Apochromatic optical correlation

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Optical correlation, or matched filtering, can now be applied more widely than before, because the light is now allowed to be totally incoherent, spatially and spectrally. Two such correlators were demonstrated recently. Their state of chromatic correction can be called achromatic, since the scaling error has two zero crossings within the visible range of wavelengths. We present a new apochromatic correlator, in which the scaling error has three zero crossings. The maximum error and the rms error are reduced by a factor of 5. Our apochromatic correlator is composed of two highly dispersive heavy flint lenses that are in contact with two diffractive lenses and two chromatic corrected refractive lenses. The uncommon combination of flint dispersion and diffractive dispersion enabled us to achieve apochromatic correction of the scaling factor of the correlator.

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VanderLugt's invention of the coherent optical matched filter system[1] created considerable excitement 35 years ago, but its impact has been less than originally expected. The reasons for this disappointment were problems with the side effects of laser light, such as coherent noise, delicate adjustment tolerances, and restriction to monochromatic objects. Indeed, the need for totally coherent laser light has ruled out many conceivable applications.

To alleviate the problems associated with coherent light, researchers have proposed and demonstrated two approaches to reducing the coherence requirement. One approach allows the object to be spatially incoherent, but the light has to be monochromatic.[2–5] The other approach requires spatial coherence but allows the light to be polychromatic.[6–11] Recently, it was shown that the two approaches can be merged so that totally incoherent illumination can be exploited.[12,13] Yet, inherent scaling variation that is due to spectral dispersion was only crudely corrected to a first-order approximation, leaving a residual scaling error with only two zero crossings within the visible range of wavelengths, analogous to the achromatic correction of achromats. Here we present a new correlator configuration in which the scaling variation is corrected more accurately, with a residual scaling error of three zero crossings within the visible range, analogous to an apochromatic correction. The scaling error is reduced by a factor of 5, leading to significant improvement in performance.

When totally incoherent light is used, the object intensity is correlated with a reference pattern, which exists indirectly as a spatial filter that can be formed as a black-and-white computer-generated Fourier hologram.[14] The correlation output intensity $I_c(x)$ is

$$I_c(x_c) = \int I_i(x_i) I_r(x_i + x_c) \mathrm{d}x_i \,, \qquad (1)$$

where $I_r(x)$ is the intensity of the reconstruction of the holographic filter, $I_i(x)$ is the input-object intensity, x_c is the coordinate in the correlation output plane, and x_i is the coordinate in the input-object plane. A peak will occur at $x_c = 0$ if I_r and I_i are matched in both shape and scale.

When one is trying to perform optical processing with polychromatic light, three problems arise. One problem is that a point source at the input plane can generate a complex amplitude in the output plane, with a quadratic phase that depends on wavelength, of the form $[I_r(x)]^{1/2}\exp[i\pi x^2/\alpha(\lambda)]$. Fortunately, this phase can be ignored, since in the spatially incoherent case we are correlating intensities, not complex amplitudes. The second problem is the longitudinal dispersion. In other words, $I_r(x;\,\lambda_1)$ and $I_r(x;\,\lambda_2)$ can be focused at different depth locations, $Z(\lambda_1) \neq Z(\lambda_2)$. The third problem is scale mismatch; i.e., the scale of the reference pattern can vary as a function of wavelength $I_r(x;\,\lambda) = I_r[x/S(\lambda)]$, where $S(\lambda)$ is the wavelength-dependent scaling factor. This problem can lead to a scaling mismatch between the reference pattern and the input, degrading the autocorrelation peak at some wavelengths.

To overcome the problems of longitudinal dispersion and scaling mismatch we consider the correlator configuration shown in Fig. 1. It is composed of two highly dispersive lenses of opposite powers with an achromatic lens between them. The filter is adjacent to the achromatic lens. Another achromat, at an appropriate location to the right of the second dispersive lens, converts the virtual white output into a real image on the detector. It was shown that longitudinal

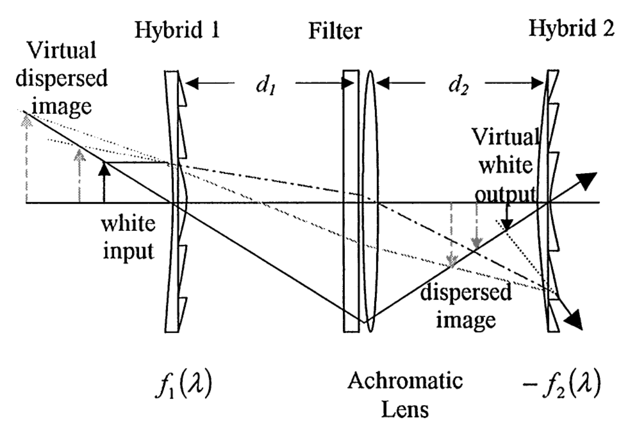

Fig. 1. Optical correlator configuration for totally incoherent white light.

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dispersion does not occur at all in this configuration if two requirements are met.[12,13] The first requirement is that distances d_1 and d_2 between the lenses be related by the imaging formula

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{f_{\text{lens}}}, \tag{2}$$

where f_{lens} is the focal length of the achromatic lens. The second requirement is that the focal lengths of the two dispersive lenses be related by

$$f_2(\lambda) = -f_1(\lambda)(d_2/d_1)^2, \tag{3}$$

where $f_1(\lambda)$ and $f_2(\lambda)$ are the focal lengths of the dispersive lenses. It is most convenient to use $d_1 = d_2 = 2f_{\text{lens}}$ and $f_2(\lambda) = -f_1(\lambda)$.

If one wishes to obtain an ideal correlator, for totally incoherent light without any scaling mismatch, the scaling factor $S(\lambda)$ should be independent of wavelength, i.e., $S(\lambda) = S_{\text{ideal}} = \text{constant}$. The dependence of $S(\lambda)$ on $f_1(\lambda)$ of the dispersive lens is[13]

$$S(\lambda) = \lambda\left[u_i + d_1 - \frac{u_i d_1}{f_1(\lambda)}\right], \tag{4}$$

where u_i is the distance of the input from the first dispersive lens and d_1 is the distance from the first dispersive lens to the achromatic lens. Hence, for an ideal correlator with no scaling mismatch error, the optical power of the dispersive lens $1/f_1(\lambda)$ should be[13]

$$\frac{1}{f_1(\lambda)} = \frac{1}{u_i} + \frac{1}{d_1} - \frac{S_{\text{ideal}}}{u_i d_1 \lambda}. \tag{5}$$

One can see that the focal power should have a wavelength dependence of the form $1/f_1(\lambda) = A + (B/\lambda)$, where A and B are constants. This kind of wavelength dependence cannot be achieved in practice with existing materials. Thus the scaling factor must include a mismatch error $\epsilon(\lambda)$, defined as $S(\lambda) = S_{\text{ideal}}[1 + \epsilon(\lambda)]$.

In the past, the ideal focal power $1/f_1(\lambda)$ of Eq. (5) was replaced with $1/f_D(\lambda) = \lambda/f_0\lambda_0$, which is the linear focal-power dispersion of a diffractive Fresnel zone plate. This replacement leads to a parabolic scaling factor, which is constant only to first order in wavelength, resulting in a parabolic scaling-mismatch error $\epsilon(\lambda)$ with two zero crossings over the entire visible spectral range, similar to the first-order achromatic correction of achromats.

In our new correlator configuration the scaling correction is significantly improved by addition of two refractive plano–convex and plano–concave singlet lenses made from highly dispersive heavy flint glass (SF6; radii of curvature, ±64.55 mm), which were used in contact with the diffractive lenses of the earlier configuration ($f_0 = \pm 160$ mm at $\lambda = 633$ nm) to form hybrid diffractive–refractive lenses. Such hybrid lenses have rather uncommon dispersion properties. Since the two lenses are in contact, the common optical power is the sum of the optical powers of the two lenses,

$$\frac{1}{f_H} = \frac{1}{f_D} + \frac{1}{f_R} = \frac{\lambda}{\lambda_0 f_0} + \frac{1}{R}[n(\lambda) - 1], \tag{6}$$

where f_H, f_D, and f_R are the focal lengths of the hybrid lens, the diffractive zone plate, and the refractive lens, respectively. $n(\lambda)$ is the refractive index of the refractive lens, and R is its radius of curvature. The rather different λ dependence of the two contributions to the hybrid focal power turned out to be useful for the design. Specifically, the diffractive lens has an optical power that is linear in wavelength, whereas the optical power of the refractive lens has a nonlinear dependence on wavelength. The nonlinear part of the refractive dispersion was what we needed to improve the order of approximation. The scale-mismatch error is now predominantly a cubic function of wavelength, so the approximation progressed like the step from an achromat (doublet) to an apochromat (triplet).

The optimal values of the radius of curvature of the refractive lens, R, and the distance of the input from the first lens, u_i, were selected according to the following procedure: First, an analytic expression for $n(\lambda)$ was obtained by use of the interpolation formula given in the Schott catalog for glass materials, for which the values of n at two specific wavelengths and the Abbe number were taken from the database of the "Oslo" Lens Design program. Then, substituting the analytic expression for $n(\lambda)$ into Eq. (6) yielded the optical power of the hybrid lens $1/f_H$ and $S(\lambda)$. Finally, by requiring the scaling factor to be the same at three specific wavelengths, $S(\lambda_1) = S(\lambda_2) = S(\lambda_3)$, and using Eqs. (4) and (6), we obtained u_i and R. The choice of wavelengths λ_1, λ_2, and λ_3 was numerically optimized to minimize the maximal scaling-mismatch error over the visible range of the spectrum, where we assume that $S_{\text{ideal}} = S(\lambda_1)$. Note that the obtained optimal scaling-mismatch error is independent of any specific parameter of the setup. It depends only on the choice of material and the wavelength range of interest. Figure 2 shows the calculated optimal scaling-mismatch error $\epsilon_2(\lambda)$ for the improved, apochromatic hybrid configuration, along with $\epsilon_1(\lambda)$, the scaling error that was obtained previously for the achromatic configuration.[13] As is evident, when the entire visible spectrum (0.4–0.7 μm)

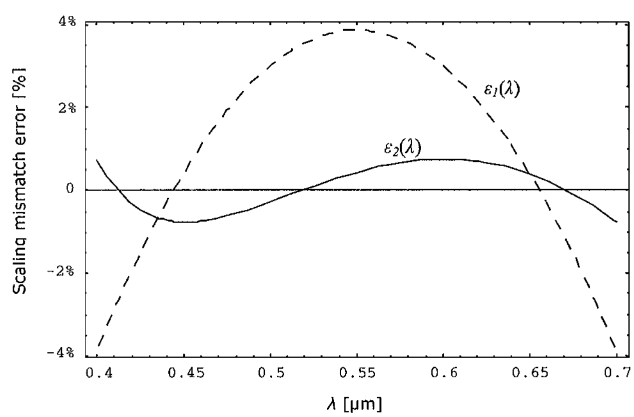

Fig. 2. Scaling-mismatch errors as functions of wavelength for $\epsilon_1(\lambda)$, the achromatic configuration, and $\epsilon_2(\lambda)$, the apochromatic configuration.

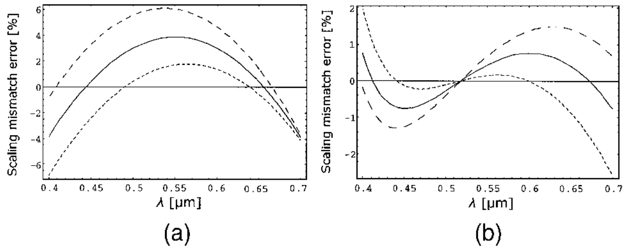

Fig. 3. Scaling-mismatch errors $\epsilon(\lambda)$ at three different longitudinal input positions u_i for (a) the achromatic correlator configuration $\epsilon_1(\lambda)$ and (b) the new hybrid apochromatic configuration $\epsilon_2(\lambda)$. In both (a) and (b) the solid curves denote the results for the optimal distance of u_i and the dashed and dotted curves denote the results for distances that differ by +7% and −7% from the optimal, respectively.

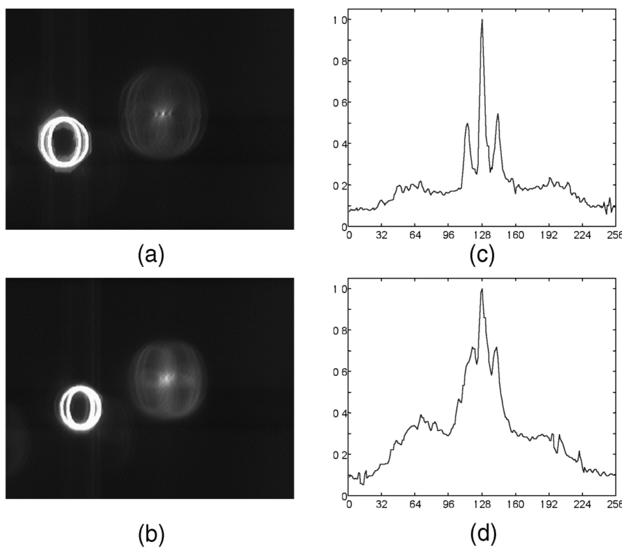

Fig. 4. Experimental autocorrelation results for the letter O. (a), (c) apochromatic configuration; (b), (d) achromatic configuration.

is covered, the maximal errors are $|\epsilon_1(\lambda)| \leq 3.9\%$, with a variance of $\sigma_1 = 2.3\%$, and $|\epsilon_2(\lambda)| \leq 0.76\%$, with a variance of $\sigma_2 = 0.526\%$. Both the variance and the maximal error values indicate a reduction of the scaling-mismatch error by a factor of ~5. The reduction factor is even greater, ~10, for a narrower spectral range of $0.486\ \mu m < \lambda < 0.656\ \mu m$.

$S(\lambda)$ depends on u_i, the distance from the input to the first hybrid lens, as indicated in Eq. (4). Evaluation of this dependence is important for estimating the tolerance of the setup to longitudinal misalignments of the input. We calculated $\epsilon_2(\lambda)$ as a function of wavelength for different distances u_i (the optimal distance and two others that differ from it by ±7%), assuming that all other parameters of the setup were fixed. The results are shown in Fig. 3. Figure 3(a) shows results from the achromatic configuration, whereas Fig. 3(b) shows results from the apochromatic configuration. As expected, when polychromatic light is used, the correlation output is rather insensitive to the longitudinal misalignment of u_i. The reason for this insensitivity is that the misalignment leads to deviations from the optimal scaling factor S_{ideal} only for small parts of the spectral range, whereas the deviation remains small for most of the spectral range; hence, as long as the misalignment is not severe, the correlation results are not degraded.

To verify the predicted improvement for the new apochromatic correlator we performed experiments. Representative results of an autocorrelation of the letter O are shown in Fig. 4. We chose this letter because it provides a critical test. The autocorrelation peak would disappear if the reference letter were magnified by more than the linewidth of the O circle. The letter X, on the other hand, is more tolerant of scaling mismatch and hence is less suitable for use in testing the performance of the correlator. Figures 4(a) and 4(c) show the autocorrelation output and the corresponding cross section along the correlation peak for the new apochromatic correlator. Figures 4(b) and 4(d) show the results obtained with the achromatic correlator for comparison. The improvement is obvious.

In conclusion, we have shown that correlation with totally incoherent white light can be significantly improved by inclusion of hybrid diffractive–refractive lenses in the correlator configuration. Such a configuration with hybrid lenses can also be exploited for other applications in which it is desired that the inherent diffraction be independent of wavelength.

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References

1. A. VanderLugt, IEEE Trans. Inf. Theory **IT-10,** 139 (1964).
2. S. Lowenthal and A. Wertz, C. R. Acad. Sci. Ser. B **266,** 542 (1968).
3. A. W. Lohmann, Appl. Opt. **7,** 561 (1968).
4. A. W. Lohmann and H. W. Werlich, Appl. Opt. **10,** 670 (1971).
5. S. Gorodeisky and A. A. Friesem, Opt. Commun. **100,** 421 (1993).
6. R. H. Katyl, Appl. Opt. **11,** 1255 (1972).
7. S. Leon and E. N. Leith, Appl. Opt. **24,** 3638 (1985).
8. P. Andrés, J. Lancis, and W. D. Furlan, Appl. Opt. **31,** 4682 (1992).
9. E. Tajahuerce, J. Lancis, V. Climent, and P. Andrés, Opt. Commun. **151,** 86 (1998).
10. G. M. Morris, Appl. Opt. **20,** 2017 (1981).
11. D. Faklis and G. M. Morris, Opt. Eng. **28,** 592 (1989).
12. P. Andrés, V. Climent, J. Lancis, G. Mínguez-Vega, E. Tajahuerce, and A. W. Lohmann, Opt. Lett. **24,** 1331 (1999).
13. A. Pe'er, D. Wang, A. W. Lohmann, and A. A. Friesem, Opt. Lett. **24,** 1469 (1999).
14. A. W. Lohmann and D. P. Paris, Appl. Opt. **6,** 1739 (1967).