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Abstract

In this paper, we have considered spatially homogeneous and anisotropic Bianchi type-III space-time filled with matter and anisotropic modified holographic Ricci dark energy in general relativity. We have solved the Einstein's field equations using the following possibilities: (i) hybrid expansion law proposed by Akarsu et al. (JCAP, 01, 022, 2014) (ii) a varying deceleration parameter considered by Mishra et al. (Int. J. Theor. Phys. 52, 2546, 2013) and (iii) a linearly varying deceleration parameter given by Akarsu and Dereli (Int. J. Theor. Phys. 51, 612, 2012). We have presented the cosmological models in each of the above cases and studied their evolutions. We have also discussed physical and kinematical properties of the models.

Key words: Bianchi type-III metric, Ricci dark energy, cosmology, general relativity, holographic dark energy.

1   Introduction

The scenario of modern cosmology is the accelerated expansion of the universe confirmed by recent observational studies ([1,2]). It is believed that the reason for the accelerated expansion of the universe could be 'dark energy'(DE) which is still a cosmological mystery. The concept of DE refers to a kind of exotic energy with negative pressure which has been proposed to explain the current accelerated expansion of the universe. The study of DE is possible through its equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ where ρ is the energy density and p is the pressure of DE. The current value of EoS parameter is not yet known. Hence many candidates have been proposed for DE. The most talked about agents driving this cosmic acceleration are supposed to be the cosmological constant (Λ), quintessence matter, the interacting DE models including Chaplygin gas ([3]) and holographic DE models ([4]). Padmanabhan [5] and Copeland et al. [6] have presented a comprehensive review of DE and DE models.

It is well known that the holographic principle ([7]) plays an important role in the black hole and string theory, which is based on the fact that in quantum gravity, the entropy of a system scales not with its volume, but with its surface area L^2. Inspired by the holographic principle, Cohen et al. [8] suggested that the vacuum energy density is proportional to the Hubble scale

$l_H \approx H^{-1}$. In this model, both the fine-tuning and coincidence problems can be alleviated, but it can not explain the cosmic accelerated expansion because that the effective equation of state for such vacuum energy is zero. Recently, Li [9] proposed that the future event horizon of the universe to be used as the characteristic length l. This holographic DE model not only presents a reasonable value for DE density, but also leads to a solution for the cosmic accelerated expansion. In fact, the choice of the characteristic length l is not unique for the holographic DE model. Gao et al. [10] assumed that the length l is given by the the inverse of Ricci scalar curvature, i.e.,$|R|^{\frac{-1}{2}}$, which is so called holographic Ricci DE model. It is argued that this model can solve the coincidence problem entirely. Thus, the properties of such holographic Ricci DE have been investigated widely ([11]; [12]). Granda et al. [13] proposed a modified Ricci DE model in which the density of DE is a function of the Hubble parameter H and its derivative with respect to time ($\dot{H}$). Chen and Jing [14] have presented a generalized DE model in which density of DE contains the second order derivative of Hubble's parameter with respect to time ($\ddot{H}$) and find that the age problem of the old objects above can be solved. The expression for energy density of this modified holographic Ricci DE (MHRDE) is defined by Chen and Jing [14] as

$$\rho_\Lambda = 3M_p^2(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}), \tag{1}$$

where M_p^2 is the reduced Planck mass, β_1, β_2 and β_3 are three arbitrary dimensionless parameters. Recently, Sarkar and Mahanta [15], Sarkar [16], Adhav et al. [17], Kiran et al. [18] have investigated minimally interacting and interacting holographic DE Bianchi models in general relativity and scalar-tensor theories of gravitation. Santhi et al. [19] have studied LRS Bianchi type-I generalized ghost pilgrim DE model in general relativity. Das and Sultana [20, 21] and Santhi et al. [22, 23] have studied some Bianchi type anisotropic MHRDE cosmological models in general relativity and scalar tensor theories.

Primordial magnetic fields can have a significant impact on the CMB anisotropy depending on the direction of field lines ([24,25]). Many people have investigated the influence of magnetic field on the dynamics of universe by analyzing anisotropic Bianchi models. Milaneschi and Fabbri [26] studied the anisotropy and polarization properties of CMB radiation in homogeneous Bianchi I cosmological model. Sharif and Zubair [27, 28] have investigated dynamics of Bianchi type universes with magnetized anisotropic DE. In spite of the fact that the present day universe is homogeneous and isotropic and is better described by Friedman-Robertson-Walker (FRW) model, it is said that Bianchi models are useful to study the anisotropies present in the early stages of evolution of the universe.

In this paper, we would like to investigate the dynamics of anisotropic Bianchi type-III model in the presence of anisotropic MHRDE and magnetic field. The paper has the following format: In section 2, we present anisotropic Bianchi type-III models and derive the dynamical

field equations which describe the evolution of the universe. In section 3, we obtain solutions to the field equations and discuss the physical properties of the models. Finally, in section 4, we summarize the results.

2   Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-III space-time described by the line element

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2x}dy^2 - C^2(t)dz^2, \tag{2}$$

where A, B and C are functions of cosmic time t only.

We assume that the universe is filled with matter and magnetized anisotropic MHRDE fluid. Here we assume that the current is flowing along x-axis so magnetic field is in the yz-plane. King and Coles [25], Jacobs [29], Sharif and Zubair [27] used the magnetized perfect fluid energy momentum tensor to discuss the effects of magnetic field on the evolution of the universe.

Here we take a more general energy-momentum tensors for matter and the magnetized anisotropic DE fluid in the following form

$$\begin{aligned}
T_{ij} &= diag[\rho_m, 0, 0, 0] \\
\overline{T}_{ij} &= diag[\rho_\Lambda + \rho_B, -\omega_\Lambda \rho_\Lambda + \rho_B, -(\omega_\Lambda + \delta_y)\rho_\Lambda - \rho_B, -(\omega_\Lambda + \delta_z)\rho_\Lambda - \rho_B]
\end{aligned} \tag{3}$$

where $\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$ is equation of state (EoS) parameter of DE, ρ_m is the energy density of the matter, p_Λ and ρ_Λ are pressure and energy density of DE respectively, ρ_B stands for energy density of magnetic field, which can be obtained from Maxwell's equation (i.e., $(F^{ij}\sqrt{-g})_{;j} = 0$). F^{ij} is the electromagnetic field tensor, δ_y and δ_z are deviations from ω_Λ on y and z directions respectively.

The Einstein's field equations are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -\frac{8\pi G}{c^4}(T_{ij} + \overline{T}_{ij}) \tag{4}$$

where R_{ij} is Ricci tensor, R is the Ricci scalar, T_{ij} is energy-momentum tensor of matter and $\overline{T}_{ij}$ is the energy-momentum tensor of magnetized anisotropic MHRDE fluid. Here we choose $8\pi G = c = 1$ (in relativistic units).

The Einstein's field equations (4) for the metric (2) with the help of (3) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\omega_\Lambda \rho_\Lambda + \rho_B \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -(\omega_\Lambda + \delta_y)\rho_\Lambda - \rho_B \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -(\omega_\Lambda + \delta_z)\rho_\Lambda - \rho_B \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = \rho_m + \rho_\Lambda + \rho_B \tag{8}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{9}$$

The energy-momentum conservation equation $(T_{ij} + \overline{T}^{ij})_{;j} = 0$, leads to two equations for the anisotropic DE and magnetic field ([25]) as

$$\dot{\rho_m} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\rho_m + \dot{\rho_\Lambda} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)(1 + \omega_\Lambda)\rho_\Lambda + \left(\delta_y\frac{\dot{B}}{B} + \delta_z\frac{\dot{C}}{C}\right)\rho_\Lambda = 0 \tag{10}$$

$$\rho_B = \frac{I}{B^2 C^2} \tag{11}$$

where overhead dot stands for ordinary differentiation with respect to t and I is constant.

3 Solutions of the field equations

Integrating equation (9) and absorbing the constant of integration into B, we obtain

$$A = B \tag{12}$$

Using equation (12), the field equations (5)-(10) form a system of four independent equations (since the equation (10) is a consequence of field equations) with seven unknown parameters A, C, ρ_m, δ_y, δ_z, ω_Λ and ρ_Λ. Hence, three additional conditions relating these parameters are required to obtain explicit solution of the system. We use the following conditions which are physically significant:

(i) We assume that shear scalar of the model is proportional to expansion scalar, which leads to ([30])

$$A = C^k. \tag{13}$$

(ii) We consider the MHRDE energy density given by equation (1) in Einstein's theory as

$$\rho_\Lambda = 3(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}), \tag{14}$$

where $H = \frac{1}{3}\left(\frac{2\dot{A}}{A} + \frac{\dot{C}}{C}\right)$ is the mean Hubble's parameter of the model.

3.1   Model-1

We consider the average scale factor is an increasing function of time ([31]; [32]) as follows:

$$a(t) = a_1 t^{\alpha_1} e^{\alpha_2 t} \tag{15}$$

where $a_1 > 0$, $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$ are constants. They referred this generalized form of scale factor to as the hybrid expansion law, being the mixture of power-law and exponential law cosmologies. We observe that (15) leads to the power law cosmology for $\alpha_2 = 0$ and to the exponential law cosmology for $\alpha_1 = 0$. Thus, the power-law and exponential law cosmologies are the special cases of hybrid expansion law cosmology. In hybrid cosmology, the universe exhibits transition from deceleration to acceleration. Recently, Santhi et al. [33] have discussed some Bianchi type generalized ghost pilgrim DE models in general relativity using hybrid expansion law.

Now using equations (12) and (13) in equation (15) we obtain the expressions for metric potentials as

$$\begin{aligned} A = B &= (a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{3k}{2k+1}} \\ C &= (a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{3}{2k+1}} \end{aligned} \tag{16}$$

Now the metric (2) can be written as

$$ds^2 = dt^2 - (a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6k}{2k+1}}(dx^2 + e^{-2x}dy^2) - (a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{2k+1}}dz^2 \tag{17}$$

Physical discussion of the model

The following physical and kinematical parameters which are very important for physical discussion of the model. From equations (14) and (16), we get the energy density of MHRDE as

$$\rho_\Lambda = 3\left[\beta_1\left(\frac{\alpha_1}{t} + \alpha_2\right) - \frac{\beta_2\alpha_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\alpha_1 + \alpha_2 t)}\right]. \tag{18}$$

From the equations (11) and (16), we have the energy density of magnetic field as

$$\rho_B = \frac{I}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}} \tag{19}$$

Now from field equations (5)-(8), (11), (16), (18) and (19) we get
the EoS parameter of MHRDE as

$$\omega_\Lambda = -\left\{ \frac{\frac{9(k^2+k+1)}{2k+1}\left(\frac{\alpha_1}{t}+\alpha_2\right)^2 - \frac{3(k+1)\alpha_1}{(2k+1)t^2} - \frac{I}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}}}{3\left[\beta_1\left(\frac{\alpha_1}{t}+\alpha_2\right) - \frac{\beta_2\alpha_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\alpha_1+\alpha_2 t)}\right]} \right\}, \tag{20}$$

the deviations from EoS parameter as

$$\delta_y = \frac{2I}{3(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}\left[\frac{\beta_2\alpha_1}{t^2} - \beta_1\left(\frac{\alpha_1}{t}+\alpha_2\right) - \frac{2\alpha_1\beta_3}{3t^2(\alpha_1+\alpha_2 t)}\right]} \tag{21}$$

$$\delta_z = \frac{(1-k)\left[\frac{9}{2k+1}\left(\frac{\alpha_1}{t}+\alpha_2\right)^2 - \frac{3\alpha_1}{(2k+1)t^2}\right] - \frac{2I}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}} + \frac{1}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6k}{2k+1}}}}{3\left[\beta_1\left(\frac{\alpha_1}{t}+\alpha_2\right) - \frac{\beta_2\alpha_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\alpha_1+\alpha_2 t)}\right]}, \tag{22}$$

and the energy density of matter can be found as

$$\rho_m = \frac{9k(k+2)}{(2k+1)^2}\left(\frac{\alpha_1}{t}+\alpha_2\right)^2 - \frac{I+(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{2k+1}}}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}} - 3\left[\beta_1\left(\frac{\alpha_1}{t}+\alpha_2\right) - \frac{\beta_2\alpha_1}{t^2} + \frac{2\alpha_1\beta_3}{3t^2(\alpha_1+\alpha_2 t)}\right]. \tag{23}$$

Thus the metric (17) together with equations (18)-(23) constitutes Bianchi type-III MHRDE
cosmological model in general relativity with hybrid expansion law.

Spatial volume of the model-1 as

$$V = ABC = (a_1 t^{\alpha_1} e^{\alpha_2 t})^3 \tag{24}$$

The Hubble's parameter

$$\begin{aligned} H &= \frac{\dot{a}}{a} \\ &= \frac{\alpha_1}{t} + \alpha_2 \end{aligned} \tag{25}$$

The scalar expansion is

$$\begin{aligned} \theta &= 3H \\ &= 3\left(\frac{\alpha_1}{t} + \alpha_2\right) \end{aligned} \tag{26}$$

The shear scalar is

$$\begin{aligned} \sigma^2 &= \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)^2 \\ &= \frac{9(k-1)^2}{2(2k+1)^2}\left(\frac{\alpha_1}{t} + \alpha_2\right)^2 \end{aligned} \tag{27}$$

The deceleration parameter is

$$
\begin{aligned}
q &= \frac{-a\ddot{a}}{\dot{a}^2} \\
&= -1 + \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2}
\end{aligned}
\tag{28}
$$

The anisotropic parameter is

$$
\begin{aligned}
A_h &= \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \\
&= \frac{6(k-1)^2}{(2k+1)^2}
\end{aligned}
\tag{29}
$$

where $H_1 = H_2 = \frac{\dot{A}}{A}$ and $H_3 = \frac{\dot{C}}{C}$ are directional Hubble's parameters.
The statefinder parameters are

$$
\begin{aligned}
r &= \frac{\dddot{a}}{aH^3} \\
&= 1 + \frac{2\alpha_1}{(\alpha_1 + t\alpha_2)^3} - \frac{3\alpha_1}{(\alpha_1 + t\alpha_2)^2}
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
s &= \frac{r-1}{3(q - \frac{1}{2})} \\
&= \frac{2\alpha_1[2 - 3(\alpha_1 + \alpha_2 t)]}{3(\alpha_1 + \alpha_2 t)(\alpha_1 - 3(\alpha_1 + \alpha_2 t)^2)}
\end{aligned}
\tag{31}
$$

We use squared speed of sound for the stability analysis of our model. It is given by

$$
v_s^2 = \frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda} = \frac{\frac{1}{(2k+1)}\left(\frac{\alpha_1}{t} + \alpha_2\right)\left[\frac{3\alpha_1(k^2+k+1)}{t^2} - \frac{I(k+1)}{(a_1 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6(k+1)}{2k+1}}}\right] - \frac{\alpha_1(k+1)}{(2k+1)t^3}}{\alpha_1\left[\frac{\beta_1}{2t^2} - \frac{\beta_2}{t^3} + \frac{\beta_3(2\alpha_1 + 3t\alpha_2)}{3(t^3(\alpha_1 + \alpha_2 t)^2)}\right]}
\tag{32}
$$

Fig. 1 represents the behavior of EoS parameter for MHRDE model. Which shows that the EoS parameter starts from high phantom values approaches to lower phantom region for the values of $\alpha_1 = 0.2, 1.0, 1.8$. However, the EoS parameter remains in the phantom region with time lapses. Hence, in these cases, we observe that the EoS parameter favors the pilgrim DE phenomenon for these three values of α_1.

From Fig. 2, we observe that the present universe with hybrid expansion law evolves with variable deceleration parameter and transition from deceleration to acceleration takes place at $t = \frac{\sqrt{\alpha_1} - \alpha_1}{\alpha_2}$. It is also clear from Fig. 2 that for $\alpha_1 \geq 1$, the model is evolving only in accelerating phase whereas for $\alpha_1 < 1$, the model is evolving from early decelerated phase to present accelerating phase.

The squared speed of sound remains positive for model-1 as shown in Fig. 3. It starts from negative values (represents instability of the model), goes towards positive maxima and
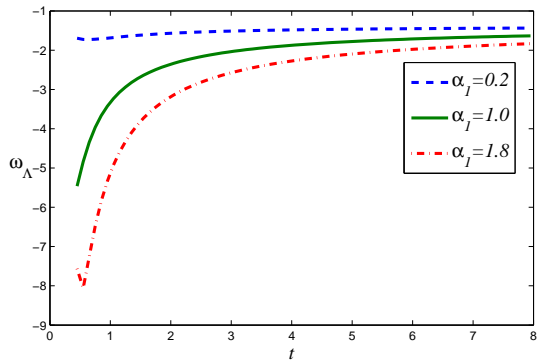
Figure 1: Plot of ω_Λ versus t (in Gyr) for $\alpha_2 = 0.7$, $k = 1.5$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-1.
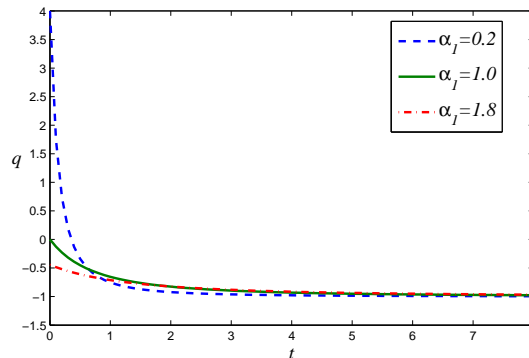

Figure 2: Plot of deceleration parameter (q) versus t (in Gyr) for $\alpha_2 = 0.7$ in model-1.
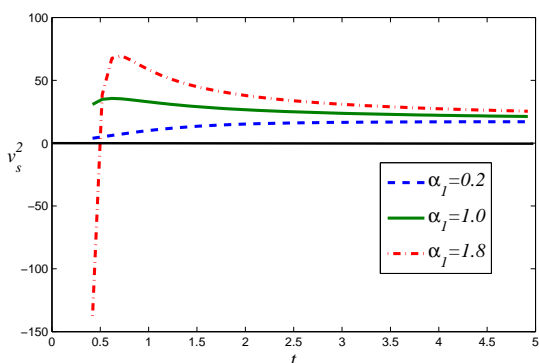

Figure 3: Plot of velocity of sound versus t (in Gyr) for $\alpha_2 = 0.7$, $k = 1.5$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-1.
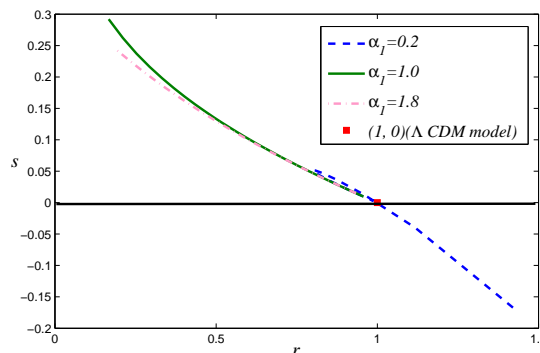

Figure 4: Plot of statefinder parameters (r, s) for $\alpha_2 = 0.7$ in model-1.

eventually decreases and approaches to positive value (exhibits stability of the model) for $\alpha_1 = 1.8$, whereas the model remains stable for $\alpha_1 = 0.2, 1.0$ throughout the evolution.

We can obtain the plane of statefinder parameters by plotting r versus s for three different choices of α_1 as shown in Fig. 4. It can be observed from the $r - s$ plane that MHRDE model correspond to ΛCDM $((r, s) = (1, 0))$ model for all three values of $\alpha_1 = 0.2, 1.0$ and 1.8. The $r - s$ plane corresponding to $\alpha_1 = 0.2$ also provide the regions of phantom $(s > 0)$ and quintessence $(r < 1)$ and Chaplygin gas model $(s < 0$ and $r > 1)$. For $\alpha_1 = 1.0$ and 1.8 the $r - s$ plane corresponding to phantom $(s > 0)$ and quintessence $(r < 1)$.

3.2 Model-2

We assume the physical variation of average scale factor ([34]) as

$$a(t) = [sinh(\alpha t)]^{\frac{1}{n}} \tag{33}$$

which yields a time dependent mean deceleration parameter (q). Pradhan [35] has studied two-fluid model from decelerating to accelerating FRW DE models using this average scale factor.

Now using equations (12) and (13) in equation (33) we obtain the expressions for metric potentials as

$$\begin{aligned} A = B &= [sinh(\alpha t)]^{\frac{3k}{n(2k+1)}} \\ C &= [sinh(\alpha t)]^{\frac{3}{n(2k+1)}} \end{aligned} \tag{34}$$

Now the metric (2) can be written as

$$ds^2 = dt^2 - [sinh(\alpha t)]^{\frac{6k}{n(2k+1)}}(dx^2 + e^{-2x}dy^2) - [sinh(\alpha t)]^{\frac{6}{n(2k+1)}}dz^2 \tag{35}$$

Physical discussion of the model

From equations (14) and (34), we get the energy density of MHRDE as

$$\rho_\Lambda = 3\alpha^2 \left[\frac{\beta_1}{n^2}coth^2\alpha t + \left(2\beta_3 - \frac{\beta_2}{n} \right) csch^2\alpha t \right] \tag{36}$$

From the equations (11) and (34), we have

$$\rho_B = \frac{I}{(sinh\alpha t)^{\frac{6(k+1)}{n(2k+1)}}} \tag{37}$$

Now from the field equations (5)-(8), (11), (34), (36) and (37), we get
the EoS parameter of MHRDE as

$$\omega_\Lambda = -\left\{ \frac{\frac{9\alpha^2(k^2+k+1)}{n^2(2k+1)^2}coth^2\alpha t - \frac{3\alpha^2(k+1)}{n(2k+1)}csch^2\alpha t - \frac{I}{(sinh\alpha t)^{\frac{6(k+1)}{n(2k+1)}}}}{3\alpha^2 \left[\frac{\beta_1}{n^2}coth^2\alpha t + \left(2\beta_3 - \frac{\beta_2}{n} \right) csch^2\alpha t \right]} \right\}, \tag{38}$$

the deviations from EoS parameter as

$$\delta_y = \frac{2I}{3\alpha^2(sinh\alpha t)^{\frac{6(k+1)}{n(2k+1)}} \left[\left(\frac{\beta_2}{n} - 2\beta_3 \right) csch^2\alpha t - \frac{\beta_1}{n^2}coth^2\alpha t \right]} \tag{39}$$

$$\delta_z = \frac{\frac{3\alpha^2(1-k)}{n(2k+1)} \left[\frac{3}{n}coth^2\alpha t - csch^2\alpha t \right] - \frac{2I}{(sinh\alpha t)^{\frac{6(k+1)}{n(2k+1)}}} + \frac{1}{(sinh\alpha t)^{\frac{6k}{n(2k+1)}}}}{3\alpha^2 \left[\frac{\beta_1}{n^2}coth^2\alpha t + \left(2\beta_3 - \frac{\beta_2}{n} \right) csch^2\alpha t \right]}, \tag{40}$$

and the energy density of matter can be written as

$$\rho_m = \frac{9\alpha^2 k(k+2)}{n^2(2k+1)^2}coth^2\alpha t - 3\alpha^2\left[\frac{\beta_1}{n^2}coth^2\alpha t + \left(2\beta_3 - \frac{\beta_2}{n}\right)csch^2\alpha t\right] - \frac{(I + (sinh\alpha t)^{\frac{6}{n(2k+1)}})}{(sinh\alpha t)^{\frac{6(k+1)}{n(2k+1)}}}.$$

(41)

Thus the metric (35) together with equations (36)-(41) constitutes Bianchi type-III MHRDE cosmological model in general relativity with time varying deceleration parameter.

Spatial volume of the model-2 as

$$V = (sinh\alpha t)^{3/n}$$

(42)

The Hubble's parameter

$$H = \frac{\alpha}{n}coth\alpha t$$

(43)

The scalar expansion is

$$\theta = \frac{3\alpha}{n}coth\alpha t$$

(44)

The shear scalar is

$$\sigma^2 = \frac{3\alpha^2(k-1)^2}{n^2(2k+1)^2}coth^2\alpha t$$

(45)

The anisotropic parameter is

$$A_h = \frac{2(k-1)^2}{(2k+1)^2}$$

(46)

The deceleration parameter is

$$q = -1 + n(sech^2\alpha t)$$

(47)

Statefinder parameters are

$$r = 1 + n(2n-3)sech^2\alpha t$$

(48)

$$s = \frac{2n(2n-3)sech^2\alpha t}{3(2nsech^2\alpha t - 3)}$$

(49)

Squared speed of sound for the stability analysis of the model-2 is given by

$$v_s^2 = \frac{1}{\left[\frac{\beta_1}{n^2} + 2\beta_3 - \frac{\beta_2}{n}\right]}\left\{\frac{3(k^2+k+1)}{n^2(2k+1)^2} + \frac{(k+1)csch\alpha t}{2n(2k+1)} + \frac{I(k+1)(sinh\alpha t)^{\frac{-6(k+1)}{n(2k+1)}}}{n(2k+1)\alpha^2 csch^2\alpha t}\right\}$$

(50)

Fig. 5 describes the behavior of EoS parameter of model-2 in terms of time t for $n = 0.5, 1.0$ and 1.5. In this model, we attain the quintessence ($\omega_\Lambda > -1$) and vacuum DE. The nature of
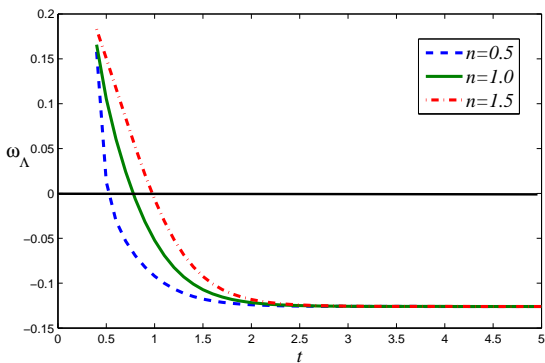
Figure 5: Plot of ω_Λ versus t (in Gyr) for $\alpha = 0.5$, $k = 1.5$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-2.
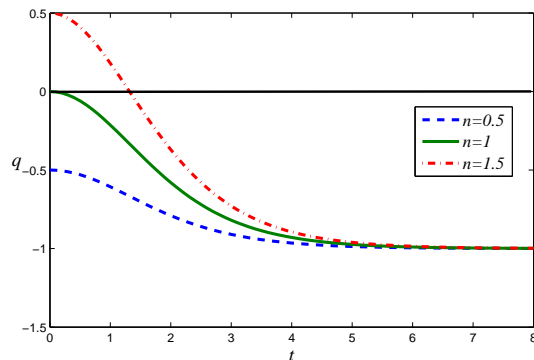

Figure 6: Plot of deceleration parameter (q) versus t (in Gyr) for $\alpha = 0.5$ in model-2.
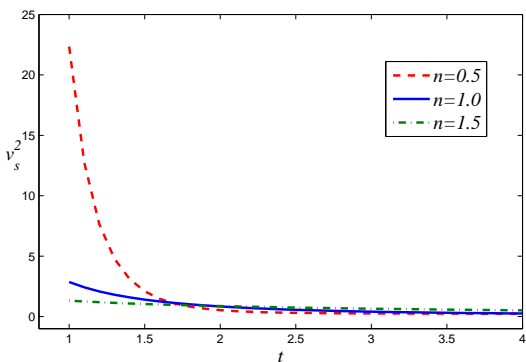

Figure 7: Plot of v_s^2 versus t (in Gyr) for $\alpha = 0.5$, $k = 1.5$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-2.
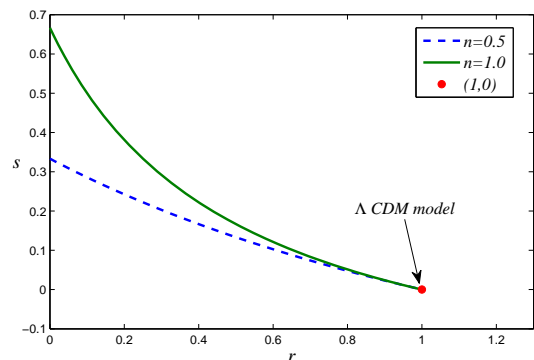

Figure 8: Plot of statefinder parameters (r, s) for $\alpha = 0.5$ in model-2.

deceleration parameter (q) versus time t is shown in Fig. 6. It is observed that for $n \leq 1$ the model is evolving only in accelerating phase and for $n > 1$ the model is evolving from early decelerated phase to present accelerating phase. Figure 7 shown the velocity of sound versus time t, it is observed that the model remains stable (i.e., $v_s^2 > 0$) for all values of n throughout the evolution of the universe. The $r - s$ plane plotted in Fig. 8. We observed that the $r - s$ plane corresponding to ΛCDM model for all three values of n. The model also represent the phantom $(s > 0)$ and quintessence $(r < 1)$ DE eras for $n = 0.5$ and 1.

3.3   Model-3

We assume that the linearly varying deceleration parameter proposed by Akarsu and Dereli [36] as

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -l_1 t + l_2 - 1 \tag{51}$$

where $l_1 \geq 0$, $l_2 \geq 0$ are constants and "a" is the average scale factor of the universe. For $l_1 = 0$, equation (51) reduces to the constant deceleration parameter ([37]). Assuming that the deceleration parameter is not a constant i.e. $l_1 \neq 0$ and solving equation (51) with a proper choice of integrating constants, we get

$$a(t) = exp\left\{ \frac{2}{l_2} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\} \quad for\ l_1 > 0; l_2 \geq 0 \tag{52}$$

Recently, Reddy et al. [38, 39] have investigated some anisotropic cosmological models with this linearly varying deceleration parameter in modified theories of gravitation.

Now using equations (12) and (13) in equation (52) we obtain the expressions for metric potentials as

$$
\begin{aligned}
A = B &= exp\left\{ \frac{6k}{m(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\} \\
C &= exp\left\{ \frac{6}{l_2(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\}
\end{aligned}
\tag{53}
$$

Now the metric (2) can be written as

$$
\begin{aligned}
ds^2 = {}& dt^2 - exp\left\{ \frac{12k}{l_2(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\}(dx^2 + e^{-2x}dy^2) \\
& - exp\left\{ \frac{12}{l_2(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\}dz^2
\end{aligned}
\tag{54}
$$

Physical discussion of the model

From equations (14) and (53), we get the energy density of MHRDE as

$$\rho_\Lambda = 6\left[\frac{2\beta_1 - 2\beta_2(l_1 t - l_2) - \beta_3(l_2^2 + (l_1 t - l_2)^2)}{t^2(2l_2 - l_1 t)^2} \right] \tag{55}$$

From the equations (11) and (34), we have

$$\rho_B = I\, exp\left\{ \frac{-12(k+1)}{l_2(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\} \tag{56}$$

Now from field equations (5)-(8), (11), (53), (55) and (56) we get
the EoS parameter as

$$\omega_\Lambda = \frac{\frac{-12((k+1)(l_1 t - l_2)+3)}{t^2(2k+1)(2l_2 - l_1 t)^2} + I\, exp\left\{ \frac{-12(k+1)}{l_2(2k+1)} arc\, tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\}}{6\left[\frac{2\beta_1 - 2\beta_2(l_1 t - l_2) - \beta_3(l_2^2 + (l_1 t - l_2)^2)}{t^2(2l_2 - l_1 t)^2} \right]}, \tag{57}$$

the deviations from EoS parameter as

$$\delta_y = \frac{I}{3exp\left\{\frac{12(k+1)}{l_2(2k+1)}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\}\left[\frac{2\beta_2(l_1t-l_2)-2\beta_1+\beta_3(l_2^2+(l_1t-l_2)^2)}{t^2(2l_2-l_1t)^2}\right]} \tag{58}$$

$$\delta_z = \frac{\frac{12(1-k)(l_1t-l_2+3)}{t^2(2k+1)(2l_2-l_1t)^2}-2I\ exp\left\{\frac{-12(k+1)}{l_2(2k+1)}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\}+exp\left\{\frac{-12k}{l_2(2k+1)}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\}}{6\left[\frac{2\beta_2(l_1t-l_2)-2\beta_1+\beta_3(l_2^2+(l_1t-l_2)^2)}{t^2(2l_2-l_1t)^2}\right]}, \tag{59}$$

and the energy density of matter can be found as

$$\begin{aligned}\rho_m &= \frac{36k(k+2)}{t^2(2k+1)^2(2l_2-l_1t)^2}-\frac{2I+exp\left\{\frac{12}{l_2(2k+1)}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\}}{exp\left\{\frac{12(k+1)}{l_2(2k+1)}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\}}\\ &\quad -6\left[\frac{2\beta_2(l_1t-l_2)-2\beta_1+\beta_3(l_2^2+(l_1t-l_2)^2)}{t^2(2l_2-l_1t)^2}\right].\end{aligned} \tag{60}$$

Thus the metric (54) together with equations (55)-(60) constitutes Bianchi type-III MHRDE cosmological model in general relativity with linearly varying deceleration parameter.

Spatial volume of the model-3 as

$$V = exp\left\{\frac{6}{l_2}arc\ tanh\left(\frac{l_1t}{l_2}-1\right)\right\} \tag{61}$$

The Hubble's parameter is

$$H = \frac{2}{t(2l_2-l_1t)} \tag{62}$$

The scalar expansion is

$$\theta = \frac{6}{t(2l_2-l_1t)} \tag{63}$$

The shear scalar is

$$\sigma^2 = \frac{12(k-1)^2}{t^2(2k+1)^2(2l_2-l_1t)^2} \tag{64}$$

The anisotropic parameter is

$$A_h = \frac{2(k-1)^2}{(2k+1)^2} \tag{65}$$

Statefinder parameters are

$$r = 1+\frac{1}{2}\left[6(l_1t-l_2)+3l_1t(l_1t-2l_2)+4l_2^2\right] \tag{66}$$

$$s = \frac{6(l_1t-l_2)+3l_1t(l_1t-2l_2)+4l_2^2}{3(-2l_1t+2l_2-3)} \tag{67}$$

Squared speed of sound of the model-3 is given by

$$
\begin{aligned}
v_s^2 &= \left\{ \frac{12}{2k+1} \left\{ l_1(k+1)(t^2(2l_2 - l_1t)^2) - 4t(2l_2 - l_1t)(l_2 - l_1t)((k+1)(l_1t - 3) + 3) \right.\right. \\
&\quad \left.\left. - \frac{12(k+1)l_2 I}{l_1 t(2k+1)(2l_2 - l_1)} exp\left\{ \frac{-12(k+1)}{l_2(2k+1)} arc\ tanh\left(\frac{l_1 t}{l_2} - 1 \right) \right\} \right\} \right\} \\
&\quad \left\{ 2\beta_2 l_1 - 2\beta_3 l_1(l_1 t - l_2)(t(2l_2 - l_1t))^2 - 4(2l_2 t - l_1 t^2)(l_2 - l_1 t) \right. \\
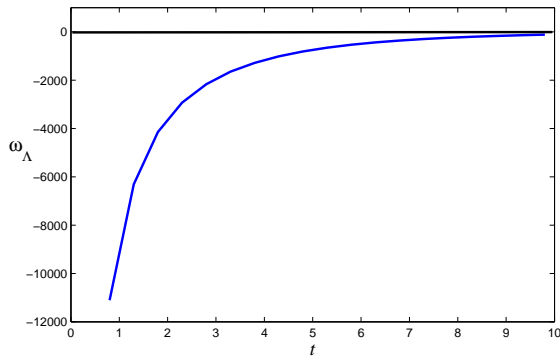&\quad \left. (2\beta_1 - 2\beta_2(l_1 t - l_2) - \beta_3(l_2^2 + (l_1 t - l_2)^2)) \right\}^{-1}. \quad (68)
\end{aligned}
$$

Figure 9: Plot of ω_Λ versus t (in Gyr) for $l_1 = 0.1$, $l_2 = 1.3$, $k = 1.5$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-3.
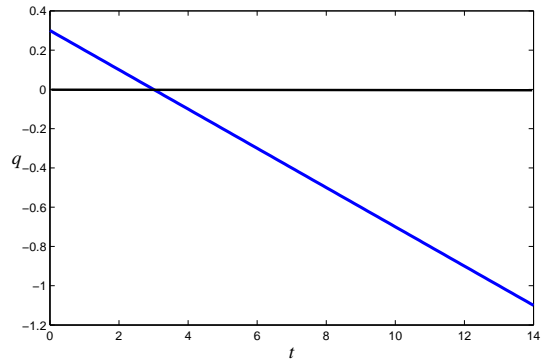

Figure 10: Plot of deceleration parameter (q) versus t (in Gyr) for $l_1 = 0.1$, $l_2 = 1.3$ in model-3.
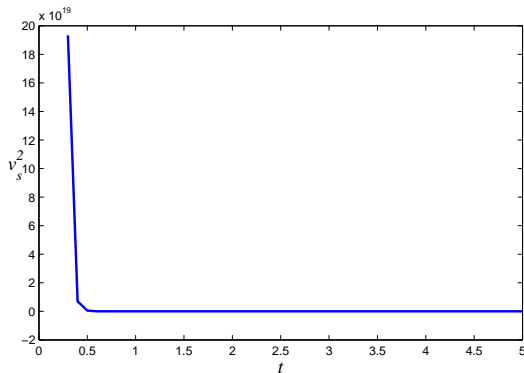

Figure 11: Plot of v_s^2 versus t (in Gyr) for $l_1 = 0.1$, $l_2 = 1.3$, $\beta_1 = 1.8$, $\beta_2 = 0.3$ and $\beta_3 = 0.8$ in model-3.
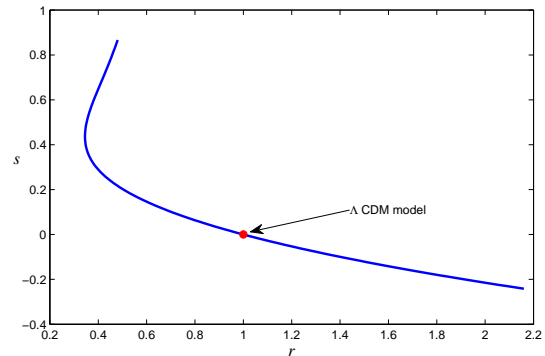

Figure 12: Plot of statefinder parameters (r, s) for $l_1 = 0.1$, $l_2 = 1.3$ in model-3.

Fig. 9 shows the behavior of EoS parameter with cosmic time t. It has been observed that the EoS parameter starts from high phantom values approaches to lower phantom region. However, the EoS parameter has remained in the phantom region with time lapses. We observe the EoS parameter favors the pilgrim DE phenomenon. The Fig. 10 shows the early deceleration and the present acceleration phase of the universe. Observational data (Ade et al. [40]) shows that the present value of deceleration parameter lies somewhere in the range $-1 \leq q < 0$. Therefore our model with linearly varying deceleration parameter consistent with the observational results. Figure 12 shows that at the cosmic time the statefinder parameters curve passes the point $[(r, s) = (1, 0)]$ which correspondence to the ΛCDM model. At this special point our MHRDE model with linearly varying deceleration parameter behaves like a ΛCDM model. The rs plane corresponding to linearly varying deceleration parameter also provide the regions of phantom $(s > 0)$, quintessence $(r < 1)$ DE models and Chaplygin gas models $(s < 0$ and $r > 1)$. It can be viewed from Fig. 11 that the MHRDE model exhibits stability in this scenario due to positive behavior of squared speed of sound.

4   Conclusions

We have obtained magnetized MHRDE Bianchi type-III cosmological models in general relativity. We have presented three different models by considering the possibilities: hybrid expansion law ([31]) of average scale factor and varying deceleration parameters ([34]; [36]) to obtain the models. The physical properties of the models are discussed and we observed the following:

The model-1 obtained by using hybrid expansion law provides very nice description of the transition from the early deceleration to present cosmic acceleration, which is an essential feature for evolution of the universe. We observe that the spatial volume of the model tends to zero at $t = 0$. Therefore, the model has point-type singularity at $t = 0$. At this epoch, all the physical and kinematical parameters diverse. As time increases, these parameters decrease. As $t \to \infty$, spatial volume becomes infinite. As $t \to \infty$, Hubble's parameter H is constant hence the universe expands forever with constant rate. The spatial volume of the models-2 and 3 vanish at $t = 0$ and increase with time. This shows that the universe starts evolving with zero volume at $t = 0$ and expand with cosmic time. The Hubble's parameter (H) and expansion scalar (θ) are infinite at $t = 0$ whereas, they are finite as $t \to \infty$, which indicates inflationary scenario. From Figs. 2, 6 and 10 we conclude that our models represent early decelerated phase to present accelerating phase. Recent observations of SNe Ia, expose that the present universe is accelerating and value of deceleration parameter lies in the range of $-1 \leq q < 0$. It follows that in our models, one can choose values of deceleration parameter consistent with recent observations.

The behavior of EoS parameter of the models-1 and 3 is shown in Figs. 1 and 9 respectively. We observe that the EoS parameter starts with comparatively high value of phantom and always remains in that region throughout the evolution of the universe. This behavior resembles the pilgrim DE, whereas, the behavior of EoS parameter of the model-2 shown in Fig. 5, is always varying in quintessence region. The contribution of magnetic field is exhibited in the expression of the physical parameters ω_Λ, ρ_m and skewness parameters. If $I = 0$, the effect of magnetic field is vanishes. In Figs. 3, 7 and 11, it is observed that MHRDE remains stable due to positive behavior of squared speed of sound. The $r - s$ planes corresponding to three models are shown in Figs. 4, 8 and 12. It is observed that the trajectories of $r - s$ plane for all three models corresponds to ΛCDM model. Also, the trajectories coincide with some well known DE models corresponding to phantom, quintessence and Chaplygin gas.

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