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Antisymmetric Treatment of Deuteron-Stripping Reaction†

——*Stripping and Heavy-Particle-Stripping Reactions*——

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The deuteron stripping reaction is reformulated by using completely antisymmetric wave functions: The amplitude of the reaction will be represented as a sum of two parts, stripping and heavy-particle-stripping amplitudes.

Quite analogous to the case of the "ordinary" stripping, the amplitude of the heavy-particle-stripping can be represented in terms of three reduced width amplitudes of bound nucleons, e.g. in (d, p) reaction, those of captured neutron and proton in the residual nucleus and that of the emitted proton in the target nucleus, or alternatively, in terms of two reduced width amplitudes of the emitted proton and captured deuteron in bound states. The latter formulation is natural generalization of Owen and Madansky's.

By making use of present formalism, one can calculate not only angular distribution but also the "magnitude" of heavy-particle-stripping cross section.

For simplicity, the Butler approximation, i.e. the cutoff Born approximation, is employed, though its generalization can be performed easily, as will be discussed in the final section.

Some qualitative discussions are given. Numerical calculations and more detailed comparison between the stripping and heavy-particle-stripping cross sections will appear in our subsequent paper.

§ 1. Introduction

When a deuteron is bombarded to a target nucleus to form the final state of a residual nucleus and free proton, one may immediately imagine the following two sorts of direct-interaction-mechanism. First the neutron in the deuteron is captured by the target nucleus, whereas the proton in the deuteron does not interact with the target nucleus, appearing at infinity as free proton. In other words, the neutron in the deuteron is "stripped" by the target nucleus to form the residual nucleus. This mechanism is called the deuteron stripping, which has been dealt with by many authors after the first work of Butler.[1] On the other hand, the target nucleus may be "stripped" by the incident deuteron. Namely, if the target nucleus is assumed to be composed of one proton, say $p2$, and the remaining core, the deuteron may capture the core to form the residual nucleus, while the proton is emitted in the final state. This is the so-called heavy-particle-stripping reaction proposed by Owen and Madansky.[2]

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In some experiments of (d, p) and (d, n) reactions, there appear peaks of angular distribution in rather backward directions, which have usually been explained in terms of heavy-particle-stripping reaction.

The theories of heavy-particle-stripping reaction so far employed, however, were concerned only with the shape of angular distribution and the magnitude of the cross section has been adjusted so as to fit the experimental data.

The main contents of the present paper may be expressed as follows. By using completely antisymmetric wave function the stripping and the heavy-particle-stripping amplitudes will be treated on an equal footing, which will be given in the next section. In sections 3 and 4, the amplitude of heavy-particle-stripping will be represented in terms of three reduced width amplitudes of bound nucleons, i.e. those of captured neutron and proton in the residual nucleus and that of the emitted proton in the target nucleus, or alternatively, in terms of two reduced width amplitudes of the emitted proton and of the captured deuteron in bound states. The latter formulation is natural generalization of Owen and Madansky's. The nuclear interaction appearing in the matrix element will be eliminated by using the Schroedinger equation of target nucleus. On the other hand, the amplitude of "ordinary" stripping is expressed, as usual, in terms of a reduced width amplitude of the captured neutron in the residual nucleus. Then one can calculate not only the angular distribution but also the magnitude of the heavy-particle-stripping reaction without explicit knowledge of the nuclear interaction. In section 5, the cross section for (d, p) and (d, n) reactions will be presented, which consists of three parts: stripping, heavy-particle-stripping and their interference terms. Finally, in section 6, some discussions will be added concerning qualitative nature between stripping and heavy-particle-stripping reactions. It is pointed out that the energy-dependence of the cross section of the heavy-particle-stripping reaction is stronger than that of the stripping reaction, so that the former reaction will not play an important role at higher energy of incident deuteron.

Detailed qualitative and quantitative discussions will be presented in a subsequent paper, in which it will be discussed especially in what condition, i.e. incident energy, Q-value, etc., the heavy-particle-stripping mechanism might be preferential to (d, p) or (d, n) reactions in comparison with the "ordinary" stripping one. The result is almost always unfavourable to the heavy-particle-stripping mechanism except the special case of low incident energy and some appropriate combination of reaction Q-value and the binding energy of the emitted proton in the target nucleus, *provided the Butler approximation is adopted.*

Other reactions, for example (α, p), may be treated in quite a similar way with slight modification which will be discussed in the derivation of the formula.

After completing the present work, it called authors' attention that quite recently Nagarajan and Banarjee have performed an antisymmetric treatment of (d, p) and (d, n) reactions. Their formulation is almost the same as our "alternative" formulation mentioned previously. If needed, the differences between their and our formulations will be noted in the footnotes of the present paper.* Their conclusion obtained through the analysis of experimental data, however, seems to be essentially different from ours; this may be due to the fact that our analysis is more systematic than theirs.**

§ 2. Antisymmetrization

As is well known, the amplitude of a nuclear rearrangement collision such as $\mathrm{A}(a, b)\mathrm{B}$ may be obtained simply as a coefficient of the asymptotic wave function in the final channel b of the total wave function which should obey the Schroedinger equation with appropriate boundary conditions.

In the antisymmetric treatment, we must further impose an additional requirement on the total wave function, i.e. the total wave function should be antisymmetric with respect to the exchange of the coordinates of arbitrary two nucleons in the system. Then, there appear various emitted particles (if distinguishable) in the channel b. The amplitude in the antisymmetric treatment, therefore, can be obtained as a linear combination of the coefficients of the asymptotic wave functions in the channel b.

In the present section, an elementary method will be employed, though completely the same result has been obtained by the general method mentioned above.

1. *Simple Example*

First, we shall deal with a simple example; antisymmetrization between the proton $p1$ of one of the constituent of the incident deuteron and a proton $p2$ in the target nucleus, which is assumed to be composed of proton $p2$ and remaining core. Further, the spins of incident and emitted particles will be disregarded.

For the sake of convenience, the interaction between the incident deuteron and the target nucleus will be denoted by

$$V(p2, C; d),$$

where the notation $p2, C$ stands for the target nucleus, i.e. $p2+$core, which may be represented as the sum of interaction between the target nucleus and the proton $p1$ and that between the target nucleus and neutron in the deuteron:

$$V(p2, C; d) = V(p2, C; n) + V(p2, C; p1).$$

More explicitly,

$$V(p2, C; n) = V(p2; n) + V(C; n),$$
$$V(p2, C; p1) = V(p2; p1) + V(C; p1).$$

Here, $V(C; n)$ stands for the interaction between the core and the neutron in the incident deuteron.

* M. A. Nagarajan and M. K. Banarjee, Nuclear Physics **17** (1960), 341. Their paper will be referred to as N-B in the following footnotes.

** Details will be discussed in the subsequent paper.

The wave function of the final state, i.e. residual nucleus+one free proton, may be expressed as

$$(1/\sqrt{2})\{e^{ik\cdot r_1}\psi_R(n,\ p2,\ C)-e^{ik\cdot r_2}\psi_R(n,\ p1,\ C)\}, \tag{2·1}$$

where r_1 and r_2 are the coordinates of proton $p1$ and $p2$, respectively, $\psi_R(n,\ p2,\ C)$ denotes the wave function of the residual nucleus which is composed of the core, proton $p2$ and the neutron.

On the other hand, the wave function of the initial state may be written, in the antisymmetric treatment, in two forms:

$$(1/\sqrt{2})\phi_d(r_n-r_1)e^{iK\cdot(r_n+r_1)/2}\psi_T(p2,\ C), \tag{2·2a}$$

and

$$-(1/\sqrt{2})\phi_d(r_n-r_2)e^{iK\cdot(r_n+r_2)/2}\psi_T(p1,\ C), \tag{2·2b}$$

where $\psi_T(p2,\ C)$ denotes the wave function of the target nucleus which is composed of the proton $p2$ and the core, ϕ_d the wave function of the internal motion of deuteron, r_n the coordinate of the neutron and K being the wave number vector of the incident deuteron. The interactions in the initial state should be taken to be

$$V(p2,\ C;\ p1)+V(p2,\ C;\ n) \tag{2·3a}$$

and

$$V(p1,\ C;\ p2)+V(p1,\ C;\ n) \tag{2·3b}$$

corresponding to the forms of the wave function of initial state (2·2a) and (2·2b), respectively.

After simple calculations, we obtain the transition matrix element in the antisymmetric treatment:

$$T=\int dr_n dr_1 dr_2 dC\{e^{-ik\cdot r_1}\psi_R{}^*(n,\ p2,\ C)-e^{-ik\cdot r_2}\psi_R{}^*(n,\ p1,\ C)\}$$
$$\times\{V(p2,\ C;\ p1)+V(p2,\ C;\ n)\}\psi_T(p2,\ C)e^{iK\cdot(r_1+r_n)/2}\phi_d(r_1-r_n), \tag{2·4}$$ *,**

where dC stands for the integration over all coordinates of the core. The range of integration with respect to r_n, r_1 and r_2 will be discussed later. The expression (2·4) has already been obtained by Owen and Madansky.[2)]

* Another, but equivalent expression has been obtained by French,[3)] using the final state interaction:

$$T\equiv\int dr_n dr_1 dr_2 dC\, e^{-ik\cdot r_1}\psi_R{}^*(n,\ p2,\ C)V(n,\ p2,\ C;\ p1)$$
$$\times\{\psi_T(p2,\ C)\,e^{iK\cdot(r_1+r_n)/2}\phi_d(r_1-r_n)-\psi_T(p1,\ C)\,e^{iK\cdot(r_2+r_n)/2}\phi_d(r_2-r_n)\}.$$

** Strictly speaking, the expression (2·4) will remain correct, only if the target nucleus is infinitely heavy and the coordinate of emitted proton is measured from its center of mass. As a result, this expression includes the error of order $1/A$ (1/(mass number of nucleus)), which leads sometimes to rather serious difference in the angular distribution of heavy-particle-stripping. The improvement will be performed in the next subsection.

Next, it may be convenient to divide the T into two parts:

$$T = T\ (\text{direct}) + T\ (\text{exchange}), \tag{2·5}$$

where

$$T\ (\text{direct}) \equiv \int d\boldsymbol{r}_n d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{C}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_1} \psi_R^*(n,\ p2,\ C)$$
$$\times \{V(p2,\ C\,;\ p1) + V(p2,\ C\,;\ n)\} \psi_T(p2,\ C) \phi_d(\boldsymbol{r}_n - \boldsymbol{r}_1)\, e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2}, \tag{2·5a}$$

and

$$T\ (\text{exchange}) \equiv -\int d\boldsymbol{r}_n d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{C} \cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2} \psi_R^*(n,\ p1,\ C)$$
$$\times \{V(p2,\ C\,;\ p1) + V(p2,\ C\,;\ n)\} \cdot \psi_T(p2,\ C) \phi_d(\boldsymbol{r}_n - \boldsymbol{r}_1)\, e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2}. \tag{2·5b}$$

The first term, (2·5a), represents the direct part, which has been dealt with by many authors[1] in the "ordinary" stripping reaction. As is well known, the initial-state-interaction $V(p2,\ C\,;\ p1) + V(p2,\ C\,;\ n)$ in T(direct) can be replaced by the final-state-interaction, i.e.

$$V(p2,\ n,\ C\,;\ p1) = V(p2,\ C\,;\ p1) + V(n\,;\ p1), \tag{2·6}$$

if the Born approximation be adopted. In the case of the Butler approximation, i.e. the cutoff Born approximation, the same relation holds also exactly, provided the cutoff radii in the initial and final states be taken properly.* The net interaction in (2·6) which produces the stripping reaction is known to be $V(n\,;\ p1)$ and the corresponding initial state interaction is given by $V(p2,\ C\,;\ n)$; the interaction between the target nucleus and the neutron in the incident deuteron.

We shall deal only with the following matrix element in T(direct), while the interaction $V(p2,\ C\,;\ p1)$ will be disregarded hereafter:

$$M\ (\text{stripping}) \equiv \int_{ext} d\boldsymbol{r}_n\, d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 d\boldsymbol{C} \cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_1} \psi_R^*(n,\ p2,\ C)$$
$$\times V(n\,;\ p1) \psi_T(p2,\ C) \phi_d(\boldsymbol{r}_1 - \boldsymbol{r}_n)\, e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2}, \tag{2·7a}$$

(by means of final state interaction),

$$\equiv \int_{ext} d\boldsymbol{r}_n\, d\boldsymbol{r}_1 \int d\boldsymbol{r}_2 d\boldsymbol{C} \cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_1} \psi_R^*(n,\ p2,\ C)$$
$$\times V(p2,\ C\,;\ n) \psi_T(p2,\ C) \phi_d(\boldsymbol{r}_1 - \boldsymbol{r}_n)\, e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2} \tag{2·7b}$$

(by means of initial state interaction),

where the symbol $\int_{ext} d\boldsymbol{r}_n d\boldsymbol{r}_1$ denotes the integration over $\boldsymbol{r}_n$ and $\boldsymbol{r}_1$ with the follow-

* The relation between the cutoff radii R_i and R_f in the initial and final states can be shown to be determined uniquely by requirement of "reciprocity".

ing restriction of radial integrals; $r_n \geq R_n$ and $r_1 \geq R_1$.* This is nothing but the Butler approximation, which may be verified from the short mean free path of deuteron in nuclear matter. For simplicity, we shall assume $R_n = R_1 \equiv R$, R being approximately equal to nuclear radius.

In the second term of (2·5), T(exchange), the interaction $V(p2, C; n) + V(p2, C; p1)$ can also be proved to be replaced by the final state interaction, i.e.

$$V(p1, n, C; p2) \equiv V(p1; p2) + V(n; p2) + V(C; p2), \qquad (2\cdot 8)$$

in a similar way as in the direct part, though the proof will not be quoted here. The interaction which produces the heavy-particle-stripping mechanism of Owen and Madansky may be safely ascribed to $V(C; p2)$ in the final state interaction and the corresponding interaction in the initial state is easily shown to be $V(C; p1) + V(C; n)$; the remaining interactions $V(n; p1) + V(p2; p1)$ will not be dealt with in the present paper. It should be added here, however, that the contribution of the interaction $V(p1; n)$ has been calculated in some detail by French,[3] obtaining the result that the angular distribution arising from this interaction shows the peaks in rather forward directions similarly to that of "ordinary" stripping reaction. The authors will not always believe that those interaction might produce such a large net contribution in the reaction because their matrix element may correspond to the so-called knock-on process, which seems to have rather small cross section contrary to the case of α-induced reaction due to very small internal energy of deuteron.

Hereafter, we shall deal with the following matrix element in T(exchange), which describes heavy-particle-stripping reaction as was previously discussed:

$$M\,(\text{H. P. Stripping}) = -\int_{ext} d\boldsymbol{r}_n\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2 \int d\boldsymbol{C} \cdot e^{-ik\cdot r_2} \psi_R^*(n, p1, C)$$
$$\times V(C; p2) \psi_T(p2, C)\ \phi_d(\boldsymbol{r}_1 - \boldsymbol{r}_n)\, e^{iK\cdot(r_1+r_n)/2}, \qquad (2\cdot 9\text{a})$$

(when we use the final state interaction),

$$= -\int_{ext} d\boldsymbol{r}_n\, d\boldsymbol{r}_1\, d\boldsymbol{r}_2 \int d\boldsymbol{C} \cdot e^{-ik\cdot r_2} \psi_R^*(n, p1, C)$$
$$\times \{V(C; p1) + V(C; n)\} \psi_T(p2, C) \phi_d(\boldsymbol{r}_1 - \boldsymbol{r}_n)\, e^{iK\cdot(r_1+r_n)/2}. \qquad (2\cdot 9\text{b})$$

(when we use the initial state interaction).

Similarly to the stripping reaction, $\int_{ext} d\boldsymbol{r}_n d\boldsymbol{r}_1 d\boldsymbol{r}_2$ denotes the integration over $\boldsymbol{r}_n$, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ with the restriction of $r_n \geq R$, $r_1 \geq R$ and $r_2 \geq R$. The first two restric-

* The restriction with respect to one radial coordinate (say, r_1) is automatically satisfied from that of another (say, r_n), only if the zero-range nuclear force is adopted for $V(p1; n)$ as is apparent from Eq. (3·6). Even for the nuclear force with finite range, the error in the differential cross section arising from the approximate restriction $r_n \leq R_n$ and $\infty > r_1 > 0$ will amount at most to 10% numerically, which was checked up by Nagasaki and one of the present authors (T. H.)[4].

tions, $r_n \geq R$ and $r_1 \geq R$ may be safely applicable as was discussed in M(Stripping). But, for $r_2 \geq R$, it may not always be verified since the mean free path of proton $p2$ in nuclear matter may be expected to be long in comparison with that of deuteron. Nevertheless, we adopt the restriction $r_2 \geq R$, though we may obtain somewhat underestimated value of M(H.P. Stripping), since the integration of r_2 over all space, i.e. $\infty > r_2 \geq 0$, leads sometimes to unusually very large value of amplitude which can be easily understood by using the bound state wave function of $p2$ in square well potential. It seems to the authors that we should use the more reliable approximation, e.g. the distorted Born approximation in order to adopt the integration of r_2 without the restriction, i.e. $\infty > r_2 \geq 0$.

2. *Improvement arising from the correct choice of coordinate system*
—— *Effect of recoil* ——

As was noticed previously, the T-matrix element (2·4) is not correct, including an error of order $1/A$, since the vector displacements of proton $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ and neutron $\boldsymbol{r}_n$ have been taken with respect to the centre of mass of the target nucleus. On the contrary, the coordinate of the emitted proton in the final state should be taken with respect to the centre of mass of the residual nucleus, which consists of the neutron and the target nucleus. In the "ordinary" stripping reaction, therefore the wave function of the emitted proton $\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}_1)$ should be correctly taken to be $\exp[-i\boldsymbol{k}\{\boldsymbol{r}_1-(M/M_f)\boldsymbol{r}_n\}]$, where M_f and M denote the reduced mass of residual nucleus and that of nucleon, respectively. Similarly, in the matrix element of heavy-particle-stripping, $\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}_2)$ should be replaced by $\exp[-i\boldsymbol{k}\cdot\{\boldsymbol{r}_2-(M_d(\boldsymbol{r}_1+\boldsymbol{r}_n)/2+M_C\boldsymbol{r}_C)/M_f\}]$, where M_d and M_C are, respectively, the reduced masses of deuteron and the core, $\boldsymbol{r}_C$ the coordinate of the core. After appropriate transformation of integral variables, we can obtain just similar expressions to (2·7) of M(Stripping) and (2·9) of M(H.P. Stripping), in which the difference appears only in the replacement of $\boldsymbol{k}\to(M_i/M_f)\boldsymbol{k}$ in M(Stripping) and $\boldsymbol{K}\to\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}$ in M(H.P. Stripping). Therefore, it will be sufficient to perform the following replacement, i.e.

$$\boldsymbol{k}\to(M_i/M_f)\boldsymbol{k} \qquad \text{in } M(\text{Stripping})$$

$$\boldsymbol{K}\to\boldsymbol{K}+(M_d/M_f)\boldsymbol{k} \qquad \text{in } M(\text{H. P. Stripping})$$

in the explicit expressions of matrix elements obtained in later sections.

The replacement in M(Stripping) can be proved to produce no serious error numerically for both magnitude and angular dependence of the differential cross section. On the other hand, that in M(H.P. Stripping) will play an important role for angular distribution, which will be discussed later.

3. *Generalization of the simple example*

Hitherto, the antisymmetrization has been performed between the proton $p1$ in the incident deuteron and a proton $p2$ in the target nucleus. Further, the spins are disregarded of the emitted and incident particles. Therefore, the results previously obtained are only the antisymmetrization with respect to space coordinate variables of two protons. When the spin coordinate is taken into account, the expression (2·4) of total amplitude remains correct, only if the spin state of $p1$ and $p2$ is symmetric, i.e. in triplet spin state. For antisymmetric spin state, i.e. singlet spin state, we must use the wave function which is symmetric in space coordinate variables. Namely, minus signs appearing in (2·1), (2·2b) and, consequently, the final expression (2·4) should be replaced by plus signs. Then, T(direct) and T(exchange) should be simply multiplied by factors α(direct) and α(exchange), respectively, which are determined by relative spin states. When we apply the single particle model without spin-dependent force or the L-S coupling shell model to target and residual nuclei, we can easily obtain these multiplication factors:

$$\alpha(\text{direct}) = \text{weight of triplet spin wave function} \\ + \text{that of singlet one},$$

$$\alpha(\text{exchange}) = \text{weight of triplet spin wave function} \\ - \text{that of singlet one}.$$

In the case of j-j coupling shell model, the situation may be rather involved, since the z-component of proton spin in the target nucleus remains no longer to be a good quantum number. Nevertheless, we might calculate the multiple factors, obtaining the same result, provided the approximation* previously employed be adopted.

Finally, we should generalize the antisymmetrization to the whole system. For this purpose, it may be sufficient to use completely antisymmetric wave functions of target and residual nuclei. Adopting the shell model, this procedure will be performed simply by means of "coefficient of fractional parentage," or shortly "c.f.p.".[5] For example, in the case of ideal j-j coupling shell model, it may be sufficient to multiply square of "c.f.p.",

$$\langle j_p{}^N(\alpha J)\{|j_p^{N-1}(\beta J')j_p\rangle$$

to T(exchange), and to perform summation over all possible states of j_p and eigen-energy. Here, we assume that the proton $p2$ is in the definite state with the total angular momentum j_p, in which the proton exists in the target nucleus. The angular momentum J' appearing in c.f.p. is just equal to the spin of the core for even-A nuclei of zero spin, since all other shells are filled up so as to form the resultant angular momentum zero. For odd-A nuclei, the spin of the core should be determined by the vector addition of angular momentum J and

* Strictly speaking, the rigorous treatment of antisymmetrization is impossible between nucleons in nuclear bound state and in positive energy state, if the Born or cutoff Born approximation be adopted. It is because the same set of quantum numbers derived from the Schroedinger equation can not be attributed to these nucleons.

the resultant angular momentum of all protons in the most outer-shell for ground states of nuclei.

Similarly, if there are several neutrons in the state which is occupied by the neutron in the incident deuteron, the following "c.f.p." should be multiplied to T(direct),

$$\langle j_n{}^m(\alpha J_n)\{|j_n{}^{m-1}(\beta J_n')j_n\rangle.$$

The applications to L-S coupling or intermediate coupling shell model, and further, more complex cases with configuration interaction will be carried out straightforwardly.

§ 3. Amplitudes of stripping and heavy-particle-stripping reactions. I
——*Elimination of nuclear interaction*——

In this section, the nuclear interaction appearing in the matrix elements of (2·7) and (2·9) will be eliminated. As a result, each matrix element will be represented as the products of reduced width amplitudes and some sorts of overlapping integrals, the calculation of which will be carried out in the next section.

Meanwhile we shall deal with the simple example discussed in the first part of the preceding section. The matrix elements of stripping and heavy-particle-stripping reaction have been given by (2·7a) and (2·9a), in terms of final state interactions:

$$M\,(\text{Stripping})=\int_{ext} d\boldsymbol{r}_n d\boldsymbol{r}_1\int d\boldsymbol{r}_2 d\boldsymbol{C}\cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_1}\psi_R{}^*(n,\ p2,\ C)$$
$$\times V(n\,;\,p1)\psi_T(p2,\ C)\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)\,e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2},\tag{2·7a}$$

and

$$M\,(\text{H. P. Stripping})=-\int_{ext} d\boldsymbol{r}_n d\boldsymbol{r}_1 d\boldsymbol{r}_2\int d\boldsymbol{C}\cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}\psi_R{}^*(n,\ p1,\ C)$$
$$\times V(C\,;\,p2)\psi_T(p2,\ C)\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)\,e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2}.\tag{2·9a}$$

First, it is necessary to present the explicit expressions of wave functions of target and residual nuclei in the external region.*

1. *Notation*

For this purpose we shall use the following notation of various angular momenta, together with each z-component being written in subsequent brackets:

(i) Nuclear spins

target nucleus $I_T(M_T)$, residual nucleus $I_R(M_R)$, core $I_C(M_C)$.

(ii) Angular momenta of nucleons

* "External region" denotes the region of $r\gtrsim R$.

	orbital angular momentum	intrinsic spin	total angular momentum
captured neutron	$l_n(m_n)$	$1/2(\nu_n)$	$j_n(\mu_n)$
proton $p1$	$l_{p1}(m_{p1})$	$1/2(\nu_{p1})$	$j_{p1}(\mu_{p1})$
proton $p2$	$l_{p2}(m_{p2})$	$1/2(\nu_{p2})$	$j_{p2}(\mu_{p2})$
captured deuteron	$l_d(m_d)$	$1(\nu_d)$	$j_d(\mu_d)$

(iii) Channel spins

$$\boldsymbol{J}_n=\boldsymbol{I}_T+1/2, \quad (M_n=M_T+\nu_n)$$
$$\boldsymbol{J}_{p2}=\boldsymbol{I}_C+1/2, \quad (M_{p2}=M_C+\nu_{p2})$$
$$\boldsymbol{J}_d=\boldsymbol{I}_C+1. \quad (M_d=M_C+\nu_d)$$

(iv) Spin wave functions

neutron	$\chi_{1/2\,\nu_n}(n)$
protons	$\chi_{1/2\,\nu_{p1}}(p1)$, $\chi_{1/2\,\nu_{p2}}(p2)$
deuteron	$\chi_{1\nu_d}(d)$; $\nu_d=\nu_n+\nu_{p1}$

Further, the binding energy of nucleon and deuteron will be denoted by

$B_n=(\hbar^2\kappa_n^2/2M)$; the binding energy of the neutron in the residual nucleus.

$B_{p2}=(\hbar^2\kappa_{p2}^2/2M)$; the binding energy of the proton $p2$ in the target nucleus.

$B_{p1}=(\hbar^2\kappa_{p1}^2/2M)$; the binding energy of the proton $p1$ in the residual nucleus.

$B_D=(\hbar^2\kappa_d^2/2M_d)$; the binding energy of the deuteron in the residual nucleus.

Since the state of residual nucleus formed by heavy-particle-stripping reaction should be the same as that formed by "ordinary" stripping reaction, the subscripts 1 and 2 in $p1$ and $p2$ need not be written explicitly, e.g. $B_{p1}=B_{p2}\equiv B_p$, $\kappa_{p1}=\kappa_{p2}\equiv\kappa_p$ etc.

2. *Nuclear wave functions*

By using the above notation, the wave function of residual nucleus appearing in (2·7a) may be decomposed into the product of wave functions of captured neutron and that of target nucleus in the external region:

$$\begin{aligned}\psi_R(n, p2, C)=&\sum_{l_n m_n \nu_n j_n \mu_n}(l_n\,1/2\,m_n\nu_n|j_n\mu_n)(j_n I_T\mu_n M_T|I_R M_R)\\&\times\psi_T(p2, C)\cdot(2M/\hbar^2 R)^{1/2}\cdot\gamma_{l_n}\cdot[h_{l_n}^{(1)}(i\kappa_n r_n)/h_{l_n}^{(1)}(i\kappa_n R)]\\&\times Y_{l_n m_n}(\Omega_{r_n})\cdot\chi_{1/2\,\nu_n}(n),\end{aligned} \qquad (3\cdot1)^{6)}$$

where $h_l^{(1)}(i\kappa r)$ are spherical Hankel functions or, for (d, n) reaction, their Coulomb analogues. γ_{l_n} is reduced width amplitude of the bound neutron, defined by

$$\gamma_{l_n}=(\hbar^2 R/2M)^{1/2}\mathcal{R}_{l_n}(R), \tag{3·2a}$$

where $\mathcal{R}_{l_n}(r)$ is the radial wave function of the neutron normalized through the internal region. It may sometimes be convenient to express γ_{l_n} in terms of dimensionless parameter θ_{l_n}, that is, in the unit of Teichman and Wigner's sum rule limit:

$$\gamma=(3\hbar^2/2MR^2)^{1/2}\theta. \tag{3·2b}$$

It should be noted, however, that the sum rule limit has not a strict meaning especially for the bound state; even for order-of-magnitude estimation, one may preferably use the formula (3·2a) to (3·2b).

Eq. (3·1) is expressed in j-j coupling scheme, because it will be convenient to the direct application of j-j coupling shell model to the nucleus. The transformation of j-j coupling scheme to other representations can be performed simply by means of the so-called transformation brackets, which is, for the transformation from j-j to channel-spin representations, given by

$$\langle 1/2\, I_C(J)lI|1/2\, l(j)I_CI\rangle=[(2J+1)(2j+1)]^{1/2}W(I_CJjl;1/2\, I).$$

Similarly, $\psi_T(p2\cdot C)$ in M(H.P. Stripping) may be represented as a product of the wave functions of bound proton $p2$ and the core in the external region.

$$\begin{aligned}\psi_T(p2,C)=&\sum_{l_{p2}m_{p2}\nu_{p2}j_{p2}\mu_{p2}I_CM_C}(l_{p2}\,1/2\; m_{p2}\nu_{p2}|j_{p2}\mu_{p2})(j_{p2}I_C\mu_{p2}M_C|I_TM_T)\\&\times\psi_C(C)(2M/\hbar^2R)^{1/2}\cdot\gamma_{l_{p2}}[h_{l_{p2}}^{(1)}(i\kappa_{p2}r_2)/h_{l_{p2}}^{(1)}(i\kappa_{p2}R)]\\&\times Y_{l_{p2}m_{p2}}(\Omega_{r2})\chi_{1/2\,\nu_{p2}}(p2).\end{aligned}\tag{3·3}$$

Further, $\psi_R(n, p1, C)$ appearing in the matrix element of heavy-particle-stripping reaction may be represented as the product of three wave functions, i.e. those of the core, the proton $p1$ and the neutron. It can be obtained directly by inserting (3·3) into the expression (3·1), in which all suffix $p2$ in (3·3) should be replaced by the suffix $p1$. Namely,

$$\begin{aligned}\psi_R(n,p1,C)=&\sum_{l_{p1}m_{p1}\tilde{\nu}_{p1}j_{p1}\mu_{p1}\tilde{l}_n\tilde{m}_n\tilde{\nu}_n\tilde{j}_n\tilde{\mu}_n\tilde{I}_C\tilde{M}_C}(l_{p1}\,1/2\; m_{p1}\tilde{\nu}_{p1}|j_{p1}\mu_{p1})(j_{p1}\tilde{I}_C\mu_{p1}\tilde{M}_C|I_TM_T)\\&\times(\tilde{l}_n\,1/2\;\tilde{m}_n\tilde{\nu}_n|\tilde{j}_n\tilde{\mu}_n)(\tilde{j}_nI_T\tilde{\mu}_nM_T|I_RM_R)\cdot\psi_C(C)\\&\times(2M/\hbar^2R)^{1/2}\gamma_{l_{p1}}(2M/\hbar^2R)^{1/2}\cdot\gamma_{\tilde{l}_n}\\&\times[h_{l_{p1}}^{(1)}(i\kappa_{p1}r_1)/h_{l_{p1}}^{(1)}(i\kappa_{p1}R)]\cdot[h_{\tilde{l}_n}^{(1)}(i\kappa_nr_n)/h_{\tilde{l}_n}^{(1)}(i\kappa_nR)]\\&\times Y_{l_{p1}m_{p1}}(\Omega_{r1})Y_{\tilde{l}_n\tilde{m}_n}(\Omega_{r_n})\cdot\chi_{1/2\,\tilde{\nu}_{p1}}(p1)\chi_{1/2\,\tilde{\nu}_n}(n).\end{aligned}\tag{3·4}$$

On the other hand, the wave function of residual nucleus may also be decomposed into the product of core and "interacting"* two-nucleon system:

* Not necessarily interacting. When the residual nucleus can be described by ideal single particle model, this is nothing but coordinate transformation.

$$\psi_R(n,\,p1,\,C)=\sum_{l_d m_d \nu_d' j_d \mu_d \tilde{I}_C \tilde{M}_C}(l_d s_d' m_d \nu_d'|j_d\mu_d)(j_d\tilde{I}_C\mu_d\tilde{M}_C|I_R M_R)$$
$$\times\psi_C(C)\cdot\phi_{np}(\boldsymbol{\omega})\cdot(2M_d/\hbar^2R')^{1/2}\cdot\gamma_{l_d}$$
$$\times[h_{l_d}^{(1)}(i\kappa_d\rho)/h_{l_d}^{(1)}(i\kappa_d R')]\cdot Y_{l_d m_d}(\Omega_\rho)\cdot\chi_{s_d'\nu_d'}(d), \qquad [3\cdot5]^*$$

where $\boldsymbol{\omega}=\boldsymbol{r}_1-\boldsymbol{r}_n$, $\boldsymbol{\rho}=(\boldsymbol{r}_1+\boldsymbol{r}_n)/2$. Here, $s_d'(\nu_d')$ is the spin of the correlating two-nucleon system, which is either 0 or 1. $\phi_{np}(\boldsymbol{\omega})$ is the wave function of correlating two nucleons near the nuclear surface.

Similar expression may be quite suitable to the case of α-induced reactions, though some (trivial) improvements, of course, are necessary for α-induced reactions. But, for deuteron-induced-reactions, it seems convenient in the present time to use the expression $(3\cdot4)$ rather than $[3\cdot5]$ because of the following reason. Firstly, when the shell model is applied to the nucleus, it may be almost impossible to show explicitly that the residual nuclear states formed by both "ordinary" stripping and heavy-particle-stripping reactions are the same. Secondly, we have not yet any knowledge of the reduced width amplitude γ_{ld}. Thirdly, we have not yet an adequate method to obtain correlating two-nucleon function $\phi_{np}(\boldsymbol{\omega})$ near the nuclear surface: The theories of nuclear many-body problems so far developed especially by Brueckner[7] have been concerned mainly with infinite nuclear matter.** Moreover, it seems likely to the authors that the nuclear reactions induced by loosely bound system, such as deuteron, should be treated in principle along the line of Eq. $(3\cdot4)$.*** Namely, intuitively speaking, capture of deuteron by a nucleus may be preferably considered to be a two-step process: First, one of the constituent of deuteron enters the nucleus and then another of constituent enters to form a residual nucleus. This has already been fully discussed and formulated in detail in the previous paper by one of the present authors (H.U.).[9]

3. *Elimination of nuclear interactions*

Next, we shall eliminate nuclear interactions appearing in the matrix elements of stripping and heavy-particle-stripping reactions, $(2\cdot7\text{a})$ and $(2\cdot9\text{a})$.

For stripping reaction, $V(p1\,;\,n)$ appearing in M(Stripping) may be eliminated simply by using the Schroedinger equation obeyed by the internal motion of deuteron:

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* We shall use square brackets [ ] instead of curly ones ( ) to denote the formula derived using Eq. $[3\cdot5]$ hereafter.

** It may be expected from the work of Weisskopf and his coworkers[8] that the correlation of two nucleons will be more important in nuclear surface than in its interior, because the "healing distance" will become longer in nuclear surface due to the weakness of the effect of Pauli principle near nuclear surface. Moreover, it seems likely to the authors that in nuclear surface three- or four-particle correlation rather than two-particle correlation may be important, which could be inferred from the careful analysis of α-induced and t-induced reactions or their reciprocal reactions.

*** Unfortunately, however, we can not analytically calculate the matrix element of heavy-particle-stripping reaction by making use of $(3\cdot4)$, that will be discussed in detail in §4 and §5.

$$V(\boldsymbol{r}_1-\boldsymbol{r}_n)\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)=-(\hbar^2/M)(8\pi\alpha)^{1/2}\delta(\boldsymbol{r}_1-\boldsymbol{r}_n), \tag{3·6}$$

where α is defined by

$$B_d=\hbar^2\alpha^2/M=2.23\text{ Mev}\;;\;\text{binding energy of deuteron.}$$

Eq. (3·6) is valid only for the deuteron wave function of Yukawa type, i.e.

$$\phi_d(r)=\alpha^{3/2}/(2\pi)^{1/2}\cdot(\alpha r)^{-1}\cdot\exp(-\alpha r).$$

If we use more realistic wave function $\phi_d(r)$, e.g. Hulthén type or phenomenological one derived by the shape independent formula, the result (3·6) will not be much altered numerically, say, at most within the difference of 10%.

On the other hand, the interaction $V(p2, C)$ appearing in the matrix element M(H.P. Stripping) may be eliminated similarly by means of the Schroedinger equation of target nucleus,

$$H_T\psi_T(p2, C)=E_T\psi_T(p2, C),$$

where H_T is the total Hamiltonian of target nucleus which may be expressed as

$$H_T=H_C+T_2+V(p2\,;C),$$

where H_C is the Hamiltonian of the core, T_2 kinetic energy operator of the proton $p2$. Then,

$$\begin{aligned} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}V(p2\,;C)\psi_T(p2, C) &= e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}\cdot[E_T-H_C-T_2]\psi_T(p_2, C)\\ &=-[B_{p2}+E_p]\cdot e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}\cdot\psi_T(p2, C) \end{aligned} \tag{3·7}$$

where
$$E_p=\hbar^2k^2/2M,$$

and

$$E_T(\text{total energy of target nucleus})-E_C(\text{total energy of the core})=-B_{p2}.$$

Here we use (3·3) for $\psi_T(p2, C)$ since in M(H. P. Stripping) it may be sufficient to use the wave function $\psi_T(p2, C)$ only in the external region.

4. *Expressions of* M(Stripping) *and* M(H. P. Stripping)

Inserting (3·1) and (3·6) into M(Stripping) and (3·3), (3·4) and (3·7) into M(H.P. Stripping), we can express those matrix elements in terms of reduced width amplitudes of bound nucleons and overlapping integrals:

$$\begin{aligned} M\,(\text{Stripping}) &= \sum_{l_n m_n \nu_n j_n \mu_n} (1/2\;1/2\;\nu_{p1}\nu_n|1\nu_d)\,(l_n\,1/2\;m_n\nu_n|\,j_n\mu_n)\\ &\times(j_nI_T\mu_nM_T|I_RM_R)\,(-)\,(\hbar^2/M)\cdot(8\pi\alpha)^{1/2}\cdot(2M/\hbar^2R)^{1/2}\cdot\gamma_{l_n}\\ &\times\int_{ext}d\boldsymbol{r}_n\cdot[h_{l_n}^{(1)}(i\kappa_n r_n)/h_{l_n}^{(1)}(i\kappa_n R)]\cdot Y_{l_n m_n}^{*}(\Omega_{r_n})\,e^{i(\boldsymbol{K}-\boldsymbol{k})\cdot\boldsymbol{r}_n}, \end{aligned} \tag{3·8}$$

and

$$M\,(\text{H. P. Stripping}) = \sum_{\substack{l_{p1} m_{p1} \tilde{\nu}_{p1} j_{p1} \mu_{p1} \cdot \tilde{l}_n \tilde{m}_n \tilde{\nu}_n \tilde{j}_n \tilde{\mu}_n \\ l_{p2} m_{p2} j_{p2} \mu_{p2} I_C M_C}} (1/2\;1/2\;\tilde{\nu}_{p1} \tilde{\nu}_n | 1 \nu_d)\,(l_{p1}\;1/2\;m_{p1} \tilde{\nu}_{p1} | j_{p1} \mu_{p1})$$
$$\times (j_{p1} I_C \mu_{p1} M_C | I_T M_T)\,(\tilde{l}_n\;1/2\;\tilde{m}_n \tilde{\nu}_n | \tilde{j}_n \tilde{\mu}_n)\,(\tilde{j}_n I_T \tilde{\mu}_n M_T | I_R M_R)$$
$$\times (l_{p2}\;1/2\;m_{p2} \nu_{p1} | j_{p2} \mu_{p2})\,(j_{p2} I_C \mu_{p2} M_C | I_T M_T)\,(B_{p2}+E_p)$$
$$\times (2M/\hbar^2 R)^{1/2} \gamma_{l_{p2}} \int_{ext} d\boldsymbol{r}_2 \cdot [h^{(1)}_{l_{p2}}(i\kappa_{p2} r_2)/h^{(1)}_{l_{p2}}(i\kappa_{p2} R)] \cdot Y_{l_{p2} m_{p2}}(\Omega_{r_2})\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}$$
$$\times (2M/\hbar^2 R) \cdot \gamma_{l_{p1}} \gamma_{\tilde{l}_n} \int_{ext} d\boldsymbol{r}_1 d\boldsymbol{r}_n \phi_d(\boldsymbol{r}_1 - \boldsymbol{r}_n) \cdot [h^{(1)}_{\tilde{l}_n}(i\kappa_n r_n)/h^{(1)}_{\tilde{l}_n}(i\kappa_n R)]$$
$$\times [h^{(1)}_{l_{p1}}(i\kappa_{p1} r_1)/h^{(1)}_{l_{p1}}(i\kappa_{p1} R)] \cdot Y^*_{\tilde{l}_n \tilde{m}_n}(\Omega_{r_n})\, Y^*_{l_{p1} m_{p1}}(\Omega_{r_1})\, e^{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2}. \qquad (3\cdot 9)^*$$

If the wave function of residual nucleus be expressed as the product of the core and the correlating two-nucleon-system as was given in [3·5], the matrix element M(H.P. Stripping) is represented as**

$$M\,(\text{H. P. Stripping}) = \sum_{l_d m_d j_d \mu_d l_{p2} m_{p2} j_{p2} \mu_{p2} I_C M_C} (l_d 1 m_d \nu_d | j_d \mu_d)\,(j_d I_C \mu_d M_C | I_R M_R)$$
$$\times (l_{p2}\;1/2\;m_{p2} \nu_{p1} | j_{p2} \mu_{p2})\,(j_{p2} I_C \mu_{p2} M_C | I_T M_T) \cdot (B_{p2}+E_p)$$
$$\times (2M/\hbar^2 R)^{1/2} \cdot \gamma_{l_{p2}} \int_{ext} d\boldsymbol{r}_2 \cdot [h^{(1)}_{l_{p2}}(i\kappa_{p2} r_2)/h^{(1)}_{l_{p2}}(i\kappa_{p2} R)] \cdot Y_{l_{p2} m_{p2}}(\Omega_{r_2})\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_2}$$
$$\times (2M_d/\hbar^2 R')^{1/2} \gamma_{l_d} \int d\boldsymbol{\omega} \cdot \phi^*_{np}(\boldsymbol{\omega})\, \phi_d(\boldsymbol{\omega})$$
$$\times \int_{ext} d\boldsymbol{\rho} \cdot [h^{(1)}_{l_d}(i\kappa_d \rho)/h^{(1)}_{l_d}(i\kappa_d R')] \cdot Y^*_{l_d m_d}(\Omega_\rho)\, e^{i\boldsymbol{K}\cdot\boldsymbol{\rho}}. \qquad [3\cdot 10]$$

§ 4. Amplitudes of stripping and heavy-particle-stripping reactions. II
——*Evaluation of overlapping integrals*——

The overlapping integrals appearing in (3·8) and (3·9) (or in [3·10]) will be calculated in this section.

1. *Preparations*

For this purpose, it seems convenient to present the following formula of integral:

$$\int_{ext} d\boldsymbol{r}\,[h_l^{(1)}(i\kappa r)/h_l^{(1)}(i\kappa R)]\, Y^*_{lm}(\Omega_r) \exp(i\boldsymbol{k}\cdot\boldsymbol{r})$$
$$= 4\pi i^l Y^*_{lm}(\Omega_k)[R/(k^2+\kappa^2)]\, J_l(k, \kappa, R), \qquad (4\cdot 1)$$

* If we use the matrix element (2·7b) and (2·9b), which have been expressed by means of initial-state-interactions, we can obtain also the same result by a similar procedure.

** This expression can be applied straightforwardly to the case of α-induced reaction, the result of which will be published later in a separate paper together with connected problems. See also the footnote of the previous page.

in which the quantization axis of angular momenta is taken to be an arbitary direction $\boldsymbol{n}$, Ω_k being solid angle of $\boldsymbol{k}$ measured from the axis $\boldsymbol{n}$, or in a slightly simpler case, in which z-direction is taken to be that of $\boldsymbol{k}$, (4·1) becomes

$$\sqrt{4\pi(2l+1)}\,i^l[R/(k^2+\kappa^2)]\,J_l(k,\kappa,R). \tag{4·2}$$

Here

$$J_l(k,\kappa,R)\equiv[kRj_{l-1}(kR)+c_l(\kappa R)j_l(kR)], \tag{4·3a}$$

and

$$c_l(\kappa R)=-i\kappa R[h^{(1)}_{l-1}(i\kappa R)/h^{(1)}_l(i\kappa R)], \tag{4·3b}$$

with the following properties:

(i) $c_l(\alpha)$ is real and positive for a real variable α.

(ii) Asymptotic form,

$$\lim_{\alpha\to\infty}c_l(\alpha)=\alpha,\quad \text{for all values of } l.$$

(iii) Recurrence relation,

$$c_{l+1}(\alpha)=\alpha^2/[c_l(\alpha)+(2l+1)], \tag{4·3c}$$

with $$c_0(\alpha)=\alpha.$$

These relations can be derived directly from the property of spherical Hankel functions with imaginary arguments.

2. *Calculations*

By the aid of the above formula, we can easily carry out the following integrations* appearing in Eqs. (3·8) and (3·9):

$$\int_{ext} d\boldsymbol{r}_n[h^{(1)}_{l_n}(i\kappa_n r_n)/h^{(1)}_{l_n}(i\kappa_n R)]\cdot Y_{l_n m_n}^*(\Omega_{r_n})\exp\{i(\boldsymbol{K}-\boldsymbol{k})\cdot\boldsymbol{r}_n\}$$
$$=4\pi i^{l_n}Y_{l_n m_n}^*(\Omega_{\boldsymbol{K}-\boldsymbol{k}})[R/\{|\boldsymbol{K}-\boldsymbol{k}|^2+\kappa_n^2\}]\,J_{l_n}(|\boldsymbol{K}-\boldsymbol{k}|,\kappa_n,R), \tag{4·4}$$

and

$$\int_{ext} d\boldsymbol{r}_2[h^{(1)}_{l_{p2}}(i\kappa_{p2}r_2)/h^{(1)}_{l_{p2}}(i\kappa_{p2}R)]\cdot Y_{l_{p2}m_{p2}}(\Omega_{r_2})\exp(-i\boldsymbol{k}\cdot\boldsymbol{r}_2)$$
$$=4\pi i^{-l_{p2}}Y_{l_{p2}m_{p2}}(\Omega_k)[R/(k^2+\kappa_{p2}^2)]\cdot J_{l_{p2}}(k,\kappa_{p2},R). \tag{4·5}$$

Further, in the expression [3·10] of M (H. P. Stripping),

$$\int_{ext} d\boldsymbol{\rho}\cdot[h^{(1)}_{l_d}(i\kappa_d\rho)/h^{(1)}_{l_d}(i\kappa_d R')]\cdot Y_{l_d m_d}^*(\Omega_\rho)\exp(i\boldsymbol{K}\cdot\boldsymbol{\rho})$$
$$=\sqrt{4\pi(2l_d+1)}\,i^{l_d}\cdot[R'/(K^2+\kappa_d^2)]\cdot J_{l_d}(K,\kappa_d,R'). \qquad [4·6]$$

---

* Hereafter, the quantization axis of the angular momenta will be taken to be the direction of $\boldsymbol{K}$. ($\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}$ in the case including the recoil effect.) Of course, it is more convenient to use the axis $\boldsymbol{K}$-$\boldsymbol{k}$, ($\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}$) if we are concerned only with the stripping reaction.

On the other hand, the integration over $\boldsymbol{r}_1$ and $\boldsymbol{r}_n$ in the expression (3·9) of M(H.P. Stripping) seems to be rather complicated, since we can not carry out the integration over $\boldsymbol{r}_1$ and $\boldsymbol{r}_n$ separately due to the appearance of $\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)$. In order to perform the integration separately, it might be necessary to expand $\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)$ in terms of Legendre polynomials $P_L(\cos\theta)$, where θ stands for the angle between $\boldsymbol{r}_1$ and $\boldsymbol{r}_n$. Unfortunately, however, the convergence of this series is rather poor in this case, because the ranges of r_1, r_n and θ which are effective in the integral are $r_1\simeq R$, $r_n\simeq R$ and $\theta=0$, as is easily seen from the integrand of (3·9). Therefore, we shall adopt the steepest decent method to handle the integral. Namely, $\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)$ will be taken out of the integral, in which the value of $\phi_d(r)$ will be fixed to $\phi_d(0)$, or more strictly speaking, an average value of $\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)$ near $r_1=R$, $r_n=R$ and $\theta=0$. Then, the integration over $\boldsymbol{r}_1$ and $\boldsymbol{r}_n$ appearing in (3·9) can be performed separately by using the formula (4·2), that is,

$$\begin{aligned}&\int_{ext} d\boldsymbol{r}_1 d\boldsymbol{r}_n\cdot\phi_d(\boldsymbol{r}_1-\boldsymbol{r}_n)\cdot[h_{l_n}^{(1)}(i\kappa_n r_n)/h_{l_n}^{(1)}(i\kappa_n R)]\cdot[h_{l_{p1}}^{(1)}(i\kappa_{p1} r_1)/h_{l_{p1}}^{(1)}(i\kappa_{p1} R)]\\&\quad\times Y_{l_n m_n}^{\ *}(\Omega_{r_n})\cdot Y_{l_{p1} m_{p1}}^{\ *}(\Omega_{r_1})\exp\{i\boldsymbol{K}\cdot(\boldsymbol{r}_1+\boldsymbol{r}_n)/2\}\\&=\phi_d(0)\cdot\sqrt{4\pi(2l_n+1)}\cdot\sqrt{4\pi(2l_{p1}+1)}\cdot i^{l_n+l_{p1}}\\&\quad\times[R/\{(K/2)^2+\kappa_n^2\}]\cdot J_{l_n}(K/2,\,\kappa_n,\,R)[R/\{(K/2)^2+\kappa_{p1}^2\}]\cdot J_{l_{p1}}(K/2,\,\kappa_{p1},\,R).\end{aligned}\tag{4·7}$$

Using the above results, we can analytically calculate the matrix element of the stripping and the heavy-particle-stripping reactions:

$$\begin{aligned}M\,(\text{Stripping})=&\sum(1/2\;1/2\;\nu_{p1}\nu_n|1\,\nu_d)\,(l_n\,1/2\;m_n\nu_n|j_n\mu_n)\\&\times(j_n I_T\mu_n M_T|I_R M_R)\,(-)\,(\hbar^2/M)\,(8\pi\alpha)^{1/2}\,(2M/\hbar^2R)^{1/2}\gamma_{l_n}\\&\times4\pi i^{l_n}Y_{l_n m_n}^{\ *}(\Omega_{K-k})\cdot[R/\{|\boldsymbol{K}-\boldsymbol{k}|^2+\kappa_n^2\}]\cdot J_{l_n}(|\boldsymbol{K}-\boldsymbol{k}|,\,\kappa_n,\,R).\end{aligned}\tag{4·8}$$

$$\begin{aligned}M\,(\text{H. P. Stripping})=&\sum(1/2\;1/2\;\tilde{\nu}_{p1}\tilde{\nu}_n|1\,\nu_d)\,(l_{p1}\,1/2\;m_{p1}\tilde{\nu}_{p1}|j_{p1}\mu_{p1})\\&\times(j_{p1}I_C\mu_{p1}M_C|I_T M_T)\,(\tilde{l}_n\,1/2\;\tilde{m}_n\tilde{\nu}_n|\tilde{j}_n\tilde{\mu}_n)\,(\tilde{j}_n I_T\tilde{\mu}_n M_T|I_R M_R)\\&\times(l_{p2}\,1/2\;m_{p2}\nu_{p1}|j_{p2}\mu_{p2})\,(j_{p2}I_C\mu_{p2}M_C|I_T M_T)\,(B_{p2}+E_p)\cdot(2M/\hbar^2R)^{1/2}\gamma_{l_{p2}}\\&\times4\pi i^{-l_{p2}}\cdot[R/(k^2+\kappa_{p2}^2)]\cdot J_{l_{p2}}(k,\,\kappa_{p2},\,R)\cdot(2M/\hbar^2R)\gamma_{l_{p1}}\gamma_{\tilde{l}_n}\cdot\phi_d(0)\\&\times\sqrt{4\pi(2l_{p1}+1)}\cdot\sqrt{4\pi(2\tilde{l}_n+1)}\cdot i^{l_{p1}+\tilde{l}_n}\cdot[R/\{(K/2)^2+\kappa_n^2\}]\cdot J_{\tilde{l}_n}(K/2,\,\kappa_n,\,R)\\&\times[R/\{(K/2)^2+\kappa_{p1}^2\}]\cdot J_{l_{p1}}(K/2,\,\kappa_{p1},\,R)\,Y_{l_{p2}m_{p2}}(\Omega_k)\,\delta_{\tilde{m}_n 0}\,\delta_{m_{p1}0},\end{aligned}\tag{4·9}$$

or in the alternative expression of M(H.P. Stripping), Eq. [3·10], in which γ_{ld} is utilized,

$$M\,(\text{H. P. Stripping}) = \sum (l_d 1 m_d \nu_d | j_d \mu_d)\,(j_d I_C \mu_d M_C | I_R M_R)$$
$$\times (l_{p2}\, 1/2\; m_{p2} \nu_{p1} | j_{p2} \mu_{p2})\,(j_{p2} I_C \mu_{p2} M_C | I_T M_T) \cdot (B_{p2} + E_p)$$
$$\times (2M/\hbar^2 R)^{1/2} \cdot \gamma_{l_{p2}} \cdot 4\pi i^{-l_{p2}} \cdot [R/(k^2+\kappa_{p2}^2)] \cdot J_{l_{p2}}(k,\, \kappa_{p2},\, R)$$
$$\times (2M_d/\hbar^2 R')^{1/2} \gamma_{l_d} \int d\boldsymbol{\omega} \cdot \phi_{np}^*(\boldsymbol{\omega})\, \phi_d(\boldsymbol{\omega}) \cdot \sqrt{4\pi(2l_d+1)}\, i^{l_d}$$
$$\times [R'/(K^2+\kappa_d^2)] \cdot J_{l_d}(K,\, \kappa_d,\, R')\, Y_{l_{p2} m_{p2}}(\Omega_k)\, \delta_{m_d 0}. \qquad [4\cdot 10]$$

§ 5. Cross sections*, **

The cross section of (d, p) or (d, n) reaction can be obtained by inserting M(Stripping) and M(H.P. Stripping) into the following formula,

$$\frac{d\sigma}{d\Omega}(d,\, p) = \frac{MM_d}{(2\pi\hbar^2)^2} \cdot \frac{k}{K} \cdot \frac{1}{3(2I_T+1)} \cdot \sum_{spin} |M\,(\text{Stripping})$$
$$+ M\,(\text{H. P. Stripping})\,|^2, \qquad (5\cdot 1)$$

where the notation $\sum_{spin}$ denotes the summation over all possible states of initial and final spin states. Before performing actual calculations, it may be convenient to divide the cross section into three parts : stripping, heavy-particle-stripping and their interference terms, that is,

$$\frac{d\sigma}{d\Omega}(d,\, p) = \frac{d\sigma}{d\Omega}(\text{Stripping}) + \frac{d\sigma}{d\Omega}(\text{H. P. Stripping})$$
$$+ \frac{d\sigma}{d\Omega}(\text{Interference}), \qquad (5\cdot 2)$$

where

$$\frac{d\sigma}{d\Omega}(\text{Stripping}) = \frac{MM_d}{(2\pi\hbar^2)^2} \cdot \frac{k}{K} \cdot \frac{1}{3(2I_T+1)} \sum_{spin} |M\,(\text{Stripping})\,|^2, \qquad (5\cdot 3\text{a})$$

$$\frac{d\sigma}{d\Omega}(\text{H. P. Stripping}) = \frac{MM_d}{(2\pi\hbar^2)^2} \cdot \frac{k}{K} \cdot \frac{1}{3(2I_T+1)} \sum_{spin} |M\,(\text{H. P. Stripping})\,|^2, \qquad (5\cdot 3\text{b})$$

$$\frac{d\sigma}{d\Omega}(\text{Interference}) = \frac{MM_d}{(2\pi\hbar^2)^2} \cdot \frac{k}{K} \cdot \frac{1}{3(2I_T+1)}$$
$$\times \sum_{spin} 2\{\text{Real part of } M\,(\text{Stripping})\; M^*\,(\text{H. P. Stripping})\}. \qquad (5\cdot 3\text{c})$$

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* The formula presented in this section are more general than those of "simple example", though the geometrical factors in §2–3 are not explicitly written. On the other hand, N-B contains no expressions of cross section, but those of matrix elements.

** The recoil effect discussed in detail in §2–2 will be explicitly included in the formula of the present section, whereas it was not written in the formula of N-B.

The summation over initial and final states combined with that over various z-components of angular momenta appearing in the marix elements can be performed by means of Racah's technique, obtaining the following results: Inserting (4·8) into (5·3a),

$$\frac{d\sigma}{d\Omega}(\text{Stripping}) = 24\cdot(\alpha R)\cdot R^2\cdot(2I_R+1)/(2I_T+1)\cdot k/K$$
$$\times\sum_{l_n} 1/\{(|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|^2+\kappa_n^2)R^2\}^2\cdot J_{l_n}^2(|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|,\ \kappa_n,\ R)\theta_{l_n}^2, \tag{5·4a}$$

and (4·9) into (5·3b),

$$\frac{d\sigma}{d\Omega}(\text{H. P. Stripping}) = \frac{9}{2}\{(1+x)(2+x)\}\cdot(\alpha R)^3\cdot R^2\cdot\frac{1}{2I_T+1}\cdot\frac{k}{K}$$
$$\times\sum i^{l_{p1}-l_{p1'}+\tilde{l}_n-\tilde{l}_n'-l_{p2}+l_{p2'}}\cdot(2\tilde{l}_n+1)^{1/2}(2\tilde{l}_n'+1)^{1/2}(2l_{p1}+1)^{1/2}(2l_{p1}'+1)^{1/2}$$
$$\times(2l_{p2}+1)^{1/2}(2l_{p2}'+1)^{1/2}(2j_{p2}+1)^{1/2}(2j_{p2}'+1)^{1/2}J_{l_{p2}}(k,\ \kappa_{p2},\ R)$$
$$\times J_{l_{p2'}}(k,\ \kappa_{p2}',\ R)\theta_{l_{p2}}\theta_{l_{p2'}}$$
$$\times 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2\}^2R^2+\kappa_n^2R^2]\cdot 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2\}^2R^2+\kappa_n'^2R^2]$$
$$\times J_{\tilde{l}_n}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_n,\ R)\cdot J_{\tilde{l}_n'}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_{n'},\ R)\theta_{\tilde{l}_n}\theta_{\tilde{l}_n'}$$
$$\times 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2\}^2R^2+\kappa_{p1}^2R^2]\cdot 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2\}^2R^2+\kappa_{p1}'^2R^2]$$
$$\times J_{l_{p1}}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_{p1},\ R)\cdot J_{l_{p1'}}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_{p1'},\ R)\theta_{l_{p1}}\theta_{l_{p1'}}$$
$$\times\sum_L(l_{p2}l_{p2}'00|L0)\,W(l_{p2}j_{p2}l_{p2'}j_{p2'};\ 1/2\ L)P_L(\cos\Omega_k)$$
$$\times\sum(-1)^{1/2+\tilde{\nu}_{p1}}(j_{p2}j_{p2'}-\tilde{\nu}_{p1}\tilde{\nu}_{p1}|L0)(1/2\ 1/2\ \tilde{\nu}_{p1}\tilde{\nu}_n|1\ \tilde{\nu}_{p1}+\tilde{\nu}_n)^2$$
$$\times(l_{p1}\ 1/2\ 0\ \tilde{\nu}_{p1}|j_{p1}\tilde{\nu}_{p1})(l_{p1}'\ 1/2\ 0\ \tilde{\nu}_{p1}|j_{p1}'\tilde{\nu}_{p1})(j_{p1}I_C\tilde{\nu}_{p1}M_C|I_TM_T)$$
$$\times(j_{p1}'I_C'\tilde{\nu}_{p1}M_C|I_TM_T)(\tilde{l}_n\ 1/2\ 0\ \tilde{\nu}_n|\tilde{j}_n\tilde{\nu}_n)(\tilde{l}_n'\ 1/2\ 0\ \tilde{\nu}_n|\tilde{j}_n'\tilde{\nu}_n)(\tilde{j}_nI_T\tilde{\nu}_nM_T|I_RM_R)$$
$$\times(\tilde{j}_n'I_T\tilde{\nu}_nM_T|I_RM_R)(j_{p2}I_C\tilde{\nu}_{p1}M_C|I_TM_T)(j_{p2}'I_C'\tilde{\nu}_{p1}M_C|I_TM_T), \tag{5·4b}$$

and (4·8) and (4·9) into (5·3c),

$$\frac{d\sigma}{d\Omega}(\text{Interference}) = (-)24\pi\cdot\{(1+x)(2+x)\}^{1/2}\cdot(\alpha R)^2\cdot R^2\cdot\frac{1}{2I_T+1}\cdot\frac{k}{K}$$
$$\times\sum i^{l_n+l_{p2}-l_{p1}-\tilde{l}_n}\cdot(2l_{p1}+1)^{1/2}\cdot(2\tilde{l}_n+1)^{1/2}J_{l_{p2}}(k,\ \kappa_{p2},\ R)\cdot\theta_{l_{p2}}$$
$$\times 1/\{|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|^2R^2+\kappa_n^2R^2\}\cdot J_{l_n}(|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|,\ \kappa_n,\ R)\theta_{l_n}$$
$$\times 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|R/2\}^2+\kappa_n^2R^2]\cdot J_{\tilde{l}_n}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_n,\ R)\theta_{\tilde{l}_n}$$
$$\times 1/[\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|R/2\}^2+\kappa_{p1}^2R^2]\cdot J_{l_{p1}}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|/2,\ \kappa_{p1},\ R)\theta_{l_{p1}}$$
$$\times\sum(1/2\ 1/2\ \nu_{p1}\nu_n|1\ \nu_d)(l_n\ 1/2\ m_n\nu_n|j_n\tilde{\nu}_n)(j_nI_T\tilde{\nu}_nM_T|I_RM_R)$$
$$\times(1/2\ 1/2\ \tilde{\nu}_{p1}\tilde{\nu}_n|1\ \nu_d)(l_{p1}\ 1/2\ 0\ \tilde{\nu}_{p1}|j_{p1}\tilde{\nu}_{p1})(j_{p1}I_C\tilde{\nu}_{p1}M_C|I_TM_T)$$

$$\times(\tilde{l}_n\, 1/2\; 0\; \tilde{\nu}_n|\tilde{j}_n\tilde{\nu}_n)\,(\tilde{j}_n I_T \tilde{\nu}_n M_T|I_R M_R)\,(l_{p2}\, 1/2\; m_{p2}\nu_{p1}|j_{p2}\tilde{\nu}_{p1})$$

$$\times(j_{p2} I_C \tilde{\nu}_{p1} M_C|I_T M_T)\, Y_{l_n m_n}^{\;*}(\Omega_{K-(M_i/M_f)k})\, Y_{l_{p2} m_{p2}}^{\;*}(\Omega_k), \tag{5·4c}$$

which does not depend on the azimuthal angle φ, since two vectors $\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}$ and $\boldsymbol{k}$ appearing in spherical harmonics and the vector $\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}$, i.e. z-axis, lie in the same plane.

Here we have adopted the Hulthén-type wave function of deuteron:

$$\phi_d(r)=\beta^{-1}\{\alpha(\alpha+\beta)(2\alpha+\beta)/(2\pi)\}^{1/2}\cdot\{e^{-\alpha r}-e^{-(\alpha+\beta)r}\}/r,$$

therefore,
$$\phi_d(0)=\alpha^{3/2}/(2\pi)^{1/2}\cdot\{(1+x)(2+x)\}^{1/2}$$

with
$$\beta=x\alpha.$$

On the other hand, using the alternative expression of M(H.P. Stripping), [4·10], we obtain the following formula:

$$\frac{d\sigma}{d\Omega}(\text{H. P. Stripping})=6\cdot(R'/R)\cdot R'^2\cdot(2I_R+1)\cdot\frac{k}{K}$$

$$\times\Big\{\int\phi_{np}^{*}(\boldsymbol{\omega})\phi_d(\boldsymbol{\omega})\,d\boldsymbol{\omega}\Big\}^2\cdot\sum i^{l_d-l_d'-l_{p2}+l_{p2}'}(-)^{I_R+1/2-I_T-1}$$

$$\times(2l_d+1)^{1/2}(2l_d'+1)^{1/2}(2j_d+1)^{1/2}(2j_d'+1)^{1/2}(2l_{p2}+1)^{1/2}(2l_{p2}'+1)^{1/2}$$

$$\times(2j_{p2}+1)^{1/2}(2j_{p2}'+1)^{1/2}J_{l_{p2}}(k,\kappa_{p2},R)\,J_{l_{p2}'}(k,\kappa_{p2}',R)\,\Theta_{l_{p2}}\Theta_{l_{p2}'}$$

$$\times 1/\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|^2R'^2+\kappa_d^2R'^2\}\cdot 1/\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|^2R'^2+\kappa_d'^2R'^2\}$$

$$\times J_{l_d}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|,\kappa_d,R')\cdot J_{l_d'}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|,\kappa_d',R')\,\Theta_{l_d}\Theta_{l_d'}$$

$$\times\sum_L(l_{p2}l_{p2}'00|L0)(l_dl_d'00|L0)\,W(l_dj_dl_d'j_d';1L)\,W(j_dI_Cj_d'I_C';I_RL)$$

$$\times W(l_{p2}j_{p2}l_{p2}'j_{p2}';1/2\,L)\,W(j_{p2}I_Cj_{p2}'I_C';I_TL)\,P_L(\cos\Omega_k), \tag{[5·5b]}$$

and

$$\frac{d\sigma}{d\Omega}(\text{Interference})=(-)32\cdot\sqrt{6}\,\pi\cdot(\alpha R)^{1/2}\cdot(R'/R)^{1/2}\cdot RR'$$

$$\times\frac{1}{2I_T+1}\,\frac{k}{K}\,\Big\{\int\phi_{np}^{*}(\boldsymbol{\omega})\phi_d(\boldsymbol{\omega})\,d\boldsymbol{\omega}\Big\}$$

$$\times\sum i^{l_n+l_{p2}-l_d}\cdot(2l_d+1)^{1/2}\cdot J_{l_{p2}}(k,\kappa_{p2},R)\cdot\Theta_{l_{p2}}$$

$$\times 1/\{|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|^2R^2+\kappa_n^2R^2\}\cdot J_{l_n}(|\boldsymbol{K}-(M_i/M_f)\boldsymbol{k}|,\kappa_n,R)\,\Theta_{l_n}$$

$$\times 1/\{|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|^2R'^2+\kappa_d^2R'^2\}\cdot J_{l_d}(|\boldsymbol{K}+(M_d/M_f)\boldsymbol{k}|,\kappa_d,R')\,\Theta_{l_d}$$

$$\times\sum(1/2\,1/2\,\nu_{p1}\nu_n|1\nu_d)\,(l_n\,1/2\,m_n\nu_n|j_n\mu_n)\,(j_nI_T\mu_nM_T|I_RM_R)$$

$$\times(l_d10\nu_d|j_d\nu_d)\,(j_dI_C\nu_dM_C|I_RM_R)\,(l_{p2}\,1/2\,m_{p2}\nu_{p1}|j_{p2}\mu_{p2})$$

$$\times(j_{p2}I_C\mu_{p2}M_C|I_TM_T)\,Y_{l_nm_n}^{\;*}(\Omega_{K-(M_i/M_f)k})\,Y_{l_{p2}m_{p2}}^{\;*}(\Omega_k). \tag{[5·5c]}$$

As the validity of the "steepest decent method" employed in (4·9) is not necessarily clear, we have performed a number of numerical calculations of cross sections (5·4b, c) and [5·5b, c], obtaining the result that the angular distributions derived from both (5·4b, c) and [5·5b, c] do not coincide with each other in the case of a low incident energy of deuteron and a large value of B_n and B_p. Therefore, we should use the formula [5·5b, c] rather than (5·4b, c) to calculate the angular distribution for such a case. The magnitude of the cross sections should, however, be estimated by means of (5·4b, c), because there are some ambiguities in (5·5b, c) such as γ_{l_d} and the overlapping integral over $\boldsymbol{\omega}$ as was discussed in detail in § 3—2.

It seems necessary to treat the integral in (3·9) without the use of "steepest decent method".

§ 6. Discussions

In this section, we shall first briefly discuss qualitative properties of the cross sections of the stripping and the heavy-particle-stripping reactions, which can be inferred directly from the formula obtained in the previous sections. Detailed discussions, especially quantitative ones, will be left to the subsequent paper.

From the angular distribution of (d, p) and (d, n) reactions, it has been conjectured that the heavy-particle-stripping mechanism becomes not to be important in comparison with the ordinary stripping one in higher incident energy of deuteron, say, above 10 Mev, since the angular distributions can be well explained by the theory of stripping reaction at those energies. On the other hand, the heavy-particle-stripping mechanism was proposed by Owen and Madansky[2] to explain the angular distributions at backward directions in low incident energies of deuteron such as $E_d=1\sim4$ Mev. In view of the above situations, therefore, the energy-dependence of heavy-particle-stripping reaction may be supposed to be stronger than that of the "ordinary" stripping reaction. Such circumstances will really be true; it can be inferred directly from the expressions of the matrix elements presented in § 4. Namely, in the matrix element of the stripping reaction (4·8) there appears one overlapping integral between an exponentially decaying wave function of nuclear bound state and an oscillating wave function of positive energy state, whereas the matrix element of the heavy-particle-stripping reaction, (4·9) or [4·10], contains two or three overlapping integrals of the same kind. As the energies of those positive energy state become high, these integrals decrease in general, because the higher the energy becomes, the rapider the wave function oscillates; this fact leads to larger cancelation in the integral. Numerical calculations will be performed in the subsequent paper.

Next, we shall discuss the angular distribution of heavy-particle-stripping reaction. It may be easily shown from Eqs. (5·4b) and [5·5b] that the angular distribution of heavy-particle-stripping will be isotropic in the case of $l_d=0$, or symmetric about 90° direction for $l_d \neq 0$, provided the approximation $(1/A)=0$ be adopted. On the other hand, the actual calculation of Owen and Madansky were confined only to the case of $l_d=0$, obtaining the peak at backward directions. Consequently, we may expect that the improvement of the approximation $(1/A)$, which was discussed in § 2–2, will play an important role in the angular distribution of the heavy-particle-stripping reaction at backward directions.

Finally, it seems necessary to add some remarks concerning the two papers: The antisymmetric treatment of (d, p) reaction has already been made by French[8] and Soga and Nakumura.[10] The former author has employed quite a similar method to that in § 2–2, whereas the latter ones have used the technique of the second quantization. Unfortunately, however, they have not divided the matrix element of (d, p) reaction clearly into the "ordinary" stripping, the heavy-particle-stripping and the extra terms, but into rather intricated parts. In consequence, they have not practically treated the matrix element corresponding to the heavy-particle-stripping reaction.

§ 7. Final comment

The formulation presented in the previous sections has been performed within the restriction of the cutoff Born approximation.

It is well-known[1] that the cutoff Born approximation may not always be reliable in the stripping reaction especially for low incident energies of deuteron. In fact, Tobocman[11] has employed "distorted wave approximation", obtaining better agreement with experiments. Consequently, it seems likely that the distortion of the plane waves plays an important role in the heavy-particle-stripping reaction. Here we shall briefly discuss the use of "distorted wave approximation" to our problem. When the distorted wave approximation is adopted in our formulation, it may be easily shown that the modification will be confined to the overlapping integrals appearing in the matrix elements, all other terms including numerical factors remaining unchanged.

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