eprints@whiterose.ac.uk
https://eprints.whiterose.ac.uk/

# Two-dimensional Antenna Array Design for Directional Modulation

Bo Zhang[1], Wei Liu[2], and Cheng Wang[1*]

[1]Tianjin Key Laboratory of Wireless Mobile Communications and Power Transmission, College of Electronic and Communication Engineering, Tianjin Normal University, Tianjin, 300387, China
(e-mail: b.zhangintj@tjnu.edu.cn; cwang@tjnu.edu.cn)
[2]Communications Research Group, Department of Electronic and Electrical Engineering, University of Sheffield, Sheffield S1 4ET, United Kingdom
(e-mail: w.liu@sheffield.ac.uk)

**Abstract.** Directional modulation (DM) has been applied to linear antenna arrays to increase security of signal transmission. However, only the azimuth angle is considered in the design, due to inherent limitation of the linear array structure, since linear antenna array lacks the ability to scan in the three dimensional (3-D) space. To solve the problem, planar antenna arrays are introduced in the design, where both the elevation angle and azimuth angle are considered. Moreover, a magnitude constraint for weight coefficients is introduced. Design examples are provided to verify the effectiveness of the proposed design.

**Keywords:** Directional modulation, magnitude constraint, planar antenna array

## 1 Introduction

Directional modulation (DM) as a physical layer security technique was introduced to keep known constellation mappings in a desired direction or directions, while scrambling them for the remaining ones [1]. In [2], a four-element reconfigurable array was designed, and the DM design can be achieved by changing elements for each symbol. Then, the genetic algorithm based on phased antenna array was introduced to DM [3], where the same carrier frequency was used for all antennas. By changing the weight coefficients properly for each symbol, DM design can be achieved, and its low bit error rate (BER) range is narrower than traditional beamforming design. In [4], directional antennas were used in the design to replace isotropic antennas, and the provided examples show that a narrower low BER range is achieved. Moreover, to solve the problem that both the eavesdroppers and the desired users will receive the same signal when they are in the same direction of the antenna array, two positional modulation (PD) schemes were proposed. One introduces a reflecting surface [5], where the multipath effect is exploited, and signals via both line of sight (LOS) and reflected paths are combined at the receiver side. The other is to use multiple antenna

arrays [6], and the principle of the design is that the eavesdropper located in the same direction as the desired user for one antenna array may not be in the same direction for another antenna array. To increase the capacity of the DM system, a multi-carrier based phased antenna array design was proposed, employing an inverse Discrete Fourier Transform (IDFT) structure [7, 8]. Another solution is to use crossed-dipole antenna array as the transmitter [9], where DM and polarisation were combined together in the proposed design. A method named dual beam DM was introduced in [10]. Different from the traditional design where inphase and quadrature (IQ) components of signals are transmitted by the same antenna, dual beam DM design is to transmit these two components by different antennas. In [11], the BER was employed for DM transmitter synthesis by linking the BER performance to the settings of phase shifters. A pattern synthesis approach was presented in [12, 13], where information pattern and interference patterns are created together to achieve DM, followed by an artificial-noise-aided zero-forcing synthesis approach in [14], and a multi-relay design in [15]. An eight-element time-modulated antenna array with constant instantaneous directivity in desired directions was proposed in [16]. The main idea of the design is that the array transmits signals without time modulation in the desired direction, while transmitting time-modulated signals in other directions.

Recently, the introduction of artificial noise has further advanced the directional modulation technology. Artificial noise (AN) can be divided into 'static' AN and 'dynamic' AN. Static AN means that the introduced AN vector is fixed, so that the constellation points for the received signal in undesired directions do not change with time. As a result, after a long period of observation, it is possible for eavesdroppers to crack the received signal. To solve this problem, 'dynamic' AN is introduced, where the AN vector is continuously updated, and the constellation of the signal in the undesired direction changes constantly, increasing the difficulty of the eavesdroppers to decode the signal correctly. For the construction of AN, two methods were introduced. One is the orthogonal vector method [17, 18], where the added AN vector is orthogonal to the steering vector of the desired direction. The other one is the AN projection matrix method [19, 20], where by designing an artificial noise projection matrix, the AN vector is projected into the zero space of the derivative of the desired direction.

However, to our best knowledge, almost all of the existing studies are focused on one dimensional DM, which is normally achieved using linear antenna arrays, and these designs lack the ability to scan in the 3-D space. For effective DM in the 3-D space, in this work, we introduce a planar antenna array based design for two-dimensional DM, where both the elevation angle and azimuth angle are studied.

The remaining part of this paper is structured as follows. A review of planar antenna array based beamforming is given in Sec. 2. DM design for a uniform planar antenna array with the corresponding formulations is presented in Sec. 3. In Sec. 4, design examples are provided, with conclusions drawn in Sec. 5.

## 2   Review of Planar Antenna Array Based Beamforming

A narrowband planar antenna array for transmit beamforming is shown in Fig. 1, which consists of $N$ equally spaced omni-directional antennas along the x-axis, and $K$ equally spaced omni-directional antennas along the y-axis. The spacings from the first antenna to its subsequent antennas along the x-axis and y-axis are represented by $d_{x,n}$ and $d_{y,k}$, respectively for $n = 0, \ldots, N-1$ and $k = 0, \ldots, K-1$. The elevation angle $\theta \in [0°, 180°]$, and azimuth angle $\phi \in [0°, 180°] \cup [0°, -180°]$. The weight coefficient for the antenna on the $n$-th position of the axis and $k$-th position of the y-axis is denoted by $w_{n,k}$ ($n = 0, \ldots, N-1$ and $k = 0, \ldots, K-1$). The steering vector of the array as a function of angular frequency $\omega$, elevation angle $\theta$ and azimuth angle $\phi$, is given by

$$\mathbf{s}(\omega, \theta, \phi) = [1, e^{j\omega(d_{x,0}\sin\theta\cos\phi + d_{y,0}\sin\theta\sin\phi)/c}, \ldots,$$
$$e^{j\omega(d_{x,0}\sin\theta\cos\phi + d_{y,K-1}\sin\theta\sin\phi)/c}, \ldots, \tag{1}$$
$$e^{j\omega(d_{x,N-1}\sin\theta\cos\phi + d_{y,K-1}\sin\theta\sin\phi)/c}]^T,$$

where $\{\cdot\}^T$ is the transpose operation, and $c$ is the speed of propagation. For a uniform planar array (UPA) with a half-wavelength spacing ($d_{x,n} - d_{x,n-1} = \lambda/2$ and $d_{y,k} - d_{y,k-1} = \lambda/2$), the steering vector of the UPA is

$$\mathbf{s}(\omega, \theta, \phi) = [1, e^{j\pi(\sin\theta\cos\phi + \sin\theta\sin\phi)}, \ldots,$$
$$e^{j\pi(\sin\theta\cos\phi + (K-1)\sin\theta\sin\phi)}, \ldots, \tag{2}$$
$$e^{j\pi((N-1)\sin\theta\cos\phi + (K-1)\sin\theta\sin\phi)}]^T.$$

All weight coefficients can be put together to form a vector represented by $\mathbf{w}$,

$$\mathbf{w} = [w_{x_0,y_o}, w_{x_0,y_1}, \ldots, w_{x_0,y_{K-1}}, \ldots, w_{x_{N-1},y_{K-1}}]^T. \tag{3}$$

Then the beam response of the array is given by

$$p(\omega, \theta, \phi) = \mathbf{w}^H \mathbf{s}(\omega, \theta, \phi), \tag{4}$$

where $\{\cdot\}^H$ represents the Hermitian transpose.

## 3   DM Design for the Uniform Planar Antenna Array

DM design is to keep the received signal following known constellation mappings in a desired direction or directions, while scrambling the phase and make the magnitude as low as possible for the rest of directions. The method to achieve DM is to find the corresponding weight vector for each symbol. For M-ary signaling, such as multiple phase shift keying (MPSK), we assume the corresponding weight vector is given by

$$\mathbf{w}_m = [w_{m,x_0,y_0}, w_{m,x_0,y_1}, \ldots, w_{m,x_0,y_{K-1}}, \ldots, w_{m,x_{N-1},y_{K-1}}]^T, \tag{5}$$
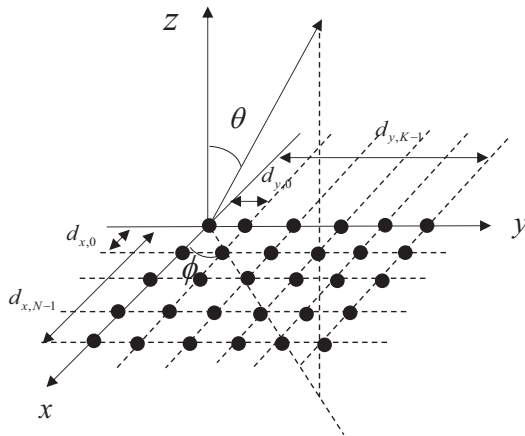
**Fig. 1.** An equally spaced planar array.

$m = 0, \ldots, M - 1$. The desired response $p_m(\omega, \theta, \phi)$ for the $m$-th constellation point, as a function of $\theta$ and $\phi$ is split into two regions: the mainlobe response and the sidelobe response, represented by $\mathbf{p}_{m,ML}$ and $\mathbf{p}_{m,SL}$, respectively. Without loss of generality, we assume there are R elevation angles sampled for each azimuth angle $\phi_v$ ($v = 0, 1, \ldots, V - 1$), and the desired directions in the 3-D space is $\theta_0, \theta_1, \ldots, \theta_{r-1}$ and $\phi_0$. Then, we have

$$\mathbf{p}_{m,ML} = [p_m(\omega, \theta_0, \phi_0), p_m(\omega, \theta_1, \phi_0), \ldots, p_m(\omega, \theta_{r-1}, \phi_0)],$$
$$\mathbf{p}_{m,SL} = [p_m(\omega, \theta_r, \phi_0), p_m(\omega, \theta_{r+1}, \phi_0), \ldots, p_m(\omega, \theta_{R-1,\phi_0}), p_m(\omega, \theta_0, \phi_1), \quad (6)$$
$$\ldots, p_m(\omega, \theta_{R-1}, \phi_1), \ldots, p_m(\omega, \theta_{R-1}, \phi_{V-1})].$$

As shown in (1), the steering vector of the array with a fixed $\theta$ and $\phi$ is the same for all $M$ constellation points. Therefore, we have steering matrix $\mathbf{S}_{SL}$ for sidelobe regions and $\mathbf{S}_{ML}$ for mainlobe directions,

$$\mathbf{S}_{ML} = [\mathbf{s}(\omega, \theta_0, \phi_0), \mathbf{s}(\omega, \theta_1, \phi_0), \ldots, \mathbf{s}(\omega, \theta_{r-1}, \phi_0)],$$
$$\mathbf{S}_{SL} = [\mathbf{s}(\omega, \theta_r, \phi_0), \mathbf{s}(\omega, \theta_{r+1}, \phi_0), \ldots, \mathbf{s}(\omega, \theta_{R-1,\phi_0}), \mathbf{s}(\omega, \theta_0, \phi_1), \quad (7)$$
$$\ldots, \mathbf{s}(\omega, \theta_{R-1}, \phi_1), \ldots, \mathbf{s}(\omega, \theta_{R-1}, \phi_{V-1})].$$

For the $m$-th constellation point, its corresponding weight coefficients can be obtained by solving the following linearly constrained optimisation problem

$$\begin{aligned} \min \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2 \\ \text{subject to} \quad & \mathbf{w}_m^H \mathbf{S}_{ML} = \mathbf{p}_{m,ML}, \end{aligned} \quad (8)$$

where $||\cdot||_2$ denotes the $l_2$ norm. The cost function in (8) is to keep the minimum difference between desired and designed sidelobe responses, and the equality constraint is to make sure that the response in the mainlobe directions exactly

takes the specified constellation values. Here, we set the desired phase response in sidelobe regions randomly and the beam responses as low as possible ($\mathbf{p}_{m,SL}$) to keep the received signal scrambled in the IQ complex plane.

Moreover, to restrain the maximum value of weight coefficient, we introduce the corresponding constraint

$$||\mathbf{w}_m||_\infty \leq \beta, \tag{9}$$

where $|| \cdot ||_\infty$ represents the L-infinity norm, and $\beta$ is the pre-defined maximum value for weight coefficients. Therefore, the DM design with the weight coefficient magnitude constraint is given by

$$
\begin{aligned}
\min \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2 \\
\text{subject to} \quad & \mathbf{w}_m^H \mathbf{S}_{ML} = \mathbf{p}_{m,ML} \\
& ||\mathbf{w}_m||_\infty \leq \beta.
\end{aligned}
\tag{10}
$$

The above problem can be solved using cvx in MATLAB, a package for specifying and solving convex problems [21, 22].
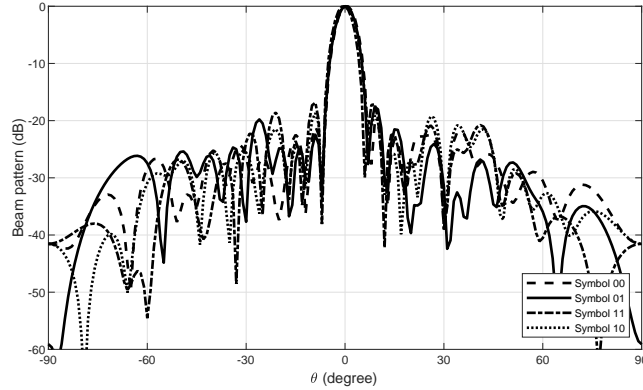
## 4 Design Examples



**Fig. 2.** Resultant beam responses based on the UPA design in (10).

In this section, we consider an $N \times K = 21 \times 20$ uniform planar antenna array with a half wavelength spacing between adjacent antennas. Without loss of generality, the desired elevation angle is $0°$ and azimuth angle is $\phi = 90°$. The sidelobe regions are $\theta_{SL} \in [5°, 90°]$ for $\phi = \pm 90°$. The desired response in the mainlobe direction is a value of one (magnitude) with $90°$ phase shift (QPSK), i.e.,

$$\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} \tag{11}$$
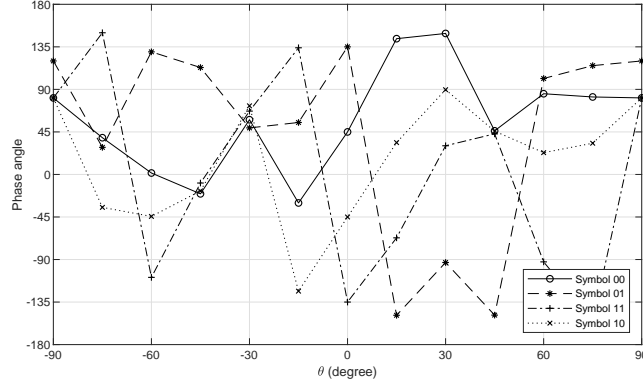
**Fig. 3.** Resultant phase responses based on the UPA design in (10).

for symbols '00', '01', '11', '10', and a value of 0.1 (magnitude) with random phase shifts over the sidelobe regions. The maximum value of weight coefficient $\beta = 0.1$. Bit error rate (BER) is also calculated based on in which quadrant the
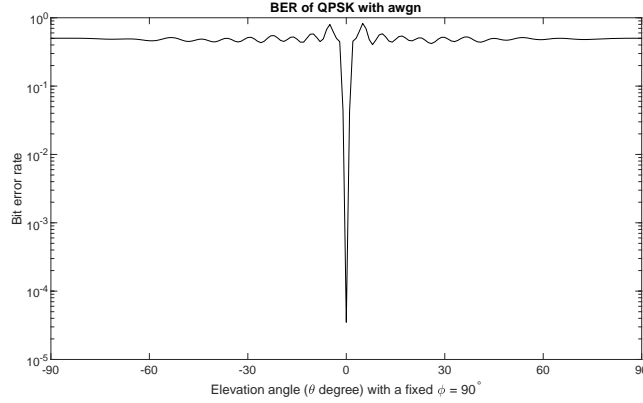


**Fig. 4.** BER based on the UPA design in (10).

received signal lies in the IQ complex plane,

$$BER = \frac{Error\ bits}{Total\ number\ of\ bits}. \tag{12}$$

Here the signal to noise ratio (SNR) is set at 12 dB in the mainlobe direction, and then with the unit average power of all randomly generated $10^6$ transmitted bits in the mainlobe, the noise variance $\sigma^2$ is 0.0631. We also assume that the additive white Gaussian noise (AWGN) level is the same for all directions, and a random noise with this power level is generated for each direction.

The resultant beam pattern in (10) for each constellation point is shown in Fig. 2. Here we can see that all main beams are exactly pointed to the desired direction $0°$ with a low sidelobe level. As shown in Fig. 3, the phase in the desired direction $0°$ is $90°$ spaced, i.e., $45°$, $135°$, $−135°$ and $−45°$ for symbols '00', '01', '11' and '10', respectively, whereas the phase is random in the sidelobe directions. Moreover, Fig. 4 shows the BER for all transmission angles. It can be seen that in the desired direction BER is down to $10^{-5}$, while it is around 0.5 in other directions, further demonstrating the effectiveness of the design.

## 5 Conclusions

Directional modulation has been applied to uniform planar antenna arrays for the first time, and two-dimensional directional modulation has been achieved effectively by the proposed design method. As shown in the provided design examples, the mainlobe is pointing to the desired direction, with a low power level for the rest of the directions; simultaneously, the transmitted signal's phase in the desired direction follows the required constellations, whereas its values in other directions are scrambled. The BER result shows that error bits received in the desired direction is the lowest, while in other directions the BER is about 0.5, indicating that it would be extremely difficult for eavesdroppers located in these regions to crack the information.

## References

1. Babakhani, A., Rutledge, D.B.,Hajimiri, A.: Near-field direct antenna modulation. *IEEE Microwave Magazine*, vol. 10, no. 1, pp. 36–46, February 2009.
2. Daly, M.P., Bernhard, J.T.: Beamsteering in pattern reconfigurable arrays using directional modulation. *IEEE Transactions on Antennas and Propagation*, vol. 58, no. 7, pp. 2259–2265, March 2010.
3. Daly, M.P., Bernhard, J.T.:, Directional modulation technique for phased arrays. *IEEE Transactions on Antennas and Propagation*, vol. 57, no. 9, pp. 2633–2640, September 2009.
4. Shi, H.Z., Tennant, A.: Enhancing the security of communication via directly modulated antenna arrays. *IET Microwaves, Antennas & Propagation.* vol. 7, no. 8, pp. 606–611, June 2013.
5. Zhang, B., Liu, W.: Antenna array based positional modulation with a two-ray multi-path model. In *Proc. Sensor Array and Multichannel signal processing workshop 2018 (SAM2018)*. Sheffield, UK, pp. 203-207, July 2018.

6. Zhang, B., Liu, W.: Positional modulation design based on multiple phased antenna arrays. *IEEE Access*, vol. 7, pp. 33 898–33 905, 2019.

7. Zhang, B., Liu, W.: Multi-carrier based phased antenna array design for directional modulation. *IET Microwaves, Antennas Propagation*, vol. 12, no. 5, pp. 765–772, April 2018.

8. Zhang, B., Liu, W., Li, Q.: Multi-carrier waveform design for directional modulation under peak to average power ratio constraint. *IEEE Access*, pp. 1–1, 2019.

9. Zhang, B., Liu, W., Lan, X.: Orthogonally polarized dual-channel directional modulation based on crossed-dipole arrays. *IEEE Access*, vol. 7, pp. 34 198–34 206, 2019.

10. Hong, T., Song, M.Z., Liu, Y.: Dual-beam directional modulation technique for physical-layer secure communication. *IEEE Antennas and Wireless Propagation Letters*, vol. 10, pp. 1417–1420, December 2011.

11. Ding, Y., Fusco, V.: Directional modulation transmitter synthesis using particle swarm optimization. in *Proc. Loughborough Antennas and Propagation Conference*, Loughborough, UK, November 2013, pp. 500–503.

12. Ding, Y., Fusco, V.: Directional modulation transmitter radiation pattern considerations. *IET Microwaves, Antennas & Propagation*, vol. 7, no. 15, pp. 1201–1206, December 2013.

13. Ding, Y., Fusco, V.: Directional modulation far-field pattern separation synthesis approach. *IET Microwaves, Antennas Propagation*, vol. 9, no. 1, pp. 41–48, 2015.

14. Xie, T., Zhu, J., Li, Y.: Artificial-noise-aided zero-forcing synthesis approach for secure multi-beam directional modulation. *IEEE Communications Letters*, vol. PP, no. 99, pp. 1–1, 2017.

15. Zhu, W., Shu, F., Liu, T., Zhou, X., Hu, J., Liu, G., Gui, L., Li, J., Lu, J.: Secure precise transmission with multi-relay-aided directional modulation. In *2017 9th International Conference on Wireless Communications and Signal Processing (WCSP)*, October 2017, pp. 1–5.

16. Zhu, Q.J., Yang, S.W., Yao, R.L., Nie, Z.P,: Directional modulation based on 4-D antenna arrays. *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 2, pp. 621–628, February 2014.

17. Ding, Y., Fusco, V.: A vector approach for the analysis and synthesis of directional modulation transmitters. *IEEE Transactions on Antennas and Propagation*, vol. 62, no. 1, pp. 361–370, January 2014.

18. Ding, Y., Fusco, V.: Vector representation of directional modulation transmitters. In *The 8th European Conference on Antennas and Propagation (EuCAP 2014)*, April 2014, pp. 367–371.

19. Hu, J., Shu, F., and Li, J.: Robust synthesis method for secure directional modulation with imperfect direction angle. *IEEE Communications Letters*, vol. 20, no. 6, pp. 1084–1087, June 2016.

20. Hu, J., Yan, S., Shu, F., Wang, J., Li, J., Zhang, Y.,: Artificial-noise-aided secure transmission with directional modulation based on random frequency diverse arrays. *IEEE Access*, vol. 5, pp. 1658–1667, 2017.

21. Grant, M., Boyd, S.: Graph implementations for nonsmooth convex programs. In *Recent Advances in Learning and Control*, ser. Lecture Notes in Control and Information Sciences, V. Blondel, S. Boyd, and H. Kimura, Eds.   Springer-Verlag Limited, 2008, pp. 95–110, `http://stanford.edu/~boyd/graph$\_$dcp.html`.

22. CVX Research: CVX: Matlab software for disciplined convex programming, version 2.0 beta. `http://cvxr.com/cvx`, September 2012.