Annihilation of shocks in forced oscillations of an air column in a closed tube (L)

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Effects of a periodic array of Helmholtz resonators on forced longitudinal oscillations of an air column in a closed tube are examined experimentally. The column is driven sinusoidally at a frequency near the lowest resonance frequency by oscillating bellows mounted on one end of the tube. Frequency responses are obtained for small and large amplitudes of the excitations. While the array lowers the resonance frequency and the peak value, its dispersive effect, i.e., the dependence of the sound speed on a frequency, can annihilate the shock effectively. [DOI: 10.1121/1.1407265]

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A periodic array of Helmholtz resonators is very effective to annihilate a shock wave (called simply a shock hereafter) in propagation of nonlinear acoustic waves in an air-filled tube.[1–3] This is made possible by the action of wave dispersion that the array yields, whereby the sound speed in the tube tends to change depending on a frequency.[4] This letter examines whether or not the array can also annihilate a shock in forced longitudinal oscillations of an air column confined in a tube. While the theory is currently being developed, the experimental findings are reported.

Forced oscillations of the air column in a tube of uniform cross-section have been studied for several decades by many authors. An extensive review is given by Ilgamov *et al.*[5] The column is commonly driven sinusoidally by a plane piston reciprocating at a frequency close to or equal to the resonance frequency of the fundamental mode. While the amplitude of displacement of the piston is small, the linear standing wave is excited with the pressure node in the center of the tube and the loops at both ends. But as the amplitude becomes large, there emerges a shock propagating back and forth, reflected from the closed end and the piston.

When the shock appears, it suppresses the maximum excess pressure by so-called acoustic saturation and also gives rise, not only to loud noise and vibrations of the tube, but also to eventual heating of the air. To generate high-amplitude oscillations, it is therefore required to annihilate the shock. Recently new researches have begun. Lawrenson *et al.*[6] and Ilinskii *et al.*[7] have developed a novel method to achieve this by varying a tube's cross-section deliberately along the axial direction and vibrating the whole tube on the shaker. Although the driving method is different from the conventional one, their idea lies in avoidance of coincidence of the frequencies of higher harmonics of the excitation with the resonance frequencies of higher modes in the tube. When they coincide as in a uniform tube, called *consonant*,[6] the energy of the fundamental mode is easily pumped up into higher modes so that the shock is formed. By changing the cross-section, the tube is made *dissonant* and the shock may be avoided. It is emphasized that even in this dissonant tube, the system remains hyperbolic and the propagation speed given by the characteristics is not different from the sound speed. On the contrary, the connection of the array of resonators to the tube changes the hyperbolic system to a dispersive one where the phase (sound) speed depends on frequency. Thanks to this dispersion, the tube of uniform cross-section is rendered *dissonant* in consequence. But remark the difference of both mechanisms in annihilation of shocks.

The experimental setup is illustrated in Fig. 1. We employ a tube of inner diameter 80 mm, of thickness 7.5 mm, and of length 3200 mm. For the sake of comparison, we prepare another tube of the same size but without the array. Each resonator has a cavity of volume $V(=4.97\times10^{-5}\,\mathrm{m}^3)$ and a throat of length $L(=35.6\,\mathrm{mm})$ and of diameter $2r$ $(=7.11\,\mathrm{mm})$. Its lossless natural frequency $\omega_0/2\pi$ is given by $\sqrt{\pi r^2 a_0^2/L_e V}/2\pi$, where a_0 is the sound speed and L_e $(=L+2\times0.82r)$ is the throat's length with the end corrections. The resonators, 64 in total, are connected to the tube with axial spacing $d(=50\,\mathrm{mm})$ and are staggered on both sides of the tube. The parameter $\kappa(=V/Ad)$ measuring the size of the array takes the value 0.198 where A denotes the cross-sectional area of the tube.

With one end of the tube closed by a flat plate, the air column under the atmospheric pressure p_0 at room temperature is driven by bellows mounted on the other end. ''By a bellows'' is meant such a cylindrical tube with a flat bottom that its lateral surface is folded into conical surfaces with

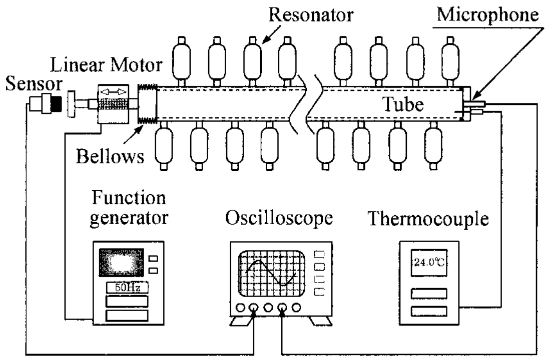

FIG. 1. Illustration of the experimental setup.

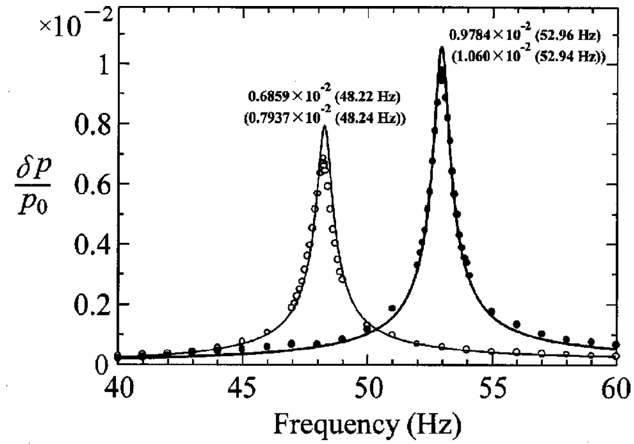

FIG. 2. Frequency responses of the fundamental mode in the tubes without the array of Helmholtz resonators and with it, respectively, for the amplitude of the displacement of the bellows 0.1 mm where the solid and blank circles indicate the experimental data measured, respectively, in the tubes without the array and with it, and the bold and thin curves represent, respectively, the amplitudes of the excess pressure Eq. (4) at the closed end; the peak values and the resonance frequencies are indicated with the theoretical values enclosed by the parentheses.

crests and troughs. Many folds of the lateral surface allow the bottom plate to move in the axial direction. The bellows is 56 mm deep in the natural state while the greatest and smallest inner diameter D_g and D_s are 110 mm and 80 mm, respectively. The bottom plate is driven at its center by a reciprocating linear motor located outside. With the depth of the bellows inclusive, the total length of the air column, l, is 3256 mm under atmospheric pressure. The displacement of the bottom plate of the bellows is measured optically by a sensor using a laser beam. Excess pressure is measured at the closed end by a microphone set flush with the flat plate at its center. The mean temperature in the tube near the closed end, T_0, is always monitored by a thermocouple.

The bellows has a merit of securing hermetic sealing whereas they have the demerit of nonuniformity in the inner diameter. To regard the displacement of the bottom plate of the bellows x_b (called simply the displacement of the bellows hereafter) as the one of the piston x_p, we make an assumption of an "equivalent volume displacement." Since a wavelength of oscillations is much longer than the depth of the bellows, the axial variations of air density in the bellows may be neglected. Then, the displacement of the bellows x_b is related to that of the piston x_p by $(1+\chi+\chi^2)x_b/3$, $\chi = D_g/D_s$.[8] For $\chi=110/80=1.375$, it follows that $x_p = 1.422x_b$.

We begin by describing the results when the amplitude of displacement of the bellows is small. In the lossless linear theory, the air column resonates at such an angular frequency of the driver ω that a half-wavelength is equal to the column's length. This gives the resonance frequency of the fundamental mode. Near resonance, however, small lossy effects come into play to shift the resonance frequency downward. Figure 2 shows the frequency response of the air column for the amplitude of displacement of the bellows fixed at 0.1 mm where the half of the peak-to-peak pressure, δp, is drawn relative to p_0 versus the driving frequency $\omega/2\pi$. Even at this level of excitation, the second harmonic appears but its magnitude is a few percent of the fundamental one at most. The solid circles indicate the data measured in the tube without the array under $T_0=24.9\,°\mathrm{C}$ and $p_0=1.007\times10^5$ Pa. The blank circles indicate the data in the tube with the array under $T_0=25.6\,°\mathrm{C}$ and $p_0=1.003\times10^5$ Pa. In the absence of the array, the frequency response has the peak value of 0.9784×10^{-2} at the resonance frequency 52.96 Hz. When the array is connected, the peak value and the frequency are lowered to 0.6859×10^{-2} and 48.22 Hz. Both the decrease in the peak value and the frequency result from the *lossless* effect of the array. The extra loss at the throats decreases the peak value. Its fraction is estimated to be merely 5.5%.

The frequency response of the second and third modes in the tube with the array is shown in Fig. 3 where the maximum acceleration, i.e., the amplitude of the displacement of the bellows times the frequency squared, is held to be a constant value at the amplitude 0.1 mm and 48.2 Hz. The blank circles indicate the experimental data. The second and third modes have, respectively, the peak values 0.2034×10^{-2} at 95.44 Hz and $T_0=25.4\,°\mathrm{C}$, and 0.070×10^{-2} at 139.5 Hz and 25.3 °C under $p_0=0.999\times10^5$ Pa. The ratio of the second resonance frequency to the fundamental one 48.2

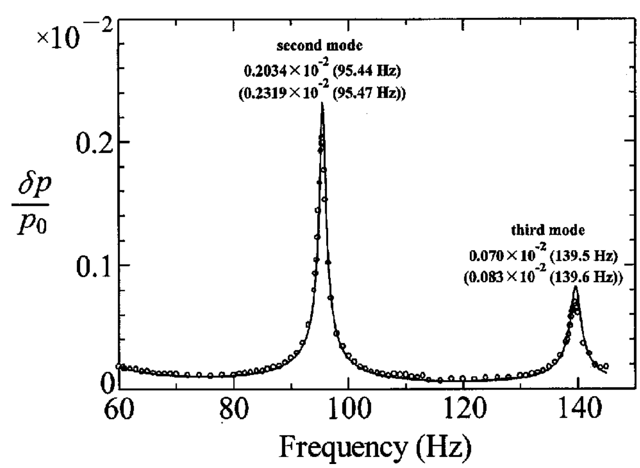

FIG. 3. Frequency response of the second and third modes in the tube with the array of Helmholtz resonators with the maximum acceleration of the bellows fixed to be the value at the amplitude 0.1 mm and 48.2 Hz where the circles represent the experimental data and the line indicates the amplitude of the excess pressure Eq. (4) at the closed end; the peak values and the resonance frequencies are indicated with the theoretical values enclosed by the parentheses.

Hz is 1.98 and very close to 2, while the ratio of the third is 2.90.

The above experimental results are explained by the linear theory, taking account of a boundary layer on the side wall of the tube. But the loss due to the diffusivity of sound is negligibly small. The oscillations are modeled by one-dimensional motions in the tube except for the boundary layer. Under the continuum approximation for the array, the excess pressure $p'(x,t)$ is governed by the following wave equation in the axial coordinate x and the time t [see Eq. (60) in Ref. 4]:

$$\frac{\partial^2 p'}{\partial t^2} - a_0^2 \frac{\partial^2 p'}{\partial x^2} + \frac{2Ca_0^2\sqrt{\nu}}{R^*}\frac{\partial^{-1/2}}{\partial t^{-1/2}}\left(\frac{\partial^2 p'}{\partial x^2}\right) = -\kappa\frac{\partial^2 p_c'}{\partial t^2}, \tag{1}$$

which is coupled with the equation for the excess pressure $p_c'(x,t)$ in the cavity [see Eq. (10) in Ref. 4]:

$$\frac{\partial^2 p_c'}{\partial t^2} + \frac{2\sqrt{\nu}}{r^*}\frac{\partial^{3/2} p_c'}{\partial t^{3/2}} + \omega_0^2 p_c' = \omega_0^2 p', \tag{2}$$

where $C = 1 + (\gamma - 1)/\sqrt{\mathrm{Pr}}$ and $c_L = (L + 2r)/(L + 2 \times 0.82r)$; γ and Pr denote, respectively, the ratio of specific heats and the Prandtl number; ν is the kinematic viscosity of air; R^* and r^* are the reduced radii of the tube and the throat, defined, respectively, by $R/(1 - r^2/2Rd)$ and r/c_L, R being the tube's radius; c_L is the factor to account for the end corrections to the throat's length.[1] Here the derivative of order $-1/2$ is defined by

$$\frac{\partial^{-1/2} p'}{\partial t^{-1/2}} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{t}\frac{1}{\sqrt{t-\tau}}p'(\tau,x)\,d\tau, \tag{3}$$

and the derivative of order 3/2 is defined by differentiating the one of order $-1/2$ twice with respect to t.

Suppose, for simplicity, a plane piston is driven sinusoidally at an angular frequency ω. Taking the origin of x at a mean position of the piston surface, let the displacement of the piston be $x_p = X_p \exp(-i\omega t)$ with a complex amplitude X_p, the real part being taken in the complex notation. The boundary conditions are imposed as $\partial p'/\partial x = i\rho_0\omega U_p \times \exp(-i\omega t)$ at $x = 0$ and $\partial p'/\partial x = 0$ at $x = l$, where ρ_0 is the mean air density and U_p is a complex amplitude of the velocity given by $-i\omega X_p$. Thus Eqs. (1) and (2) are solved as follows:

$$p' = i\frac{\rho_0\omega\cos[k(x-l)]}{k\sin kl}U_p\exp(-i\omega t), \tag{4}$$

and $p_c' = p'/\mathcal{D}_r$ with $\mathcal{D}_r = 1 - \omega^2/\omega_0^2 - (1+i)\sqrt{2\nu/\omega}(\omega/\omega_0)^2/r^*$, where a wave number k is determined by $k = (\omega/a_0)\sqrt{(1+\kappa/\mathcal{D}_r)/\mathcal{D}_R}$, with $\mathcal{D}_R = 1 - C(1+i)\sqrt{2\nu/\omega}/R^*$. The derivative of order $-1/2$ of $\exp(-i\omega t)$ is given by $[(1+i)/\sqrt{2\omega}]\exp(-i\omega t)[=(-i\omega)^{-1/2}\exp(-i\omega t)]$.

In the lossless limit $(\nu \to 0)$, k is reduced to $(\omega/a_0)\sqrt{1+\kappa/(1-\omega^2/\omega_0^2)}$. The resonance conditions $kl = m\pi$ $(m = 1,2,3,\ldots)$ for p' give a pair of frequencies $\omega_m^{\pm}$ for a fixed value of m as solutions to the following quadratic equation in ω^2: $\omega^4 - [(1+\kappa)\omega_0^2 + \omega_m^2]\omega^2 + \omega_0^2\omega_m^2 = 0$, where $\omega_m (= m\pi a_0/l)$ give the lossless resonance frequencies of the mth mode in the absence of the array. For a small value of κ, the pair of the frequencies are evaluated asymptotically as $\omega_m^{\pm} = \omega_l[1 \pm \kappa/2(1 - \omega_m^2/\omega_0^2) + O(\kappa^2)]$, with $\pm$ vertically ordered and $l = 0$ for the upper sign and $l = m$ for the lower one. Each $\omega_m^{\pm}$ gives the lossless resonance frequency. By connecting the array, ω_1^- becomes lower than ω_1 because $\omega_1 < \omega_0$. When the lossy effects are taken into account, ω_1^- is further shifted down slightly.

To compare the experimental data against the theory, accurate values of the density ρ_0, viscosity $\mu(=\rho_0\nu)$ and thermal conductivity k_T in $\mathrm{Pr}(=\mu c_p/k_T)$ are required, c_p being the specific heat. Using the mean temperature T_0, μ and k_T are calculated by Sutherland's formula and the one in Ref. 9 while ρ_0 is calculated from p_0 and T_0 by the equation of state for ideal gas with gas constant 2.870×10^2 J/kg K. The values of c_p and γ are taken constant as 1.007 kJ/kg K and 1.402, respectively, and a_0 is calculated by $331.5 + 0.61T_0$ m/s for T_0 measured in the degree Celsius where an effect of humidity (around 60%) is ignored.

In Fig. 2, the bold and thin curves represent the amplitude of p'/p_0 at $x = l$ calculated by Eq. (4) in the tubes without the array and with it, respectively. Here X_p is set equal to $1.422X_b$ where $|X_b|$, the amplitude of displacement of the bellows, is 0.1 mm. For the bold curve, we use $a_0 = 346.7$ m/s, $\rho_0 = 1.177$ kg/m^3, $\nu = 1.561 \times 10^{-5}$ m^2/s, and $\mathrm{Pr} = 0.7092$ at 24.9 °C while for the thin curve, $a_0 = 347.1$ m/s, $\rho_0 = 1.170$ kg/m^3, $\nu = 1.573 \times 10^{-5}$ m^2/s, and $\mathrm{Pr} = 0.7089$ at 25.6 °C so that $\omega_0/2\pi = 242.6$ Hz. The theory gives the peak value 1.060×10^{-2} at 52.94 Hz in the tube without the array, while 0.7937×10^{-2} at 48.24 Hz in the tube with the array. The experimental data agree well with the theory except for just on resonance where the maximum values measured are smaller in both cases. In Fig. 3, the curve represents the amplitude of p'/p_0 at $x = l$ with $\omega^2|X_b|$ fixed to be $(2\pi \times 48.2)^2 \times 0.1$ mm/s^2. The quantities in the parentheses are the theoretical values for the respective peaks. Good agreement between the experiment and the theory can be observed.

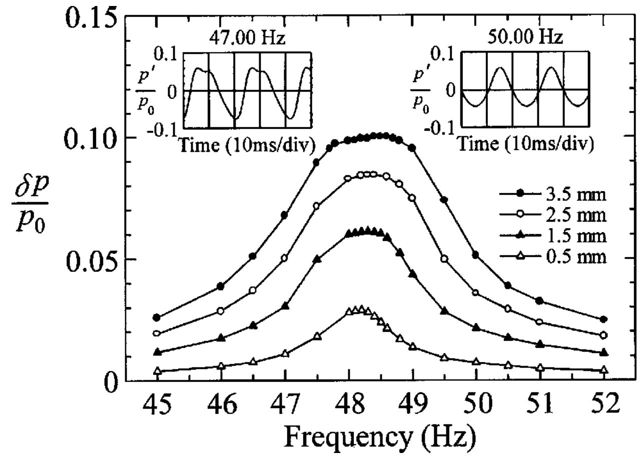

FIG. 4. Nonlinear frequency response of the fundamental mode in the tube with the array of Helmholtz resonators for the large amplitudes of displacement of the bellows $|X_b|$ indicated where the right and left insets display the temporal profiles of the excess pressure at the closed end when the column is driven at $|X_b| = 3.5$ mm and 50.00 Hz and 47.00 Hz, respectively.

We now drive largely the air column in the tube with the array. Figure 4 shows the frequency response with $|X_b|$ fixed

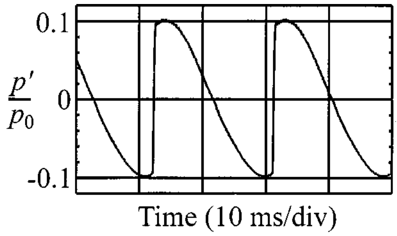

FIG. 5. Temporal shock profile in the excess pressure p' relative to p_0 at the closed end of the tube without the array of Helmholtz resonators.

at 0.5, 1.5, 2.5, and 3.5 mm, respectively, where $T_0 = 25.6\,°\mathrm{C}$ and $p_0 = 1.007 \times 10^5$ Pa. The half of the peak-to-peak pressure, δp, is plotted relative to p_0 against $\omega/2\pi$. As the driving amplitude increases, the curves become asymmetric with a flatter peak, and the resonance frequencies become slightly higher. Incidentally, the second harmonics in the displacement of the bellows tend to appear. Its magnitude is below 4% at the greatest excitations. For $\delta p/p_0 = 0.1$, the sound pressure level attains 170 dB. But the pressure profile remains free from the shock. In the tube without the array, the shock appears even for smaller excitations. In fact, when the air column is driven at the amplitude $|X_b| = 2.423$ mm and 52.96 Hz, the shock is observed. Figure 5 shows the temporal profile of the excess pressure p', relative to p_0, measured at the closed end. In passing, this is the case just on resonance because each "*N*" profile is anti-symmetric with respect to the point $p'/p_0 = 0$.[10]

By contrast, Fig. 6 shows the temporal profiles of p'/p_0 at the closed end of the tube with the array when the column is driven at $|X_b| = 3.5$ mm and $\omega/2\pi = 48.50$ Hz. The profile is smooth but not symmetric with its peak. When the frequency is increased to 50.00 Hz, the profile becomes symmetric, which is shown in the right inset of Fig. 4. When the frequency is lowered to 47.00 Hz, the peaks are flattened as in the left inset. Such a flat peak appears in a very narrow range of frequencies around 47.0 Hz. Over the frequencies in between, the profiles are similar to the asymmetric one shown in Fig. 6. As the frequencies are set further away from the resonance frequency, the profiles tend to take a symmetric form but are smaller in magnitude. Hence, the array of Helmholtz resonators is also shown to be effective to annihilate the shock in the case of oscillations as well. This provides another new method to generate high-amplitude and shock-free oscillations of a gas column.

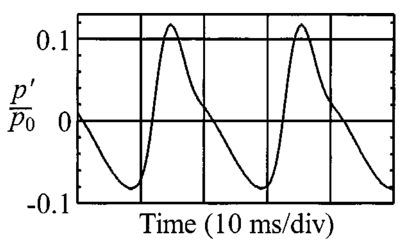

FIG. 6. Temporal shock-free profile in the excess pressure p' relative to p_0 at the closed end of the tube with the array of Helmholtz resonators.

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[8] When a piston is displaced by x_p into the tube, it displaces volume of Ax_p. When the bottom plate of the bellows is displaced by x_b, it displaces volume of $\pi(D_s^2 + D_s D_g + D_g^2)x_b/12$, if we assume the bellows to shrink or expand uniformly. By equating both volume with $A = \pi D_s^2/4$, we derive the equivalent displacement of the bellows.

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