Antenna-Pattern Diversity Versus Space Diversity for Use at Handhelds

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***Abstract*—This paper investigates diversity for dual-antenna systems operating in indoor environments. First, an approximated equation of the diversity gain is derived for different combining techniques. These theoretical results show that the two-term approximation, as generally used in the literature [5], is too rough an estimate. Consequently, a new six-term approximation is derived. Next, it is demonstrated by a comparison of theoretical and experimental diversity gain values that, due to mutual coupling between the two antennas in practice, the diversity gain will not approach 0 dB if the distance between the two antennas approaches zero. Finally, it is concluded from measurements at 900 MHz that antenna-pattern diversity is a better choice than space diversity for use at handhelds.**

***Index Terms*—Correlated fading, correlation, diversity gain, maximal ratio combining, pattern diversity, selection combining.**

I. Introduction

NOWADAYS, mobile communication plays a very important role in human life. The number of mobile applications is still growing, e.g., global system for mobile communications (GSM), short message service (SMS), mobile-Internet, etc. Unfortunately, the quality of the mobile radio link can be poor, particularly in urban environments. In such environments, severe multipath propagation may occur. This propagation phenomenon affects the signal-to-noise ratio (SNR) and introduces signal distortion. This altogether results in a poor audio signal for analog systems and a high bit-error rate for digital communications systems. In order to diminish the vulnerability of mobile systems against the destructive interference of multipath radiowaves, diversity techniques can be applied. There are several forms of diversity, such as space, frequency, time, and antenna-pattern diversity. The traditional space diversity uses multiple antennas separated by a distance d from each other.

In diversity systems, there generally are three linear methods that are used to combine the antenna signals, viz.,

- selection combining,
- maximal ratio combining,
- equal gain combining.

Since selection and maximal ratio combining yield the lower and upper bound for the improvement of system performance, these two combining techniques will be discussed in this paper. A figure of merit for the improvement of diversity receivers can be the diversity gain.

It strongly depends on the correlation between the antenna signals; see also [12]. A high-diversity gain asks for a low signal correlation. Therefore, this paper starts with a theoretical analysis of the diversity gain as a function of signal correlation (Section II). It will be demonstrated that the two-term approximation that is often made in the derivation of a closed-form expression for the diversity gain is inaccurate. Therefore, a new M-terms approximation will be derived, which gives more accurate results. Subsequently, a measurement system is described to obtain the diversity gain as a function of experimental antenna spacing in an indoor environment at 900 MHz (Section III). These practical diversity gain values will be compared with the theoretical values.

Since in handhelds the antenna spacing has to be relatively small, the correlation between the received antenna signals might be very high, resulting in a lower diversity gain. This problem can be solved by using antenna-pattern diversity. Vaughan [8] proposed switched parasitic elements to realize antenna-pattern diversity. In his configuration, the parasitic elements can either be terminated to ground or left open. In this paper, the parasitic element will be terminated with different impedances in such a manner that the diversity gain is optimized. Switching between different impedances results in different far-field radiation patterns. In this way, it is possible to maximize diversity gain by taking advantage of the mutual coupling between closely spaced antenna elements. In this paper, the design of such an antenna will be described (Section IV). Performance of the novel antenna-pattern diversity and the classical space diversity for different antenna spacings will be compared (Section V). Finally, the conclusions are summarized (Section VI).

II. Analysis of the Diversity Gain

In order to improve the reception in mobile radio links, multiple antennas can be used. This may be a system with multiple antennas at the base station as well as multiple antennas at the handheld. In this paper, only antenna diversity at the handheld will be considered and the handheld will be called "receiver" throughout this paper. Although there are more techniques to improve the reception, such as coding or advanced modulation schemes, these techniques will not be covered in this paper. Once using multiple receive antennas, the question of what is the improvement compared to a single receive antenna arises. Diversity gain is a well-known figure of merit to show the improvement of the diversity receiver. Intuitively, the diversity gain is maximal when the different received

Manuscript received May 21, 2001; revised January 15, 2003, December 9, 2003, February 4, 2004, and February 20, 2004.

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Digital Object Identifier 10.1109/TVT.2004.830156

antenna signals are completely uncorrelated and are minimal when the opposite holds. Signal variation in the first situation mentioned is generally denoted as uncorrelated fading [5], [15]. More often, the correlation coefficient will be higher than zero, which is denoted as correlated fading. The behavior of correlated fading on the statistics of the radio channel has been investigated previously in [1], [3], [5], and [15]. They made the assumption that the fading at each diversity branch is Rayleigh distributed and that the SNR is the same for all branches. Furthermore, the angles-of-arrival (AoAs) of the multipath waves are supposed to be uniformly distributed. Using these assumptions, the cumulative distribution functions (cdfs) of the SNR can be derived. For a two-branch receiver, the cdfs of SNR are derived next [15].

In the case of the selection combining method

$$P_s(\gamma \leqslant \gamma_s) = 1 - \exp\left(-\frac{\gamma_s}{\Gamma}\right)[1 - Q(a,b) + Q(b,a)] \quad (1)$$

where

$$Q(a,b) = \int_b^\infty \exp\left(-\frac{1}{2}(a^2+x^2)\right) I_0(ax)x\,dx$$
$$b = \sqrt{\frac{2\gamma_s}{\Gamma(1-\rho^2)}}, \qquad a = b\rho. \quad (2)$$

Properties of the Q-function can be found in Appendix I.

In the case of the maximal ratio combining method

$$P_r(\gamma \leqslant \gamma_r) = 1 - \frac{1}{2\rho}\left[(1+\rho)\exp\left(-\frac{\gamma_r}{\Gamma(1+\rho)}\right) - (1-\rho)\exp\left(-\frac{\gamma_r}{\Gamma(1-\rho)}\right)\right] \quad (3)$$

where

$$\gamma \triangleq \frac{\text{momentarily mean signal power per branch}}{\text{mean noise power per branch}} = \frac{r^2}{2N}$$

is the instantaneous SNR and

$$\Gamma \triangleq \frac{\text{mean signal power per branch}}{\text{mean noise power per branch}} = \mathrm{E}[\gamma] = \frac{\sigma^2}{N}$$

is the time average SNR and $\mathrm{E}[\cdot]$ denotes the expectation operator and ρ stands for the magnitude of the complex cross-covariance of the two fading Gaussian signals. This means that ρ^2 approximates the normalized envelope covariance of the two signals.

As a reference, we consider Rayleigh fading in a single-receiver system; the γ's are exponentially distributed. Therefore, the probability density function (pdf) of γ is given by

$$p(\gamma) = \frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right) \quad (4)$$

and the associated cdf is

$$P(\gamma \leqslant \gamma_o) = 1 - \exp\left(-\frac{\gamma_o}{\Gamma}\right). \quad (5)$$

Now that the probability functions are known, it is possible to derive formulas for the *diversity gain* (DG). Consider a certain outage $P(\gamma \leqslant \gamma_o) = \alpha$ with which the γs are below a certain threshold γ_o. Then, it is possible to calculate the γ_o/Γ's. Without diversity, this will lead to

$$\gamma_o = -\Gamma\log(1-\alpha). \quad (6)$$

In order to calculate the γ_ss in case of selection combining, the inverse of (1) has to be calculated. Unfortunately, no closed-form expression for the inverse function could be found. Therefore, the power-series expansion of (1) will be used to derive the inverse function (see Appendix II), which is

$$P_s(\gamma \leqslant \gamma_s) \doteq (1-\rho^2)\sum_{m=2}^{M} a_m(\rho) b^{2m} \quad (7)$$

where

$$a_m(\rho) = \frac{(-1)^m}{m!}\sum_{k=1}^{\lfloor m/2\rfloor}\rho^{2k-2}\sum_{j=k}^{\lfloor m/2\rfloor} c_{m,j}$$
$$c_{m,j} = \sum_{l=2j}^{m}\left(-\frac{1}{2}\right)^l \binom{m}{l}\binom{l}{j}\epsilon_{l-2j}$$
$$\epsilon_{l-2j} = \begin{cases} 1, & \text{if } l-2j = 0 \\ 2, & \text{if } l-2j = 1,2,\ldots. \end{cases} \quad (8)$$

Taking into account the first two terms of the power expansion, which is common practice in the literature [5], results in the two-term approximated cdf of (1) and yields

$$P_s(\gamma \leqslant \gamma_s) \doteq \frac{\gamma_s^2}{\Gamma^2(1-\rho^2)}. \quad (9)$$

In obtaining accurate results, γ_s/Γ should be much smaller than $(1-\rho^2)$. Taking the inverse of (9) yields

$$\gamma_s = \pm\Gamma\sqrt{\alpha(1-\rho^2)}. \quad (10)$$

Because this quotient must be positive, only the positive part of (10) is a valid solution. In the case of maximal ratio combining, the inverse of (3) is also needed. As in the case of selection combining, no closed expression could be found. So, again the power series expansion is used (see Appendix II)

$$P_r(\gamma \leqslant \gamma_r) \doteq (1-\rho^2)\sum_{m=2}^{M}\frac{(-1)^m}{m!}c_m(\rho)d^{2k} \quad (11)$$

where

$$d = \sqrt{\frac{\gamma_r}{\Gamma(1-\rho^2)}}$$
$$c_m = \sum_{l=1}^{\lfloor 1/2m\rfloor}\binom{m-1}{2l-1}\rho^{2l-2}.$$

If only the first two terms of the power-series expansions are taken into account, the two-term approximated cdf becomes

$$P_r(\gamma \leqslant \gamma_r) \doteq \frac{\gamma_r^2}{2\Gamma^2(1-\rho^2)} \quad (12)$$

with its inverse function

$$\gamma_r = \pm\Gamma\sqrt{2\alpha(1-\rho^2)}. \quad (13)$$

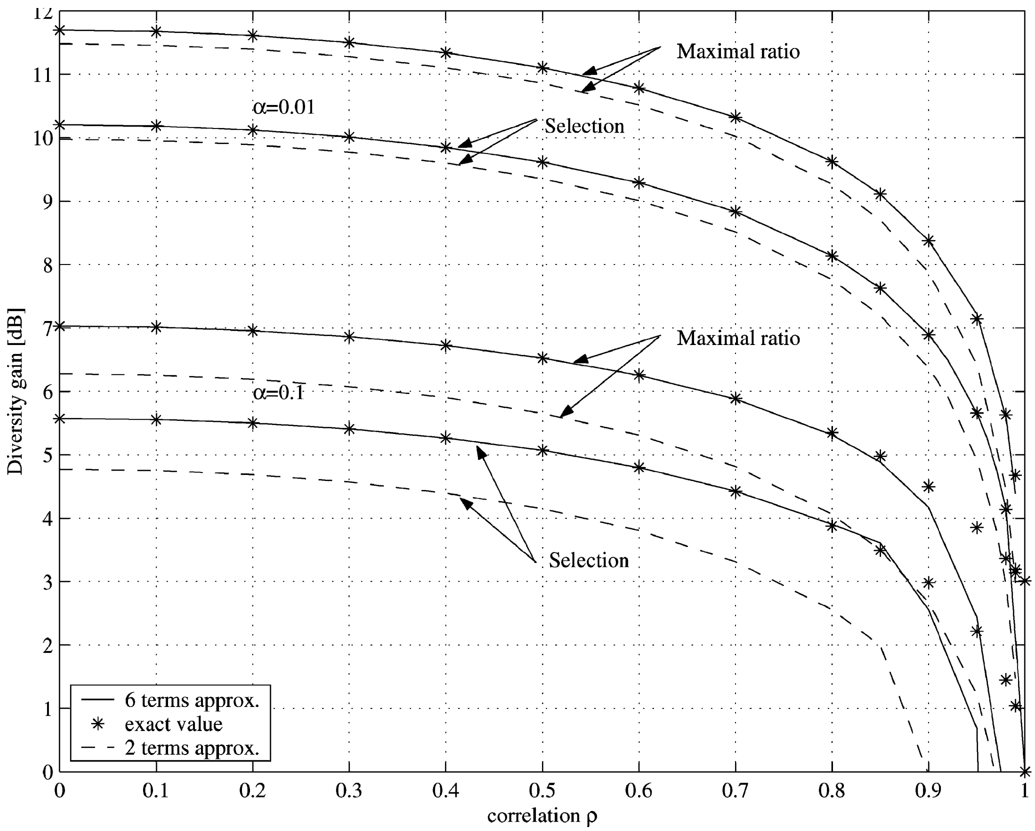

Fig. 1. Diversity gain versus correlation for maximal ratio combining and selection for two antennas, including six-term approximation.

Also in this case, only the positive part of (13) gives a valid solution. Using these formulas, the DG that is defined as

$$\mathrm{DG} = \frac{\gamma_i(\alpha)}{\gamma_o(\alpha)} \tag{14}$$

or expressed in decibels as

$$\mathrm{DG} = 10\log_{10}(\gamma_i(\alpha)) - 10\log_{10}(\gamma_o(\alpha)) \tag{15}$$

can be derived in terms of SNR. The subscript i can be either s or r, denoting selection or maximal ratio, respectively. Substituting (6), (9), and (12) into (14) results in the following two-term approximated formula for the diversity gain:

$$\mathrm{DG}_s(\alpha,\rho) \doteq 10\log_{10}\left(\frac{\sqrt{\alpha(1-\rho^2)}}{-\log(1-\alpha)}\right)_{\gamma_s/\Gamma \ll (1-\rho^2)} \tag{16}$$

in the case of selection diversity and

$$\mathrm{DG}_r(\alpha,\rho) \doteq 10\log_{10}\left(\frac{\sqrt{2\alpha(1-\rho^2)}}{-\log(1-\alpha)}\right)_{\gamma_s/\Gamma \ll (1-\rho^2)} \tag{17}$$

in the case of maximal ratio combining. These two-term approximated diversity gain values as a function of ρ are shown in Fig. 1.

To obtain an accurate diversity gain for $\rho \leq 0.7$, it appears that M should be at least 6. The resulting diversity gain is also plotted in Fig. 1 together with the exact values (M equals infinity, see Appendix II) as a reference. Fig. 1 clearly shows that the two-term approximation, as often used in the literature [5], becomes inaccurate when the outage α becomes larger than 0.01. Nevertheless, (16) and (17) show the general behavior of the diversity gain in the case of correlated fading. It is also clear from the figure that the six-term approximation becomes inaccurate when ρ approaches 1. A correlation of approximately one is, however, unimportant because it will almost never occur in mobile application. Fortunately, the curves remain relatively flat up to a correlation coefficient of about 0.7, with a substantial drop in the diversity gain afterward. This means that, in practice, some correlation may be tolerated without sacrificing diversity gain. Finally, it can be concluded from Fig. 1 or directly from a comparison of (16) and (17), that maximal ratio combining performs about 1.5 dB (square root of 2) better than selection diversity.

For our application, antenna diversity at handhelds, selection diversity was chosen, because of the much higher cost to implement maximal ratio combining. Therefore, the next section on measured diversity gain considers only selection diversity.

III. Measured Diversity Gain Using Space Diversity

As shown in the previous section, the correlation coefficient will influence the diversity gain. Since in practice a certain correlation coefficient cannot easily be imposed, a variation of the distance between two antennas will be used to change the correlation coefficient indirectly. For instance, a large distance implies a low correlation coefficient and vice versa. This section starts with a description of the measurement setup, followed by an explanation of how those diversity gain values can be obtained from the measurements that are subsequently used for a comparison with the theoretical values derived in the previous section.

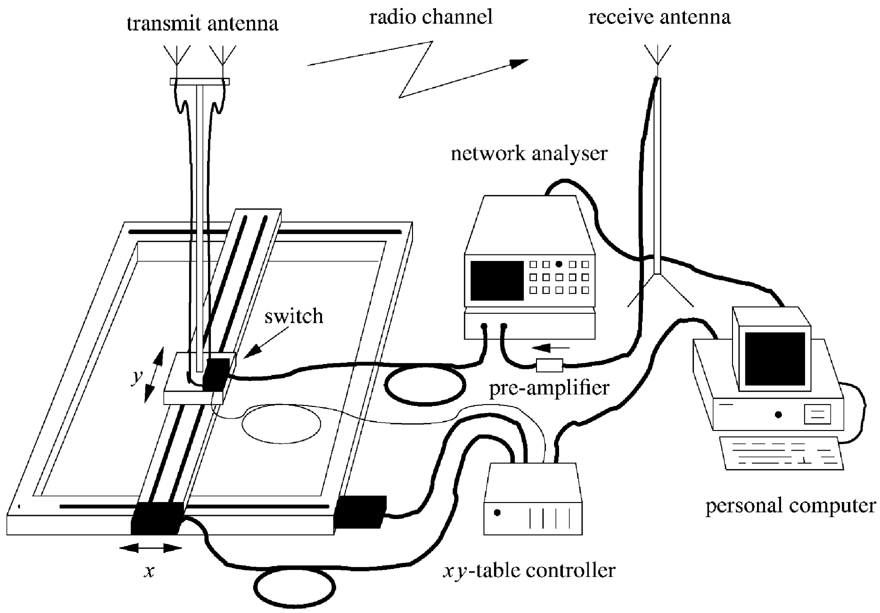

Fig. 2. Measurement setup.

A. Measurement Setup

In order to explore selection diversity, the radio channel characteristics are measured with the setup shown in Fig. 2, which consists of a xy table (or scanner), by which a dual antenna system can be moved in a horizontalplane over a distance of 100 cm in the x direction and 80 cm in the y direction. The movement is controlled via a xy table controller. The antenna signals are fed to a network analyzer and, while the frequency-swept measurement is in progress, the online analyzed data (both amplitude and phase) will be sent to the hard disc of the personal computer (PC). There, the data can be further analyzed offline in order to obtain the desired channel characteristics and from these the required diversity gain values. The antenna switch is an HF-terminated switch with low losses. Like the scanner, the switch is controlled by the xy table controller. Once the data arrive at the PC, they are saved into a matrix format having a dimension $K \times F$ where K denotes the total number of spatial observations points on the scanner and F is the total number of frequency points. In our case, $K = 5551$ and $F = 201$. The carrier frequency of these measurements was 900 MHz and the bandwidth equals 100 MHz. The measured data have been normalized by its mean power.

The measurement setup was placed in a rich scattering indoor environment of 20 m^2, surrounded by walls made of plasterboard with wire glass windows and in each corner reinforced concrete pillars. Inside this room, there are several laboratory tables and metal furniture (cabinets, measurement equipment, and trolleys). During the measurements, the wooden doors were left open. The floor and ceiling are made of reinforced concrete.

B. Space Diversity

In this section, the diversity gain of selection combining will be measured in a frequency band around 900 MHz. The measurements took place in an indoor environment under nonline-of-sight conditions. The transmit antenna is a half-wavelength monopole and the two receive antennas are half-wavelength dipoles. All antennas are vertically polarized. The scanner was placed somewhere in a room where a lot of multipath waves are expected, which will remain stationary during the measurements. It was verified that the radio channel was indeed stationary.

TABLE I
THEORETICAL DIVERSITY GAINS FOR SPACE DIVERSITY HAVING DIFFERENT ANTENNA SPACINGS. ALL DG FIGURES ARE EXPRESSED IN DECIBELS. CORRELATION IS OBTAINED AFTER CLARKE

d/λ	ρ_{Clarke}	DG (outage=0.01)	DG (outage=0.1)
0.01	0.99	1.0	0
0.1	0.82	8.0	3.9
0.25	0.22	10.1	5.5
0.50	0.093	10.1	5.6

As mentioned before, the correlation coefficient can be varied by changing the spacing between the antennas. The theoretical relation between antenna spacing and signal correlation was first investigated by Clarke [2] and resulted in the so-called Clarke functions. The assumptions for obtaining these Clarke functions are that the AoAs of the multipath waves are uniformly distributed and that the antennas are isotropic radiators. Then, the relation between the correlation coefficient and the antenna spacing is given by the zeroth order Bessel function to the power two. In Table I, the relation between antenna spacing and diversity gain is shown for different outages. When using a spacing of 0.01λ, the diversity gain approaches zero [cf. (15)]. Once the measurements have been performed, the diversity gain is calculated from the cumulative distribution functions of the SNR (see Fig. 5). For the sake of convenience, the experimental diversity gain values are tabulated in Table II for an outage of both 0.01 and 0.1. The effective correlation coefficients are obtained with help of Fig. 1 by means of finding the corresponding correlation coefficient at a measured diversity gain. The table shows that the difference between the diversity

TABLE II
MEASURED DIVERSITY GAINS FOR SPACE DIVERSITY HAVING DIFFERENT ANTENNA SPACINGS, ALL DG FIGURES ARE EXPRESSED IN DECIBELS. EFFECTIVE CORRELATIONS ARE OBTAINED (CF. FIG. 1)

	DG (outage=0.01)		DG (outage=0.1)	
d/λ	$\rho_{\rm eff}$	measured	$\rho_{\rm eff}$	measured
0.01	0.97	2	0.95	1
0.10	0.94	6	0.89	3
0.25	0.81	8	0.53	5
0.50	0.66	9	0.19	5.5

gain for antenna spacings of 0.5λ and 0.25λ is small. This corresponds to the flat shape of the curves shown in Fig. 1 for correlation coefficients smaller than 0.7. For larger correlation coefficients, the diversity gain drops dramatically, which is also confirmed by the measurements at antenna spacings of 0.1λ and 0.01λ. Furthermore, it appears that, for the two largest antenna spacings, the measured diversity gains lie below the theoretical values. This may be explained by the fact that, during the measurements, the assumptions made by Clarke were not completely met. AoA estimations based on the measured data clearly demonstrated that its distribution function was not uniform.

In the case of the smallest antenna spacing of 0.01λ, it is observed from the table that there is still a diversity gain of 1 dB left while theory predicts 0 dB. This can be explained by the fact that, in theory, the mutual coupling between the antennas has been neglected. This means that the antenna patterns of the two antennas are not the same (isotropic) any longer, which violates another assumption made in the derivation of the Clarke functions. Due to the different antenna patterns, the two antennas receive different sets of multipath waves, resulting in a reduced correlation of the two antenna signals. Referring to mobile handhelds, which become smaller and smaller, the use of space diversity on handhelds should intuitively result in a very small diversity gain. According to Clarke's function, the correlation between the received signals of two antennas with a mutual distance of 0.1λ (3.6 cm at 900 MHz) is 0.82. But, fortunately, it now appeared that, due to the mutual coupling between the antennas, it is possible to have a larger diversity gain. How this effect can be further exploited will be discussed in the next section on antenna-pattern diversity.

IV. ANTENNA-PATTERN DIVERSITY

Antenna-pattern diversity can be realized in different ways. One method is by combining the signals of two antennas in such a way that two different (ideally orthogonal) antenna patterns occur. This method was considered by Dietrich *et al.* for their comparative study of different diversity techniques [6]. In their study, which was performed simultaneously with our study [7], two dipoles connected to a 90° hybrid with a fixed spacing of 0.25 wavelength were used [2]. The antenna-pattern diversity method considered in our study and described in this section is based on the idea of switched parasitic antennas, as proposed by Vaughan [8]–[10] and Scott [11]. This antenna concept allows a much smaller antenna spacing and provides multiple antenna patterns with a single radio frequency (RF) signal port without the need for RF switches or phase shifters in the direct RF path. In the previous section, it appeared that the radiation pattern of

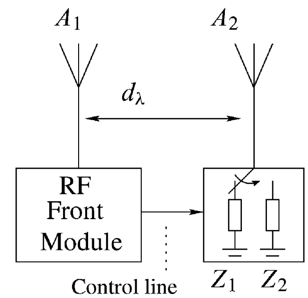

Fig. 3. Parasitic switching concept with two dipoles.

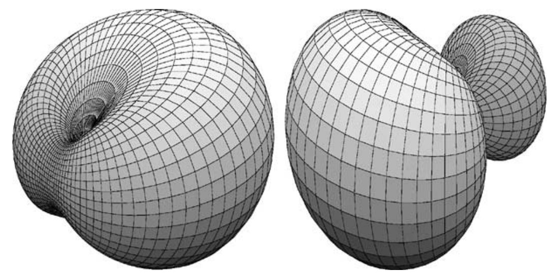

Fig. 4. Electrical far-field patterns at $Z_1 = -100j$ (left) and $Z_2 = -20j$ (right).

an antenna is modified by placing a second antenna close to it. Vaughan showed that it is possible to change the pattern of the active antenna by terminating the parasitic antenna to the ground or by leaving it open. Then, the diversity gain is obtained by selecting the best antenna pattern. In this paper, the diversity gain will be further improved by optimizing the two impedance values. Diversity gain can be explained by the fact that different antenna patterns receive different sets of multipath waves. This gain will be largest if the two sets are completely independent that can, in theory, be realized with two antenna patterns that have zero overlap. To judge whether the antenna patterns are similar or different, the antenna correlation ρ_a will be evaluated. As an example, the vertically polarized dual dipole antenna configuration, as used in the previous section, will be analyzed with an antenna spacing of $d = 0.1\lambda$ (see Fig. 3). Suppose that the parasitic antenna A_2 is connected to a two-state switch. In state one, an impedance Z_1 is connected, resulting in a far-field component $E_1(\theta)$ for the active antenna A_1, with θ being the azimuth angle. In the second state, a variable impedance Z_2 is connected to antenna A_2, producing another far-field $E_2(\theta)$. Subsequently, the antenna correlation is calculated as defined in [13] and [14].

The far-field antenna patterns were calculated by means of the numerical electromagnetic code numerical electromagnetic code (NEC). Evaluating the antenna correlation for a large number of Z_1, Z_2 combinations, the lowest antenna correlation was found for $Z_1 = -100j$ and $Z_2 = -20j$ having at the same time a good matching. With this optimal set of impedances, the antenna correlation equals 0.39. Note that the two impedances are, in fact, imaginary in order to avoid impedance losses. In order to create a feeling for a low antenna-correlation coefficient, the two radiation patterns are visualized in Fig. 4. It can be observed that these antenna patterns have indeed a minimal overlap. This antenna system was built and in the next section its performance will be analyzed.

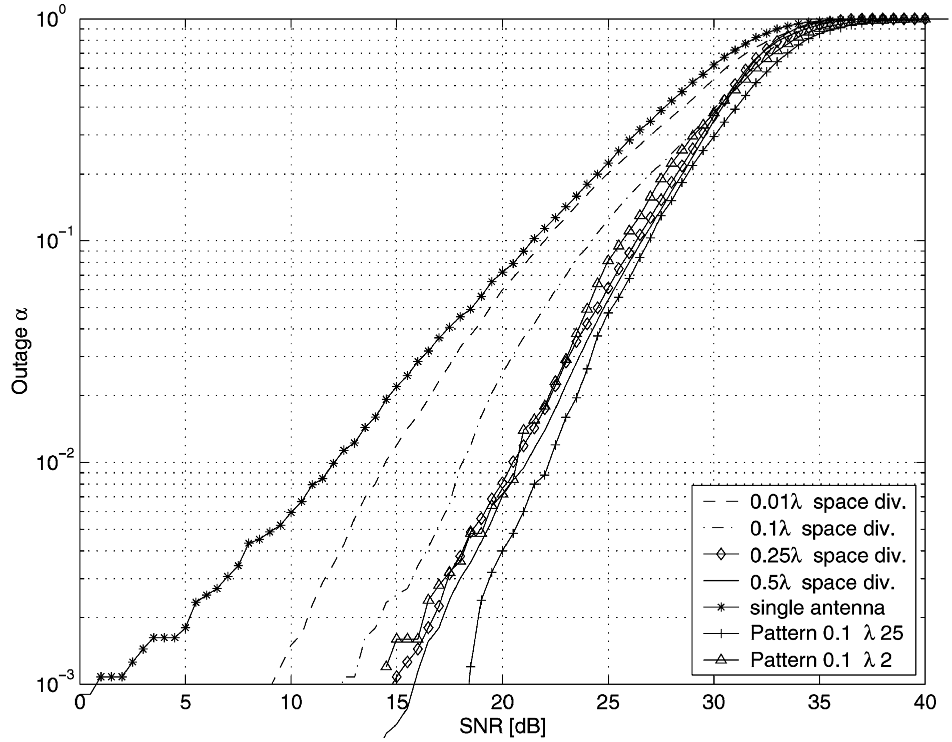

Fig. 5. Measured cdfs for dual selection combining for different antenna spacings and antenna-pattern diversity.

V. Diversity Gain of Space Diversity Versus Antenna-Pattern Diversity

In this section, the performance of antenna-pattern diversity will be analyzed with the help of cdfs of measured SNRs. For a fair comparison with space diversity, the same measurement setup was used (see Section III-A). In the preceeding section, it was concluded that it is possible to realize a low correlation between the far-field radiation pattern by selecting an optimal combination of impedance values for the parasitic antenna. Instead of having two states, the number of states has been extended to 25. This was realized in practice by connecting a varicap to the terminals of the parasitic antenna, which was controlled by a programmable voltage source.

The reason to have more steps is that the calculated impedances obtained from theory might not give an optimal decorrelation in practice. This can be explained from the definition of the antenna correlation, which includes the AoA distribution of the multipath waves. In the theoretical optimization, the assumption was made that the AoA is uniformly distributed. Fig. 5 shows the resulting cdfs of measured SNRs for both space diversity and antenna-pattern diversity. From the curves for antenna-pattern diversity and space diversity with identical antenna spacing (0.1λ), it appears that antenna-pattern diversity having 25 states outperforms space diversity with 6 dB at an outage of 0.01 and 3 dB at an outage of 0.1. Having 25 states is impractical. Therefore, the 25 states were reduced to 2. This reduction in the number of states was obtained by finding the two states where the correlation between the two antenna signals is at its lowest. Fig. 5 shows that this reduction to two states results in a loss of 1.5 dB at an outage 0.01 and 1 dB at an outage 0.1 with respect to 25 states. This is a very promising result because, compared with space diversity, antenna-pattern diversity has the advantage that no switching occurs on the RF path and that it is easier to implement into handhelds.

The two optimal states were found at 900 MHz. Consider another frequency, which lies 25 MHz below 900 MHz having the same states as found at 900 MHz. This resulted in a 1-dB drop in the diversity gain for both outages. The latter can be regarded as transmit diversity. For instance (suppose the GSM standard), during receive the optimal states are determined. Those states can also be used in the transmission mode without loosing a certain amount of diversity gain. Therefore, antenna-pattern diversity appears to be the best choice.

VI. Conclusion

From Section II, it is concluded that the two-term approximation, as generally used in literature to derive the diversity gain of dual-antenna systems operating in an indoor environment, is inaccurate. Therefore, a new six-term approximation is derived and recommended. It is demonstrated in Section III that, due to mutual coupling between the two antennas, the diversity gain will not become 0 dB if the distance between the antennas approaches zero.

It was shown in Section IV that, in the case of switch parasitic antennas, a low antenna-pattern correlation coefficient is possible, having a good match at the same time. Furthermore, it was demonstrated that the number of impedance states of the parasitic antenna can be reduced to two without affecting the antenna-pattern diversity significantly. These two impedances were optimized in the receive mode, but can also be used in the transmit mode without loosing too much antenna-pattern diversity. Finally, it is concluded from Section V that antenna-pattern diversity is a better choice than space diversity for use at handhelds.

APPENDIX I
Q_M FUNCTION AND RELATED IDENTITIES

The definition of the generalized Q_M function is

$$Q_N(a,b) \triangleq \int_b^\infty \left(\frac{x}{a}\right)^{N-1} e^{-1/2(a^2+x^2)} I_{N-1}(ax)\, x\, dx. \tag{18}$$

The case $N = 1$ is known as the Marcum's Q-function. In this paper, the generalized Q-function will be referred to simply as the Q-function. The relevant identities of the Q-function are summarized as

$$Q(a,b) = \int_b^\infty e^{-1/2(a^2+x^2)} I_0(ax)\, x\, dx \tag{19}$$
$$Q(0,b) = e^{-(-b^2/2)} \tag{20}$$
$$Q(a,0) = 1 \tag{21}$$
$$Q(a,b) + Q(b,a) = 1 + e^{-1/2(a^2+b^2)} I_0(ab). \tag{22}$$

APPENDIX II
POWER SERIES EXPANSIONS

$$P_s(\gamma \leqslant \gamma_s) = 1 - \exp\left(-\frac{\gamma_s}{\Gamma}\right)[1 - Q(a,b) + Q(b,a)] \tag{23}$$

where

$$b = \sqrt{\frac{2\gamma_s}{\Gamma(1-\rho^2)}}, \qquad a = b\rho.$$

Substitution of the new variables $\gamma_s/\Gamma = 1/2(1-\rho^2)b^2$ and $a = b\rho$ into (23) yields

$$P_s(b) = 1 - \exp\left(-\frac{1}{2}(1-\rho^2)b^2\right)[1 - Q(\rho b, b) + Q(b, \rho b)]. \tag{24}$$

Due to the lengthy expressions, $P_s(\gamma \leqslant \gamma_s)$ will be replaced by $P_s(b)$. With the help of (22), it is possible to eliminate one of the Q-functions. Eliminating $Q(b,\rho b)$ results in

$$\begin{aligned} P_s(b) = 1 - \exp(-b^2) I_0(\rho b^2) \\ -2\exp\left(-\frac{1}{2}(1-\rho^2)b^2\right)(1 - Q(\rho b, b)). \end{aligned} \tag{25}$$

By using Schnidman's [16] elegant form for the $1 - Q(\cdot,\cdot)$ term

$$1 - Q(a,b) = \exp\left(-\frac{1}{2}(1+\rho^2)b^2\right) \sum_{r=N}^{\infty} \left(\frac{1}{\rho^2}\right)^{r/2} I_r\left(\rho\, b^2\right) \tag{26}$$

and substituting this formula into (25) results in

$$\begin{aligned} P_s(b) &= 1 - \exp\left(-b^2\right)\left(I_0(\rho b^2) + 2\sum_{r=1}^{\infty} \rho^{-r} I_r(\rho b^2)\right) \\ &= 1 - \exp\left(-b^2\right) \sum_{r=-\infty}^{\infty} \rho^{-|r|} I_r(\rho b^2). \end{aligned} \tag{27}$$

Furthermore, the modified Bessel function of order r can be replaced by its power-series expansion

$$P_s(b) = 1 - \exp\left(-b^2\right) \sum_{r=-\infty}^{\infty} \rho^{-|r|} \left(\frac{1}{2}\rho b^2\right)^{|r|} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}(\rho b^2)^2\right)^k}{k!(k+|r|)!}. \tag{28}$$

After some mathematical manipulations, the cdf $P_s(b)$ can be written in the power series

$$P_s = (1-\rho^2) \sum_{m=2}^{\infty} a_m(\rho) b^{2m} \tag{29}$$

where

$$\begin{aligned} a_m(\rho) &= \frac{(-1)^m}{m!} \sum_{k=1}^{\lfloor m/2 \rfloor} \sum_{j=k}^{\lfloor m/2 \rfloor} c_{m,j} \cdot \rho^{2k-2} \\ c_{m,j} &= \sum_{l=2j}^{m} \left(-\frac{1}{2}\right)^l \binom{m}{l} \binom{l}{j} \epsilon_{l-2j} \\ \epsilon_{l-2j} &= \begin{cases} 1, & \text{if } l - 2j = 0 \\ 2, & \text{if } l - 2j = 1, 2, \ldots \end{cases} \end{aligned} \tag{30}$$

where $\lfloor \cdot \rfloor$ denotes the largest integer smaller than the argument.

For the case of maximal ratio combining, the exponential functions will be replaced by its power-series expansions. This results in the following formula:

$$P_r(\gamma \leqslant \gamma_r) = (1-\rho^2) \sum_{m=2}^{\infty} b_m(\rho) d^{2m} \tag{31}$$

where

$$\begin{aligned} b_m(\rho) &= \frac{(-1)^m}{m!} \sum_{k=1}^{\lfloor 1/2m \rfloor} \binom{m-1}{2k-1} \rho^{2k-2} \\ d &= \sqrt{\frac{\gamma_r}{\Gamma(1-\rho^2)}}. \end{aligned}$$

ACKNOWLEDGMENT

The authors would like to thank Dr. A. J. E. Janssen for the mathematical derivation of the power-series expansions. They would also like to thank B. Vandewiele for his assistance in developing the software for automating the measurement setup.

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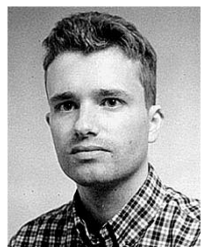

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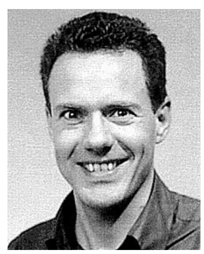

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