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ANISOTROPIC SPECTRAL MAGNITUDE ESTIMATION FILTERS FOR NOISE REDUCTION AND IMAGE ENHANCEMENT

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ABSTRACT

This paper describes an algorithm for noise reduction and enhancement of images which is able to take into account anisotropies of signal as well as of noise. Processing is based on subjecting each image to a block DFT, followed by comparing each observed magnitude coefficient to the expected noise standard deviation for it. Depending on this comparison, each coefficient is attenuated the more, the more likely it is that it contains only noise. In addition, the attenuation is made dependent on whether or not the observed coefficient contributes to an oriented prominent structure within the processed image block. Orientation as well as the distinctness with which it occurs are detected in the spectral domain by an inertia-like matrix. Orientation information is additionally exploited to selectively enhance oriented structures, thus only marginally increasing noise as compared to isotropic enhancement.

1. INTRODUCTION

Estimation of spectral amplitude (or magnitude) from noisy observations is a widely reported approach to the restoration of speech signals from noise [1, 2]. Based on a division of the observed noisy signal into overlapping short time intervals which are then subjected to the (Discrete) Fourier transform, the central idea is to attenuate the observed frequency coefficients G_k depending on their *instantaneous* signal-to-noise ratios (SNR) r_k^2, i.e.

$$\hat{F}_k = G_k \cdot h(r_k) \ , \quad \text{where } r_k^2 = |G_k|^2/\Phi_n(k) \ , \qquad (1)$$

with $\Phi_n(k)$ denoting an estimate of the noise power spectrum (NPS), and $\hat{F}_k$ the estimate for the noise-free spectral coefficient F_k. For noise reduction, the *attenuation function* $h(r)$ varies between zero and one, and increases monotonically with the square root of the instantaneous SNR r. The overall effect is attenuating spectral coefficients likely to represent mainly noise. The precise shape of $h(r)$ depends on noise and signal models, and the chosen objective function [3, 4].

In [5], one spectral amplitude estimation method — termed *power spectral subtraction* — was extended to two dimensions and applied to noisy real-world images, with the processing now based on overlapping image blocks. In our own experience [6], such spectral domain techniques compare very favourably with spatial domain filters when applied to noisy medical images acquired with low x-ray doses. In such images, noise is signal-dependent and exhibits a lowpass-shaped, potentially anisotropic power spectrum [6]. For known parameter settings of the imaging system, a noise model provides an estimate for the noise standard deviation for each coefficient, which, together with the observed coefficient's magnitude, determines the attenuation applied to the coefficient (Fig. 1). However, whereas there exists

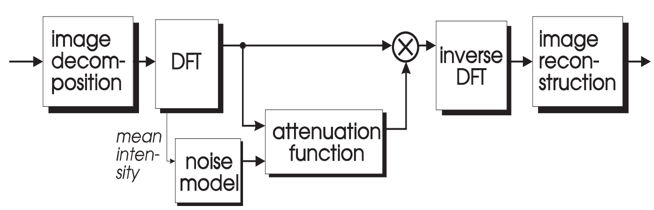

Fig. 1. Block diagram of noise reduction by spectral magnitude estimation.

a variety of anisotropic spatial-domain noise reduction filters which can adapt to potential local orientation in the image [7, 8], the described spatial frequency domain methods are isotropic in the sense that the same attenuation function is applied to all coefficients G_k regardless of their position relative to local orientation. To improve the performance of spectral amplitude estimation particularly with respect to perceptually important oriented patterns formed by lines and edges, and to allow selective enhancement of these patterns, this paper describes a new estimation algorithm which integrates local orientation. Specifically, our algorithm takes into account the prior knowledge $\mathcal{P}$ that each block *can* exhibit an arbitrary orientation, which in the spectral domain results in a concentration of energy along the line perpendicular to the spatial orientation and passing through the origin. The central idea is to apply less attenuation to coefficients along this line than to other ones, with this behaviour being the more pronounced, the more distinct the local orientation. We will bring the prior knowledge to bear within a Bayesian framework similar to the one described in [9] for classification. Rather than employing the maximum a posteriori criterion as in [9], we will base our *numerical* estimation problem on the minimum mean square error (MMSE) criterion.

2. ORIENTATION DETECTION

We evaluate local orientation in the spectral domain by means of an 2×2 *inertia* matrix J [10], the eigenvectors of which determine in a least squares sense the two axes along which concentration of energy is strongest (local orientation) and least, respectively. [1]. The eigenvalues along these lines measure how well this concentration is pronounced. For our algorithm, we have carried out three modifications:

- To increase robustness against noise, calculation of the inertia matrix is based on the signal-to-noise ratios r_k rather than on the observed coefficients G_k alone. Doing so has the additional advantage that orientation detection becomes adaptive to potential noise anisotropies.
- The inertia matrix is rotated and normalized such that its eigenvalues range between $+1$ (along local orientation axis) and -1 (along the axis perpendicular to local orientation). These values are reached for optimal concentration of energy along the local orientation axis.
- To weight all coefficients identically independent of distance from the origin, all coefficients are "projected" onto the unit circle, i.e. the "mass" or spectral energy is thought to be concentrated on the unit circle.

For a block of $N \times N$ coefficients, the non-normalized inertia matrix is given by

$$A = \begin{pmatrix} \sum\limits_{j,k} \frac{j^2}{j^2+k^2} r_{jk}^2 & \sum\limits_{j,k} \frac{jk}{j^2+k^2} r_{jk}^2 \\ \sum\limits_{j,k} \frac{jk}{j^2+k^2} r_{jk}^2 & \sum\limits_{j,k} \frac{k^2}{j^2+k^2} r_{jk}^2 \end{pmatrix} , \quad (2)$$

with $r_{jk}^2,\ j,k = -N/2+1, \ldots, N/2-1$, denoting the instantaneous SNR values. Note that, apart from the mechanical analogy, this matrix can also be interpreted probabilistically: Define a random variable (J, K) that selects a spectrum location with $P(J = j, K = k) \propto r_{jk}^2$. Then A is proportional to the covariance matrix of the corresponding random unit vectors $(J, K)/\sqrt{J^2 + K^2}$. Eigenvalue normalization is then carried out by

$$B = \frac{2}{\text{Trace}(A)} A - I = \begin{pmatrix} b_1 & b_2 \\ b_2 & -b_1 \end{pmatrix} . \quad (3)$$

The eigenvalues of this matrix are $d = \pm\sqrt{b_1^2 + b_2^2}$, with the corresponding eigenvectors pointing parallel and perpendicular to the dominant direction in the spectrum. If all non-zero coefficients are ideally concentrated along the orientation axis, we have $d = \pm 1$, whereas an ideally isotropic spectrum results in $d = 0$. In our aproach, the eigenvectors are not computed explicitly. Referring to Fig. 2, we rather evaluate at each location $v = (j, k)^T$ in the 2D spectral domain the expression

$$g(v) = \frac{v^T B v}{v^T v} \quad . \quad (4)$$

The value of $g(v)$ is the larger, the smaller the angular distance δ of v to the local orientation axis $\hat{v}$, which needs not to be known explicitly. The maximum $|d|$ of $g(v)$ is the larger, the more distinct the local orientation.

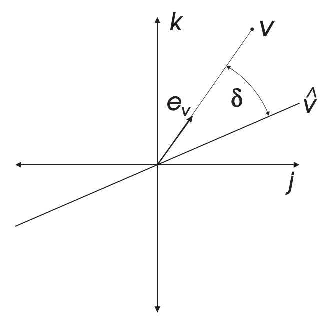

Fig. 2. 2-D spatial frequency domain for each block. $v = (j, k)$ is the coordinate of an observed coefficient G_v, and $\hat{v}$ denotes the local orientation axis. e_v is the unit vector pointing to v.

3. THE NOISE REDUCTION ALGORITHM

The prior knowledge $\mathcal{P}$ expressed through the modified inertia matrix B is brought to bear within an MMSE approach, which also exploits the energy compaction and decorrelation properties of the DFT. Modelling the coefficients as (complex) Gaussian distributed (cf. [4, 3]), they are also independent, and can be estimated individually by $\hat{F}_v = E[F_v|G_v, \mathcal{P}]$ (*marginal* MMSE estimation). Without orientation information, this leads to an attenuation function to be used in (1) given by [6]

$$h(r) = (1 + \lambda \exp(-\alpha r^2))^{-1} \quad , \quad (5)$$

[1] In [10], this approach to local orientation detection is transformed to the spatial domain by means of Parseval's theorem, leading to the same expression of local orientation as in [11], where it is shown that local orientation is the axis with greatest intensity variance.

with α being a weighting factor similar to the one used in generalized Wiener filters. The parameter λ can be decomposed into $\lambda = \lambda_0 \cdot \Pr(H_0(v))/[1 - \Pr(H_0(v))]$, with $\Pr(H_0(v))$ being the a priori probability of $G(v)$ to contain noise only. λ_0 depends on the signal and noise variances, but is regarded here as a free parameter balancing noise reduction and signal preservation. The orientation information $\mathcal{P}$ can now be taken into account by varying $\Pr(H_0(v))$ through $g(v)$ in (4). To selectively reduce attenuation along the orientation, we define a "selectivity" function

$$M(v) = \max[g(v), 0]^8/|d|^7 \quad , \tag{6}$$

which ranges in $[0, 1]$ and rises sharply in the vicinity of the dominant orientation, where the value 1 is reached only for ideally orientated patterns. Normalization by $|d|^7$ makes sure that the distinctness of orientation is still linearly dependent on $|d|$. Assuming that $\Pr(H_0(v))$ decreases sharply when approaching the orientation axis, one can model

$$\Pr(H_0(v))/[1 - \Pr(H_0(v))] = (1 - M(v)) \quad , \tag{7}$$

which, when integrated into λ in (5), results in the desired anisotropic attenuation function plotted in Fig. 3. Alternatively or additionally, one could assume a variable α as motivated by the "signal equivalent" approach [7, 8], e.g.

$$\alpha = \alpha(v) = \alpha_0/[1 - M(v)] \quad . \tag{8}$$

For $M(v) = 1$, both (7) and (8) prevent all attenuation along the (then ideal) orientation axis.

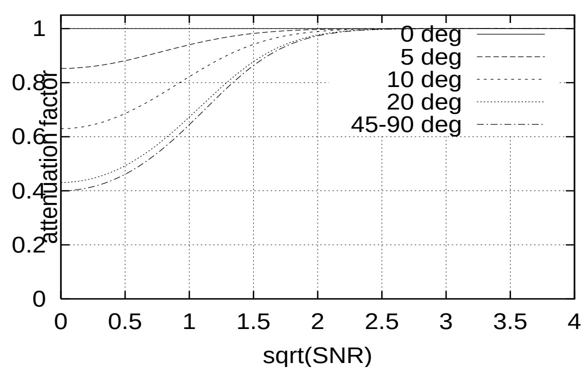

Fig. 3. A family of attenuation curves plotted versus the square root of the instantaneous SNR r. For a given SNR, attenuation increases with the angular distance between v and the orientation. Plot based on eqs. (5) and (7) for $\lambda_0 = 1.5$, $\alpha = 1$.

4. DIRECTIONAL ENHANCEMENT

Availability of orientation information also enables selective enhancement of oriented patterns. Generally, sharpness can be improved by band or high pass filtering. Isotropic filtering, however, results in a corresponding increase of noise. Restricting the enhancement to the dominant orientation, noise amplification can be kept moderate due to the limited number of coefficients involved and their relatively good SNR. Let $BP(|v|)$ denote a suitable isotropic band pass transfer function radially selecting those frequencies that are important for the appreciation of image sharpness, like

$$BP(|v|) = \sin^2\left(\sqrt{2} \cdot \pi \cdot \frac{|v|}{N}\right) \quad , \tag{9}$$

with $|v| = \sqrt{j^2 + k^2}$. Combining the radial function $BP(|v|)$ with the angular selectivity function $M(v)$ given in (6) by

$$BP_a(v) = 1 + M(v) \cdot BP(|v|) \tag{10}$$

leads to the anisotropic transfer function $BP_a(v)$ depicted in Fig. 4.

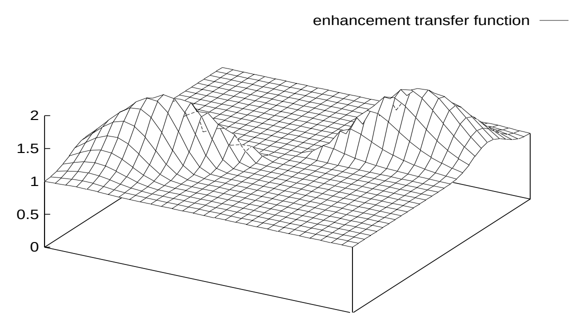

Fig. 4. Example of an anisotropic enhancement transfer function, plotted for ideal orientation, i.e. assuming a maximum value of one for $M(v)$.

5. RESULTS AND SUMMARY

Both anisotropic noise reduction and enhancement were applied to the low dose x-ray image depicted in Fig. 5. Intensity dependence and spectral distribution of the noise power in this image were known (cf. "noise model" in Fig. 1). As Fig. 6 shows, noise could be appreciably suppressed, while visually important oriented patterns are well preserved, most prominently the guide wire. Processing was based on a block size of 32×32 pixels, with an overlap of 16 pixels between adjacent blocks. As in [6], the FFT window was a separable 2D Hanning window. Orientation dependence was introduced into (5) by (7), with $\alpha = 5$, and $\lambda_0 = 1.5$.

In summary, we have developed a new anisotropic spectral magnitude estimation algorithm for restoration of noisy

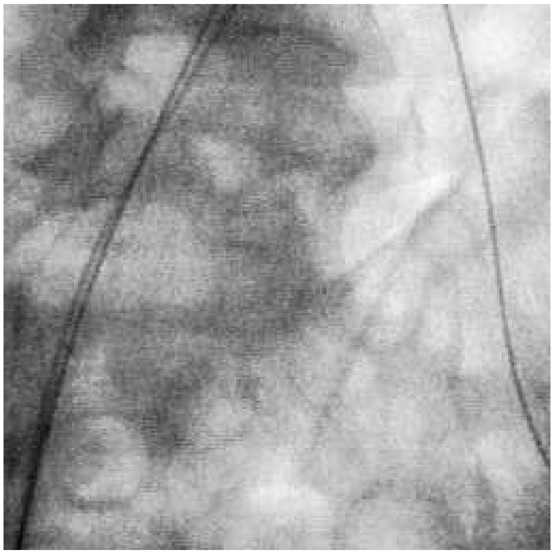

Fig. 5. Part of an original noisy medical low dose x-ray image, depicting a thin guidewire inserted into a patient's vascular system.

images by a combination and extension of the approaches of [5] and [10] based on an estimation-theoretic framework. Direction and strength of locally dominant orientation were quantified by an inertia-like matrix. Noise in images restored by this algorithm is considerably reduced, with perceptually important detail information well preserved. Additionally, enhancement of image sharpness without sacrificing image quality with respect to noise can be achieved by selectively emphasising coefficients along the orientation axis.

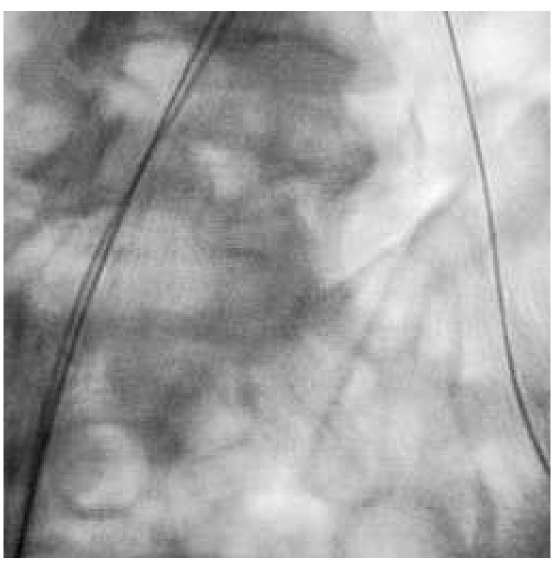

Fig. 6. Image of Fig. 5 processed by anisotropic noise reduction and enhancement.

6. REFERENCES

[1] J. S. Lim, A. V. Oppenheim, "Enhancement and bandwidth compression of noisy speech," *Proc. IEEE* 67 (12), 1586–1604, 1979.

[2] P. Vary, "Noise suppression by spectral magnitude estimation – mechanism and limits," *Signal Processing* 8 (4), 387–400, 1986.

[3] Y. Ephraim, D. Malah, "Speech enhancement using a minimum mean-square error short-time spectral amplitude estimator," *IEEE Trans. Acoust. Speech Sig. Proc.* 32 (6), 1109–1121, 1984.

[4] R. J. McAulay, M. L. Malpass, "Speech enhancement using a soft-decision noise suppression filter," *IEEE Trans. Acoust. Speech Sig. Proc.* 28 (2), 137–145, 1980.

[5] J. S. Lim, "Image restoration by short space spectral subtraction," *IEEE Trans. Acoust. Speech Sig. Proc.* 28 (2), 191–197, 1980.

[6] T. Aach, D. Kunz, "Spectral estimation filters for noise reduction in x-ray fluoroscopy imaging," *EUSIPCO-96*, Trieste, Sept. 10–13, 1996 (in print).

[7] J. F. Abramatic, L. M. Silverman, "Nonlinear restoration of noisy images," *IEEE Trans. Patt. Anal. Mach. Intell.* 4 (2), 141–149, 1982.

[8] H. E. Knutsson, R. Wilson, G. H. Granlund, "Anisotropic nonstationary image estimation and its applications: Part I – restoration of noisy images," *IEEE Trans. Comm.* 31 (3), 388–397, 1983.

[9] R. M. Haralick, "Decision making in context," *IEEE Trans. Patt. Anal. Mach. Intell*, vol. 5, no. 4, pp. 417–429, 1983.

[10] J. Bigün, G. H. Granlund, "Optimal orientation detection of linear symmetry," in *Proc. IEEE 1st Int'l Conf. Comput. Vis.*, London, 433–438, June 1987.

[11] M. Kass, A. Witkin, "Analyzing oriented patterns ," *Comput. Vis. Graph. Im. Proc.* 37, 362–385, 1987.