

Lawrence Berkeley National Laboratory

Recent Work

Title

ANGULAR DISTRIBUTION OF UNDULATOR POWER FOR AN ARBITRARY DEFLECTION
PARAMETER K

Permalink

<https://escholarship.org/uc/item/1cq4g16c>

Author

Kim, K.-J.

Publication Date

1985-08-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

LAWRENCE
BERKELEY LABORATORY

JUL 23 1985

LIBRARY AND
DOCUMENTS SECTION

Presented at the International Conference on X-ray and VUV Synchrotron Radiation Instrumentation, Stanford, CA, July 29 - August 2, 1985; and to be published in Nuclear Instruments and Methods in Physics Research

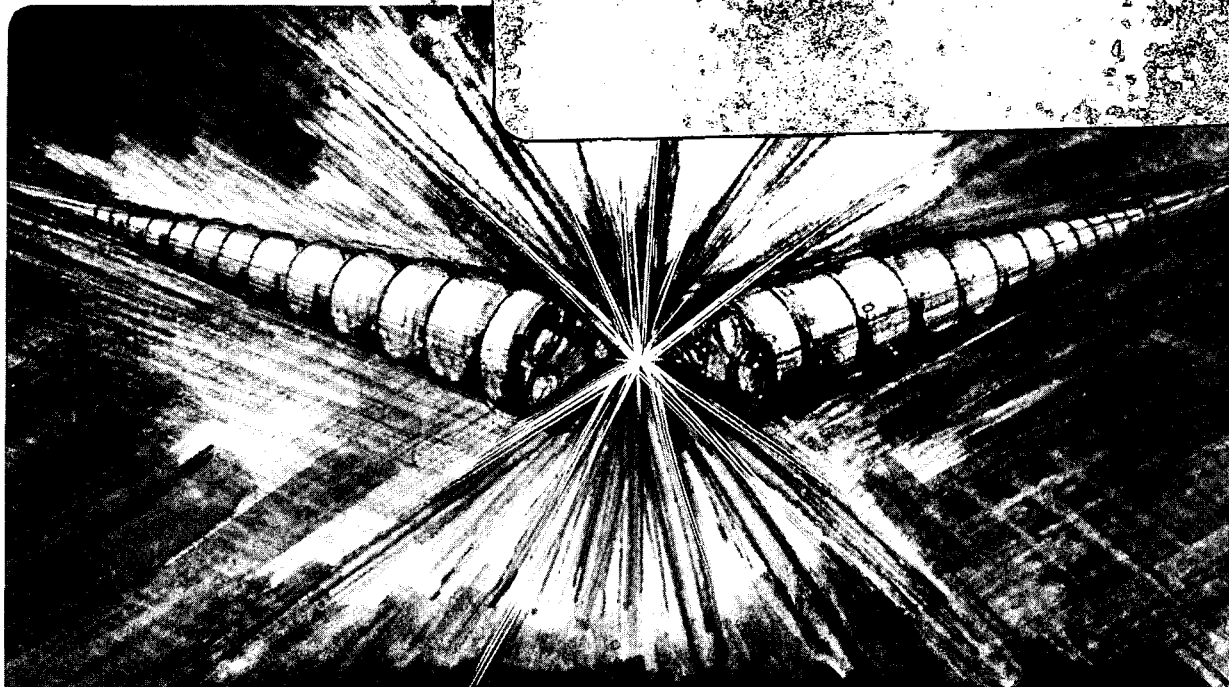
ANGULAR DISTRIBUTION OF UNDULATOR POWER FOR AN ARBITRARY DEFLECTION PARAMETER K

K.-J. Kim

August 1985

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks*



LBL-20167
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

ANGULAR DISTRIBUTION OF UNDULATOR POWER FOR
AN ARBITRARY DEFLECTION PARAMETER K^*

Kwang-Je Kim

Center for X-ray Optics, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

August 1985

*This work was supported by the Office of Basic Energy Sciences,
U.S. Department of Energy, under Contract # DE-AC03-76SF00098.

ANGULAR DISTRIBUTION OF UNDULATOR POWER FOR.
AN ARBITRARY DEFLECTION PARAMETER K^*

Kwang-Je Kim

Center for X-ray Optics, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

ABSTRACT

A calculation of the angular distribution of power generated from an undulator, integrated over all frequencies, is presented. The result, valid for any arbitrary value of the deflection parameter K , reduces to the known expressions in the cases $K \rightarrow \infty$ and $K \rightarrow 0$.

*This work was supported by the Office of Basic Energy Sciences,
U.S. Department of Energy, under Contract # DE-AC03-76SF00098.

One of the important considerations in the design of optical elements for future, advanced synchrotron radiation sources is the angular distribution of frequency-integrated power $dP/d\Omega$ from undulators. The expression for $dP/d\Omega$ is well-known in the limiting cases $K \rightarrow \infty$ (the wiggler limit) and $K \rightarrow 0$ (the weak undulator limit), where K is the usual deflection parameter given by

$$K = 0.934 \lambda_u (\text{cm}) B_0 (\text{T}) \quad (1)$$

Here λ_u is the period of the undulator and B_0 is the peak magnetic field. In this paper an exact expression for $dP/d\Omega$, valid for an arbitrary K , is derived and discussed.

Schwinger [1] gave a general formula for $dP/d\Omega$ for an arbitrary electron trajectory. For electrons following a sinusoidal trajectory, as shown in Fig(1), his expression reduces to

$$\frac{dP}{d\Omega} \equiv \frac{d^2P}{d\theta d\psi} = P_T \frac{21\gamma^2}{16\pi K} G(K) f_K(\gamma\theta, \gamma\psi) \quad (2)$$

Here P_T is the total power (integrated over angles and frequencies) given by

$$P_T = \frac{N}{6} Z_0 I \frac{2\pi e c}{\lambda_u} \gamma^2 K^2 \quad (3)$$
$$= 0.633 E^2 (\text{GeV}) B_0^2 (\text{T}) L (\text{m}) I (\text{A}) (\text{kW}),$$

where

N = number of the undulator periods

Z_0 = vacuum impedance (377 Ohms)

I = electron current

e = electron charge

c = velocity of light

γ = electron's relativistic energy/rest energy

E = electron's energy

L = length of the undulator

G(K) is a normalization factor given by

$$G(K) = K \frac{(K^6 + \frac{24}{7} K^4 + 4K^2 + \frac{16}{7})}{(1 + K^2)^{7/2}} \quad (4)$$

Finally $f_K(\gamma\theta, \gamma\psi)$ is a factor which gives the angular dependence as follows:

$$f_K(\gamma\theta, \gamma\psi) = \frac{16K}{7\pi G(K)} \int_{-\pi}^{\pi} d\alpha \sin^2\alpha \left(\frac{1}{D^3} - \frac{4(\gamma\theta - K\cos\alpha)^2}{D^5} \right), \quad (5)$$

where

$$D = 1 + (\gamma\psi)^2 + (\gamma\theta - K\cos\alpha)^2. \quad (6)$$

The function f_K is normalized so that

$$f_K(0,0) = 1 \quad (7)$$

The integral in Eq(5) can be evaluated using the contour integration technique.

The result is complicated for the general case, and it is better to use numerical integration of Eq(5). For the special case $\theta = 0$, however, the contour integration method yields the following expression:

$$f_K(0, \gamma\psi) = \frac{1}{7G(K)(1+(\gamma\psi)^2)^{5/2}} \left[\frac{12 + 16\epsilon}{(1+\epsilon)^{3/2}} - \frac{5 + 16\epsilon + 16\epsilon^2}{(1+(\gamma\psi)^2)(1+\epsilon)^{7/2}} \right], \quad (8)$$

where

$$\epsilon = \frac{1 + (\gamma\psi)^2}{K^2} \quad (9)$$

When $K \rightarrow \infty$, Eq(5) becomes

$$K \rightarrow \infty: f_K(\gamma\theta, \gamma\psi) = \sqrt{1 - (\gamma\theta/K)^2} \left\{ \frac{1}{(1+(\gamma\psi)^2)^{5/2}} + \frac{5(\gamma\psi)^2}{7(1+(\gamma\psi)^2)^{7/2}} \right\} \quad (10)$$

The factor in the curly bracket above is familiar from the expression for the angular distribution of power from a bending magnet [1], while the factor

$\sqrt{1 - (\gamma\theta/K)^2}$ arises because the magnetic field observed from a horizontal angle θ is reduced from B_0 by the same factor. In the limit $K \rightarrow 0$, one can derive

from Eq(5) that

$$K \rightarrow 0: f_K(\gamma\theta, \gamma\psi) = \frac{1 + 2\gamma^2(\psi^2 - \theta^2) + (\gamma^2(\theta^2 + \psi^2))^2}{(1 + \gamma^2(\theta^2 + \psi^2))^5} \quad (11)$$

This expression was first derived by Hofmann [2] by considering an oscillating dipole moving with relativistic velocity.

The functions $f_K(0, \gamma\psi)$ and $f_K(\gamma\theta, 0)$ are plotted for values of K ranging between 0.25 and ∞ in Fig(2) and Fig(3). One observes in both cases that the curves for $K \geq 1$ quickly approach the curves for the limiting case corresponding to $K = \infty$.

From Eq(2) and Eq(7), the power density in the forward direction is

$$\begin{aligned} \frac{dP}{d\Omega}(0,0) &= P_T \frac{21 \gamma^2}{16\pi K} G(K) & (12) \\ &= 10.84 B_0(T) E^4(\text{GeV}) I(A) N G(K) (W/(mr)^2) \end{aligned}$$

The function $G(K)$, which is given by Eq(4), is plotted in Fig(4). Its limiting behavior is

$$\begin{aligned} G(K) &\rightarrow 1 \quad (K \rightarrow \infty) \\ &\rightarrow \frac{16}{7} K \quad (K \rightarrow 0) \end{aligned} \quad (13)$$

If one approximates $dP/d\Omega$ by a rectangular distribution, then

$$\begin{aligned} \frac{dP}{d\Omega} &= \frac{P_T}{(\Delta\psi)(\Delta\theta)}, \quad -\frac{\Delta\psi}{2} \leq \psi \leq \frac{\Delta\psi}{2}, \quad -\frac{\Delta\theta}{2} \leq \theta \leq \frac{\Delta\theta}{2} & (14) \\ &= 0, \text{ otherwise.} \end{aligned}$$

From Fig(2), one finds that $\Delta\psi \sim \gamma^{-1}$. The magnitude of $\Delta\theta$ can be obtained from Eqs(12), (13) and (14). The result is

$$\begin{aligned} \Delta\psi &\sim \frac{1}{\gamma} \quad (\text{all } K) \\ \Delta\theta &\sim \frac{1}{\gamma} \quad (K \rightarrow 0) \\ &\sim \frac{2K}{\gamma} \quad (K \rightarrow \infty) \end{aligned} \quad (15)$$

$\Delta\psi$ and $\Delta\theta$ in the above can roughly be identified as the FWHM angular widths of the power distribution.

At a given frequency, the angular distribution of power per unit frequency

interval $d^2P/d\omega d\Omega$ is concentrated in narrow rings in angular space [3]. The radii of these rings change as the frequency changes. Thus, $dP/d\Omega$, which is obtained by integrating $d^2P/d\omega d\Omega$ over ω , is a smooth function of θ and ψ . This is shown in Fig(5).

Of particular importance is the behavior of $d^2P/d\omega d\Omega$ near $\omega = n\omega_1$, where ω_1 is the fundamental frequency of undulator radiation given by

$$\omega_1 = \frac{2\pi c}{\lambda_u} \frac{1+K^2/2}{2\gamma^2} \quad (16)$$

At these frequencies, $d^2P/d\omega d\Omega$ develops a peak around the forward direction $\theta = \psi = 0$. For $n=1$, the rms angular widths of the central cone are

$$\begin{aligned} (\Delta\psi)_c &= \sqrt{\frac{1+K^2/2}{2N\gamma^2} + \sigma_y'^2}, \\ (\Delta\theta)_c &= \sqrt{\frac{1+K^2/2}{2N\gamma^2} + \sigma_x'^2} \end{aligned} \quad (17)$$

Here σ_x' and σ_y' are the rms angular divergences of the electron beam in the horizontal and vertical directions, respectively. In an undulator beam line, a pinhole will select out the central cone, which contains the useful photon flux. The power transmitted through the central cone is

$$P_{cen} = \frac{dP}{d\Omega} (0,0) \times 2\pi(\Delta\psi)_c(\Delta\theta)_c \quad (18)$$

For synchrotron radiation sources, σ_x' and σ_y' are usually smaller than γ^{-1} , and the effects of the beam divergence on $dP/d\Omega$ are negligible. On the other hand, these angles are comparable to the central radiation cone of the fundamental radiation of an undulator, and thus must be taken into account.

Power generated from undulators for various synchrotron radiation sources, as well as other useful data, can be found in Reference [4].

References

- [1] J. Schwinger, Phys. Rev. 75 (1949)1912
- [2] A. Hofmann, Nucl. Inst. Methods 152 (1978)17
- [3] See, for example, D. Attwood, K. Halbach and K-J. Kim, Science 228
(1985)1265
- [4] D. Attwood and K-J. Kim, these proceedings

Figure Captions

- Fig(1) The electron trajectory and coordinate system. The electron moves on a sinusoidal trajectory in horizontal plane x-z. θ and ψ are the angles of observation in the horizontal and vertical directions, respectively.
- Fig(2) The behavior of the function $f_K(0, \gamma\psi)$
- Fig(3) The behavior of the function $f_K(\gamma\theta, 0)$
- Fig(4) The behavior of the function $G(K)$
- Fig(5) Illustration of how $dP/d\omega d\Omega$ adds up to $dP/d\Omega$.

Coordinate system

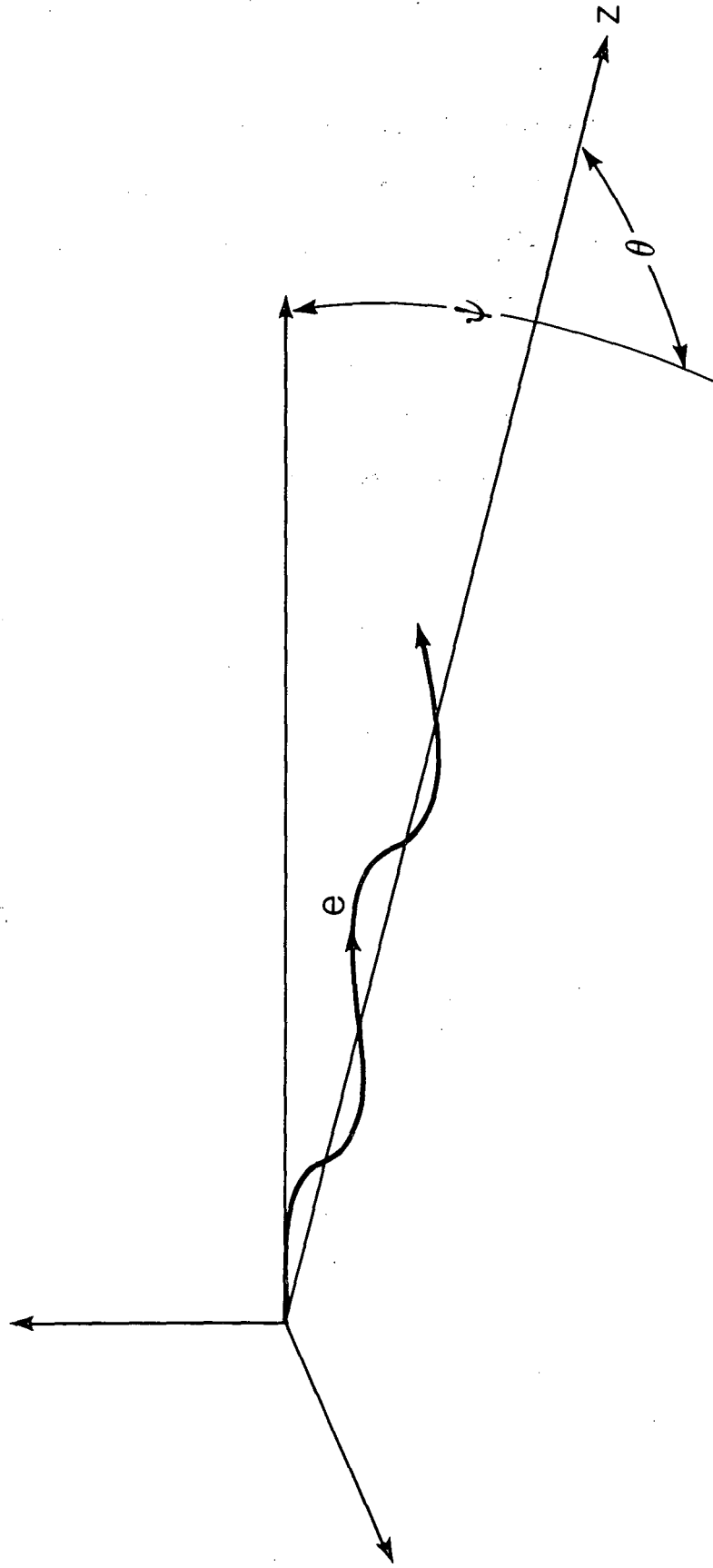


Figure 1

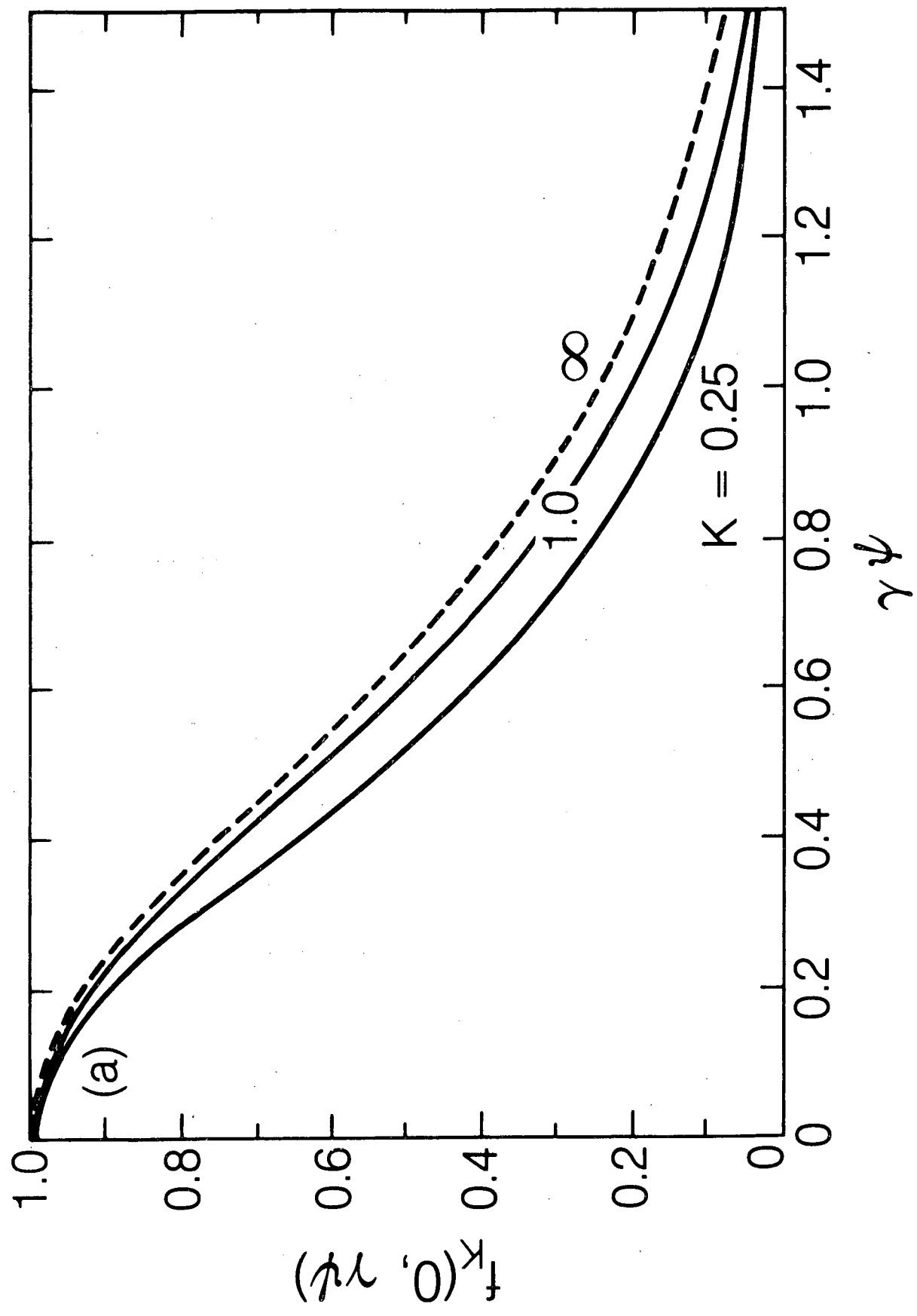


Figure 2

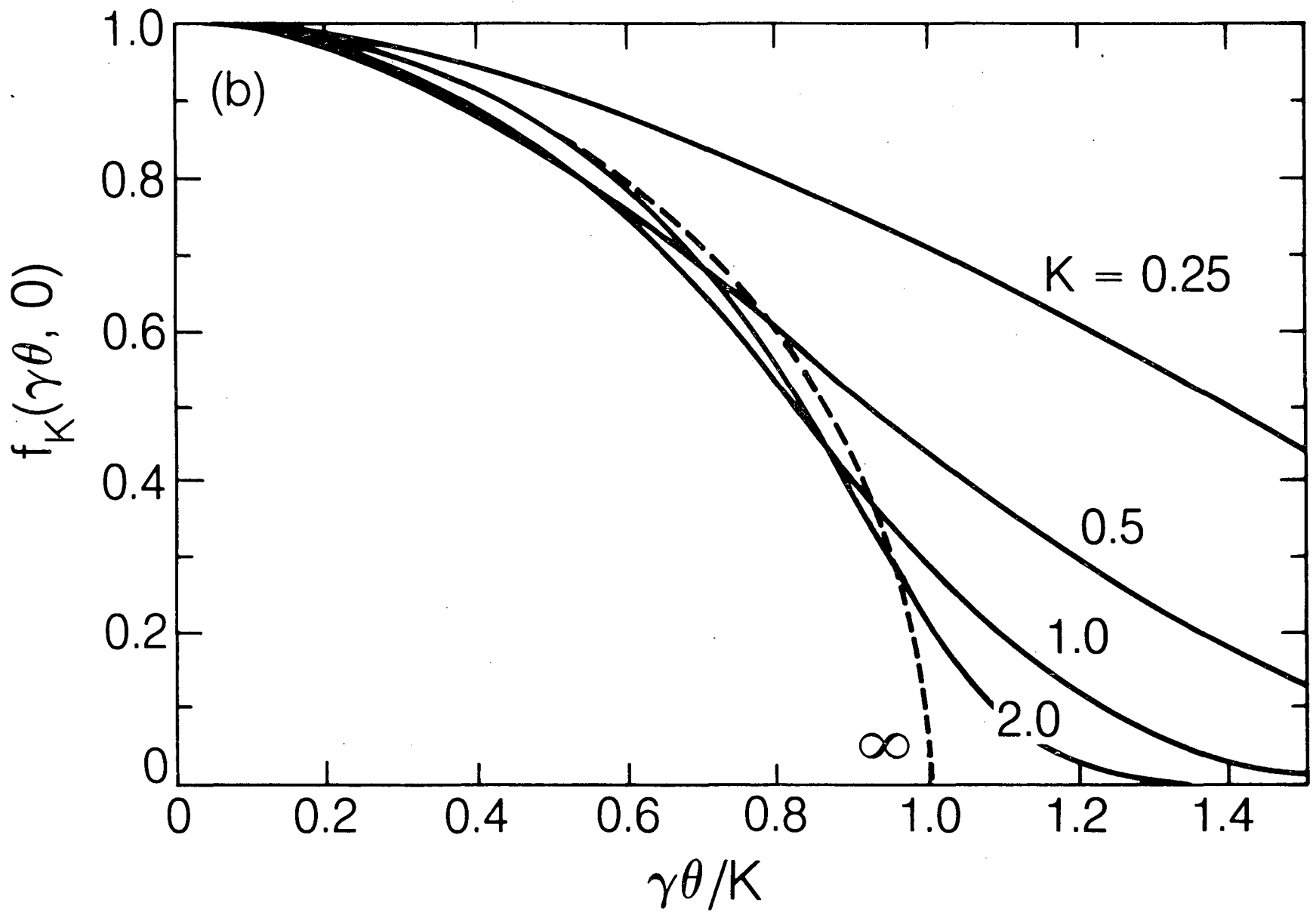


Figure 3

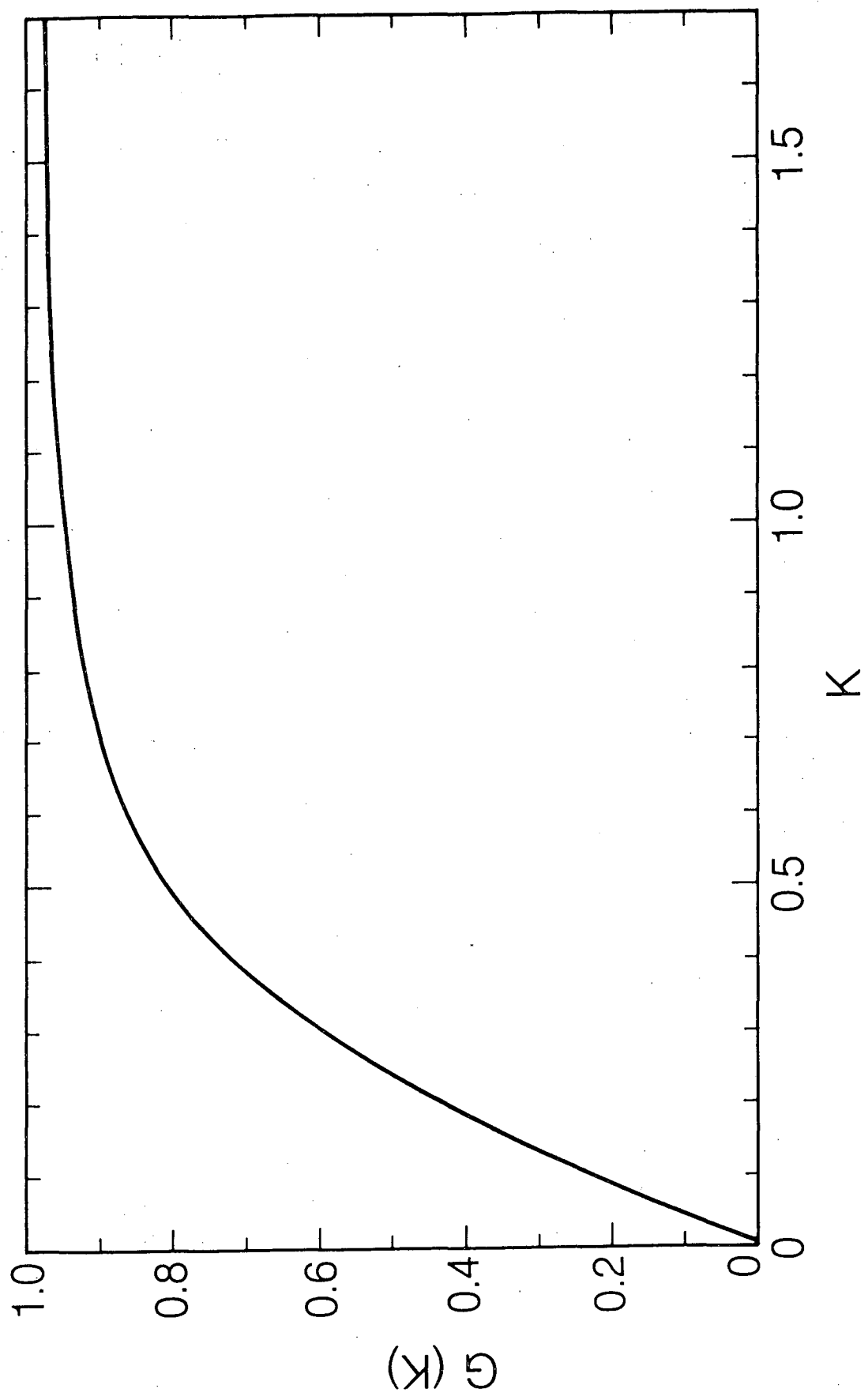


Figure 4

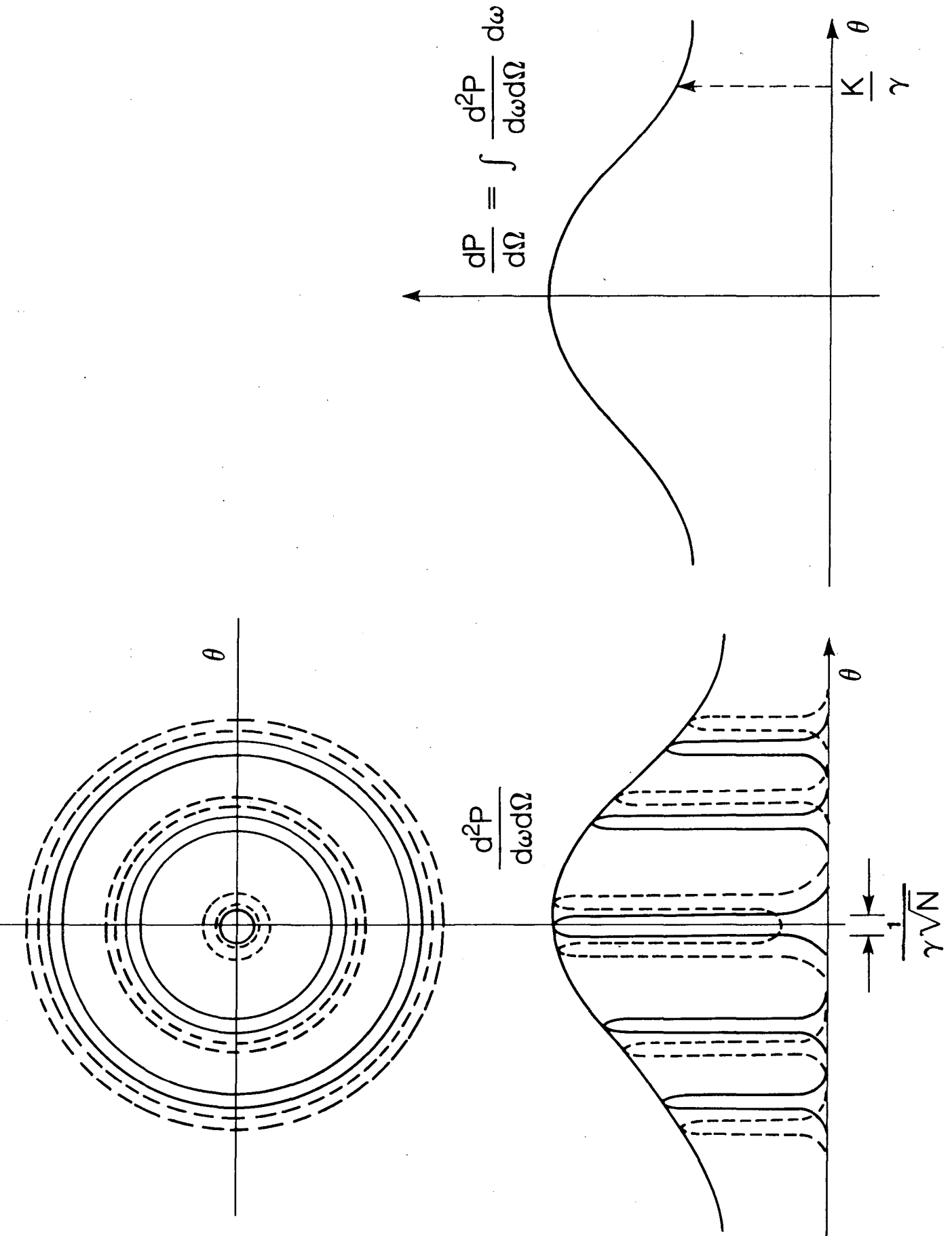


Figure 5

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

*LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720*