Research article

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Anomalous Bessel vortex beam: modulating orbital angular momentum with propagation

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Abstract: Zero-order and higher-order Bessel beams are well-known nondiffracting beams. Namely, they propagate with invariant profile (intensity) and carry a fixed orbital angular momentum. Here, we propose and experimentally study an anomalous Bessel vortex beam. Unlike the traditional Bessel beams, the anomalous Bessel vortex beam carries decreasing orbital angular momentum along the propagation axis in free space. In other words, the local topological charge is inversely proportional to the propagation distance. Both the intensity and phase patterns of the generated beams are measured experimentally, and the experimental results agree well with the simulations. We demonstrate an easy way to modulate the beam's topological charge to be an arbitrary value, both integer and fractional, within a continuous range. The simplicity of this geometry encourages its applications in optical trapping and quantum information, and the like.

Keywords: orbital angular momentum; fractional topological charge; optical vortex beam; diffraction.

1 Introduction

In 1992, Allen et al. recognised that vortex beams with helical phase fronts, described by a transverse phase structure of $\exp(i\ell\varphi)$, carry an orbital angular momentum (OAM) of $\ell\hbar$ per photon [1], where φ is the azimuthal angle, ℓ is the topological charge of the field, and $\hbar$ is Planck's constant divided by 2π. The extrinsic nature of the OAM of photons offers an additional degree of freedom, and optical vortex beams have found numerous applications in optical micromanipulation [2], free-space communication [3], and quantum information [4]. It is noted that the phase front of a vortex beam winds by $2\pi\ell$ on a closed path around the axis, and the winding number is also possibly a noninteger [5]. When the winding number ℓ is a fractional number, the vortex beam has a fractional topological charge. In recent years, much attention has been paid to vortex beams with fractional topological charges [5–10], which are of importance to multiple applications in quantum optics, quantum information, and microparticle transportation guiding [11–13].

Bessel beams, one of the most common vortex beams, has attracted increasing interest due to their nondiffracting and self-healing properties. Thus far, generation and propagation properties of a fractional Bessel vortex beam (FBVB) have been studied extensively both theoretically and experimentally [13–19]. Using a spatial light modulator (SLM), Tao et al. [14] experimentally generated a FBVB in 2003. Soon afterwards, the self-reconstruction and nondiffracting properties of FBVB were examined [15–17].

On the other hand, it is well known that a vortex beam generally carries a fixed OAM during propagation. However, more recently, increasing interest has been paid to controlling the topological charge of a vortex beam during propagation [20, 21]. Using a class of nondiffracting frozen waves, Dorrah et al. [21] demonstrated that the topological charge of the beam can be controlled along the propagation direction. Recent studies show that the degree of longitudinal control of OAM may find applications in remote sensing, dense data communications, and others [20–23]. Although it is noted that the method to produce a Bessel vortex beam with decreasing OAM, that is, the topological charge is inversely proportional to the propagation distance, has not been studied yet.

In this study, we combine the two ideas to produce an anomalous Bessel vortex beam. Note that this beam

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is entirely different from the traditional Bessel beams or FBVBs discussed before. The most appealing property of the anomalous Bessel beam is that the local topological charge varies during propagation. The experimental results demonstrate that such a beam carries a continuously decreasing OAM, both integer and fractional multiple of $\hbar$, along the propagation direction in free space.

2 Principle and methods

We know that a zeroth-order Bessel beam can be thought of as the Fourier transform of an annular slit. Therefore, for the first time, Durnin et al. [24] observed a Bessel beam by placing an annular slit in the back focal plane of a converging lens. Due to the circular symmetry of the slit, we obtain a bright spot in the centre of the diffraction pattern, that is, zero-order Bessel beam. However, if the annular slit is changed into a spiral slit, then the transmitting wavelets from different parts of the spiral slit undergo different optical paths and reach the centre of the observation plane. In other words, we can introduce a continuous phase shift using a spiral slit, and then a phase singularity can be seen on the beam axis [25]. Recently, spiral slits [26–29] and plasmonic nanosieves [30, 31] have been adopted to produce plasmonic vortices and manipulate OAM. Cho et al. [27] showed that we can sculpture the fractional plasmonic vortex by modifying the structure of the spiral slit or tuning the operating wavelength. Wang et al. [28] theoretically analysed a method for fractional plasmonic vortex sculpturing, utilising the radial phase gradient induced by incident Laguerre–Gaussian beam. Here, we show that a Fermat's spiral slit can generate a novel anomalous vortex beam, and the OAM can be manipulated with propagation easily.

We start by studying the proposed method theoretically. A spiral slit located in $z=0$ plane is plotted in Figure 1, where r_0 is the initial radius of the spiral slit, r_α and α are the radial and the angular coordinates, respectively. Let us suppose that ρ_α is the distance from point $(r_\alpha, \alpha, 0)$ to the centre of observation plane $(0, 0, z)$. Then, if a plane wave is incident on the screen, the transmitted wavelets from two close points, (r_α, α) and $(r_{\alpha+\Delta\alpha}, \alpha+\Delta\alpha)$, along the slit with a little increment of angle $\Delta\alpha$ undergo different optical paths and arrive at the centre of the observation plane $(0, 0, z)$. The phase difference at the point $(0, 0, z)$ can be written as $\Delta\theta=2\pi\Delta\rho/\lambda$, where $\Delta\rho=\rho_{\alpha+\Delta\alpha}-\rho_\alpha$, and λ is the wavelength of the plane wave. It is noted that if $\Delta\rho=\ell\,\lambda\Delta\alpha/2\pi$, then, the total phase shift caused by the spiral slit can be calculated as $\int_0^{2\pi} d\theta=\frac{2\pi}{\lambda}\int_0^{2\pi}\frac{\ell\lambda}{2\pi}d\alpha=2\pi\ell$, where ℓ is a constant. This means that the phase front of the beam in the centre of the observation plane winds by $2\pi\ell$ on a closed path around the axis, namely, there will be an optical vortex in the centre of the observation plane [32], and the topological charge of which is ℓ. It is noted that the topological charge ℓ is dependent on distance z and describes the vortex located at the beam centre; therefore, it is a local topological charge. To meet this requirement for forming a helical phase front, the structure of a spiral slit can be written as

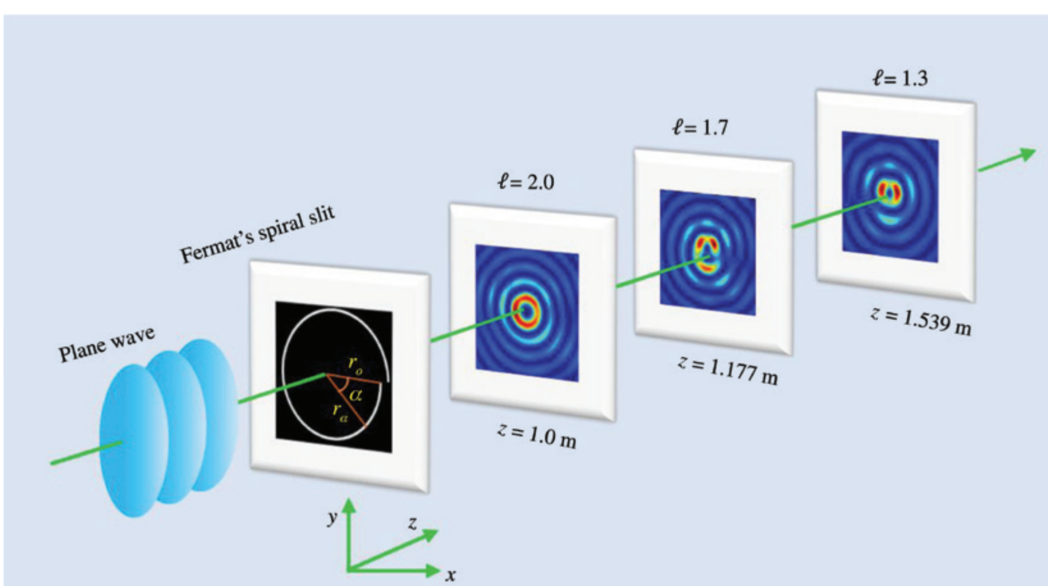

Figure 1: Geometry and notation of the spiral slit for achieving an anomalous Bessel vortex beam.

$$r_\alpha = \left(r_0^{\,2} + \frac{\ell z \lambda \alpha}{\pi} \right)^{1/2}. \tag{1}$$

The aforementioned analysis shows that if there is a spiral slit as described in Eq. 1, namely, a Fermat's spiral, then we can get a vortex beam with topological charge ℓ at observation plane z. Interestingly, Eq. 1 indicates that for a given spiral slit, the value of $\ell z \lambda$ can be regarded as a constant, which indicates that we can get a decreasing topological charge ℓ with the increasing propagation distance z. In other words, for a given optical beam (the wavelength is fixed), the topological charge ℓ of the generated vortex beam is inversely proportional to the propagation distance z. In the Fresnel approximation, the complex amplitude of the diffracted beam at a distance z can be obtained from the diffraction integral [33]

$$\mathrm{E}(x,y,z) = \frac{\mathrm{Exp}(ikz)}{i\lambda z} \int\int_{-\infty}^{+\infty} T(x,y)\mathrm{Exp}\left(\frac{ik[(x-x')^2+(y-y')^2]}{2z} \right) \mathrm{d}x'\mathrm{d}y', \tag{2}$$

where $T(x,y)$ denotes the aperture function of the spiral slit.

3 Results and discussion

According to the analysis in Section 2, we know that one can obtain a vortex beam using a Fermat's spiral. If we want to obtain an anomalous Bessel vortex beam, the spiral slit must meet another requirement: the gap of the spiral slit is much smaller than its radius, that is, $\rho_{2\pi}-\rho_0 \ll \rho_0$. If so, the spiral slit can be regarded as an annular slit approximately; accordingly, the field distribution of the generated vortex beam can be described by a Bessel function at some specific propagation distances. The simulations of the intensity and the phase patterns for different propagation distances are shown in Figure 2A. The spiral slit parameters are set to $\lambda = 532$ nm, $\ell = 2$, $z = 1$ m, $r_0 = 3$ mm, and $d = 0.07$ mm, where d is the width of the slit. Although the slit width d does not appear in Eq. 1, it

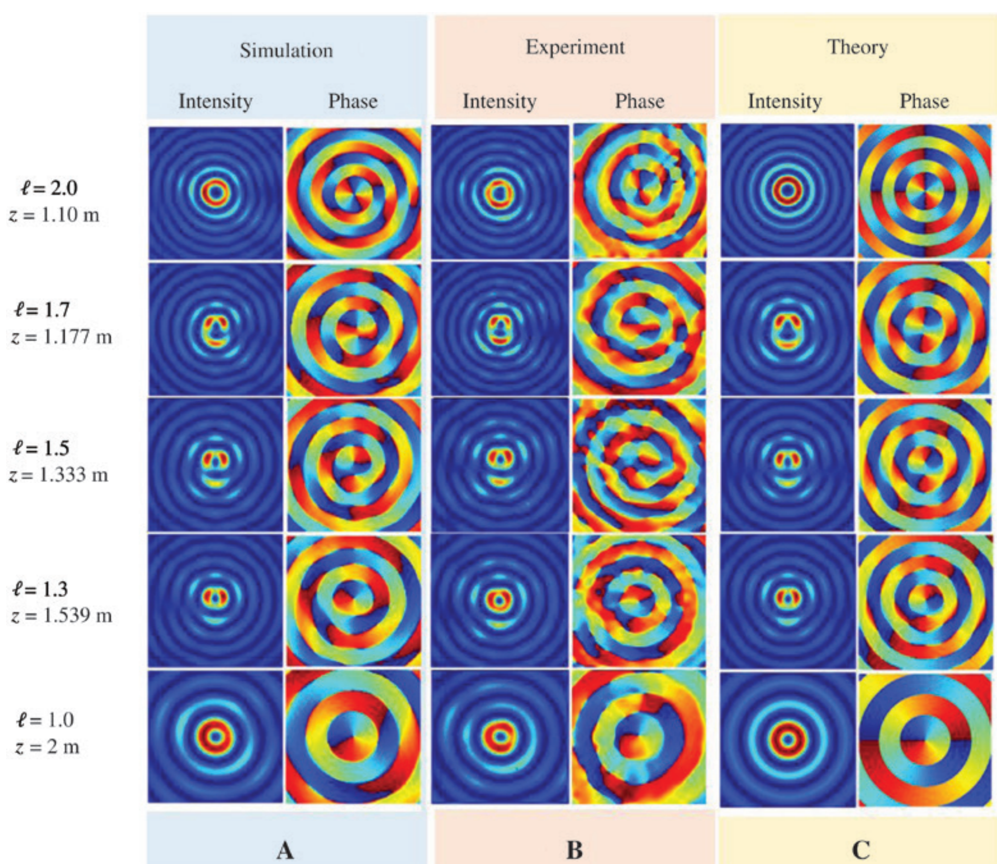

Figure 2: Intensity and the phase patterns of vortex beams with different topological charge at different distance z.
(A) Simulations based on the spiral slit; (B) experimental results; (C) theoretical results based on Eq. 9 from the article by Gutiérrez-Vega and López-Mariscal [17].

plays an important role in generating anomalous Bessel vortex beams. Similar to the annular aperture for producing Bessel beams [24], the width of the slit should be much smaller than the radius, that is, $d \ll \rho_0$. Figure 2A shows that we can obtain a Bessel beam with topological charge $\ell = 2$ in the plane $z = 1$ m. Interestingly, we can see that the topological charge of the vortex beam decreases continually with increasing propagation distance, both integer and fractional. Furthermore, when the topological charges are not an integer, the intensity patterns are no longer rotationally symmetrical rings. Figure 2B shows the corresponding experimental results for different propagation distances, which agree well with the simulations in Figure 2A. Therefore, we can see clearly that the topological charges of the vortex beam are inversely proportional to the propagation distance.

To verify the integer and FBVBs generated by a Fermat's spiral slit, the intensity and phase patterns of the exact FBVB with the same topological charges, using the analytical expression Eq. 9 in the article by Gutiérrez-Vega and López-Mariscal [17], are shown in Figure 2C. The comparison shows excellent agreement between the results obtained by the proposed method and the analytical results based on the work of Gutiérrez-Vega and López-Mariscal [17]. Figure 2 indicates that using a Fermat's spiral slit, we can obtain an FBVB with decreasing topological charge during propagation, namely, an anomalous Bessel vortex beam. Furthermore, it is noted that the topological charge decreases from 2 to 1 within a 1-m distance. Although if the spiral slit is designed for producing a vortex beam with topological charge $\ell = 2$ at $z = 0.5$ m, then we will get a vortex beam with charge $\ell = 1$ at $z = 1$ m. That is, the topological charge decreases from 2 to 1 within a 0.5-m distance. Therefore, we can engineer the OAM decreasing rate by tuning the structure of the spiral slit.

Figure 3 shows the experimental setup for generating an anomalous Bessel vortex beam and measuring its intensity and phase patterns. The laser beam, which is expanded by a beam expander, illuminates a Fermat's spiral slit and is displayed by SLM1. The output beam can be split into the transmitted and reflected beams by the first beam splitter (BS1). The distance between BS1 and SLM2 is equal to that between BS1 and the beam profile analyser. The intensity profile generated by the anomalous Bessel vortex beam is captured by the beam profile analyser. The phase pattern of the generated anomalous Bessel vortex beam can be measured by BS2, SLM2, and CCD based on a new method that we proposed recently [34].

Furthermore, from Eq. 1, we can see that the geometric structure of the spiral slit is dependent on the wavelength as well. Therefore, for a given Fermat's spiral slit, if we change the wavelength of incident beams, we can obtain different topological charges at the same propagation distance. Simulations of the intensity and phase patterns with different wavelengths and the same propagation distance are shown in Figure 4A, in which the parameters are set to $z = 1$ m, $r_0 = 3.17$ mm, and $d = 0.028$ mm. The corresponding experimental results are shown in Figure 4B. From Figure 4, we can see that the topological charge of the vortex beam decreases with increasing wavelength.

In the previous scenario, we have demonstrated the possibility of modulating the topological charge as a vortex beam propagates. It should be emphasised that the topological charge modulation discussed here does not violate the conservation of angular momentum. To fully understand this phenomenon, let us briefly review one

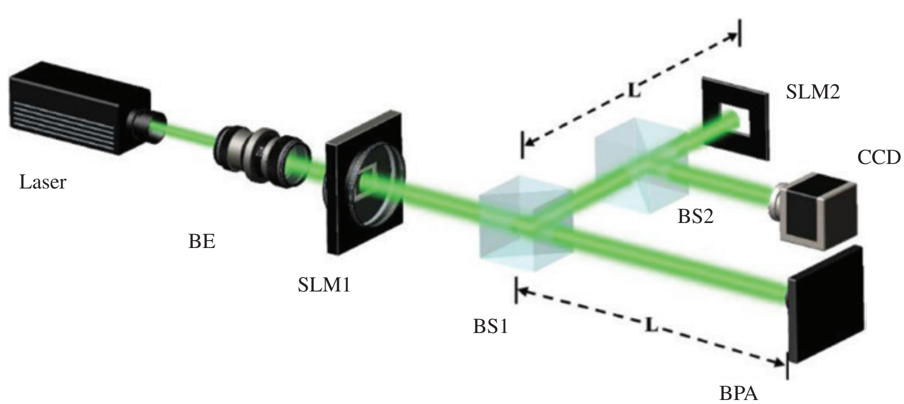

Figure 3: Experimental setup for generating an anomalous Bessel vortex beam and measuring its phase patterns.
BE, beam expander; SLM1 and SLM2, spatial light modulators; BS1 and BS2, beam splitters; BPA, beam profile analyser; CCD, charge-coupled device.

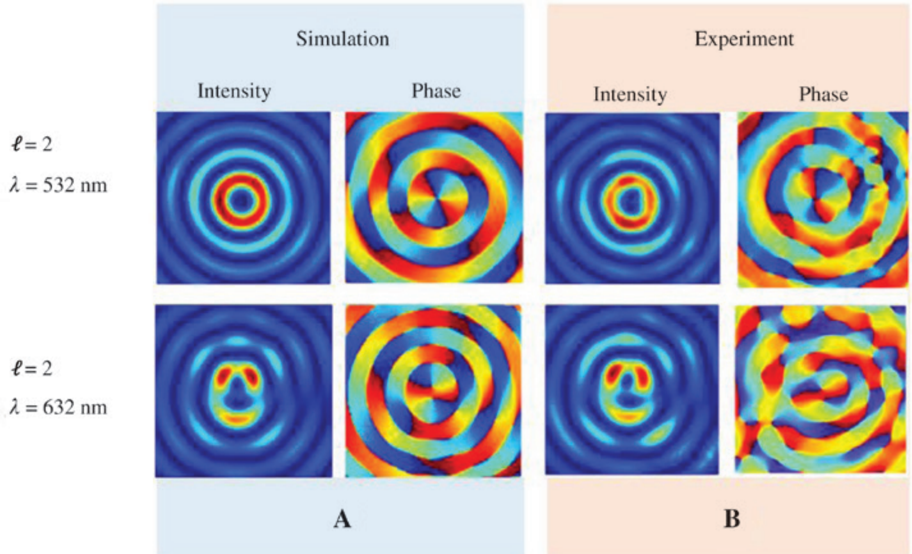

Figure 4: Simulations and experimental results of anomalous Bessel vortex beams with different wavelengths at same propagation distance. (A) Simulations; (B) experimental results.

of the most popular methods for the generation of vortex beams from a plane wave, that is, using spiral zone plates [35]. It is known that if a spiral zone plate is designed for producing a vortex with topological charge ℓ, then, such a plate can produce a series of vortex beams, with topological charge $\pm\ell$, $\pm 2\ell$, $\pm 3\ell\ldots$, at different propagation planes. It is noted that, here, the topological charges refer to the local one in the beam centre rather than the whole beam. Because the vortex beam generated by this method can be regarded as two parts, the dominant structure located in the central region and the background has a lower intensity spread over a larger area in the outer region. In other words, the generated beam is a mixture of vortex beams with different OAMs, only one of which is focused in the centre in a specific propagation plane, and others are out of focus and are in the background [35]. Therefore, the total angular momentum in any plane remains constant. It is worth noting that what we discussed in the previous scenario is the local topological charge near the propagation axis. Similar to a spiral zone plate, the spiral slit proposed in our study just changes the distribution of OAMs in different propagation planes, but the total OAMs of the entire optical field in each propagation plane remains unvaried. That is, whereas the local topological charge of the beam in the central part changes, charges in the outer part change accordingly and the total momentum is always conserved [20, 21]. Therefore, for the most part, what we can modulate is the local OAM rather than total OAM. For some special cases, the local OAM may even change sign whereas the total OAM of the beam remains constant [36]. Moreover, we know that the rotation rate of particles, trapped within a vortex beams, is dependent on the transfer of local angular momentum rather than total angular momentum [37].

4 Conclusion

In summary, we have studied a simple method for the generation of anomalous Bessel vortex beams both numerically and experimentally. An appealing propagation property of such a beam is that the local topological charge of the central beam is inversely proportional to the propagation distance, and the OAM deceasing rate can be engineered by tuning the structure of the spiral slit. We have demonstrated that one can control the OAM longitudinally by a simple spiral slit, which may find applications in optical trapping, quantum communications, dense data communications, and so on. Moreover, in this study, we retrieved the phase pattern experimentally; therefore, we can obtain the topological charge intuitively, rather than by analyzing the intensity patterns of the vortex beam indirectly [38, 39].

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