ANGULAR SIZE-FLUX DENSITY RELATION FOR EXTRAGALACTIC RADIO SOURCES

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SUMMARY

A comparison has been made of the maximum angular extents of 163 weak extragalactic radio sources observed by the method of lunar occultation at Ootacamund with those for 199 strong sources from the 3CR complete sample. The angular sizes of the 62 sources with flux densities $S_{408} \ge 16.5$ Jy (1 Jy = 10⁻²⁶ W m⁻² Hz⁻¹) in the All-sky catalogue of Robertson were also considered. Over a flux density range of about 0.3-100 Jy at 408 MHz, it is found that the median value of angular size $\theta_{\rm m}$ increases with flux density $S_{\rm m}$ and is related approximately as $\theta_{\rm m} \propto S^{1/2}$. A similar relation is found if the identified QSOs are excluded from the source samples. These results indicate that statistically the weaker sources are located farther away. Since both θ and $S_{\rm m}$ are dependent on world models, their relationship is of value for cosmological investigations.

I. INTRODUCTION

In relativistic cosmologies, the angular diameter of a source of constant size first decreases with increasing redshift z, reaches a minimum around $z \sim 1$ to 2 and increases thereafter for larger redshifts (Hoyle 1959; Sandage 1961). In the steady state model on the other hand, a minimum angular size is reached asymptotically as z approaches infinity. The correlation between the angular size and the redshift for QSOs has been investigated by Miley (1968, 1971), Legg (1970) and Wardle & Miley (1974). It was found that the maximum angular size of QSOs decreased inversely with redshift up to $z \sim 2.5$, as expected in a static Euclidean universe. Since their result was not consistent with the predictions of cosmological models based on constant source sizes, it was suggested that the sizes of QSOs were smaller at earlier epochs. The cosmological origin of the redshifts is, however, not vet clear for QSOs (Burbidge 1973). It is therefore important to investigate a corresponding relationship for extragalactic radio sources excluding QSOs. But optical identification and redshift determination is difficult and highly incomplete for galaxies with $z \gtrsim 0.3$. An alternative possibility is to study the correlation between flux density and angular size. Although this introduces scatter in the data due to the wide range of source luminosities, we can use complete and unbiased samples of radio sources.

High resolution observations for several hundred weak radio sources have recently been made at 327 MHz by the method of lunar occultation using the Ooty radio telescope (Swarup et al. 1971a). In this paper we compare the structures of the Ooty lunar-occultation (OTL) sources with those of 3CR sources observed by the Cambridge group at 1407 MHz using the One-Mile radio telescope. In order to

improve the statistical significance of inferences for the intense sources and to increase the range of flux densities covered, we have also studied structures of all 62 sources with flux densities $S_{408} \ge 16.5$ Jy in the 'All-sky' catalogue of Robertson (1973). The study has established an angular size vs flux density relation covering a flux density range of nearly 300: 1. This relation along with angular size counts for the 3CR sources has been used as a sensitive test of steady-state vs evolving cosmological models by Kapahi (1975a).

2. THE ANGULAR SIZE DATA

2.1 Ooty lunar-occultation sources

Observations of over 600 extragalactic sources have been made so far (1970–71 and 1973–74) at Ootacamund by the method of lunar occultation. Most of these sources have a flux density between 0.25 and 5.0 Jy at 327 MHz. Analysis has been completed for 280 sources observed during 1970 and 1971 (Swarup *et al.* 1971b; Kapahi, Joshi & Kandaswamy 1973a; Kapahi *et al.* 1973b; Joshi *et al.* 1973; Kapahi, Joshi & Sarma 1974; Gopal-Krishna & Subrahmanya, to be published). A revised analysis of sources in List 1 (Swarup *et al.* 1971b) has indicated that sources OTL 1627–27 and 2109–18 are doubles with angular separations of 24" and 92" respectively.

The restoration of a lunar occultation record gives a strip brightness distribution across a source in a direction normal to the Moon's limb at the point of occultation. All the 280 sources were scanned along at least two position angles. About a third of these were scanned along three to seven directions. The angular resolution achieved β was about I" to 4" for most of the sources with $S_{327} \gtrsim 1$ Jy, and about 2" to 10" for weaker sources. Higher values of β were used for about 10 per cent of the sources, mostly those with extended structure. The angular extent of each source was determined from the restored brightness distributions for which the peak value was about 5-7 times the rms noise, although outputs of higher β giving lower noise were also used sometimes to ensure that any secondary components were not missed. In view of the limited signal to noise ratio, most of the sources were considered unresolved if the half-power widths of the restored distributions were less than about 1.3 or 1.4 times β . For sources with flux density ≥2 Jy, the angular resolution achieved was limited to ~1" due to receiver bandwidth and therefore a source could be resolved to a value comparable to β as a result of the higher signal-to-noise ratio.

The value of the 'largest angular size' (LAS) of a radio source depends on the shape of its brightness distribution. For a double or multicomponent source LAS is taken here to be the angular separation between its outermost peaks. For a complex source, LAS is the overall extent found after correction for beambroadening. For a partially resolved source, LAS refers to the half-power width of the source along the major axis, on the assumption that the source has a gaussian distribution of brightness. For core and halo sources, half-power widths of halos have been considered but only a few such structures have been found.

If the source is scanned along a limited number of directions, none of which coincides with its major axis, the measured value of LAS will in general be smaller than its actual value. In order to minimize this error, we have selected only those of the 280 sources for which the extreme position angles of the strip scans across the source were separated by more than 30° but less than 150° (position angles of

scans being considered to lie only between $o-180^{\circ}$ rather than $o-360^{\circ}$). Also, sources with galactic latitude $|b| < 10^{\circ}$ were excluded, leaving a sample of 163 OTL sources (Kapahi 1975b). This sample does not form a complete sample in the usual sense of containing all sources above a certain flux density and within a definite area of the sky. Nevertheless, for investigating angular size vs flux density relation, the 163 sources constitute an unbiased random sample in the flux density range of about $o\cdot25-5$ Jy at 327 MHz because sources have been selected irrespective of their structure.

The flux densities of OTL sources measured at Ootacamund at 327 MHz during 1970–71 were based on the KPW scale (Kellermann, Pauliny-Toth & Williams 1969) and had rms errors of \sim 20 per cent for $S_{327} \sim 1$ Jy and \sim 30 per cent for weaker sources. In order to reduce the errors and for comparison of OTL sources with 3CR sources for which flux densities are available at 408 MHz instead of 327 MHz, flux densities of 112 out of 163 OTL sources were measured at 408 MHz with the Molonglo Cross. Wyllie's (1969) scale was adopted and the rms errors were \sim 8 per cent (Swarup & Sutton, to be published). For 16 of the OTL sources, S_{408} values were taken from the Bologna catalogue (Colla et al. 1972). These were multiplied by 1.06 to be consistent with Wyllie's scale. For the remaining 35 sources, S_{408} values were calculated from Ootacamund S_{327} values by multiplying them with a ratio S_{408}/S_{327} determined for different flux density ranges from the observations of 112 sources made at Molonglo. Since the frequency ratio is only 1.24, errors due to dispersion in spectral index (\pm 0.2 in 0.75) should be negligible.

In only a few cases were significant differences found between the positions of the 112 OTL sources measured at Molonglo with a 3' beam and the positions measured by occultations. Thus it seems unlikely that secondary weaker components of double sources were missed in the structures determined at Ootacamund.

2.2 The 3CR sample

In the revised 3C catalogue (Bennett 1962), the 200 radio sources which have $\delta > +10^{\circ}$ and $|b| > 10^{\circ}$ are believed to form a complete sample of sources with $S_{178} \geqslant 9$ Jy in 4.25 sr of the sky (Longair & Pooley 1969; Mackay 1971). Excluding 3C 326 which is probably a galactic source (Mackay 1971), the 3CR 'complete sample' consists of 199 extragalactic radio sources. Improved measurements indicate that several of these sources have flux densities between 3.70 and 8.80 Jy at 178 MHz (Kellermann *et al.* 1969), but on the other hand it is possible that a few sources with $S_{178} \geqslant 9$ Jy have been missed in the 3CR catalogue. Since such changes are only of a minor nature we have included all 199 sources in our analysis.

The 3CR complete sample is based on flux density measurements at 178 MHz, whereas the angular size data for these sources are available only at the higher frequencies. But the overall extents of radio sources are nearly independent of the observing frequencies between 408 and 2695 MHz, although the flux ratios of components of many sources change with frequency due to differences in their spectral indices (Macdonald, Kenderdine & Neville 1968; Mackay 1971; Wilkinson, Richards & Bowden 1974). We have therefore assumed that largest angular size of the sources is the same between 178 and 2695 MHz.

Structures of all the 199 sources have been determined at 1407 MHz by the Cambridge group (Macdonald, Kenderdine & Neville 1968; Mackay 1969; Elsmore & Mackay 1970) using the One-Mile radio telescope with a resolution of

23'' in RA and 23'' cosec δ in declination. It was possible to resolve the radio sources if their angular size exceeded $\sim 7''$ in RA and 7'' cosec δ in declination. Although the angular sizes were not determined accurately in the range 7-20'' at 1407 MHz, there are no significant differences between these estimates and accurate values found for some of these sources with higher resolutions as provided by the One-Mile radio telescope at 2.7 and 5 GHz (Branson *et al.* 1972). For resolved and partially resolved sources we have therefore adopted angular size values determined at 1407 MHz.

For 59 sources only an upper limit to the angular size could be determined at 1407 MHz; for 25 of these the upper limits lie between 7 and 12", for 16 between 12.5 and 20", for 14 between 21 and 30" and for four between 31 and 40". For 40 of the 59 sources, the angular size values were available from observations made at different frequencies between 327 MHz to 5 GHz with other high resolution interferometers or from lunar occultations. For the remaining 19 sources upper limits to the angular sizes based on Cambridge data at 1407 MHz lie between 7 and 12" for nine sources, 12.5 and 20" for five sources, 21 and 30" for three sources and 31 and 36" for the other two sources. The angular size values for the 40 sources at frequencies closest to 1407 MHz have been obtained from the following papers:

3C 43, 318 (Bash 1968); 305 (Branson et al. 1972); 48, 67, 245, 270.1 (Clark & Hogg 1966); 153, 371 (Hogbom & Carlsson 1974); 49 (Lang 1971); 9, 138, 147, 181, 208, 268.4, 277.1, 286, 287, 288.1, 454, 454.3 (Miley 1971); 196 (Mitton 1970); 216, 268.3, 295, 309.1, 345, 380 (Wilkinson 1972); 186, 191 (Wilkinson et al. 1974); 236 (Willis, Strom & Wilson 1974); 16, 19, 184, 190, 208.1, 241, 326.1, 460 (Wraith 1972). Sources 3C 83.1B and 84A in the Perseus cluster have been considered as separate sources and their angular sizes taken to be those of the halo components at 408 MHz (Ryle & Windram 1968). Only 3C 225B is included and is considered unrelated to 3C 225A (Kapahi, Joshi & Gopal-Krishna 1972). The size of 3C 274 has been taken as the size of the halo at 408 MHz (Macdonald et al. 1968).

At 178 MHz, the flux densities were taken mostly from the list of Kellermann et al. (1969), and at 408 MHz from the three papers that give observations made with the One-Mile telescope; these values were multiplied by factors of 1.09 and 1.10, respectively to be consistent with Wyllie's scale (Roger, Bridle & Costain 1973; Robertson 1973). Flux densities of the unrelated components of 3C 66 and 3C 442 (Mackay 1971; Wills & Wills 1974) have been subtracted. For 3C 208 and 3C 208.1 flux densities are based on occultation observations at 327 MHz (Gopal-Krishna, private communication).

Optical identification data are taken from the three papers that give observations made with the One-Mile telescope and from the compilation by Véron & Véron (1974). Deep surveys by Kristian, Sandage & Katem (1974) and Longair & Gunn (1975) have also been considered. Probable or doubtful identifications have been ignored.

2.3 All-sky catalogue

There are 160 sources in the 'All-sky' catalogue by Robertson (1973) with $S_{408} \ge 10$ Jy based on Wyllie's scale; the catalogue covers 10·1 sr of sky and excludes sources with $|b| < 10^{\circ}$ as well as regions of the Magellanic clouds. Angular structure data are available in various catalogues for only 150 of these 160 sources.

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We have considered only 62 sources with $S_{408} \ge 16.5$ Jy, of which 26 are common to the 3CR sample. Angular sizes for the remaining 36 sources were taken from Bridle *et al.* (1972), Cooper, Price & Cole (1965), R. D. Ekers (1969), Fomalont (1968), Fomalont & Moffet (1971), Schwartz, Cole & Morris (1973) and Wall & Cole (1973). Optical identification data were obtained from J. A. Ekers (1969).

3. THE ANGULAR SIZE vs FLUX DENSITY RELATION

3.1 Scatter diagram and histograms

Fig. 1 shows a plot on a logarithmic scale of largest angular size θ against flux density for all 199 3CR sources and 163 OTL sources. Although there is a large spread in the values of θ at any S, it is clear from the figure that angular sizes of

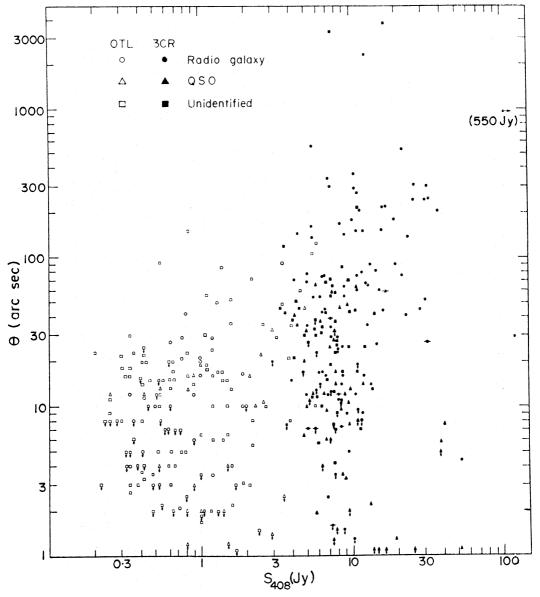


Fig. 1. Scatter diagram of 'largest angular size' θ of radio sources against their flux density at 408 MHz. The symbol $- \bullet -$ indicates that optical identification is from deep surveys by Kristian et al. (1974) or Longair & Gunn (1975).

radio sources decrease with decreasing flux density. This correlation is also evident from the histograms of angular sizes presented for eight different flux density ranges in Fig. 2. It is also seen that unidentified sources and quasi-stellar objects (QSOs), shown by squares and triangles respectively in Fig. 1, have generally smaller angular sizes than the identified radio galaxies shown by circles.

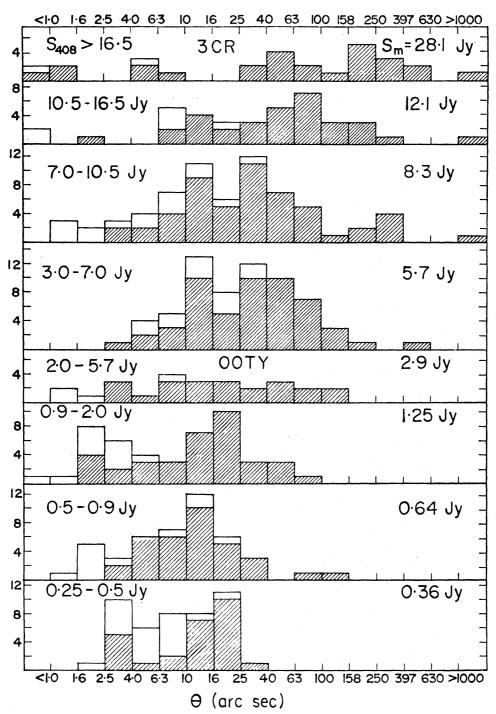


Fig. 2. Number of radio sources of different angular sizes θ for eight flux density ranges. The median value of flux densities for each range is given on the right-hand corner. The hatched and blank portions in the histograms respectively show numbers of sources for which definite values of θ and only upper limits of θ were available.

3.2 Relation between median value of angular size and flux density

The angular sizes of these radio sources vary from less than 1" to about 4000". A few of the large diameter sources may have been missed in these surveys, particularly in the lower flux density ranges. Also, if one of the components of a double source is weak, the source could be wrongly classified as a single source. Further, many sources are not resolved due to instrumental limitations and only an upper limit can be placed on their diameter. The data, however, permit the derivation of a quantitative relation between the median angular size $\theta_{\rm m}$ and the median value of flux density $S_{\rm m}$. Median values were found for several different ranges of flux density by plotting cumulative numbers of sources against θ for each range and fitting a smooth curve through the data points.

For the OTL sources, the measured values of $\theta_{\rm m}$ are likely to be somewhat smaller than the actual values since these sources have been scanned along a few position angles only. Let us first consider the case when the sources are scanned along two position angles separated by angles 2α which vary uniformly from $2\alpha_1$ to $2\alpha_2$. If ψ is the angle between the major axis of a given source and the line bisecting angle 2α , and ψ has a uniform probability distribution between $-\pi/2$ to $\pi/2$, the mean (or similarly median) values of θ are smaller than the actual values by a factor K given by

$$K^{-1} = rac{\displaystyle\int_{lpha_1}^{lpha_2} \int_0^{\pi/2} \cos\left(\psi - lpha
ight) d\psi \; dlpha} {\displaystyle\int_{lpha_1}^{lpha_2} \int_0^{\pi/2} d\psi \; dlpha}.$$

For 163 OTL sources we find that α has a nearly uniform distribution between $\alpha_1 = 15^{\circ}$ and $\alpha_2 = 75^{\circ}$, in which case $K = 1\cdot16$. However, the factor K would be smaller if the sources are scanned along intermediate position angles also. Further, no such correction factors are required for the double and the complex sources because their angular extents are given by the separations of the outermost peaks which would mostly be determined truly and unambiguously even if the sources are scanned along two position angles only. We have consequently multiplied the measured median values of the OTL sources by factors varying from $1\cdot05$ for the largest flux density range to $1\cdot14$ for the lowest range.

In Fig. 3, the median value of θ has been plotted against the median value of S for four flux density ranges of OTL sources, four ranges of 3C sources and three ranges of sources from the All-sky catalogue with $S_{408} \ge 16.5$ Jy. The 26 most intense sources in the 3CR catalogue with $S_{408} \ge 16.5$ Jy are common to the 62 All-sky sources with $S_{408} \ge 16.5$ Jy. Fig. 4 shows a corresponding plot after excluding all the identified QSOs; in this case 3C and All-sky sources were divided into three ranges of flux density. The rms errors were determined for each point from the frequency distributions (histograms) of the angular size (Fig. 2) and of the flux density. The rms error is given by $\sqrt{n/(2f_{\rm p})}$ in units of the class interval of log θ or log S, where n is the total number of sources and $f_{\rm p}$ is the ordinate value of the frequency distribution at the median value (Yule & Kendall 1950). Errors in S were found to be negligible for all the ranges compared to errors in θ .

Since the resolution provided by the One-Mile radiotelescope is poor for low declination sources, we have also determined $\theta_{\rm m}$ values for 3CR sources after excluding sources with $\delta < 30^{\circ}$ for $S_{408} \leq 9.5$ Jy for the case of all 199 sources and $S_{408} \leq 13.8$ Jy for the case without QSOs. No appreciable differences were found

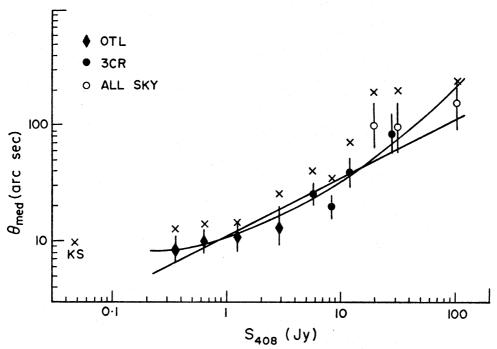


Fig. 3. A plot of the median value of angular size θ_m against the median value of flux density S_{408} for all sources. Error bars are shown by vertical lines. Crosses show 67 percentile values.

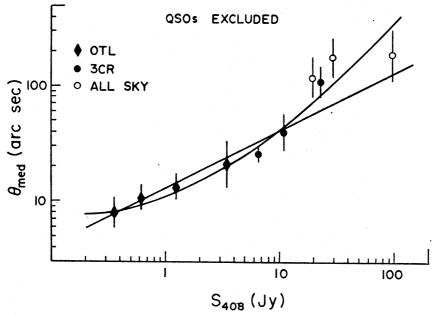


Fig. 4. A plot of the median value of angular size θ_m against the median value of flux density S_{408} , excluding QSOs.

from the values given in Figs 3 and 4. It may be noted that the median value of θ is of the same order as the resolution achieved for the lower flux density ranges of 3C as well as OTL sources. This would give an overestimate of the size of a double source by about 20 per cent if it is only partially resolved. Hence we have also determined 67 percentile values as shown by crosses in Fig. 3. These values are appreciably higher than the resolutions of the instruments used and show a trend

similar to that of the median values; their statistical errors are only about 10 per cent higher than those of median values. In Fig. 3 we have also shown by a cross marked KS, the 67 percentile value of angular sizes for a complete sample of 53 sources with $S_{1415} > 10^{-2}$ Jy based on a survey at 1415 MHz by Katgert & Spinrad (1974). The median value of flux density has been extrapolated to 408 MHz assuming a spectral index of 0.52. This datum point should be treated with caution for two reasons. First, the KS survey is at a frequency much higher than that of the 3C and OTL surveys, and hence contains an excess of flat spectrum sources which are likely to be of small diameter. Secondly, the instrumental resolution is not as high as that achieved in the occultation measurements.

4. DISCUSSION AND COSMOLOGICAL INFERENCES

In Euclidean space, $\theta \propto S^{1/2}$ because $\theta \propto 1/r$ and $S \propto 1/r^2$ where r is the distance to the source. A least squares fit was made to the data of $\log \theta_{\rm m}$ vs $\log S_{\rm m}$ with a slope of 0.5 and is shown by solid lines in Figs 3 and 4. For $S_{408} \ge 16.5$ Jy only the 62 All-sky sources were considered. According to a χ^2 test, the parabolic curves in the figures give a better fit, but the data show no departure from straight lines at the 5 per cent level of significance. It appears that the slope of the $\log \theta_{\rm m}$ vs $\log S_{\rm m}$ data for the four ranges of OTL sources is smaller than that for 3C sources. However, departure from a straight line is influenced mostly by (a) the points for the highest flux density range which have relatively large statistical errors and (b) by the two points for the lowest flux density range for which $\theta_{\rm m} \sim \beta$.

Figs 3 and 4 show that statistically the angular size of radio sources decreases with decreasing flux density indicating a distance effect. It is reasonable, therefore, to assume that the weaker OTL radio sources are located statistically much farther away than the stronger 3C radio sources. Hoyle & Burbidge (1970) have pointed out that there is no correlation between S and z for the 3CR radio galaxies. But their conclusion is biased by observational selection as redshifts have been measured mostly for the brighter galaxies which are likely to be located nearby. As shown in Table I, redshifts have been measured for only 16 radio galaxies out of 129 3CR sources with $S_{408} <$ 10 Jy as compared to 34 radio galaxies out of 70 sources with $S_{408} >$ 10 Jy. Table I also shows that there exists a correlation between z, S and θ ; however, it is difficult to estimate statistical errors in view of the selection effects. It should be noted that z has been measured for 35 of 42 identified radio galaxies out of 56 3CR sources with $\theta > 60$ °, but for only 15 of 53 identified radio galaxies out of 143 3CR sources with $\theta < 60$ °.

About 30 per cent of OTL sources have been optically identified, 17 per cent with galaxies and 13 per cent with QSOs (M. N. Joshi, to be published). The number-magnitude histogram for these QSOs peaks well above the sensitivity

Table I

Median values of z, S and θ for identified 3CR radio galaxies

Flux density at	Total	Number of radio galaxies identified 2 measured			Median values $S_{\rm m}$ (Jy)	θ _m (" arc)
408 MHz (Jy)	number of sources			$z_{ m m}$		
3-10	129	48	16	0.11	6	66
> 10	70	47	34	0.06	14	150

limits of the Sky Survey prints indicating that most QSOs have been identified. Further, no new types of optical objects have been found for these weak radio sources as compared to the types of optical objects identified with 3C sources. By comparing with the median S_m , θ_m and z_m for the 3CR sources, it seems reasonable to assume therefore that a majority of the unidentified radio sources with flux densities of ~ 1 Jy and $\theta_m \sim 10''$ are not nearby objects, associated with some new class of optically faint galaxies, but are distant objects with $z \ge 0.5$ for which associated galaxies are fainter than the sensitivity limits of the Sky Survey prints. Indeed we find that the angular sizes of the unidentified radio sources are appreciably smaller than those of the identified radio galaxies, suggesting that the former are located farther away.

It is also interesting to note that the 3C QSOs have a median value of θ of about 8" (see Fig. 1), which is close to the value of $\theta_{\rm m}$ for all OTL sources indicating that both are located at larger distances than the other 3C sources, on the assumption that QSOs as a class have physical sizes similar to those of the radio galaxies. The median value of θ for the 16 QSOs in the OTL lists is only 4", again revealing a distance effect.

As shown by Kapahi (1975a), a combination of the angular size-flux density relation with angular size counts for the 3CR sample is not consistent with the steady-state theory, and indicates that the density and/or average luminosity of radio sources were higher and the linear sizes smaller at the earlier epochs than at present.

5. CONCLUSIONS

From a comparison of the largest angular sizes of 163 sources observed at Ootacamund with those of 199 sources from the 3CR catalogue and 62 sources from the All-sky catalogue, we conclude that:

- (i) The median values of angular sizes are well correlated with flux densities over a range of 0·3–100 Jy at 408 MHz and are related approximately as $\theta_{\rm m} \propto S^{1/2}$. The same result is found if the identified QSOs are excluded. The slope of the $\theta_{\rm m}(S)$ curve for the occultation sources is flatter than for the 3CR and All-sky sources, and there is an indication that $\theta_{\rm m}$ approaches asymptotically a value of $\sim 6-8''$ as the flux density decreases. In view of its value for cosmological investigations, it is important to determine this relation accurately for weaker sources. Also measurements of sizes of radio sources with $S_{408} \sim 5-15$ Jy located in the southern sky would reduce the uncertainties at the higher flux density levels.
- (ii) The median value of angular sizes for identified QSOs in the 3C list is about 8", which is the same as that for the weaker OTL radio sources.
- (iii) Generally, the sizes of unidentified radio sources and QSOs are appreciably smaller than those of the identified radio galaxies.
- (iv) The observed relation between $\theta_{\rm m}$ and S indicates that the sources of smaller flux densities are located statistically at larger distances.

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REFERENCES

Bash, F. N., 1968. Astrophys. J. Suppl. Ser., 16, 373.

Bennett, A. S., 1962. Mem. R. astr. Soc., 68, 163.

Branson, N. J. B. A., Elsmore, B., Pooley, G. G. & Ryle, M., 1972. Mon. Not. R. astr. Soc., 156, 377.

Bridle, A. H., Davis, M. M., Fomalont, E. B. & Lequeux, J., 1972. Astr. J., 77, 405.

Burbidge, G. R., 1973. Nature Phys. Sci., 246, 17.

Clark, B. G. & Hogg, D. E., 1966. Astrophys. J., 145, 21.

Colla, G., Fanti, C., Fanti, R., Ficarra, A., Formiggini, L., Gandolfi, E., Lari, C., Marano, B., Padrielli, L. & Tomasi, P., 1972. Astr. Astrophys. Suppl., 7, 1.

Cooper, B. F. C., Price, R. M. & Cole, D. J., 1965. Aust. J. Phys., 18, 589.

Ekers, J. A., ed., 1969. Austr. J. Phys. Astrophys. Suppl., No. 7.

Ekers, R. D., 1969. Austr. J. Phys. Astrophys. Suppl., No. 6.

Elsmore, B. & Mackay, C. D., 1970. Mon. Not. R. astr. Soc., 146, 361.

Fomalont, E. B., 1968. Astrophys. J. Suppl. Ser., 15, 203.

Fomalont, E. B. & Moffet, A. T., 1971. Astr. J., 76, 5.

Hogbom, J. A. & Carlsson, I., 1974. Astr. Astrophys., 34, 341.

Hoyle, F., 1959. Paris Symposium on radio astronomy, p. 529, ed. R. N. Bracewell, Stanford University Press, Stanford.

Hoyle, F. & Burbidge, G. R., 1970. Nature, 227, 359.

Joshi, M. N., Kapahi, V. K., Gopal-Krishna, Sarma, N. V. G. & Swarup, G., 1973. Astr. J., 78, 1023.

Kapahi, V. K., 1975a. Mon. Not. R. astr. Soc., 172, 513.

Kapahi, V. K., 1975b. PhD Thesis, Tata Institute of Fundamental Research, Bombay.

Kapahi, V. K., Joshi, M. N. & Gopal-Krishna, 1972. Astrophys. Lett., 11, 155.

Kapahi, V. K., Joshi, M. N. & Kandaswamy, J., 1973a. Astrophys. Lett., 14, 31.

Kapahi, V. K., Joshi, M. N., Subrahmanya, C. R. & Gopal-Krishna, 1973b. Astr. J., 78, 673.

Kapahi, V. K., Joshi, M. N. & Sarma, N. V. G., 1974. Astr. J., 79, 515.

Katgert, J. K. & Spinrad, H., 1974. Astr. Astrophys., 35, 393.

Kellermann, K. I., Pauliny-Toth, I. I. K. & Williams, P. J. S., 1969. Astrophys. J., 157, 1.

Kristian, J., Sandage, A. & Katem, B., 1974. Astrophys. J., 191, 43.

Lang, K. R., 1971. IAU Highlights of Astronomy, 2, 626, Reidel Publishing Co.

Legg, T. H., 1970. Nature, 226, 65.

Longair, M. S. & Gunn, J. E., 1975. Mon. Not. R. astr. Soc., 170, 121.

Longair, M. S. & Pooley, G. G., 1969. Mon. Not. R. astr. Soc., 145, 121.

Macdonald, G. H., Kenderdine, S. & Neville, A. C., 1968. Mon. Not. R. astr. Soc., 138, 259.

Mackay, C. D., 1969. Mon. Not. R. astr. Soc., 145, 31.

Mackay, C. D., 1971. Mon. Not. R. astr. Soc., 154, 209.

Miley, G. K., 1968. Nature, 218, 933.

Miley, G. K., 1971. Mon. Not. R. astr. Soc., 152, 477.

Mitton, S., 1970. Mon. Not. R. astr. Soc., 149, 101.

Robertson, J. G., 1973. Austr. J. Phys., 26, 403.

Roger, R. S., Bridle, A. H. & Costain, C. H., 1973. Astr. J., 78, 1030.

Ryle, M. & Windram, M. D., 1968. Mon. Not. R. astr. Soc., 138, 1.

Sandage, A. R., 1961. Astrophys. J., 133, 355.

Schwartz, U. J., Cole, D. J. & Morris, D., 1973. Austr. J. Phys., 26, 661.

Swarup, G., Sarma, N. V. G., Joshi, M. N., Kapahi, V. K., Bagri, D. S., Damle, S. H., Ananthakrishnan, S., Balasubramanian, V., Bhave, S. S. & Sinha, R. P., 1971a. *Nature*, *Phys. Sci.*, 230, 185.

Swarup, G., Kapahi, V. K., Sarma, N. V. G., Gopal-Krishna, Joshi, M. N. & Rao, A. P., 1971b. Astrophys. Lett., 9, 53.

Véron, M. P. & Véron, P., 1974. Astr. Astrophys. Suppl., 18, 309.

Wall, J. V. & Cole, D. J., 1973. Austr. J. Phys., 26, 881.

Wardle, J. F. C. & Miley, G. K., 1974. Astr. Astrophys., 30, 305.

Wilkinson, P. N., 1972. Mon. Not. R. astr. Soc., 160, 305.

Wilkinson, P. N., Richards, P. J. & Bowden, T. N., 1974. Mon. Not. R. astr. Soc., 168, 515.

- Willis, A. G., Strom, R. G. & Wilson, A. S., 1974. Nature, 250, 625. Wills, B. V. & Wills, D., 1974. Astrophys. J., 190, L97.
- Wraith, P. K., 1972. Mon. Not. R. astr. Soc., 160, 283.
- Wyllie, D. V., 1969. Mon. Not. R. astr. Soc., 142, 229. Yule, U. G. & Kendall, M. G., 1950. An introduction to the theory of statistics, p. 425, Charles Griffin & Co. Ltd, London.