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Anomaly Cancellations in the Type I D9-D$\overline{9}$ System and the $USp(32)$ String Theory

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We check some consistency conditions for the D9-D$\overline{9}$ system in type I string theory. The gravitational anomaly and gauge anomaly for $SO(n) \times SO(m)$ gauge symmetry are shown to be cancelled when $n - m = 32$. In addition, we find that a string theory with $USp(n) \times USp(m)$ gauge symmetry also satisfies the anomaly cancellation conditions. After tachyon condensation, the theory reduces to a tachyon-free $USp(32)$ string theory, though there is no spacetime supersymmetry.

§1. Introduction

To this time, most research on D-branes in string theory has been carried out in supersymmetric configurations. The BPS property of branes protects the system from quantum corrections and provides a nice perspective to go beyond perturbation in the weakly coupled regime. In particular, the stability of the BPS D-branes is one of the key properties in testing various dualities in string theories and supersymmetric gauge theories.

However, fortunately or unfortunately, the real world is not supersymmetric, at least in the low energy scale, and we should engage ourselves in the study of non-supersymmetric theories sooner or later. Even if one postpones consideration of the phenomenological aspects, there are various interesting features in the non-BPS configurations of branes, as should be the case, since most of the states in string theory are non-BPS.

Recently, the research on the non-BPS configurations of D-branes in string theory has entered a new stage. It was discussed in Refs. 1) and 2) that D-branes can be constructed as bound states of brane-anti-brane systems and several new non-BPS D-branes were found using this construction. In this paper, we mainly consider the D9-D$\overline{9}$ system in type I string theory. As shown in Ref. 2), lower dimensional D-branes in type I string theory can be constructed by arranging non-trivial Chan-Paton bundles for the D9- and/or D$\overline{9}$-branes. This construction leads to an interpretation in K-theory and it has been shown that the possible D-branes in type I string theory can be classified by KO-groups.[2)]

We will check some consistency conditions for the D9-D$\overline{9}$ system in this paper. The gauge group of the D9-D$\overline{9}$ system is $SO(n) \times SO(m)$, which is potentially anomalous. The gravitational and mixed anomalies may also arise in this system. We will show that these anomalies are all cancelled when $n - m = 32$. Interestingly,

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there is another solution of the anomaly cancellation conditions, which suggests the existence of a consistent D9-$\overline{\text{D9}}$ system with $USp(n) \times USp(m)$ gauge symmetry. After tachyon condensation, the theory reduces to a tachyon-free $USp(32)$ string theory, though there is no spacetime supersymmetry.

This paper is organized as follows. In §2 we analyze the effective field theory of the D9-$\overline{\text{D9}}$ system. We guess the contents of massless fermions in the theory and check that the gravitational and gauge anomalies as well as the mixed anomalies are all cancelled by the Green-Schwarz mechanism. In §3, we make systematic analyses in perturbative string theory and give some results that are consistent with the analyses given in §2. In §4, we investigate the general formulation of the D9-$\overline{\text{D9}}$ system and check the anomaly cancellations in stringy calculations. In §5, we discuss some properties of the $USp(32)$ string theory.

§2. Analyses in the effective field theory

2.1. *Green-Schwarz mechanism in the type I D9-$\overline{\text{D9}}$ system*

In this subsection, we determine the massless fermions in the type I D9-$\overline{\text{D9}}$ system, imposing the Green-Schwarz anomaly cancellation conditions. Related analyses in the type IIB D9-$\overline{\text{D9}}$ system are given in Ref. 3).

Let us briefly review the Green-Schwarz anomaly cancellation conditions in type I string theory.[4),*)] We consider the case in which the gauge group is $SO(n)$. The gaugino belongs to the adjoint representation of the gauge group and contributes to the anomaly. The gravitational anomaly cancellation requires

$$496 = \frac{1}{2}\, n(n-1), \tag{2·1}$$

where the right-hand side is the dimension of the adjoint representation of $SO(n)$. Equation (2·1) implies the well-known result $n = 32$. Then the rest of the anomaly is proportional to

$$\begin{aligned} I_{12} \propto -\frac{1}{15}\,\mathrm{Tr}\,F^6 + \frac{1}{24}\,\mathrm{Tr}\,F^4\,\mathrm{tr}\,R^2 - \frac{1}{960}\,\mathrm{Tr}\,F^2(4\,\mathrm{tr}\,R^4 + 5(\mathrm{tr}\,R^2)^2) \\ + \frac{1}{8}\,\mathrm{tr}\,R^2\,\mathrm{tr}\,R^4 + \frac{1}{32}(\mathrm{tr}\,R^2)^3, \end{aligned} \tag{2·2}$$

where F is the field strength 2-form of $SO(n)$ and R is the curvature 2-form. One of the important identities for the gauge anomaly cancellation is

$$\mathrm{Tr}\,F^6 = (n-32)\,\mathrm{tr}\,F^6 + 15\,\mathrm{tr}\,F^2\,\mathrm{tr}\,F^4. \tag{2·3}$$

Here we denote traces in the adjoint representation of $SO(n)$ by the symbol 'Tr', while traces in the fundamental representation are denoted 'tr'. In order to cancel the gauge anomaly, the coefficient of $\mathrm{tr}\,F^6$ on the right-hand side of (2·3) must vanish. This condition is also satisfied when $n = 32$. Then all of the terms in (2·2) are

*) We mainly follow the description in §13.5 of Ref. 5).

cancelled by the counterterm

$$\Delta\Gamma = \int BX_8 - \left(\frac{2}{3}+\alpha\right)\int(\omega_{3L}-\omega_{3Y})X_7, \tag{2·4}$$

where B is the 2-form field and α is an adjustable parameter. The quantity X_8 is given as

$$X_8 = \mathrm{tr}\,F^4 - \frac{1}{8}\,\mathrm{tr}\,F^2\,\mathrm{tr}\,R^2 + \frac{1}{8}\,\mathrm{tr}\,R^4 + \frac{1}{32}(\mathrm{tr}\,R^2)^2, \tag{2·5}$$

and ω_{3L}, ω_{3Y} and X_7 are defined by

$$\mathrm{tr}\,R^2 = d\omega_{3L}, \qquad \mathrm{tr}\,F^2 = d\omega_{3Y}, \qquad X_8 = dX_7, \tag{2·6}$$

modulo exact forms. The counterterm (2·4) induces an anomaly of the form

$$I_{12} \propto (\mathrm{tr}\,R^2 - \mathrm{tr}\,F^2)X_8, \tag{2·7}$$

which exactly cancels the anomaly (2·2).

Now, consider type I string theory with n D9-branes and m D$\overline{9}$-branes. The gauge group is $SO(n)\times SO(m)$. The ordinary type I string theory corresponds to the case in which $n=32$ and $m=0$. Since several observations suggest that we can create or annihilate Dp-D$\overline{\mathrm{p}}$ pairs without changing the physical context,[1),2)] it is natural to assume that the coefficients of the anomalies only depend on $n-m$. Then the condition corresponding to (2·1) becomes

$$496 = \frac{1}{2}\,(n-m)(n-m-1) \tag{2·8}$$
$$= \frac{1}{2}\,n(n-1) + \frac{1}{2}\,m(m+1) - mn, \tag{2·9}$$

which implies $n-m=32$. Equation (2·9) suggests that the 9-9 fermions λ and the $\overline{9}$-$\overline{9}$ fermions $\widetilde{\lambda}$ are positive chirality spinors, which belong to the adjoint representation of $SO(n)$ and the second rank symmetric tensor representation of $SO(m)$, respectively, while the 9-$\overline{9}$ or $\overline{9}$-9 fermions ψ are negative chirality spinors, which belong to the bifundamental representation of $SO(n)\times SO(m)$ (Table I).

Let us show that all the anomalies are cancelled when we choose these fermions. We denote the field strength 2-forms of $SO(n)$ and $SO(m)$ gauge fields by F_1 and F_2, respectively. We have the identities

$$\mathrm{Tr}_1\,F_1^6 = (n-32)\,\mathrm{tr}_1\,F_1^6 + 15\,\mathrm{tr}_1\,F_1^2\,\mathrm{tr}_1\,F_1^4, \tag{2·10}$$
$$\mathrm{Tr}_2\,F_2^6 = (m+32)\,\mathrm{tr}_2\,F_2^6 + 15\,\mathrm{tr}_2\,F_2^2\,\mathrm{tr}_2\,F_2^4, \tag{2·11}$$

Table I. Fermions in the D9-D$\overline{9}$ system.

string	fermion	rep. of $SO(n)\times SO(m)$	chirality
9-9 string	λ	$(\begin{smallmatrix}\square\\\square\end{smallmatrix},1)$	$+$
$\overline{9}$-$\overline{9}$ string	$\widetilde{\lambda}$	$(1,\square\square)$	$+$
9-$\overline{9}$, $\overline{9}$-9 string	ψ	$(\square,\square)$	$-$

$$\mathrm{Tr}_1\, F_1^4 = (n-8)\,\mathrm{tr}_1\, F_1^4 + 3(\mathrm{tr}_1\, F_1^2)^2, \tag{2·12}$$
$$\mathrm{Tr}_2\, F_2^4 = (m+8)\,\mathrm{tr}_2\, F_2^4 + 3(\mathrm{tr}_2\, F_2^2)^2, \tag{2·13}$$
$$\mathrm{Tr}_1\, F_1^2 = (n-2)\,\mathrm{tr}_1\, F_1^2, \tag{2·14}$$
$$\mathrm{Tr}_2\, F_2^2 = (m+2)\,\mathrm{tr}_2\, F_2^2, \tag{2·15}$$

where we label traces in the representations of $SO(n)$ and $SO(m)$ by the subscripts 1 and 2, respectively. We denote traces in the adjoint representation of $SO(n)$ by 'Tr_1', while 'Tr_2' represents the traces in the second rank symmetric tensor representation of $SO(m)$. Traces in the fundamental representations of $SO(n)$ and $SO(m)$ are denoted 'tr_i' $(i = 1, 2)$. Collecting all the contributions of the fermions λ, $\widetilde{\lambda}$ and ψ, the term $\mathrm{Tr}\, F^6$ in (2·2) is replaced by

$$\begin{aligned} \mathrm{Tr}\, F^6 \to\ & \mathrm{Tr}_1\, F_1^6 + \mathrm{Tr}_2\, F_2^6 - m\,\mathrm{tr}_1\, F_1^6 - n\,\mathrm{tr}_2\, F_2^6 \\ & - \binom{6}{2} (\mathrm{tr}_1\, F_1^2\,\mathrm{tr}_2\, F_2^4 + \mathrm{tr}_1\, F_1^4\,\mathrm{tr}_2\, F_2^2), \end{aligned} \tag{2·16}$$

where the first two terms in (2·16) are the contributions of the 9-9 fermion λ and the $\overline{9}$-$\overline{9}$ fermion $\widetilde{\lambda}$, while the other terms are the contributions of the 9-$\overline{9}$ fermion ψ. Using the identities (2·10) and (2·11), it is easy to check that the coefficients of $\mathrm{tr}_1\, F_1^6$ and $\mathrm{tr}_2\, F_2^6$ in (2·16) vanish if and only if $n - m = 32$. Similarly, $\mathrm{Tr}\, F^4$ and $\mathrm{Tr}\, F^2$ in (2·2) are replaced by

$$\begin{aligned} \mathrm{Tr}\, F^4 \to\ & \mathrm{Tr}_1\, F_1^4 + \mathrm{Tr}_2\, F_2^4 - m\,\mathrm{tr}_1\, F_1^4 - n\,\mathrm{tr}_2\, F_2^4 \\ & - \binom{4}{2} \mathrm{tr}_1\, F_1^2\,\mathrm{tr}_2\, F_2^2, \end{aligned} \tag{2·17}$$
$$\mathrm{Tr}\, F^2 \to \mathrm{Tr}_1\, F_1^2 + \mathrm{Tr}_2\, F_2^2 - m\,\mathrm{tr}_1\, F_1^2 - n\,\mathrm{tr}_2\, F_2^2. \tag{2·18}$$

Then, using the identities (2·10)–(2·15), the anomaly (2·2) can be written as

$$I_{12} \propto (\mathrm{tr}\, R^2 - \mathrm{tr}_1\, F_1^2 + \mathrm{tr}_2\, F_2^2) X_8', \tag{2·19}$$
$$\begin{aligned} X_8' =\ & \mathrm{tr}_1\, F_1^4 - \mathrm{tr}_2\, F_2^4 - \frac{1}{8}(\mathrm{tr}_1\, F_1^2 - \mathrm{tr}_2\, F_2^2)\,\mathrm{tr}\, R^2 \\ & + \frac{1}{8}\,\mathrm{tr}\, R^4 + \frac{1}{32}(\mathrm{tr}\, R^2)^2. \end{aligned} \tag{2·20}$$

This anomaly can be cancelled by the counterterm

$$\varDelta\Gamma = \int B X_8' - \left(\frac{2}{3} + \alpha\right) \int (\omega_{3L} - \omega_{3Y_1} + \omega_{3Y_2}) X_7', \tag{2·21}$$

where we have defined $\mathrm{tr}_i\, F^2 = d\omega_{3Y_i}$ and $X_8' = dX_7'$. The gauge invariant combination of the field strength of the 2-form field B is now

$$H = dB + \omega_{3L} - \omega_{3Y_1} + \omega_{3Y_2}. \tag{2·22}$$

2.2. *Coupling to the tachyon fields*

There are tachyon fields in the D9-D$\overline{9}$ system. After tachyon condensation, the D9-D$\overline{9}$ brane pairs are expected to vanish, and the field content of the theory turns out to be the same as that of type I $SO(32)$ string theory. Let us show that the fermion contents given in Table I are also suitable to explain the brane-anti-brane pair annihilation.

The tachyon fields $T_{i\bar{j}}$ belong to the bifundamental representation of $SO(n) \times SO(m)$. We denote $i,\ j = 1, \cdots, n$ as the vector indices of $SO(n)$ and $\bar{i},\ \bar{j} = 1, \cdots, m$ as the vector indices of $SO(m)$. The following Yukawa interactions are consistent with the symmetry:

$$\mathcal{L}_Y \sim \overline{\psi}^{i\bar{i}} \lambda^{ij} T_{j\bar{i}}, + \overline{\psi}^{i\bar{i}} \widetilde{\lambda}^{\bar{i}\bar{j}} T_{i\bar{j}}, \tag{2·23}$$

where $\overline{\psi} = \psi^T \Gamma^0$.

The tachyon VEV is of the form

$$\begin{array}{cc} \longleftarrow \quad n \quad \longrightarrow & \\ (T_{i\bar{j}}) = \left(\begin{array}{ccc} * & & \\ & \ddots & \\ & & * \end{array} \right) & \begin{array}{c} \uparrow \\ m \\ \downarrow \end{array} \\ \leftarrow 32 \rightarrow \leftarrow \ m \ \rightarrow & \end{array} \tag{2·24}$$

up to gauge symmetry. It is plausible to assume that the symbols $*$ in (2·24) are all non-zero, although we do not know the precise form of the tachyon potential. Then, the Yukawa terms (2·23) will induce mass terms for ψ, $\widetilde{\lambda}$ and λ^{ij} ($i > 32$ or $j > 32$). The number of the components of $\widetilde{\lambda}$ and λ^{ij} ($i > 32$ or $j > 32$) are $\frac{1}{2}m(m+1)$ and $\frac{1}{2}m(m-1) + 32m$, respectively, and the sum is just enough to be paired with ψ, which has $mn = m^2 + 32m$ components. As a result, the massless components of the fermions after the tachyon condensation are λ^{ij} ($i,\ j = 1, \cdots, 32$), which belong to the adjoint representation of the unbroken $SO(32)$ gauge group, as expected.

§3. Analyses in string theory

3.1. *Physical states in the type I D9-D$\overline{9}$ system*

The D$\overline{9}$-brane is a 9-brane with -1 units of R-R charge. It can be obtained by flipping the sign of the R-R charge of a D9-brane. Using this fact, we can compute the vacuum amplitudes in the D9-D$\overline{9}$ system, from which the physical spectrum can be extracted, as discussed in Ref. 1). As a preliminary step, let us first collect here the one-loop vacuum amplitudes for the 9-9 strings. [6] There are the contributions from the NS sector and the Ramond sector, which are denoted as $Z^{\rm NS}$ and $Z^{\rm R}$. We decompose these terms into the contributions of NS-NS exchange and R-R exchange in the closed string channel, denoted $Z_{\rm NS\,NS}$ and $Z_{\rm R\,R}$. We also label the contributions from the cylinder diagram and the Möbius strip diagram for the amplitudes with superscripts as $Z^{(\rm C_2)}$ and $Z^{(\rm M_2)}$:

$$Z_{\text{9-9}} = Z^{\rm NS}_{\text{9-9}} + Z^{\rm R}_{\text{9-9}}, \tag{3·1}$$

$$Z^{\rm NS}_{\text{9-9}} = Z^{\rm NS\ (C_2)}_{\rm NS\,NS} + Z^{\rm NS\ (M_2)}_{\rm NS\,NS} + Z^{\rm NS\ (C_2)}_{\rm R\,R} + Z^{\rm NS\ (M_2)}_{\rm R\,R}, \tag{3·2}$$

$$Z^{\rm R}_{\text{9-9}} = Z^{\rm R\ (C_2)}_{\rm NS\,NS} + Z^{\rm R\ (M_2)}_{\rm NS\,NS} + Z^{\rm R\ (C_2)}_{\rm R\,R} + Z^{\rm R\ (M_2)}_{\rm R\,R}, \tag{3·3}$$

$$
\begin{aligned}
Z_{\mathrm{NS\,NS}}^{\mathrm{NS}\ (\mathrm{C}_2)} &= \int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{NS}}\left(\exp(-Hl)\right), & (3\cdot4)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}\eta(it)^{-8}Z_0^0(it)^4, & (3\cdot5)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}(q^{-1/2}+8+O(q^{1/2})), & (3\cdot6)\\
Z_{\mathrm{NS\,NS}}^{\mathrm{NS}\ (\mathrm{M}_2)} &= \int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{NS}}\left(\Omega(1+(-1)^F)\exp(-Hl)\right), & (3\cdot7)\\
&= -inV_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}\frac{Z_1^0(2it)^4Z_0^1(2it)^4}{\eta(2it)^8Z_0^0(2it)^4}, & (3\cdot8)\\
&= -inV_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}(16+O(q)), & (3\cdot9)\\
Z_{\mathrm{R\,R}}^{\mathrm{NS}\ (\mathrm{C}_2)} &= \int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{NS}}\left((-1)^F\exp(-Hl)\right), & (3\cdot10)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}\eta(it)^{-8}\left(-Z_1^0(it)^4\right), & (3\cdot11)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}(-q^{-1/2}+8+O(q^{1/2})), & (3\cdot12)\\
Z_{\mathrm{R\,R}}^{\mathrm{NS}\ (\mathrm{M}_2)} &= 0, & (3\cdot13)\\
Z_{\mathrm{NS\,NS}}^{\mathrm{R}\ (\mathrm{C}_2)} &= -\int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{R}}\left(\exp(-Hl)\right), & (3\cdot14)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}\eta(it)^{-8}\left(-Z_0^1(it)^4\right), & (3\cdot15)\\
&= in^2V_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}(-16+O(q)), & (3\cdot16)\\
Z_{\mathrm{NS\,NS}}^{\mathrm{R}\ (\mathrm{M}_2)} &= 0, & (3\cdot17)\\
Z_{\mathrm{R\,R}}^{\mathrm{R}\ (\mathrm{C}_2)} &= -\int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{R}}\left((-1)^F\exp(-Hl)\right) = 0, & (3\cdot18)\\
Z_{\mathrm{R\,R}}^{\mathrm{R}\ (\mathrm{M}_2)} &= -\int_0^\infty \frac{dl}{8l}\,\mathrm{Tr}_{\mathrm{R}}\left(\Omega(1+(-1)^F)\exp(-Hl)\right), & (3\cdot19)\\
&= +inV_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}\frac{Z_1^0(2it)^4Z_0^1(2it)^4}{\eta(2it)^8Z_0^0(2it)^4}, & (3\cdot20)\\
&= +inV_{10}\int_0^\infty \frac{dt}{4t}(8\pi^2\alpha' t)^{-5}(16+O(q)), & (3\cdot21)
\end{aligned}
$$

where $q=e^{-2\pi t}$, and

$$Z_0^0(it) = q^{-1/24}\prod_{m=1}^{\infty}(1+q^{m-1/2})^2, \qquad (3\cdot22)$$

$$Z_1^0(it) = q^{-1/24}\prod_{m=1}^{\infty}(1-q^{m-1/2})^2, \qquad (3\cdot23)$$

$$Z_0^1(it) = 2q^{1/12}\prod_{m=1}^{\infty}(1+q^m)^2. \qquad (3\cdot24)$$

Then we have

$$Z^{\rm NS}_{9\text{-}9} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(8\cdot\frac{1}{2}n(n-1)+O(q)\right), \qquad (3\cdot25)$$

$$Z^{\rm R}_{9\text{-}9} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(-8\cdot\frac{1}{2}n(n-1)+O(q)\right). \qquad (3\cdot26)$$

The contributions of the $\overline{9}$-$\overline{9}$ strings can be obtained by replacing n by m and flipping the sign of the R-R charge of the D9-brane. In the one-loop vacuum amplitudes, we should flip the sign of $Z^{\rm R\,(M_2)}_{\rm R\,R}$, since the contribution of the R-R exchange between the D9-brane boundary state and the cross-cap state is proportional to the R-R charge of the D9-brane. Thus we obtain

$$Z^{\rm NS}_{\overline{9}\text{-}\overline{9}} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(8\cdot\frac{1}{2}m(m-1)+O(q)\right), \qquad (3\cdot27)$$

$$Z^{\rm R}_{\overline{9}\text{-}\overline{9}} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(-8\cdot\frac{1}{2}m(m+1)+O(q)\right). \qquad (3\cdot28)$$

Equation (3·27) is the contribution of the $SO(m)$ gauge fields, and (3·28) is the contribution of the massless fermions. Equation (3·28) suggests that the fermions belong to a second rank symmetric tensor representation of the gauge group $SO(m)$, as discussed in the last section. To confirm this observation in a more systematic way, note that we have taken an opposite Ω projection in the Ramond sector, which corresponds to the sign flip $Z^{\rm R\,(M_2)}_{\rm R\,R} \to -Z^{\rm R\,(M_2)}_{\rm R\,R}$. The action of Ω on the massless fermions is

$$\Omega\,|\,s\,;ij\,\rangle\,(\widetilde{\lambda}_s)_{ij} = -\,|\,s\,;ij\,\rangle\,\gamma^{-1}_{jj'}(\widetilde{\lambda}_s)_{j'i'}\gamma_{i'i}, \qquad (3\cdot29)$$

where $\gamma_{ij}=\delta_{ij}$ for $SO(m)$ theory and $\gamma_{ij}=i\mathbb{J}_{ij}$ for $USp(m)$ theory. For the Ramond sector of the $\overline{9}$-$\overline{9}$ string, we take the states with $\Omega=-1$ as the physical states, and thus (3·29) implies

$$\widetilde{\lambda} = \widetilde{\lambda}^T. \qquad (3\cdot30)$$

The one-loop vacuum amplitudes for $\overline{9}$-9 and 9-$\overline{9}$ strings are obtained by replacing n^2 with $2nm$ in the cylinder diagrams and flipping the sign of the contributions of the R-R exchange between the D9-brane and D$\overline{9}$-brane boundary states. Then we have

$$Z^{\rm NS}_{9\text{-}\overline{9},\overline{9}\text{-}9} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(mn\cdot q^{-1/2}+O(q^{1/2})\right), \qquad (3\cdot31)$$

$$Z^{\rm R}_{9\text{-}\overline{9},\overline{9}\text{-}9} = iV_{10}\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-5}\left(-8\cdot mn+O(q)\right). \qquad (3\cdot32)$$

From (3·31), we conclude that there are mn tachyon fields in the open string channel. Equation (3·32) is the contribution of mn massless fermions. It may be useful to write down the physical state conditions for 9-$\overline{9}$ and $\overline{9}$-9 strings, as given in Ref. 1). The sign flip $Z_{\rm RR}^{(\rm C_2)} \to -Z_{\rm RR}^{(\rm C_2)}$ means that we have taken an opposite GSO projection for 9-$\overline{9}$ and $\overline{9}$-9 strings, i.e., $(-1)^F = -1$. Since we have

$$\Omega^2 = (-1)^F, \tag{3·33}$$

in the NS sector, we should take the states with $\Omega = \pm i$ as physical states in the NS sector. We choose the convention $\Omega = i$ in the following. Since the action of Ω on the NS ground state is given by

$$\Omega\,|\,0; i\bar{j}\,\rangle_{\rm NS} = i\,(\gamma_{\overline{9}})_{\bar{j}\bar{j}'}\,|\,0; \bar{j}'i'\,\rangle_{\rm NS}\,(\gamma_9^{-1})_{i'i}, \tag{3·34}$$

where $i,\ i' = 1, \cdots, n$ are D9-brane Chan-Paton indices and $\bar{j},\ \bar{j}' = 1, \cdots, m$ are D$\overline{9}$-brane Chan-Paton indices. In the present case, since the gauge group is the SO group, we have $(\gamma_9)_{i'i} = \delta_{i'i}$ and $(\gamma_{\overline{9}})_{\bar{j}\bar{j}'} = \delta_{\bar{j}\bar{j}'}$. The tachyon field created by the $\overline{9}$-9 and 9-$\overline{9}$ strings are combined as

$$T = \begin{pmatrix} & T_{\bar{i}j} \\ T_{i\bar{j}} & \end{pmatrix}, \tag{3·35}$$

which is an $(n{+}m){\times}(n{+}m)$ Hermitian matrix. Imposing the physical state condition $\Omega = i$, we have

$$T^T = \gamma^{-1} T \gamma, \tag{3·36}$$

where

$$\gamma = \begin{pmatrix} \gamma_9 & \\ & \gamma_{\overline{9}} \end{pmatrix}, \tag{3·37}$$

leaving nm components as the physical tachyon.

For the Ramond sector ground states, the operator $(-1)^F$ is equivalent to the chirality operator Γ. Thus, if we take an opposite GSO projection for 9-$\overline{9}$ and $\overline{9}$-9 strings, the chirality of the fermions created by these string is opposite to the chirality of 9-9 and $\overline{9}$-$\overline{9}$ fermions. This result is consistent with the anomaly cancellation conditions discussed in the previous section.

3.2. *The R-R tadpole cancellation in the type I D9-D$\overline{9}$ system*

The R-R tadpole cancellation is one of the most important constraints in a consistent string theory. In the D9-D$\overline{9}$ system, the R-R tadpole cancellation requires the condition $n - m = 32$, which we encountered in the previous section, (2·9). Though this condition can be easily understood by counting the R-R charges of D9-branes, D$\overline{9}$-branes and an O9$^-$-plane, it would be instructive to demonstrate the explicit calculation in our framework.

The divergences due to the R-R tadpole can be extracted by the modular transformation in one-loop vacuum amplitudes $Z_{\rm RR}$. Using the identities

$$\eta(it) = t^{-1/2}\eta(i/t), \qquad Z^{\alpha}_{\beta}(it) = Z^{\beta}_{\alpha}(i/t), \tag{3·38}$$

and defining $s = \pi/t$ for the cylinder and $s = \pi/4t$ for the Möbius strip, we have

$$Z_{\mathrm{R\,R}}^{\mathrm{NS}\ (\mathrm{C}_2)} = n^2 \cdot \frac{iV_{10}}{8\pi(8\pi^2\alpha')^5} \int_0^\infty ds \left(-16 + O(e^{-2s})\right), \tag{3·39}$$

$$Z_{\mathrm{R\,R}}^{\mathrm{R}\ (\mathrm{M}_2)} = +2^6 n \cdot \frac{iV_{10}}{8\pi(8\pi^2\alpha')^5} \int_0^\infty ds \left(16 + O(e^{-2s})\right). \tag{3·40}$$

There are also the contributions from the Klein bottle diagram,

$$Z_{\mathrm{R\,R}}^{(\mathrm{K}_2)} = 2^{10} \cdot \frac{iV_{10}}{8\pi(8\pi^2\alpha')^5} \int_0^\infty ds \left(-16 + O(e^{-2s})\right). \tag{3·41}$$

The contributions from the $\overline{9}$-9, 9-$\overline{9}$ and $\overline{9}$-$\overline{9}$ strings can be obtained similarly.

The results are summarized in Table II, where we have suppressed the divergent factor

$$\frac{iV_{10}}{8\pi(8\pi^2\alpha')^5} \int_0^\infty ds \left(-16 + O(e^{-2s})\right). \tag{3·42}$$

Table II. The divergences due to R-R tadpole.

string	$Z_{\mathrm{R\,R}}^{(\mathrm{C}_2)}$	$Z_{\mathrm{R\,R}}^{(\mathrm{M}_2)}$	$Z_{\mathrm{R\,R}}^{(\mathrm{K}_2)}$
9-9 string	n^2	$-2^6 n$	
$\overline{9}$-$\overline{9}$ string	m^2	$+2^6 m$	
9-$\overline{9}$, $\overline{9}$-9 string	$-2mn$		
closed string			2^{10}

The total contribution is

$$n^2 + m^2 - 2mn - 2^6 n + 2^6 m + 2^{10} = (n - m - 32)^2, \tag{3·43}$$

which is cancelled if and only if $n - m = 32$, as expected.

3.3. *The $USp(n) \times USp(m)$ theory*

The analyses in §2 can also be applied to the case in which the gauge group is the symplectic group. The 9-9 strings will create fields in the adjoint representation of $USp(n)$, which is equivalent to the second rank symmetric tensor representation. Then, we can guess the fermions of the theory as in Table III.

The condition corresponding to (2·9) is satisfied if $m - n = 32$. In addition, if we interchange n and m, the identities (2·10)–(2·15) are also satisfied for the $USp(n) \times USp(m)$ gauge theory, and all the anomalies are cancelled in the manner as discussed in §2.1.

The interpretation in string theory is as follows. We fill the spacetime with n D9-branes and m D$\overline{9}$-branes, and we take the Sp-type Ω projection. It is easy to repeat the analyses of §§3.1 and 3.2. For example, taking into account that γ in (3·29) is $i\mathbb{J}$, the condition corresponding to (3·30) is now

$$\mathbb{J}\widetilde{\lambda} = -(\mathbb{J}\widetilde{\lambda})^T, \tag{3·44}$$

implying that the $\overline{9}$-$\overline{9}$ fermions $\widetilde{\lambda}_{ij} \equiv \mathbb{J}_{ik}\widetilde{\lambda}^k{}_j$ belong to the second rank anti-symmetric tensor representation of $USp(m)$, as expected.

Table III. Fermions in the Sp-type D9-D$\overline{9}$ system.

string	fermion	rep. of $USp(n) \times USp(m)$	chirality
9-9 string	λ	$(\square\!\square, 1)$	$+$
$\overline{9}$-$\overline{9}$ string	$\widetilde{\lambda}$	$(1, \begin{smallmatrix}\square\\\square\end{smallmatrix})$	$+$
9-$\overline{9}$, $\overline{9}$-9 string	ψ	$(\square, \square)$	$-$

Table IV. The divergences due to R-R tadpole in *Sp*-type theory.

string	$Z^{(\mathrm{C}_2)}_{\mathrm{R\,R}}$	$Z^{(\mathrm{M}_2)}_{\mathrm{R\,R}}$	$Z^{(\mathrm{K}_2)}_{\mathrm{R\,R}}$
9-9 string	n^2	$+2^6 n$	
$\overline{9}$-$\overline{9}$ string	m^2	$-2^6 m$	
9-$\overline{9}$, $\overline{9}$-9 string	$-2mn$		
closed string			2^{10}

The divergences due to the R-R tadpole are summarized in Table IV. The total contribution is again cancelled when $m - n = 32$.

We can also derive this result by counting R-R charges. The *Sp*-type Ω projection can be understood as an effect of an $\mathrm{O9}^+$-plane filling the space-time. In ordinary type I string theory, there is an $\mathrm{O9}^-$-plane, which induces the *SO*-type Ω projection, and 32 D9-branes are needed to cancel the R-R 10-form charge. In the *Sp*-type theory, however, since the sign of the R-R charge of the $\mathrm{O9}^+$-plane is opposite to that of the $\mathrm{O9}^-$-plane, we need 32 D$\overline{9}$-branes. Therefore, in the system with n D9-branes and m D$\overline{9}$-branes, we must impose the condition $m - n = 32$ to cancel the R-R charge.

§4. General formulation of the D9-D$\overline{9}$ system

4.1. *Generalization to arbitrary amplitudes*

Adding D9-D$\overline{9}$ pairs in type I or type IIB string theory can be understood as adding additional open strings in the theory. We have observed from the vacuum amplitudes that the $\overline{9}$-9 and 9-$\overline{9}$ strings have the opposite GSO projection, and the Ramond sector of the $\overline{9}$-$\overline{9}$ strings have the opposite Ω projection as the ordinary 9-9 strings. This observation should be confirmed in arbitrary amplitudes, as required by the unitarity of the *S*-matrix.

When we compute the amplitudes in superstring theory, we must sum over spin structures of the world-sheet. The spin structures are characterized by the boundary conditions for the world-sheet fermions. When we move the fermions around a non-trivial cycle of the world-sheet, the sign of the fermions can be flipped. We represent the sign flip by including a cut in the world-sheet. The cut may end at a boundary of the world-sheet or a position where a Ramond vertex operator is inserted. There are holomorphic and anti-holomorphic sectors, and the spin structures are chosen for each sector. If the world-sheet has no boundary, the spin structures for the holomorphic and anti-holomorphic sectors are chosen independently. However, if the world-sheet has boundaries, holomorphic sectors and anti-holomorphic sectors are related at the boundaries by the open string boundary conditions. Accordingly, if the cuts end at the boundary in the holomorphic sector, the same holds in the anti-holomorphic sector. We refer to the boundary, at which odd numbers of cuts end, as an "R-R boundary". In §3, we have assigned an extra minus sign for each R-R boundary with D$\overline{9}$-brane Chan-Paton indices in the vacuum amplitudes. This prescription can be easily generalized to the case with arbitrary numbers of boundaries without open string vertex operators. However, a problem may arise when there are $\overline{9}$-9 or 9-$\overline{9}$ string vertex operators at a boundary of the world-sheet. In this case, the boundary is broken into pieces with D9-brane and D$\overline{9}$-brane Chan-Paton indices, and it is ambiguous which sign should be assigned. In order to resolve this problem, we propose that an extra minus sign should be assigned for each endpoint of the cut at the boundary with D$\overline{9}$-brane Chan-Paton indices. The position of the cuts can be continuously moved without changing any physical quantities, and hence we must show that the amplitudes are not changed when we move the cut ending at the boundary across the 9-$\overline{9}$ or $\overline{9}$-9 string vertex operators (Fig. 1).

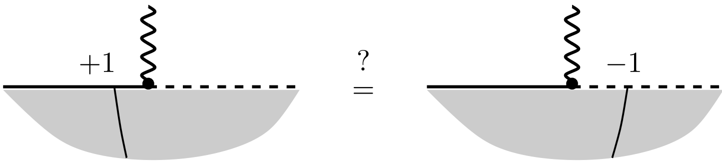
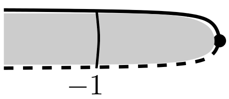


Fig. 1. The solid and dashed boundaries are equipped with D9 and D$\overline{9}$-brane Chan-Paton indices, respectively. We assign an extra minus sign for each endpoint of the cut at the D$\overline{9}$-brane boundary.

Before solving this problem, let us confirm that the 9-$\overline{9}$ and $\overline{9}$-9 strings have the opposite GSO projection. In the open string channel, making a cut parallel to the spatial direction of the open string corresponds to the insertion of the operator $(-1)^F$ in the operator formalism. If the open string is a 9-$\overline{9}$ or $\overline{9}$-9 string, one of the endpoints of the cut is at the D$\overline{9}$-boundary, and thus we should assign an extra -1 factor. Therefore, in our prescription, making a cut parallel to the spatial direction of the 9-$\overline{9}$ or $\overline{9}$-9 string corresponds to the insertion of $-(-1)^F$ in the operator formalism, as desired (Fig. 2).



$$\mathcal{V}_T \quad = \quad -1 \times \left\langle \quad \cdots \quad (-1)^F\, \mathcal{V}_T \right\rangle .$$

Fig. 2.

Now consider the cut ending at the D9-boundary, as illustrated in the left-hand side of Figs. 1 and 3. This cut can be deformed as in the right-hand side of Fig. 3.

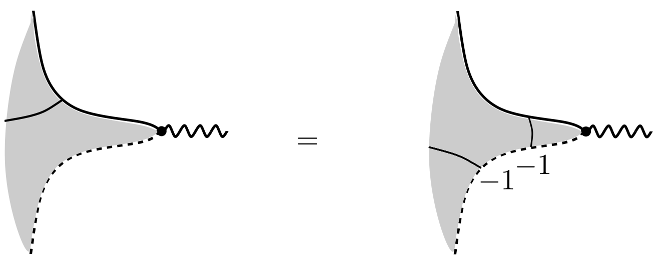
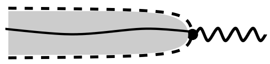


Fig. 3.

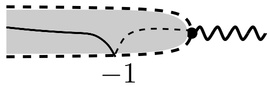
Then, if the 9-$\overline{9}$ string vertex operator in the figure is projected to satisfy the physical condition $-(-1)^F = 1$, the amplitude is equivalent to the right-hand side of Fig. 1.

This is the desired result in order for the amplitudes to be invariant under continuous deformations of the cut in the world-sheet.

Next we wish to reconfirm that the Ramond sector of the $\overline{9}$-$\overline{9}$ string has the opposite Ω projection. The open string vertex operator in the Ramond sector creates a cut in the world-sheet of either the holomorphic or anti-holomorphic sector.*) Let us consider the case in which the vertex operator $\mathcal{V}_R$ creates a cut in the holomorphic sector. Then $\Omega\,\mathcal{V}_R$ will create a cut in the anti-holomorphic sector. In order to connect the cut consistently, it should end at the boundary, and hence we need an extra -1 factor (Fig. 4). Summing up these terms, we have the projection $\Omega = -1$, as desired.



$$= \langle \;\cdots\; \mathcal{V}_R \rangle\,.$$



$$= -1 \times \langle \;\cdots\; \Omega\,\mathcal{V}_R \rangle\,.$$

Fig. 4. The solid and dashed cuts in the world-sheet are the cuts in the holomorphic and anti-holomorphic sectors, respectively.

Note that the cut created by the Ramond sector vertex operator could be taken in the anti-holomorphic sector. Then the sign of the amplitude will be flipped. But this is not a problem, since the overall sign of the amplitude is unphysical.

4.2. *Anomaly cancellations in string theory*

To confirm our prescription, let us show that the gauge anomaly is cancelled in the D9-D$\overline{9}$ system. There are three types of diagrams that contribute to the anomaly,[7),5)] planer, non-orientable, and non-planer orientable diagrams, as depicted in Figs. 5–7.

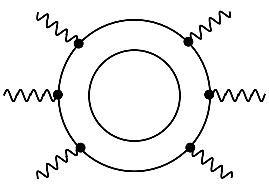


Fig. 5. Planer.

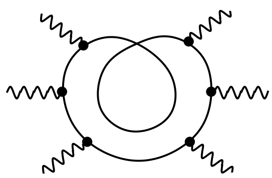


Fig. 6. Non-orientable.

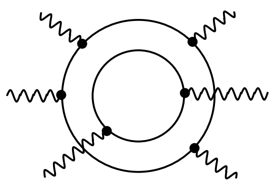


Fig. 7. Non-planer.

We consider these diagrams as open string one-loop diagrams. Then, only the Ramond sector of the open strings with the $(-1)^F$ insertion will contribute to the anomaly, since the parity-conserving terms are not anomalous. As explained in Refs. 7) and 5), the non-planer diagrams are not divergent and do not contribute to the anomaly. Hence we consider the sum of the contribution from the planer and non-orientable diagrams. Let us consider the case that all the external gauge

*) If necessary, we deform the cut so that it does not lie along the boundary of the world-sheet.

fields are created by 9-9 strings. The case with $\overline{9}$-$\overline{9}$ gauge fields can be treated similarly. The $(-1)^F$ insertion will create a cut in the world-sheet, and hence we need an extra -1 factor for each D$\overline{9}$-brane boundary in the planer diagrams. Thus the contributions from the planer diagrams are proportional to $n-m$, where n and m are the numbers of D9 and D$\overline{9}$-branes, respectively. The explicit calculations are exactly the same as those given in Ref. 7), except for this factor. The result is

$$\mathcal{A} = (\,n - m + 32\,l\,)\,G, \tag{4·1}$$

where G is a non-vanishing factor, for which we do not need detailed structure, and l is given as

$$l = \begin{cases} +1, & USp(n), \\ \ \ 0, & U(n), \\ -1, & SO(n). \end{cases} \tag{4·2}$$

As a result, the anomaly is cancelled when $n-m = 32\,l$; namely, $n = m$ for the type IIB D9-D$\overline{9}$ system, $n - m = 32$ for the type I D9-D$\overline{9}$ system, and $m - n = 32$ for the D9-D$\overline{9}$ system of USp-type theory.

§5. Discussion of the $USp(32)$ theory

As the D9-D$\overline{9}$ pairs are expected to vanish after the tachyon condensation, it is interesting to examine the case with $m = 32$ and $n = 0$, which is the tachyon-free case. This theory contains closed strings and open strings with $USp(32)$ Chan-Paton indices. The formulation is almost the same as that of the type I $SO(32)$ string theory, except we take the opposite Ω projection for the Ramond sector of the open strings, and accordingly, we associate an extra minus sign for each R-R boundary of the world-sheet in calculating the amplitudes.

Unlike the type I $SO(32)$ theory, the $USp(32)$ theory does not have a space-time supersymmetry, since the fermion λ, created by the open strings, belongs to the second rank anti-symmetric tensor representation of $USp(32)$ and cannot be a supersymmetric partner of the $USp(32)$ gauge field. Therefore, there is no reason for the vacuum energy to be cancelled. Indeed, there is a divergence in the vacuum amplitude due to the NS-NS tadpole, that needs to be cancelled by the Fischler-Susskind mechanism.[8)] This induces the cosmological constant term $\sqrt{-g}\,e^{-\phi}$ in the effective action.

The K-theory analyses given in Ref. 2) suggest that there are D1, 3, 4, 5, 9-branes in this theory (Table V). The analyses in Ref. 9) show that D1-branes and D5-branes have Sp and SO-type Chan-Paton indices, respectively. We can also understand this fact using the isomorphism of K-theory groups $KSp(\mathbb{R}^n) \simeq KO(\mathbb{R}^{n\pm4})$. Suppose

Table V. D-branes in the $USp(32)$ theory.

k	10	9	8	7	6	5	4	3	2	1	0
$KSp(\mathbb{R}^k)$			$\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$				$\mathbb{Z}$
p-brane	-1	0	1	2	3	4	5	6	7	8	9

Table VI. D-branes in the D5-$\overline{\text{D5}}$ system.

k	6	5	4	3	2	1	0
$KO(\mathbb{R}^k)$			$\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
p-brane	-1	0	1	2	3	4	5

that we wish to construct lower dimensional D-branes in the D5-$\overline{\text{D5}}$ system, as given in Refs. 1) and 2). In order to obtain the same D-branes as in Table V, the suitable K-group for the D5-$\overline{\text{D5}}$ system is $KO(\mathbb{R}^k)$, rather than $KSp(\mathbb{R}^k)$ (Table VI). Thus, we conclude that the Chan-Paton indices for D5-branes are of the SO-type. A similar argument for D1-branes shows that D1-branes have Sp-type Chan-Paton indices.

The analysis of the spectrum of open strings ending on the D1-branes is similar to the ordinary type I D-strings, which is given in Ref. 10). Consider n D1-branes lying along the x^0, x^9 direction in the $USp(32)$ string theory. Note that n should be even, since the gauge group on the D1-branes is $USp(n)$.

1-1 strings in the NS sector create an $USp(n)$ gauge field A_μ (which can be gauged away) and eight massless scalar fields X^i $(i = 1, \cdots, 8)$, which belong to the second rank anti-symmetric tensor representation of the gauge group. World-sheet massless fermions created by the 1-1 strings in the Ramond sector are S^a_+ and $S^{\hat{a}}_-$ $(a, \hat{a} = 1, \cdots, 8)$. The subscripts $+$ and $-$ represent the chirality of the world-sheet Lorentz group $SO(1,1)$, and the superscripts a and $\hat{a}$ are indices of the 8_s and 8_c spinor representations of the transverse spacetime Lorentz group $SO(8)$. S^a_+ is a right-moving fermion that belongs to the adjoint representation of $USp(n)$, while $S^{\hat{a}}_-$ is a left-moving fermion that belongs to the second rank anti-symmetric tensor representation of the gauge group. 1-$\overline{9}$ and $\overline{9}$-1 strings will create a left moving fermion λ^I_- $(I = 1, \cdots, 32)$, which belongs to the fundamental representation of $USp(n)$. The superscript I comes from the Chan-Paton indices associated with 32 D$\overline{9}$-branes.

Table VII. Massless spectrum on D1-branes.

	$USp(n)$	$SO(8)$	$USp(32)$
A_μ	$\square\!\square$	1	1
X^i	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	8_v	1
S^a_+	$\square\!\square$	8_s	1
$S^{\hat{a}}_-$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	8_c	1
λ^I_-	$\square$	1	$\square$

Let us consider the minimal case $n = 2$. In this case, the gauge group is $USp(2) \simeq SU(2)$, and the second rank anti-symmetric tensor representation is a singlet. The action is

$$S = \int d^2\sigma \left(-\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu X^i)^2 + S^a_+[D_-, S^a_+] + S^{\hat{a}}_- \partial_+ S^{\hat{a}}_- + \lambda^I_- D_+ \lambda^I_- \right), \tag{5·1}$$

where we have defined $D_\pm = \partial_\pm + iA_\pm$. In analogy to the argument in S-duality for type I and heterotic string theory,[10] it has been suggested that this action is the action of the fundamental string in the heterotic version of $USp(32)$ string theory. It would be interesting to investigate the detailed structure of this theory.

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