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ANOMALOUS HUBBLE EXPANSION AND INHOMOGENEOUS COSMOLOGICAL MODELS

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SUMMARY

It is shown that the inhomogeneous cosmological models can be applied to the particular case of the Local Supercluster of Galaxies (LSG). Here it is considered as an expanding region inside a vacuole which is itself embedded in a Friedmann model. The conditions deduced from Einstein's equations for this scheme are in good agreement with the observational data.

1. INTRODUCTION

It is a work of Rubin, Ford & Rubin (1) that has bounced the controversy on the so-called 'anomalous' extragalactic redshifts. Some evidence for a local anisotropy in the Hubble expansion had already been given by Rubin (2) and also by de Vaucouleurs (3). Rubin *et al.* used the observations on 74 ScI galaxies and concluded that the Hubble constant is not isotropic. This result was confirmed by Jaakkola *et al.* (4) with other types of sources, and by Karoji & Moles (5) with Markarian galaxies.

A quite opposite conclusion was arrived at by Sandage & Tammann (6). They discussed the distance calibration and found that 'the local velocity field is as regular, linear, isotropic and as quiet as it can be mapped with the present material' (7).

Nevertheless, analysing again the sample of galaxies studied by Sandage & Tammann, with the same data, de Vaucouleurs (8) shows that the relation between the red-shift z (or the symbolic velocity V) and the distance D

$$cz \equiv V = H.D \tag{1}$$

—or the equivalent relation between log z and the magnitude m—is not isotropic. The value of Hubble's 'constant', deduced from (1), depends on the distance D (9). According to de Vaucouleurs, the deviations from the linear relation (equation 1) confirm the model that he had proposed (3) for the Local Supercluster of Galaxies (LSG). But, for Pecker (10), inside the LSG, the diameter of which is roughly 30 Mpc, the value of Hubble's parameter H would be

$$H_{\text{LSG}} \sim 100 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{2}$$

Outside the LSG, for more distant objects, H would get its cosmological value

$$H_{\text{cosmological}} \sim 50 \text{ km s}^{-1} \text{ Mpc}^{-1}. \tag{3}$$

It seems that inside the Local Supercluster, the redshifts are greater than farther away, in the remaining Universe. It is possible to account for this excess in redshifts with a theory of tired light, and in particular with that proposed recently (11).

On the other hand, we shall examine here the possibility of interpreting the results summarized by (2) and (3) with a non-homogeneous cosmological model.

We shall keep the classical framework of relativistic cosmology, but give up the assumption of homogeneity.

In the next section, we describe the scheme adopted. To simplify the mathematical treatment, we suppose spherical symmetry. We consider the LSG as a spherical expanding condensation inside a vacuole, itself embedded in the cosmological expanding fluid. It is possible, in that case, to get an *exact* and *complete* solution of Einstein's equations, deducible from Tolman's solution.

In Section 3, we discuss the mathematical properties of our solution. As the model here adopted is dust-filled in the condensed LSG and in the embedding cosmological fluid, it is possible to match the Schwarzschild metric of the vacuole, at the two boundaries, with the two Friedmann models, interior and exterior. We give, here, explicitly, the junction conditions deduced from the two fundamental forms of the boundaries.

In the last section, the solution and the junction conditions are applied to the interpretation of the anomalous redshifts obtained in the case of the LSG. It is shown that, in spite of the great simplification of the scheme here adopted, it is possible to account for a Hubble parameter greater inside the LSG than outside, in the cosmological fluid. The characteristics of our model fit, very satisfactorily, the numerical results obtained by observations: Hubble's parameter, size and density of the LSG, cosmological density.

So, we have demonstrated here that we can have a greater expansion inside a local condensation with a density greater than that of the outside Universe.

2. THE GENERAL RELATIVITY TREATMENT

For simplicity, let us consider a model with spherical symmetry, and the following scheme:

(a) Region (1): a central condensation, with radius R_1 described by an expanding Friedmann model. This is a very crude scheme of the LSG which, in fact, is flattened and rotating as well as expanding.

(b) Region (2): an intermediate zone surrounding the preceding material distribution. We shall suppose that this region is empty, so that its dimensions be minimum. It is the vacuole-model, already introduced by many authors (12).

(c) Region (3): the expanding universe in which the preceding vacuole is embedded.

As usual for cosmologic solutions, the material in (1) and (3) will be envisaged as an ideal fluid without pressure, the corresponding energy-tensor being

$$T^{\alpha\beta} = \rho u^{\alpha} u^{\beta}. \tag{4}$$

It is well known that the field inside the vacuole (2) is given by the static Schwarzschild solution. What becomes time-dependant in this model, are the two boundaries R_1 and R_2 of this region.

Among the numerous spherically symmetric solutions of Einstein's equations for dust-filled inhomogeneous models, a very convenient one is Tolman's solution

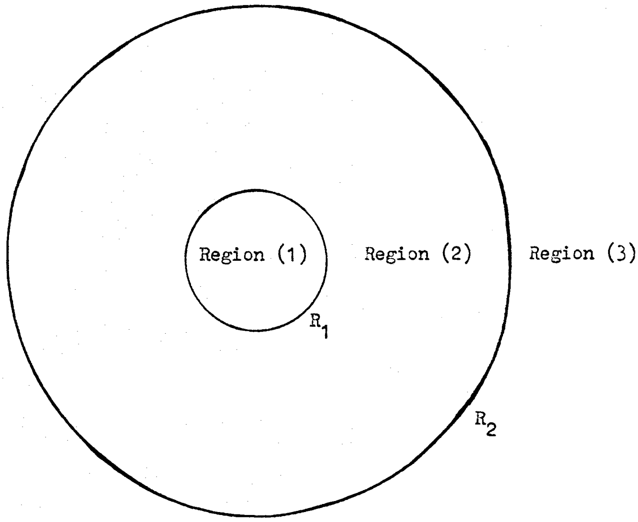

FIG. 1. *A model with spherical symmetry* (see text for details).

(**13**). Using a co-moving coordinate system, it may be written

$$ds^2 = dt^2 - \frac{R'^2}{1+f(r)} dr^2 - R^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{5}$$

f is an arbitrary function of r only, and $R(r, t)$ must satisfy, according to the field equations

$$\dot{R}^2 = f(r) + \frac{F(r)}{R}. \tag{6}$$

$F(r)$ is another arbitrary function of r. Here $'$ means $\partial/\partial r$ and $\cdot$ means $\partial/\partial t$.

The proper density is related to the metric by

$$8\pi\rho = \frac{F'}{R'R^2}. \tag{7}$$

Bondi (**14**) investigated various physical properties of these inhomogeneous systems. Bonnor (**15**) studied their evolution.

We shall, here, limit ourselves to the peculiar case

$$f(r) = 0 \tag{8}$$

which corresponds to the *parabolic models* (**14**) (**15**). The exact solution corresponding to the scheme adopted has been determined, in this case (8), by Papapetrou (**16**).

Integrating (6), one gets, for parabolic models

$$R^3 = \frac{9}{4} F(r)[t - T(r)]^2, \tag{9}$$

where $T(r)$ is an arbitrary function, an integration 'constant'.

It is easily seen from (5), that R is then, at $t = cte$, the distance from the origin ($r = 0$) to a point of coordinate r.

We can deduce from (4), (5), (7) and (9) that the mass of dust inside a sphere with co-moving radius r is

$$M(r) = \int_0^r dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \sqrt{-g}\, g^{\alpha\beta} T_{\alpha\beta} = \frac{F(r)}{2}. \tag{10}$$

It is possible (**16**) to choose the arbitrary functions $F(r)$ and $T(r)$ in such a way that our scheme will be completely determined. Its main features are the following ones:

$$\left.\begin{array}{ll}\text{Region (1):} & 0 \leqslant r < r_1; \quad T(r) = T_0 = cte > 0; \quad F(r) = \beta r^3 \\ & R^3 = \dfrac{9}{4}\beta r^3[t-T_0]^2; \quad 8\pi\rho_1 = \dfrac{4}{3(t-T_0)^2} \\ \text{Region (2):} & r_1 \leqslant r < r_2; \quad T'(r) < 0; \quad F = c_{te} = \beta r_1^3 \\ & R^3 = \dfrac{9}{4}\beta r_1^3[t-T(r)]^2; \quad \rho_2 = 0 \\ \text{Region (3):} & r_2 \leqslant r; \quad T = 0; \quad F(r) = \alpha r^3 \\ & R^3 = \dfrac{9}{4}\alpha r^3 t^2; \quad 8\pi\rho_0 = \dfrac{4}{3t^2}\end{array}\right\} \tag{11}$$

3. THE SOLUTION

The above results show that the interior condensation—region (1)—and the imbedding exterior universe—region (3)—are two Einstein–de Sitter models (that is to say two Friedmann models with a flat 3-space). This comes from the choice (8) which leads to a null curvature for the 3-space (see also Bondi (**14**)), and from the choice (11) ($T = cte$, 0; $F(r) = Q(r^3)$) that gives homogeneous Friedmann models in (1) and (3).

The transition zone (2) is the Schwarzschild vacuole ($\rho_2 = 0$).

It is now necessary to examine the matching problem at the boundaries ($r = r_1$ and $r = r_2$) of the vacuole. That this problem (matching between the Schwarzschild and Friedmann metrics) has a solution follows from the well-known theorem of Misner & Sharp (**17**), independently demonstrated also by Bel & Hamoui (**18**): All spherically symmetric solutions of Einstein's equations with perfect fluid are locally joinable to Schwarzschild metric provided that the pressure vanishes on a time-like hypersurface.

We are here interested by the junction conditions to be satisfied on the time-like hypersurfaces $S_1(r = r_1)$ and $S_2(r = r_2)$.

Sufficient boundary conditions are the following: the field $g_{\alpha\beta}$ goes over continuously up to the first derivatives for $r = r_1$ and $r = r_2$. But this is *not necessary* because a discontinuity of the $g_{\alpha\beta}$ and their first derivatives may be caused by a discontinuity of the respective systems of coordinates and not by a discontinuity of the fields.

So we shall impose as boundary conditions to be satisfied on S_1 and S_2 that the first and second fundamental forms of the two hypersurfaces $S(S_1$ and $S_2)$, calculated from the metrics on the two sides of S shall be identical—Cocke; Bonnor & Faulkes (**19**). These junction conditions reduce to

$$\beta r_1^3 = \alpha r_2^3. \tag{12}$$

The mass of the condensation (1) is then such that

$$M(r_1) = \frac{F(r_1)}{2} = \frac{\beta r_1^3}{2} = \frac{4}{3}\pi R_1^3\rho_1 = \frac{4}{3}\pi R_2^3\rho_0. \tag{13}$$

The last relation (where ρ_0 is the cosmological density) comes from the matching condition (12). It has already been pointed out in other models (**20**). From this very relation, one can easily show that the dimensions of the transition zone (2) are minima when this region is empty ($\rho_2 = 0$).

4. PHYSICAL INTERPRETATION. APPLICATION TO THE LOCAL SUPERCLUSTER OF GALAXIES

Tolman (**13**) had already shown that, in the case of combination of uniform distributions (as is the case in Fig. 1), the dust in each Friedmann zone behaves as in a completely homogeneous model, without reference to the behaviour of other parts of the model. So, the general relations of the Friedmann models are still valid here for each zone, and in particular the equation

$$\frac{4\pi G}{3}\rho = q_0 H^2, \tag{14}$$

where H is Hubble's parameter and q_0 the acceleration factor. For dust-filled homogeneous models with flat 3-space—as are regions (1) and (3)—the value of the acceleration factor is

$$q_0 = 0{\cdot}5 \tag{15}$$

when Einstein's field equations are used with a vanishing cosmological term (**21**). Consequently, taking into account (2) and (3), one gets from (14) and (15)

$$\frac{(\rho_1)_{\rm LSG}}{\rho_0} = \frac{(H)^2_{\rm LSG}}{(H_0)^2} = 4. \tag{16}$$

With the value (3) of $H_0 = H_{\text{cosmological}}$, value also adopted by Sandage & Tammann (**7**), we get from (14)—taking account of (15)—for the density of matter in the Universe

$$\rho_0 = \rho_{\text{cosmological}} \sim 0{\cdot}5 \, . \, 10^{-29}\ \text{g cm}^{-3} \tag{17}$$

Let us suppose that the mass of the Virgo Cluster is about

$$M_{\rm V.C.} \sim 2 \, . \, 10^{14}\, M_\odot, \tag{18}$$

this value, very controversial, depending essentially on the method used for its determination. We can then consider that the mass of the Local Supercluster is approximately

$$M(r_1) \equiv M_{\rm LSG} \sim 2 \, . \, 10^{15}\, M_\odot. \tag{19}$$

The mean density ρ_1 of the LSG, as deduced from (13) is in good agreement with that obtained taking into account (16) and (17) if

$$R_1 \sim 12\ \text{Mpc}, \tag{20}$$

a value which fits well de Vaucouleurs' estimation.

The junction conditions (12) or (13) then lead to the following value for the radius of the vacuole

$$R_2 \sim 19{\cdot}2\ \text{Mpc}. \tag{21}$$

Let us, at last, examine the propagation of light in this model, from a source with radial coordinate r_1 to a terrestrial observer located at the centre of the condensation. This is only a first approximation because of our effective position in the Local Supercluster. One can show (**14**) that, in our model (11), we have for a light-ray

$$\theta = cte, \qquad \phi = cte, \qquad t = Y(r) \tag{22}$$

with

$$\frac{dY(r)}{dr} = -R'\{r, Y(r)\}. \tag{23}$$

The shift of spectral lines, z_1, for a source with radial coordinate r_1, is given by

$$\log(1+z_1) = -\log(1-v_1) - \int_0^{r_1} \frac{M'dr}{r(1-v)} \tag{24}$$

where

$$v(r) = \left(\frac{\partial R}{\partial t}\right)_{r, Y(r)}. \tag{25}$$

Let us emphasize the very important significance of the variable R in these inhomogeneous models, as pointed out by Bondi (14). $R(r, t)$ is equivalent to the luminosity distance, from the origin, of a source (r, θ, ϕ) at time t. That is to say: $R(r, t)$ is proportional to the square root of the ratio of absolute and corrected apparent luminosity, the square of the Doppler shift $(1+z)$ being the correcting factor. Then, since t measures the proper time of each co-moving particle, $\partial R/\partial t$ is its velocity.

In the expression (24) of the shift of spectral lines appear two terms: the first one is a velocity shift and the second one is an Einstein gravitational redshift. The kinematical shift is due to the relative motion of the source and the observer; its sign depends on the sign of v_1. The Einstein shift depends not only on the gravitational conditions at the source and at the observer, but also on the intervening matter; it is towards the red in most cases. Region (2) being empty ($M'_{(2)} = 0$) does not contribute to this term. And we can show that this gravitational shift may be neglected with respect to the velocity shift in regions (1) and (3) of this model. Indeed when applied to the LSG, the calculation gives

$$\frac{z_{\text{Einstein}}}{z_{\text{velocity}}} \leqslant 1{,}2 . 10^{-3}. \tag{26}$$

We are thus justified in regarding the observed redshifts as velocity-shifts with

$$z_1 \sim v_1. \tag{27}$$

As R is equivalent to the luminosity-distance and $v = \partial R/\partial t$, is the velocity of the source at coordinate r, we can write, using the definition (1)

$$H = \frac{\text{velocity}}{\text{distance}} = \frac{\dot{R}}{R}, \tag{28}$$

a relation that mimics the current definition of the Hubble parameter in the theory of uniform model universes. In homogeneous cosmology, R is the scale-factor and an arbitrary function of t only.

In the scheme here adopted, $R = R(r, t)$ is the luminosity–distance from the origin of a source with radial coordinate r at time t.

Using (9), the relation (28) becomes

$$H = \frac{2}{3(t-T)}. \tag{29}$$

In region (1), we have

$$H_{(1)} = \frac{2}{3(t-T_0)} \sim 100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{30}$$

In region (3), we obtain

$$H_{(3)} = \frac{2}{3t} \sim 50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{31}$$

Consequently,

$$T_0 = \frac{t}{2} = \frac{1}{2}(6\pi G\rho_0)^{-1/2} \sim 2.10^{17}\,\mathrm{s}, \tag{32}$$

when one takes into account (11) and (17), t is the present time of observation.

So, the different characteristics of this model have been determined. This scheme is in quite good agreement with the observational data concerning the Local Supercluster, in spite of its great simplification. A better fitting could be obtained with a more realistic model.

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