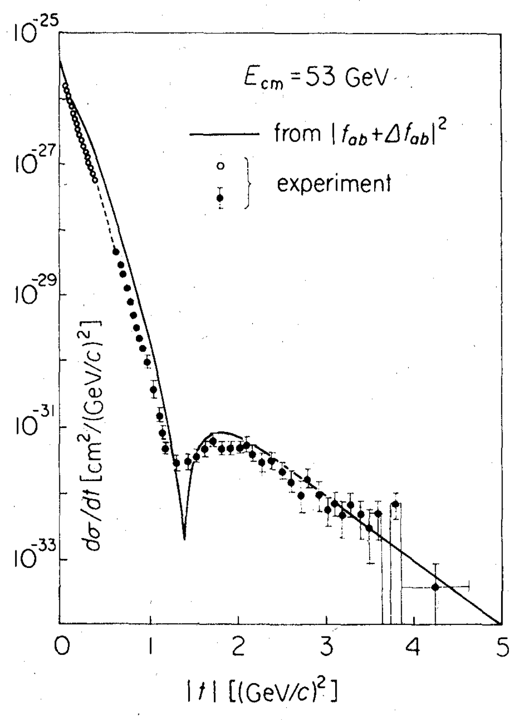

Fig. 7. The angular distribution of p-p elastic scattering at $E_{cm}=53$ GeV, with the correction for the absorption at large impact parameters.

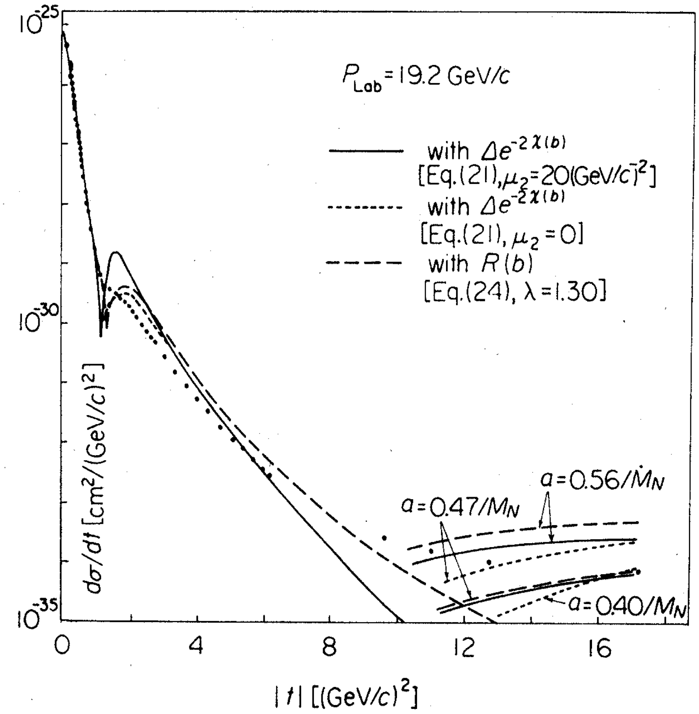

Fig. 8. The angular distribution of p-p elastic scattering at $P_{\text{Lab}}=19.2$ GeV/c, with the correction for the absorption at large impact parameters.

$|t|$'s represent the angular distributions due to $|f'_{hc}|^2$ and $|f_{ab}+\Delta f_{ab}|^2$, respectively, calculated with $\mu_1=12$, $\mu_2=20$ and $a=15$ (GeV/c)$^{-2}$, and $\mu'=0.550$ and $\alpha'=0.175$, together with the values of γ, β, β', ν and ν' given above. With these values of the parameters, we have $\sigma_{tot}=39.5$ mb. The dotted lines in Fig. 8 show similar ones obtained with $\mu_1=10$ (GeV/c)$^{-2}$, $\mu_2=0$, $\mu'=0.399$ and $\alpha'=0.111$, together with the same values of the other parameters as above. In the latter case, we have $\sigma_{tot}=39.5$ mb, too.

We have also calculated numerically the angular distribution resulting from the absorption coefficient given in Eq. (16) multiplied by the reduction factor $R(b)$,

$$R(b)=e^{-\mu'(b^2/2\mu)^\lambda/\{1+(b^2/2\mu)^\lambda\}}, \tag{24}$$

with which $(f_{ab}+f'_{ab})$ cannot be analytically integrated. The broken-dotted lines in Fig. 8 show the result obtained with $\mu=30$ (GeV/c)$^{-2}$, $\lambda=1.30$ and $\mu'=1.60$, for which we have $\sigma_{tot}=40.0$ mb. As is seen from the comparison of Fig. 3 and Fig. 8, the value of the radius of the hard core is almost unaltered by the correction given by Eq. (21) or Eq. (24) to the absorption coefficient at large b's. In Fig. 9, the absorption coefficient with the correction given by Eq. (24) with the same values of the parameters as above is shown for the case of $P_{\text{Lab}}=19.2$ GeV/c.

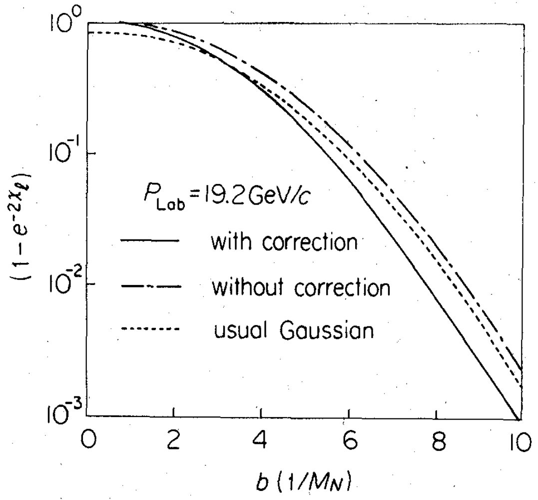

Fig. 9. The absorption coefficient versus the impact parameters b. (M_N^{-1}=the nucleon wave length).

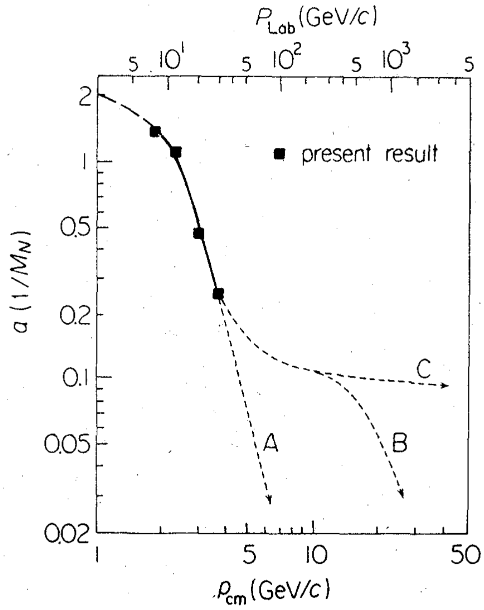

Fig. 10. The variation of the radius of the hard core a with the incident momentum P_{Lab} or p_{cm}.(M_N^{-1}=the nucleon Compton wave length.)

In Fig. 10, the general trend of the variation of the radius of the hard core a is shown in units of the nucleon Compton wave length $1/M_N$, as a function of incident momentum. It is seen that these values of a shown in Fig. 10 join smoothly to the value $a=2/M_N$ at low energies. It is also seen from Fig. 10 that at $P_{Lab}\simeq 30$ GeV/c there can be a hard core of radius of twice the Compton wave length of a particle with a mass of about 10 M_N.

§4. Conclusion

The radius a shown in Fig. 10 gives us an upper limit for the radius of the hard core at high energies, in the sense that in our present analysis we have taken into account only the absorption process for the p-p elastic differential cross section at non-large $|t|$. If nuclear force outside the hard core, which is not taken into account in the present analysis, is attractive, however, it will reduce the phase shifts due to the hard core. In that case, we get for the radius of the hard core effectively a smaller value than it is. Our present result means, therefore, that the hard core in nuclear force at low energies should not necessarily disappear completely in tens GeV region of incident laboratory energy, as it does in the non-relativistic quark model.

In connection with the theory of nucleon structure described in §2, it is very interesting to know whether the radius of hard core varies at higher energies somewhat like the dotted curve A or B or C in Fig. 10. In the case of A, it cannot be known at what energy the nucleon structure itself is excited. In the case of B, it can be known that the nucleon structure itself is excited in 10^2 to 10^3 GeV region. Such distinction will be obtained from analyses of p-p scattering at large angles in Serpukhov, NAL and CERN ISR energy region. In doing such analyses, it will be necessary for us, as is pointed out in the introduction, to proceed step-by-step, by investigating various possibilities strategically, not forgetting Sakata's saying that "even the neutrino is an inexhaustible entity."[45]

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