Progress of Theoretical Physics, Vol. 22, No. 3, September 1959

Applicability Conditions of the Hydrodynamical Model of Multiple Production of Particles from the Point of View of Quantum Field Theory

Chikashi ISO,* Kenju MORI** and Mikio NAMIKI***

**Research Institute for Fundamental Physics, Kyoto University, Kyoto*
***Department of Physics, Tokyo University of Education, Tokyo*
****Department of Applied Physics, Waseda University, Tokyo*

(Received June 10, 1959)

In the framework of quantum field theory, it is attempted to investigate whether the hydrodynamical description is applicable to the meson cloud produced in extremely high energy collision of nucleons as considered in Landau's theory of the multiple production of particles. The applicability conditions of the hydrodynamical model consist of local equilibrium and conditions for the possibilities of defining the local system in the meson cloud, which are prepared by the methods based on quantum statistical mechanics of irreversible processes. These conditions are examined by comparison of the correlation lengths and the relaxation times of the meson fluid with a characteristic length and time, in which the thermodynamical parameters, the temperature for example, of the fluid decrease or increase by an appreciable amount on a macroscopic scale. From such examinations, it may be concluded that the hydrodynamical model holds almost everywhere except in the front part of the cloud after the whole cloud spreads over a region whose size is the order of the correlation length. It is, however, emphasized that the interactions in the initial cloud directly after collision and in the front part of the expanding cloud can never be described by any statistical law or hydrodynamics. The fact that the front particles are never in any thermal equilibrium suggests that they remember some features of initial high energy interactions in the very small cloud. In other words, it is inferred that the distributions (for example, K/π ratio and the momentum or angular distribution) of the front particles may inform us about the interactions at very small distances. On the other hand the influences of initial interactions on the remaining cloud are only taken into account through the initial boundary conditions for the hydrodynamical equation. In addition to the above discussions, it is pointed out that the assumption of the perfect fluid used by Landau is not so good; it turns out that one can expect an increment of the number of particles through the final interactions. Finally it is discussed whether these characteristics may be consistent with the recent experiments.

§ 1. Introduction

The statistical laws in thermal equilibrium or the hydrodynamical equations are often used in theories of the multiple production of particles in nucleon-nucleon collisions. Such theories are to be grounded in the statistical mechanics of irreversible processes as presented in the previous paper[1]. Following its prescriptions, it should be confirmed that interactions in the system relax most disturbances and revive thermal equilibrium in a reasonably short time. Furthermore, in the hydrodynamical model, one must define a small cell which may be regarded as a point in the continuous meson fluid in question. In every such cell, it must be shown that good relaxation phenomena occur, in other words, local equilibrium holds.

A few years ago Blokhintsev[2)] pointed out by discussions using the uncertainty principle that the momentum density could not be defined, in a small cell, compatible with the hydrodynamical description of the meson cloud. We however, think that such a small cell is not an isolated system as treated in Blokhintsev's criticism, but must be considered to have furious interactions with the surrounding cloud of high density and high temperature. Following statistical mechanics, it is natural that one should determine the smallest size of the above-mentioned cells by the longest of the various correlation lengths in the presence of interactions with the surrounding cells and self-interactions. Otherwise, the physical quantities in a cell change with some correlation to surrounding cells, so that an individual cell can not be considered as a local system in the fluid. If the correlation lengths are much smaller than the linear dimensions of the cloud, it becomes possible to define many local systems in the cloud and, consequently, to use the notion of the mass flow or the local velocity formulated in I. Local equilibrium can be expected for systems with sufficiently short relaxation times. Strictly speaking, the correlation length and the relaxation times should be compared with the characteristic length and time, in which the thermodynamical parameters, the temperature for example, of the meson fluid vary by an appreciable amount on a macroscopic scale. Moreover, in order to examine the assumption of the perfect fluid used by Landau, we must show the smallness of the transport coefficients, for example, the heat conductivity, the coefficients of shear and bulk viscosities, of the meson fluid. This can be performed by estimating quantities like the Reynolds number. As will be seen later, the transport coefficients can also be used as a measure of fluctuations of the transported quantities associated with them.

To summarize the above arguments, we must examine the following three applicability conditions of Landau's model: (i) the correlation lengths of the cloud must be much smaller than the linear dimensions of the system and the characteristic length for a macroscopic change of the temperature, (ii) the relaxation time of the clould must be much shorter than the characteristic time for a macroscopic change of the temperature, and (iii) the transport coefficients must be small. Furthermore Landau has assumed $3p=\varepsilon$ for the equation of state of the meson fluid. Then the applicability conditions of Landau's model should be supplemented by examining such an equation of state. It is the purpose of the present paper to perform these examinations.

In Appendix A it is shown that the correlation lengths are as small as $(1/T)$ in the meson fluid with temperature T.* Since T decreases from a high initial value to a low final one, this guarantees in part the validity of defining a small

* The units $\hbar=c=k$ (Boltzmann constant) $=1$ are used through the present note.

cell in the meson fluid after the size of the system has exceeded the correlation lengths. The exceptional case occurs in the earlier stage of expansion where the Lorentz contraction brings about too flat an initial shape of the cloud, whose thickness ($\sim(1/T^2)$) is smaller than the correlation length ($\sim(1/T)$). This can be easily seen without detailed calculations. Most of the remaining discussions in this note will be devoted to estimations of the relaxation times and transport coefficients and to examination of the conditions mentioned above.

In § 2, by replacing the Heisenberg equation of the meson field with a Langevin-like equation, we formulate the semi-phenomenological interaction Hamiltonian representing the furious interactions in the meson cloud. The fluctuation-dissipation theorem is used to characterize the interaction Hamiltonian. In § 3, the temperature dependences of the various transport coefficients, and the relaxation times associated with them, are determined. In § 4 we discuss whether the temperature dependences of the relaxation times and examination of other conditions permit us to use a hydrodynamical description of the meson cloud consistent with the space-time variations in temperature obtained from Landau's model. In § 5, discussions are presented of the information to be obtained from extremely high energy phenomena and the consistency of the results in this paper with some recent experiments. Appendix A is concerned with the estimation of the correlation length and the validity of defining the mass flow or the local velocity. In Appendix B, the method of the Green's function of one meson in a medium is presented. In Appendix C the detailed calculation of the relaxation times and the transport coefficients is explained.

§ 2. Fluctuation-dissipation theorem and interaction Hamiltonian

We now consider an appropriate interaction Hamiltonian to represent furious interactions in a meson cloud of high density and high temperature. In the perturbation theory with an elementary interaction (such as $\lambda\phi^4$, for example), the calculations to the lowest order are only justified for a dilute meson gas. Consequently, it is of convenience for practical calculations to derive a semi-phenomenological Hamiltonian from the exact one, for the purpose of treating furious interactions in a compact form. We shall make use of the fluctuation-dissipation theorem on the analogy to the theory of the Brownian motion.

The interacting meson field* ϕ obeys the operator equation

$$(\Box - m^2)\phi = F \tag{2·1}$$

in the Heisenberg representation, m being the meson mass. Here F consists of the absorption and creation operators of one and more mesons, nucleon pairs and other particles. From analogous discussions of the vacuum field theory, it may be expected that the operator ϕ or F can be divided into two parts, one representing

* For simplicity we deal exclusively with the neutral field.

asymptotically one clothed meson in the medium of the meson cloud and the other corresponding to the remaining part. We may denote the one clothed meson part by $\Pi\cdot\phi$ and the remaining part by f, respectively, where Π is a c-number operator. Naturally, in the asymptotic sense the operator f is only associated with two and more mesons and other particles, and the difference of the numbers of creation operators and annihilation operators is more than 1. The field equation (2·1) is rewritten as

$$(\Box - m^2 - \Pi)\phi = f. \tag{2·2}$$

The c-number operator Π can be connected with the Green function, for one clothed meson in the medium of the meson cloud, defined by

$$\begin{aligned} G(\boldsymbol{x}, t\,;\, \boldsymbol{x}', t') &= \langle \mathrm{T}(\phi(\boldsymbol{x}, t)\phi(\boldsymbol{x}', t'))\rangle \\ &= \mathrm{T}_r\{\rho \mathrm{T}(\phi(\boldsymbol{x}, t)\phi(\boldsymbol{x}', t'))\}, \end{aligned} \tag{2·3}$$

where ρ is the density matrix of the system and T stands for Wick's chronological operator. From (2·2) and (2·3), we easily find that the Green function G satisfies the equation

$$(\Box - m^2 - \Pi)G = 1, \tag{2·4}$$

because of $\mathrm{T}_r\{\rho\mathrm{T}(\phi f)\} = 0$. Here the symbol 1 means the delta function $\delta(\boldsymbol{x}-\boldsymbol{x}')\times\delta(t-t')$. The c-number function G or Π is to be calculated from the exact Hamiltonian including the elementary interactions. The real part of Π is nothing but the effective increment of meson mass, while its imaginary part implies the dissipation of one clothed meson in probability.

Since the effects of f in the exact equation are expected to become random on a rough time scale, neglecting the interval of order (the correlation length/the light velocity) because it contains furious changes in the meson cloud of high density and high temperature, one may replace the definite operator f by a fluctuating external source (or sink) function f which represents the random interaction between the cloud and the large surrounding heat bath or the cloud itself. The equation (2·2) with such a random f is considered to be analogous to the Langevin equation in the theory of Brownian motion. That is to say, such a Langevin-like equation should be regarded as an asymptotic one valid only when one disregards the fine interactions during each time interval whose width is the correlation time $(\xi_0/1)$, ξ_0 being the correlation length and 1 the light velocity. The fluctuating source f has a statistical character given by the fluctuation-dissipation theorem. The equation of form (2·2) with the external source f can be derived from the interaction Hamiltonian

$$\mathcal{H} = \int f(\boldsymbol{x}, t)\phi(\boldsymbol{x})\, d^3\boldsymbol{x} \tag{2·5}$$

in the Schrödinger representation. This takes the place of the Hamiltonian representing the fluctuating interactions between the local system and the surrounding fluid, or the fluctuating self-interactions. Thus the power (or the dissipation of energy per unit time) due to the action of f becomes

$$\int \left\langle f(\boldsymbol{x}, t) \frac{\partial \phi(\boldsymbol{x}, t)}{\partial t} \right\rangle d^3\boldsymbol{x}. \tag{2·6}$$

This fact permits us to regard $\partial\phi/\partial t$ and f as the thermodynamical flow and deriving force, respectively. Hence the ratio of the Fourier transform of f to that of $\partial\phi/\partial t$ can be treated as the " impedance " of the system, so that the fluctuation-dissipation theorem[3) can be described as follows;

$$\langle \bar{f}(\boldsymbol{k}, \omega)\bar{f}^*(\boldsymbol{k}', \omega')\rangle = \coth\frac{|\omega|}{2T}(\mathrm{Im}\,\bar{\Pi}_{k\omega})\,\delta(\boldsymbol{k}-\boldsymbol{k}')\,\delta(\omega-\omega'), \tag{2·7}$$

where $\bar{\Pi}$ is the Fourier transform of Π and $\bar{f}(\boldsymbol{k}, \omega)$ that of $f(\boldsymbol{x}, t)$;

$$\bar{f}(\boldsymbol{k}, \omega) = \frac{1}{\sqrt{(2\pi)^4}}\iint e^{-i\boldsymbol{k}\cdot\boldsymbol{x}+i\omega t} f(\boldsymbol{x}, t)\, d^3\boldsymbol{x}\, dt.$$

This theorem holds as far as f can be regarded as a random function, i. e., the oscillations in the interval $t<\xi_0$ can be disregarded. Hence the theorem (2·7) must be used in the frequency range $|\omega| \lesssim (1/\xi_0)$. In other words, the right-hand side of (2·7) must be multiplied by a cut-off factor for the range $|\omega| \gtrsim (1/\xi_0)$. Since $\xi_0 \simeq (1/T)$ as seen in Appendix A, the frequency range of (2·7) becomes the interval from $(-T)$ to $(+T)$, so that we can always approximate the factor $\coth(|\omega|/2T)$ by $(2T/|\omega|)$. Thus one gets

$$\langle \bar{f}(\boldsymbol{k}, \omega)\bar{f}^*(\boldsymbol{k}', \omega')\rangle \simeq \frac{2\zeta T}{|\omega|}\,\delta(\boldsymbol{k}-\boldsymbol{k}')\,\delta(\omega-\omega'), \tag{2·8}$$

in which we have put $\zeta = \mathrm{Im}\,\bar{\Pi}_{k\omega}$. On the other hand, we are only concerned with free mesons in the range $|\omega| \lesssim m$, so that $\zeta \simeq 0$ for $|\omega| \lesssim m$.

In the present formalism based on (2·7) or (2·8), the parameter ζ is only a phenomenological one unless we calculate it by means of the Green function within the framework of the exact Hamiltonian. Of course, we know the way to obtain the parameter ζ from the exact Hamiltonian. Although it is very difficult to calculate ζ exactly, it may be of some significance to obtain ζ from the exact Hamiltonian by conventional perturbation theory. Thus we get the rough formula (see Appendix B)

$$\begin{aligned} \zeta &\simeq 2\pi^2 g_s^2 T^2 && \text{(ps-coupling)} \\ &\simeq \pi^2 (g_v/m)^2 T^4 && \text{(pv-coupling)} \end{aligned} \tag{2·9}$$

to the lowest order of the coupling constant (g_s or g_v) for the meson-nucleon system. Here we have used the approximation $T \gg m$ and M, M being the nucleon mass. At first sight one may distinguish the types of elementary interaction by the T-dependence of ζ or the autocorrelation function $\langle ff^*\rangle$. Nevertheless, such a difference might be washed out due to the damping effect that may multiply ζ by a factor of the form $[1+A(g_v/m)^2T^2]^{-1}$ which brings the T-dependence of ζ in the case of the pv-coupling to that in the case of the ps-coupling for high temperatures. Moreover, we may get the real part of Π of order T^2, so that the meson wave would propagate with effective mass $\sim T$.

It is true that the theory is relatively simple if one calculates the various quantities by making use of the interaction Hamiltonian (2·5), but it may be more convenient to formulate the theory by introducing an effective Hamiltonian in parallelism to the familiar form $\lambda\phi^4$ of elementary interaction. For this purpose we replace f in (2·5) by $\lambda\langle\phi^2\rangle\phi\chi$, where $\langle\phi^2\rangle\simeq 4\pi T^2(T\gg m)$ is a density-like quantity of the meson fluid and χ represents the fluctuating potential. If we normalize the autocorrelation function of χ as follows;

$$\langle\bar{\chi}(\boldsymbol{k},\omega)\bar{\chi}^*(\boldsymbol{k}',\omega')\rangle=\delta(\boldsymbol{k}-\boldsymbol{k}')\delta(\omega-\omega'), \tag{2·10}$$

then the dimensionless quantity (λT^2) becomes

$$(\lambda T^2)^2\simeq\zeta T^{-2}/32\pi^3, \tag{2·11}*$$

(λT^2) is nothing but the (dimensionless) effective coupling constant of interactions between the meson field and the heat bath (i.e. the meson fluid as a medium). It is noted that (λT^2) is a slowly varying function, as $\ln(T/m)$, of T, but (λT^2) vanishes for $T\lesssim m$. The interaction Hamiltonian has then the form

$$\mathcal{H}=\int\lambda\langle\phi^2\rangle\phi^2(\boldsymbol{x})\chi(\boldsymbol{x},t)\,d^3\boldsymbol{x} \tag{2·12}$$

in the Schrödinger representation.

§ 3. Relaxation phenomena in the meson cloud

We have shown in the first section that one can take a small cell, whose size is $(1/T)$ at least, as a local system of the meson cloud in question. We now talk about the relaxation phenomena in a small cell in its rest coordinate system, in which one can use the non-covariant expression obtained in I for the various quantities. It is of practical convenience to calculate the various transport coefficients and the relaxation times associated with them in a large system (having the volume V) in which all thermodynamical parameters, i.e., temperature, pressure and so on, are everywhere just the same slowly varying functions with the same constant gradients as those in the local system at a given point. In what follows, it is assumed that $T\gg m$.

The heat conductivity κ is defined by Fourier's law

* We have replaced the factor $(1/\omega)$ in the right-hand side of (2·8) by $(1/T)$ without an appreciable change of order, because such a factor appears in the integrals of form $\int^T d\omega\frac{1}{\omega}e^{-\omega/T}\cdots$ in the calculation of various quantities.

$$q=-\kappa\frac{1}{T}\nabla T, \tag{3·1}$$

where q is the heat flow. In the previous paper I we have obtained the formula

$$\kappa=\int_V K(x-x')\,d^3x', \tag{3·2}$$

where

$$K(x-x')=\frac{1}{T}\int_0^\infty \langle\{g_1(x',0),\ g_1(x,t)\}\rangle_0\,dt. \tag{3·3}$$

Here the operator $g_1(x,t)$ is a component of the momentum density operator

$$g_i(x,t)=\phi^{(+)}(x,t)\frac{\overrightarrow{\nabla}_i-\overleftarrow{\nabla}_i}{2i}\ \frac{\overrightarrow{\nabla}_t-\overleftarrow{\nabla}_t}{-2i}\phi^{(-)}(x,t) \tag{3·4}$$

and the symbol $\langle\{A,B\}\rangle_0$ stands for

$$\langle\{A,B\}\rangle_0=T_r\{\rho_0\tfrac{1}{2}(AB+BA)\}-\text{vacuum term},$$

where ρ_0 is the density matrix in thermal equilibrium with temperature T. The operator $(1/\sqrt{2})\phi^{(\pm)}$are the plus and minus frequency parts of ϕ. As is easily shown, the function K is a function of the difference $x-x'$, so that the quantity κ is independent of x. Hence we rewrite (3·2) as

$$\kappa=\frac{1}{VT}\int_0^\infty \langle\{\mathcal{P}_1(0),\ \mathcal{P}_1(t)\}\rangle_0\,dt, \tag{3·5}$$

where $\mathcal{P}$ is a component of the total momentum operator

$$\mathcal{P}_i(t)=\int_V g_i(x,t)\,d^3x$$

of the meson cloud. Since the integrand of (3·5) is the response function for an unit pulse, its damping time is nothing but the relaxation time for disturbing the temperature, i.e., the relaxation time associated with the heat conduction.

Concerning the shear and bulk viscosities, we have the well-known phenomenological equation

$$t_{ik}=p_s\delta_{ik}-\eta_{(s)}[(\nabla_i u_k+\nabla_k u_i)-\tfrac{2}{3}\delta_{ik}(\nabla\cdot u)]-\eta_{(v)}\delta_{ik}(\nabla\cdot u) \tag{3·6}$$

among the stress t_{ik}, the static pressure p_s, and the deformation velocity* $\nabla_i u_k$, where $\eta_{(s)}$ and $\eta_{(v)}$ are the coefficients of shear and bulk viscosities, respectively. The formulas for $\eta_{(s)}$ and $\eta_{(v)}$ are given by

$$\eta_{(s)}=\frac{1}{2VT}\int_0^\infty \langle\{\mathcal{J}_{12}(0),\ \mathcal{J}_{12}(t)\}\rangle_0\,dt, \tag{3·7}$$

* u_k is the local velocity.

$$\eta_{(v)}=\frac{1}{3VT}\int_0^\infty\langle\{\mathcal{T}_{11}(0),\ \mathcal{T}_{11}(t)\}\rangle_0 dt, \tag{3·8}$$

where the operator

$$\mathcal{T}_{ik}(t)=\int_V T_{ik}(\boldsymbol{x},t)\,d^3\boldsymbol{x} \tag{3·9}$$

is a component of the total stress tensor operator. Here T_{ik} is connected with the meson field as follows;

$$T_{ik}(\boldsymbol{x},t)=\phi^{(+)}(\boldsymbol{x},t)\frac{\overrightarrow{\nabla}_i-\overleftarrow{\nabla}_i}{2i}\frac{\overrightarrow{\nabla}_k-\overleftarrow{\nabla}_k}{2i}\phi^{(-)}(\boldsymbol{x},t). \tag{3·10}$$

We shall show details of the calculation of κ exclusively and write only the results for $\eta_{(s)}$ and $\eta_{(v)}$. Following the perturbation theory, the integrand of (3·5) can be developed in the series

$$\begin{aligned}\langle\{\mathcal{P}_1(0),\ \mathcal{P}_1(t)\}\rangle_0=&\sum_{n=1}^\infty(-i)^n\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-1}}dt_n\\&\times\langle\{\mathcal{P}_1(0),[[\cdots[[\mathcal{P}_1(0),\mathcal{H}_{\rm I}(t_1)],\mathcal{H}_{\rm I}(t_2)],\cdots],\mathcal{H}_{\rm I}(t_n)]\}\rangle_0,\end{aligned} \tag{3·11}$$

where $\mathcal{H}_{\rm I}(t)$ is the interaction Hamiltonian in the interaction representation. Taking the randomness of $\mathcal{H}_{\rm I}(t)$ into account, the series (3·11) can be easily summed up and becomes the simple expression[4]

$$\langle\{\mathcal{P}_1(0),\ \mathcal{P}_1(t)\}\rangle_0=\langle[\mathcal{P}_1(0)]^2\rangle_0\exp\{-\int_0^t dt'(t-t')\mathcal{J}(t')\}, \tag{3·12}$$

where

$$\mathcal{J}(t')=\frac{1}{\langle[\mathcal{P}_1(0)]^2\rangle_0}\langle\{[\mathcal{P}_1(0),\mathcal{H}_{\rm I}(t')],[\mathcal{H}_{\rm I}(0),\mathcal{P}_1(0)]\}\rangle_0. \tag{3·13}$$

Strictly speaking, the exponent of (3·12) must be supplemented with the infinite series consisting of higher order terms with respect to the even powers of $\mathcal{H}_{\rm I}$. However, one may expect the effects of these terms to be some modification of the effective coupling constant like, for example, the damping effects.

In terms of the Fourier transform $\overline{\mathcal{J}}(\omega)$ of $\mathcal{J}(t')$, the time-integral in the exponent of (3·12) can be rewritten as

$$\frac{1}{\sqrt{2\pi}}\int d\omega\,\overline{\mathcal{J}}(\omega)\,\Delta(\omega,t), \tag{3·14}$$

in which

$$\Delta(\omega,t)=\frac{1}{(i\omega)^2}[e^{-i\omega t}-1+i\omega t] \tag{3·15}$$

$$\overline{\mathcal{J}}(\omega)=\frac{1}{\langle[\dot{\mathcal{P}}_1(0)]^2\rangle_0}\langle\{[\dot{\mathcal{P}}_1(0),\ \overline{\mathcal{H}}_1(\omega)],[\mathcal{H}_1(0),\ \dot{\mathcal{P}}_1(0)]\}\rangle_0. \quad (3\cdot16)$$

Here $\overline{\mathcal{H}}_1(\omega)$ is the Fourier transform of $\mathcal{H}_1(t')$. Let us define a characteristic time σ of the interaction by

$$\frac{1}{\sigma^2}=\frac{2}{\pi}\mathcal{J}(0)=\frac{-2}{\pi}\cdot\frac{\langle[\dot{\mathcal{P}}_1(0),\ \mathcal{H}_I(0)]^2\rangle_0}{\langle[\dot{\mathcal{P}}_1(0)]^2\rangle_0}. \quad (3\cdot17)$$

Since the time σ characterizes the initial behavior of (3·14), we can proceed to evaluation of the integral (3·14) in the following way. If $\sigma\Delta\omega\gg1$ for the spectral width $\Delta\omega$ of $\overline{\mathcal{J}}(\omega)$, then one gets the asymptotic formula $\Delta(\omega,t)\simeq\pi t\delta(\omega)$ because the integral (3·14) contains a number of oscillations and, consequently,

$$\int d\omega\overline{\mathcal{J}}(\omega)\Delta(\omega,t)\simeq t\left(\sqrt{\frac{\pi}{2}}\overline{\mathcal{J}}(0)\right).$$

In this case the Lorentzian type of relaxation occurs in the relaxation time

$$\tau=\left(\sqrt{\frac{\pi}{2}}\overline{\mathcal{J}}(0)\right)^{-1}. \quad (3\cdot18)$$

Inversely, if $\sigma\Delta\omega\ll1$, then we have

$$\frac{1}{\sqrt{2\pi}}\int d\omega\overline{\mathcal{J}}(\omega)\Delta(\omega,t)\simeq\frac{t^2}{2}\mathcal{J}(0)=\frac{\pi}{4\sigma^2}t^2,$$

because $\Delta(\omega,t)\simeq t^2/2$. Hence this is just the Gaussian type of relaxation, whose relaxation time is nothing but the characteristic time σ of the interaction defined by (3·17).

After some calculations (see Appendix C) one gets the formula for the T-dependence of σ and τ as follows;

$$\left.\begin{aligned}\sigma&\simeq\frac{\pi}{\sqrt{10}(\lambda T^2)}\cdot\frac{1}{T},\\ \tau&\simeq\frac{1}{5(\lambda T^2)^2}\cdot\frac{1}{T},\end{aligned}\right\} \quad (3\cdot19)$$

where we have used the interaction Hamiltonian (2·12) and the fluctuation-dissipation theorem (2·10) and (2·11). The spectral width $\Delta\omega$ is of order $(1/\xi_0)\sim T$ as is expected from the spectral intensity of the autocorrelation function (see discussions given under (2·7)). Thus we have

$$\sigma\Delta\omega\simeq\frac{\pi}{\sqrt{10}}(\lambda T^2)^{-1}. \quad (3\cdot20)$$

This is used as a criterion to judge whether the relaxation phenomena is Lorentzian or Gaussian, according as $\sigma\Delta\omega\gg1$ or $\ll1$. Because of $(\pi/\sqrt{10})\simeq1$, this criterion depends critically on the numerical value of the effective coupling constant (λT^2).

If $(\lambda T^2)\ll 1$, that is, we are concerned with the case of weak coupling; then the relaxation is Lorentzian and the relaxation time is naturally τ. In this case there holds the relation

$$\tau \gg \sigma \gg \hat{\xi}_0 .$$

If $(\lambda T^2)\gg 1$ in the case of strong coupling, we have the Gaussian type of relaxation and the relaxation time $\sigma \ll \hat{\xi}_0$. In this case, however, the effective coupling constant λT^2 would be reduced to a value of order 1 due to the strong damping effect. This is very plausible. In fact, if σ were much smaller than $\hat{\xi}_0$, the different regions with a size of order σ would attain thermal equilibrium independently of each other in local system. This is inconsistent with the notion of the correlation length. Thus we may as well consider σ to be of order $\hat{\xi}_0$ due to the strong damping effect on the effective coupling constant (λT^2).

Although the calculations which lead to the formula (2·9) are very rough, one may estimate order of magnitude by using (2·9) with a value of g_s or g_v consistent with one obtained from the low energy meson physics. Thus it is reasonable to put the T-dependence of the relaxation time τ_0 (τ or σ) in the form

$$\tau_0 \simeq a/T, \quad a \text{ being of order } 1. \tag{3·21}$$

This value of τ_0 means that the mechanism of relaxation depends critically on the numerical values of a (that is, $(\lambda T^2)^{-2}$ or $(\lambda T^2)^{-1}$) and is perhaps intermediate between Lorentzian and Gaussian. At the end of § 2, we have remarked that the effective coupling constant (λT^2) will tend to zero as T approaches to m. Thus, the Lorentzian type of relaxation occurs for $T\simeq m$, as expected in dilute meson gases. Futher it is noted that $(\lambda T^2)^2$ may depend on T as $\ln(T/m)$.

For the relaxation times associated with the shear viscosity, we have the formulas

$$\left.\begin{aligned} \sigma' &\simeq \frac{\pi}{\sqrt{10}(\lambda T^2)}\cdot\frac{1}{T}, \qquad &\text{(Gaussian)},\\ \tau' &\simeq \frac{1}{5(\lambda T^2)^2}\cdot\frac{1}{T}. \qquad &\text{(Lorentzian)}. \end{aligned}\right\} \tag{3·22}$$

The relaxation times associated with the bulk viscosity are given by the formulas

$$\left.\begin{aligned} \sigma'' &\simeq \frac{\pi}{\sqrt{10}(\lambda T^2)}\cdot\frac{1}{T}, \qquad &\text{(Gaussian)},\\ \tau'' &\simeq \frac{1}{5(\lambda T^2)^2}\cdot\frac{1}{T}. \qquad &\text{(Lorentzian)}. \end{aligned}\right\} \tag{3·23}$$

As is easily understood, one may use (3·21) as the common formula for the relaxation times in the above three cases.

Now we can readily evaluate the time-integral in the formula for the heat conductivity κ as follows;

$$\kappa=\frac{1}{VT}\langle[\mathcal{P}_1(0)]^2\rangle_0\tau_0\simeq\frac{4a}{\pi^2}T^3 \tag{3·24}$$

for either type (Gaussian or Lorentzian).* Here we have used the expression

$$\langle[\mathcal{P}_1(0)]^2\rangle_0\simeq\frac{4}{\pi^2}T^5 V \tag{A·17}$$

(see Appendix C). Similarly we get the formulas

$$\left.\begin{aligned}\eta_{(s)}&=\frac{1}{2VT}\langle[\mathcal{J}_{12}(0)]^2\rangle_0\tau_0\simeq\frac{2a}{5\pi^2}T^3,\\ \eta_{(v)}&=\frac{1}{3VT}\langle[\mathcal{J}_{11}(0)]^2\rangle_0\tau_0\simeq\frac{4a}{5\pi^2}T^3\end{aligned}\right\} \tag{3·25}$$

for the coefficients of shear and bulk viscosities, respectively. These expressions for the transport coefficients show that their values can also be used as a measure of magnitude of fluctuations of the related quantities.

§ 4. Discussions on applicability conditions of Landau's model

Now we examine the applicability conditions of Landau's model presented in § 1. They are to be satisfied by the temperature of the meson cloud in question. Here we first formulate these conditions in the form of inequalities among the several quantities, such as the linear dimensions d of the system, the characteristic length x_0 and time t_0 for the macroscopic changes of T, the Reynolds number R, the correlation length ξ_0, the relaxation time τ_0 and so on. We shall use the solutions[5] obtained by Landau and others as the functions representing the dependences of T on space and time. It is noted that such solutions contain a single space variable and a time variable.

The characteristic length x_0 and time t_0 are defined by the relations

$$\frac{1}{x_0}=\frac{1}{T}\sqrt{(\Delta_{\mu\nu}\nabla_\nu T)^2}, \tag{4·1a}$$

$$\frac{1}{t_0}=\frac{1}{T}|DT|, \tag{4·1b}$$

respectively. Here we have used the abbreviations

$$\Delta_{\mu\nu}=\delta_{\mu\nu}+U_\mu U_\nu,\quad D=U_\mu\nabla_\mu,$$

where $U_\mu=(\boldsymbol{u}/\sqrt{1-u^2},\ i/\sqrt{1-u^2})$ and $\nabla_\mu=(\nabla,\ 1-i\cdot\nabla_t)$ are the local four velocity and the four vector of differentiation, respectively, $\boldsymbol{u}$ being the local velocity. Because of the projection character of $\Delta_{\mu\nu}$ onto the space-like direction, $\Delta_{\mu\nu}\nabla_\nu$ means the

---

* Note that $\int_0^\infty e^{-t/\tau_0}dt=\tau_0$ and $\int_0^\infty e^{-\pi t^2/4\tau_0^2}dt=\tau_0$.

differential operator with respect to the space variable x' in the local rest system, while D is the invariant differential operator with respect to the time variable t' in the local rest system. Thus we can rewrite (4·1a) and (4·1b) as follows;

$$\frac{1}{x_0}=\left|\frac{1}{\sqrt{1-u^2}}\frac{1}{T}\left[\frac{\partial T}{\partial x}+u\frac{\partial T}{\partial t}\right]\right|, \tag{4·1a'}$$

$$\frac{1}{t_0}=\left|\frac{1}{\sqrt{1-u^2}}\frac{1}{T}\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right]\right|, \tag{4·1b'}$$

in terms of the coordinate variables x and t in the center-of-mass system.

To examine the assumption of the perfect fluid, we must obtain the Reynolds number R for viscosity and the number K for heat conduction analogous to R. For this purpose let us divide the energy-momentum density tensor $T_{\mu\nu}$ into the perfect fluid part and the part due to irreversible processes as follows;

$$T_{\mu\nu}=T_{\mu\nu}^{(p)}+T_{\mu\nu}^{(i)}, \tag{4·2}$$

where

$$\begin{aligned} T_{\mu\nu}^{(p)}&=p_s\delta_{\mu\nu}+(p_s+\mathcal{E})\,U_\mu U_\nu,\\ T_{\mu\nu}^{(i)}&=U_\mu q_\nu+U_\nu q_\mu-P_{\mu\nu}. \end{aligned} \tag{4·3}$$

Here $\mathcal{E}$, q_μ and $P_{\mu\nu}$ are the invariant energy density, the heat flow and the viscous stress tensor, respectively. The part $T_{\mu\nu}^{(i)}$ is moreover decomposed in the way

$$T_{\mu\nu}^{(i)}=T_{\mu\nu}^{(h)}+T_{\mu\nu}^{(v)}, \tag{4·4}$$

where $T_{\mu\nu}^{(h)}$ and $T_{\mu\nu}^{(v)}$ correspond to the heat conduction and the viscosity, respectively, as follows;

$$T_{\mu\nu}^{(h)}=U_\mu q_\nu+U_\nu q_\mu,\quad T_{\mu\nu}^{(v)}=-P_{\mu\nu}. \tag{4·5}$$

Here we can use the phenomenological equations

$$q_\mu=-\kappa\left[\frac{1}{T}\varDelta_{\mu\nu}\nabla_\nu T+DU_\mu\right] \tag{4·6}$$

$$\begin{aligned} P_{\mu\nu}=&\,\eta_{(v)}\varDelta_{\mu\nu}(\nabla_\alpha U_\alpha)\\ &+\eta_{(s)}[\varDelta_{\mu\alpha}\varDelta_{\nu\beta}(\nabla_\alpha U_\beta+\nabla_\beta U_\alpha)-\tfrac{2}{3}\varDelta_{\mu\nu}(\nabla_\alpha U_\alpha)] \end{aligned} \tag{4·7}$$

for q_μ and $P_{\mu\nu}$. However, it is possible to discard the shear viscosity in $P_{\mu\nu}$ because the quantities $\nabla_1 U_2$ and $\nabla_2 U_1$ vanish in the one-dimensional motion of the fluid in Landau's model. By making use of the above tensors, we can define the invariant R and K by the following ratio:

$$R=\frac{\varDelta_{\mu\nu}T_{\nu\mu}^{(p)}}{|\varDelta_{\mu\nu}T_{\nu\mu}^{(v)}|}=\frac{\mathcal{E}}{3\eta_{(v)}|\nabla_\alpha U_\alpha|}, \tag{4·8}$$

$$K=\frac{\varDelta_{\mu\nu}T_{\nu\mu}^{(p)}}{\sqrt{(\varDelta_{\mu\nu}T_{\nu\beta}^{(h)}U_\mu)^2}}=\frac{\mathcal{E}}{\kappa\sqrt{[(1/T)\varDelta_{\mu\nu}\nabla_\mu T+DU_\mu]^2}}. \tag{4·9}$$

Here it is noted that $\varDelta_{\mu\nu}T_{\nu\mu}^{(p)}=3p_s=\mathcal{E}=U_\mu T_{\mu\nu}^{(p)}U_\nu$ due to the equation of state, $3p_s=\mathcal{E}$.

We interpret briefly the behaviors of the solutions[5] of the hydrodynamical equation for T in Landau's model. The space-time distribution of T consists of the simple wave occupying the front part and the remaining wave occupying the back region, which is called the non-trivial region. In the non-trivial region, the dependence of T on x and t is well described by the approximate solution

$$\left.\begin{aligned}&\ln\left(\frac{T}{T_0}\right)=-\frac{1}{3}\left[\ln\left(\frac{t+x}{\varDelta}\right)+\ln\left(\frac{t-x}{\varDelta}\right)-\sqrt{\ln\left(\frac{t+x}{\varDelta}\right)\ln\left(\frac{t-x}{\varDelta}\right)}\right]\\ &\text{and}\\ &u=\frac{x}{t},\end{aligned}\right\}\qquad(4\cdot9)$$

where $\varDelta$ is the initial thickness of the cloud given by

$$\varDelta=2.5/T_0^2,\qquad(4\cdot10)^*$$

T_0 being the initial temperature of the system assumed by Fermi and Landau. Instead of x and t, it is more convenient to use the variables T and α defined by the equations

$$\left.\begin{aligned}\frac{t+x}{\varDelta}&=\left(\frac{T}{T_0}\right)^{-\frac{3}{1+\alpha-\sqrt{\alpha}}},\\ \frac{t-x}{\varDelta}&=\left(\frac{t+x}{\varDelta}\right)^{\alpha}=\left(\frac{T}{T_0}\right)^{-\frac{3\alpha}{1+\alpha-\sqrt{\alpha}}}.\end{aligned}\right\}\qquad(4\cdot11)$$

The variable α runs over the interval from $\alpha=1$ to $\alpha\sim0$ (but $\neq0$), according as the point restricted to the surface $T=$const. passes through the non-trivial region from its central part to its front part. In the region of the simple wave, we have the exact solutions

$$\left.\begin{aligned}&\frac{T}{T_0}=\left(\frac{t-x}{t+x}\cdot\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^{1/2\sqrt{3}}\\ &\text{and}\\ &u=\frac{t+\sqrt{3}\,x}{\sqrt{3}\,t+x}.\end{aligned}\right\}\qquad(4\cdot12)$$

Here it is convenient to introduce the variables T and β, defined by

$$\left.\begin{aligned}\frac{t+x}{\varDelta(\sqrt{3}-1)}&=\left(\frac{T}{T_0}\right)^{\frac{2\sqrt{3}}{\beta-1}},\\ \frac{t-x}{\varDelta(\sqrt{3}+1)}&=\left(\frac{t+x}{\varDelta(\sqrt{3}-1)}\right)^{\beta}=\left(\frac{T}{T_0}\right)^{\frac{2\sqrt{3}\beta}{\beta-1}},\end{aligned}\right\}\qquad(4\cdot13)$$

* In what follows, we shall use the unit $m=1$.

where β varies from $\beta=7-4\sqrt{3}$ to $\beta=-\infty$ according as the point restricted to the surface T=const. moves from the back boundary of the simple wave region to the wave front.

Now we can formulate the conditions (i), (ii) and (iii) presented in § 1 in the following inequalities:

$$\text{(i)}\quad d\gg\xi_0 \quad\text{and}\quad x_0\gg\xi_0, \tag{4·14a}$$

$$\text{(ii)}\quad t_0\gg\tau_0, \tag{4·14b}$$

$$\text{(iii)}\quad R\gg1 \quad\text{and}\quad K\gg1. \tag{4·14c}$$

These conditions will be examined in the following subsections.

4·1. *Definition of the local system*

In this subsection we discuss the conditions (4·14a) for the possibility of defining a local system. As is discussed in § 1, the first condition of (4·14a) becomes most serious in the initial cloud directly after collision because of its flatness due to Lorentz contraction. Using $\xi_0=(a'/T)$ obtained in Appendix A, (4·14a) becomes

$$T_0\ll 2.5/a'. \tag{4·15}$$

For several values of a' (which is of order 1), (4·15) becomes the following inequalities

$$\left.\begin{array}{lll} T_0\ll 2.5 \text{ or } E_{\text{lab.}}\ll 4\text{ Bev} & \text{if } a'=1, \\ T_0\ll 2 \quad\text{or } E_{\text{lab.}}\ll 100\text{ Bev} & \text{if } a'=0.5, \\ T_0\ll 10 \quad\text{or } E_{\text{lab.}}\ll 1000\text{ Bev} & \text{if } a'=0.25. \end{array}\right\} \tag{4·16}$$

At any rate it is clear that (4·16) is not satisfied at extremely high energies. Thus it is hardly acceptable to regard the initial cloud produced in extremely high energy collisions as a sort of fluid. On the other hand, the assumption that the initial cloud is in thermal equilibrium as a whole is, of course, not self-consistent because of finiteness ($\lesssim$ light velocity) of the transmission velocity of disturbances. The features as a fluid will appear only after its thickness exceeds the correlation length $\xi_0\simeq(a'/T)$. In the initial period before some fluid features appear in the cloud, the interactions in the cloud are governed by another law apart from hydrodynamics. The results of such interactions are to be taken into account as the initial boundary conditions for the hydrodynamical equation to describe the subsequent expansion of the cloud. In other words the initial boundary conditions ought to be accepted as partial reflection of high energy interactions in the initial cloud.

The total energy E of a local system with volume V is

$$E=\frac{6.49}{2\pi^2}T^4V,$$

while the mean square deviation is

$$\Delta E = \frac{2\sqrt{3}}{\pi} T^{5/2} V^{1/2}.$$

Thus we get the fractional fluctuation

$$\frac{\Delta E}{E} = \frac{4\pi\sqrt{3}}{6.49} T^{-3/2} V^{-1/2}.$$

For $V=(A/T)^3$, the condition, $\Delta E/E \ll 1$, of small fluctuation becomes

$$A \gg \left(\frac{4\pi\sqrt{3}}{6.49}\right)^{3/2} \simeq 1.$$

This is automatically satisfied by the local system whose size is much larger than ξ_0. The condition $\Delta E/E \ll 1$ is nothing but that considered by Blokhintsev.

In the course of expansion, (4·15) will be satisfied as the system spreads. There we must examine the additional condition, that is, the second of (4·14a). From (4·1a′), (4·9), (4·11), (4·12) and (4·13), one can easily obtain the characteristic length x_0 for the macroscopic change of T as follows;

$$x_0 = \frac{15\sqrt{\alpha}}{1-\alpha} \frac{1}{T_0^2} \left(\frac{T_0}{T}\right)^{\frac{3(\alpha+1)}{2(1+\alpha-\sqrt{\alpha})}} \quad \text{(non-trivial region)}, \tag{4·17a}$$

$$= \frac{5}{T_0^2} \left(\frac{T_0}{T}\right)^{\frac{\sqrt{3}(\beta+1)}{\beta-1}} \quad \text{(simple wave region)}. \tag{4·17b}$$

Thus we get the condition, $x_0 \gg \xi_0$, in the forms

$$\frac{x_0}{\xi_0} = \frac{15\sqrt{\alpha}}{(1-\alpha)a' T_0} \left(\frac{T_0}{T}\right)^{\frac{1+\alpha+2\sqrt{\alpha}}{2(1+\alpha-\sqrt{\alpha})}} \gg 1 \quad \text{(non-trivial region)}, \tag{4·18a}$$

$$= \frac{5}{a' T_0} \left(\frac{T_0}{T}\right)^{\frac{\beta(\sqrt{3}+1)+(\sqrt{3}-1)}{1-\beta}} \gg 1 \quad \text{(simple wave region)}. \tag{4·18b}$$

We first discuss the condition (4·18a) in the non-trivial region. The upper limit T_1 of temperature allowed by (3·18a) is expressed by

$$\left(\frac{T_1}{T_0}\right) = \left[\frac{15\sqrt{\alpha}}{(1-\alpha)a' T_0}\right]^{\frac{2(1+\alpha-\sqrt{\alpha})}{1+\alpha+2\sqrt{\alpha}}}. \tag{4·19}$$

The power in the right-hand side of (4·19) is always a positive number less than 1, so that T_1 increases with increasing T_0 (increasing incident energy). In fact, (4·19) becomes, for several values of α,

$$\left.\begin{aligned} T_1/T_0 &= \infty && \text{for } \alpha=1, \\ T_1/T_0 &= [51.96/a'T_0]^{0.508} && \text{for } \alpha=\tfrac{3}{4}, \\ T_1/T_0 &= [10/a'T_0]^{0.667} && \text{for } \alpha=\tfrac{1}{4}, \\ T_1/T_0 &= [5.62/a'T_0]^{0.875} && \text{for } \alpha=\tfrac{1}{9}. \end{aligned}\right\} \tag{4·20}$$

From this we can see that T_1 decreases as the observation point moves from the central part ($\alpha=1$) to the front ($\alpha\simeq0$) in the non-trivial region. But, even at the front ($\alpha\simeq0$), one can always find a value of T smaller than T_1. This fact may guarantee the validity, in the non-trivial region, of the hydrodynamical description applied to the expansion of the meson cloud produced in extremely high energy collisions.

The condition $x_0\gg\xi_0$ becomes more severe in the simple wave region. Since the power of (T_0/T) in the right-hand side of (4·18b) is not always positive, we must write (4·18b) as

$$\left.\begin{aligned}&\left(\frac{5}{a'T_0}\right)^{\frac{1-\beta}{\beta(\sqrt{3}+1)+(\sqrt{3}-1)}}\gg\frac{T}{T_0}\quad\text{for } 7-4\sqrt{3}>\beta>-2+\sqrt{3},\\&\left(\frac{5}{a'T_0}\right)\gg1\quad\text{for } \beta=-2+\sqrt{3},\\&\left(\frac{5}{a'T_0}\right)^{\frac{\beta-1}{\beta(\sqrt{3}+1)+(\sqrt{3}-1)}}\ll\frac{T}{T_0}\quad\text{for } -2+\sqrt{3}>\beta,\end{aligned}\right\}\tag{4·21}$$

where all the powers are positive. Although we may find a value of T allowed by the first condition of (4·21) for the range $(7-4\sqrt{3}>\beta>-2+\sqrt{3})$, one never finds T satisfying the last condition of (4·21) for the range $(\beta\leq-2+\sqrt{3})$, for very high values of T_0. Consequently, it is concluded that the hydrodynamical description of the meson cloud breaks down in the neighbourhood of the wave front.

4·2. *Local equilibrium*

Here we examine the condition $t_0\gg\tau_0$ (4·14b) for local equilibrium. The examination is quite similar to discussions given in the preceding subsection for the condition $x_0\gg\xi_0$. The characteristic time t_0 defined by (4·1b′) is given by the formula

$$t_0=\frac{15\sqrt{\alpha}}{(4\sqrt{\alpha}-\alpha-1)T_0^2}\left(\frac{T_0}{T}\right)^{\frac{3(\alpha+1)}{2(1+\alpha-\sqrt{\alpha})}}\quad\text{(non-trivial region)},\tag{4·22a}$$

$$=\frac{5\sqrt{3}}{T_0^2}\left(\frac{T_0}{T}\right)^{\frac{\sqrt{3}(\beta+1)}{1-\beta}}\quad\text{(simple wave region)}.\tag{4·22b}$$

Thus the condition (4·14b) becomes

$$\frac{t_0}{\tau_0}=\frac{15\sqrt{\alpha}}{(4\sqrt{\alpha}-\alpha-1)aT_0}\left(\frac{T_0}{T}\right)^{\frac{1+\alpha+2\sqrt{\alpha}}{2(1+\alpha-\sqrt{\alpha})}}\quad\text{(non-trivial region)},\tag{4·23a}$$

$$=\frac{5\sqrt{3}}{aT_0}\left(\frac{T_0}{T}\right)^{\frac{(\sqrt{3}+1)\beta+(\sqrt{3}-1)}{1-\beta}}\quad\text{(simple wave region)}.\tag{4·23b}$$

As is easily seen, the temperature dependence of (t_0/τ_0) is just the same as that of x_0/ξ_0, that is,

$$\frac{t_0}{x_0}=\left(\frac{\tau_0}{\xi_0}\right)\left(\frac{1-\alpha}{4\sqrt{\alpha}-\alpha-1}\right) \quad \text{(non-trivial region)}, \qquad (4\cdot24\text{a})^*$$

$$=\left(\frac{\tau_0}{\xi_0}\right)\sqrt{3} \quad \text{(simple wave region)}. \qquad (4\cdot24\text{b})^*$$

This means that the condition $t_0 \gg \tau_0$ can hardly be satisfied in the same region in which the hydrodynamical description has already broken down due to the condition $x_0 \gg \xi_0$. Since the physical content of the condition $t_0 \gg \tau_0$ is that particles in a local system are in local equilibrium, the front particles free from the condition $t_0 \gg \tau_0$ are, of course, not in thermal equilibrium. Consequently, it is concluded that the front particles remember the high energy initial interactions in a very small region' and that the distributions of the front particles give us some knowledge about interactions at very small distances.

To illustrate the α-dependence of the condition $t_0 \gg \tau_0$ in the nontrivial region, it is convenient to define the upper limit T_1' of T allowed from (4·23) by

$$\frac{T_1'}{T_0}=\left[\frac{15\sqrt{\alpha}}{(4\sqrt{\alpha}-\alpha-1)aT_0}\right]^{\frac{2(1+\alpha-\sqrt{\alpha})}{1+\alpha+2\sqrt{\alpha}}}. \qquad (4\cdot25)$$

For several values of α, one gets

$$\left.\begin{aligned} T_1'/T_0 &= (7.5/aT_0)^{0.5} && \text{for } \alpha=1,\\ T_1'/T_0 &= (7.6/aT_0)^{0.508} && \text{for } \alpha=\tfrac{3}{4},\\ T_1'/T_0 &= (10/aT_0)^{0.667} && \text{for } \alpha=\tfrac{1}{4},\\ T_1'/T_0 &= (22.5/aT_0)^{0.875} && \text{for } \alpha=\tfrac{1}{9}.\end{aligned}\right\} \qquad (4\cdot26)$$

Although the α dependence of T_1' is inverse to that of T_1 (see (4·20)), the validity of the hydrodynamical description is not altered in the non-trivial region.

The Table I contains the values of (T_1'/T_0) in the non-trivial region for $a=1$, 0.5 and for $T_0=10\sim100$. In the simple wave region, one gets, for reasonable values of a, the upper limits of temperature:

Table. I

		$a=1$			$a=0.5$		
E_{lab}	T_0 \ α	1	1/4	1/9	1	1/4	1/9
10^{12} ev	10	0.87	1	>1	>1	>1	>1
10^{14} ev	25	0.55	0.54	0.91	0.78	0.87	>1
10^{15} ev	50	0.34	0.34	0.50	0.55	0.55	0.91
10^{16} ev	100	0.27	0.22	0.27	0.34	0.34	0.50

* Note that $(\tau_0/\xi_0)=(a/a')\simeq1$.

$$\begin{aligned} \text{if} \quad a=1, \quad & T=8.66 \quad && \text{for } \beta=7-4\sqrt{3}, \\ & T_0=8.66 && \text{for } \beta=-2+\sqrt{3}, \\ \text{if} \quad a=0.5, \quad & T=17.32 && \text{for } \beta=7-4\sqrt{3}, \\ & T_0=17.32 && \text{for } \beta=-2+\sqrt{3}, \end{aligned}$$

4·3. *Assumption of the perfect fluid*

It is the purpose of this subsection to examine the assumption of the perfect fluid used by Landau. To do this, we must estimate the Reynolds number R and the number K defined by (4·8) and (4·9), respectively.

By using the formulas[5)]

$$\varepsilon=\frac{6.49}{2\pi^2}T^4 \tag{4·27}$$

for the invariant energy density ε and (3·25) for $\eta_{(v)}$, one easily obtains the formula

$$R=\frac{3.38}{aT_0}\left(\frac{T_0}{T}\right)^{\frac{1+\alpha+2\sqrt{\alpha}}{2(1+\alpha-\sqrt{\alpha})}} \tag{4·28}*$$

for the Reynolds number in the non-trivial region. For several values of α, (4·28) becomes

$$\left.\begin{aligned} R&=\frac{3.38}{aT_0}\left(\frac{T_0}{T}\right)^2 && \text{for } \alpha=1, \\ &=\frac{3.38}{aT_0}\left(\frac{T_0}{T}\right)^{1.97} && \text{for } \alpha=\tfrac{3}{4}, \\ &=\frac{3.38}{aT_0}\left(\frac{T_0}{T}\right)^{1.5} && \text{for } \alpha=\tfrac{1}{4}, \\ &=\frac{3.38}{aT_0}\left(\frac{T_0}{T}\right)^{1.14} && \text{for } \alpha=\tfrac{1}{9}. \end{aligned}\right\} \tag{4·29}$$

The formula (4·28) is closely related with the ratio (t_0/τ_0) as follows;

$$R=10.14\left(\frac{4\sqrt{\alpha}-\alpha-1}{15\sqrt{\alpha}}\right)\left(\frac{t_0}{\tau_0}\right), \tag{4·30}$$

where the coefficient of (t_0/τ_0) varies from 0.5 to 0.2 as α decreases from 1 to $\frac{1}{9}$. In the simple wave region we have the formula

$$R=0.45\left(\frac{t_0}{\tau_0}\right) \tag{4·31}$$

for the Reynolds number. In every region the condition $R\gg1$ is somewhat more

* It is to be noted that this formula is obtained under the assumption $T\gg m$.

severe than the condition $(t_0/\tau_0)\gg 1$. In fact, we obtain the value $R=1\sim 10$, in the non-trivial region, for the values $T_0\simeq 100$ and $(T_0/T)\simeq 10$. In the simple wave region, R becomes smaller than in the non-trivial region. Thus we may infer that the assumption of the perfect fluid is not so good as supposed by Landau.

In the non-trivial region, it is easily proved that

$$DU_\mu=0.$$

Hence one gets the formula

$$K=\mathcal{E}x_0/\kappa=2.43(\xi_0/\tau_0)(x_0/\xi_0), \tag{4·32}$$

where (4·1a), (3·24) and (4·27) have been used. Apart from the factor (ξ_0/τ_0) of order 1, the examination of the condition $K\gg 1$ is almost that of the condition $(x_0/\xi_0)\gg 1$. Although we may have somewhat larger values of K than R, it is hardly acceptable to neglect the irreversible process due to heat conduction. In the simple wave region, we can show that the heat flow vanishes, that is,

$$q_\mu=-\kappa\left(\frac{1}{T}\Delta_{\mu\nu}\nabla_\nu T+DU_\mu\right)=0.$$

This means that the motion of the fluid is adiabatic. Thus we obtain an infinite K, but it is, of course, impossible to regard the meson cloud as a perfect fluid due to the smallness of R.

4·4. *Production of entropy*

In the preceding subsection, we have obtained rather small values for R and K, so that entropy must be produced by the final interactions described by the hydrodynamical equation. Such a production of entropy results in an increment of the number of particles produced in the final interactions. Here we shall calculate only the production of entropy in the non-trivial region.

Now the thermodynamical equation of the entropy balance is expressed in the form

$$\nabla_\mu S_\mu^{(\mathrm{irr})}=\frac{1}{T}\left[\kappa\left(\frac{1}{T}\Delta_{\mu\nu}\nabla_\nu T+DU_\mu\right)^2+\eta_{(v)}(\nabla_\alpha U_\alpha)^2\right], \tag{4·33}$$

where $S_\mu^{(\mathrm{irr})}$ is the entropy density four-current produced in irreversible processes. Denoting the total produced entropy by Σ_{irr}, we get

$$\begin{aligned}\Sigma_{\mathrm{irr}}&=\iint(\nabla_\mu S_\mu^{(\mathrm{irr})})\,d^3\boldsymbol{x}\,dt\\&=\iint(\nabla_\mu S_\mu^{(\mathrm{irr})})\,dx\,dt.\end{aligned} \tag{4·32}*$$

Dividing Σ_{irr} into the heat part $\Sigma_{\mathrm{irr}}^{(h)}$ and the viscosity part $\Sigma_{\mathrm{irr}}^{(v)}$, and transforming

* Note that $\int d^3\mathbf{x}\cdots=\left(\frac{1}{m}\right)^2\int dx\cdots$ in the one-dimensional motion of the fluid and $m=1$.

the integration variable x and t to another pair of variables T and α, we have the integrals

$$\Sigma_{\mathrm{irr}}^{(h)}=\iint\frac{1}{T}\kappa\left(\frac{1}{T}\Delta_{\mu\nu}\nabla_{\nu}T+DU_{\mu}\right)^{2}\frac{\partial(x,t)}{\partial(T,\alpha)}dT\,d\alpha, \qquad (4\cdot33\mathrm{a})$$

$$\Sigma_{\mathrm{irr}}^{(v)}=\iint\frac{1}{T}\eta_{(v)}(\nabla_{\alpha}U_{\alpha})^{2}\frac{\partial(x,t)}{\partial(T,\alpha)}dT\,d\alpha, \qquad (4\cdot33\mathrm{b})$$

where $\partial(x,t)/\partial(T,\alpha)$ is the Jacobian of the transformation from (x,t) to (T,α) and is given by

$$\frac{\partial(x,t)}{\partial(T,\alpha)}=\frac{9\Delta^{2}}{2T_{0}}(1+\alpha-\sqrt{\alpha})^{-2}\left(\frac{T}{T_{0}}\right)^{-1-\frac{3(1+\alpha)}{1+\alpha-\sqrt{\alpha}}}\ln\left(\frac{T_{0}}{T}\right). \qquad (4\cdot34)$$

The intervals of the integrations are the range from 1 to $\alpha_0(\simeq 0)$ for α and the range from the initial value T_i to the final one T_f for T. We can use $\alpha_0\simeq\beta_0=7-4\sqrt{3}=0.072$ to a good approximation, because the equations defining α and β approach each other in the front region as time goes on. $T_f=1$ will be used as an extrapolation, while the upper limit* T_1' of temperature, allowed by the condition $t_0\gg\tau_0$, will be identified with T_i. (Here we disregard the slight dependence of T_1' on α.)

In the non-trivial region, we have the formulas

$$\left.\begin{aligned}&\left(\frac{1}{T}\Delta_{\mu\nu}\nabla_{\nu}T+DU_{\mu}\right)^{2}=x_0^{-2},\\ &(\nabla_{\alpha}U_{\alpha})=\frac{1}{\Delta}\left(\frac{T}{T_{0}}\right)^{\frac{3(\alpha+1)}{2(1+\alpha-\sqrt{\alpha})}}.\end{aligned}\right\} \qquad (4\cdot35)$$

Substituting (3·24), (3·25), (4·34), (4·35) and (4·17a) into (4·33a) and (4·33b), one obtains

$$\Sigma_{\mathrm{irr}}^{(h)}\simeq\frac{1.14\,aT_0^{2}}{\pi^{2}}\left(\frac{T_i}{T_0}\right)^{2}\left[1+\ln\left(\frac{T_0}{T_i}\right)^{2}\right], \qquad (4\cdot36\mathrm{a})$$

$$\Sigma_{\mathrm{irr}}^{(v)}\simeq\frac{3.96\,aT_0^{2}}{\pi^{2}}\left(\frac{T_i}{T_0}\right)^{2}\left[1+\ln\left(\frac{T_0}{T_i}\right)^{2}\right]. \qquad (4\cdot36\mathrm{b})$$

It is important that, roughly speaking, the entropy produced in irreversible processes is proportional to T_i^2, or T_0^2 for constant (T_i/T_0). Because the total entropy given by Fermi or Landau is proportional to T_0, it is possible that Σ_{irr} exceeds the original entropy of the fluid part of the meson cloud. The entropy production means that the initial energy of the fluid part dissipates into new degrees of freedom, in other words, into new produced particles. Consequently we may expect an increment in the number of particles due to the final interactions.

* The choice $T_i=T_1'$ is not significant unless $T_1'\lesssim T_0$. When $T_1'\gtrsim T_0$, one should use T_0 as T_i.

4·5. *Equation of state*

One of the most important assumptions used by Landau is that the equation of state can be put in the form

$$3p=\mathcal{E}. \tag{4·37}$$

As has been shown in I, the form of the equation holds exactly only when particles in meson cloud interact with each other through the first kind of interactions, i. e., interactions having dimensionless coupling constants. The second kind* of interactions does not necessarily lead to the equation $3p=\mathcal{E}$, but, in general, to the equation

$$3p=\mathcal{E}+\lambda\langle\Xi\rangle, \tag{4·38}$$

where $\lambda\Xi$ is the interaction Hamiltonian apart from a numerical factor. Here, suppose that λ is the coupling constant with the dimension $[L^l]$. Then the additional term $\lambda\langle\Xi\rangle$ becomes

$$\lambda\langle\Xi\rangle=\begin{cases}\mathcal{E}\lambda T^l\times(\text{the power series of }(\lambda T^l))\text{ for }l=\text{even},\\ \mathcal{E}\lambda^2T^{2l}\times(\text{the power series of }(\lambda^2T^{2l}))\text{ for }l=\text{odd}\end{cases} \tag{4·39}$$

at extremely high temperatures, from the view-point of perturbation theory. At first sight we feel as if the violation of the equation $3p=\mathcal{E}$ is serious. However, there is a possibility that the effects of the above power series may appear as damping effects. It is very plausible, though it is difficult to derive a definite conclusion from the exact calculations. If so, (4·39) may be reduced to

$$\lambda\langle\Xi\rangle=\begin{cases}\dfrac{\mathcal{E}A(\lambda T^l)}{1+B(\lambda T^l)} & \text{for } l=\text{even},\\[2ex] \dfrac{\mathcal{E}A'(\lambda^2T^{2l})}{1+B'(\lambda^2T^{2l})} & \text{for } l=\text{odd},\end{cases} \tag{4·40}$$

where A, A', B and B' are numerical factors of the order 1. The interaction part of $\mathcal{E}$ would have the same dependence on T. Thus, since $\lambda\langle\Xi\rangle$ attains the same T-dependence as $\mathcal{E}$, that is,

$$\lambda\langle\Xi\rangle\propto\mathcal{E}$$

at extremely high temperatures, the equation of state (4·38) has the form

$$3p=(1+C)\mathcal{E} \tag{4·41}$$

* In I the authors talked as if there were always essential differences between derivative coupling and non-derivative coupling in the second kind of interactions. This is not necessarily so, because such differences would vanish at extremely high energies. The differences appear in the case of moderate temperatures and moderate densities.

Furthermore it may be noted that the additional term in (4·38) vanishes exactly in the case of pv-coupling of the nucleon-neutral meson system. This is a direct result due to the equivalence theorem.

Here C is a constant factor of the order 1. Consequently, this modification of the equation of state would result in some changes of the discussions about the hydrodynamical motion of the fluid, for example, the change of the sound velocity and the different power law of $\mathcal{E}$ from T^4.

§ 5. Concluding remarks

In the last section, we have investigated the consistency of Landau's model applied to the meson cloud which is produced in high energy nucleon-nucleon collision, with the applicability conditions derived from the statistical mechanics of irreversible processes. The results obtained are summarized in the following way: (i) The interactions in the initial cloud can not be described by any hydrodynamical equation. The interactions are, in part, reflected in the initial boundary condition for the subsequent hydrodynamical expansion. (Such a boundary condition might be different from that assumed by Landau.) (ii) After the cloud spreads over a region whose size is the order of the correlation length, the hydrodynamical description of the cloud is valid almost everywhere except in the front part of the cloud. Here it must be emphasized that the front particles are never in thermal equilibrium and consequently they remember the initial interactions in the very small region. The front particles would be subject to quite different distribution laws from those of the fluid particles, which are given by hydrodynamics or statistical mechanics. (iii) The assumption of the perfect fluid is not so good as expected by Landau, so that the number of particles increases as a result of the irreversible motion in the fluid part of the cloud. (iv) The equation of state, $3p=\mathcal{E}$, holds exactly if the interactions are the first kind (having dimensionless coupling constant), while it is necessary to modify this equation as $3p=\text{const.}\times\mathcal{E}$ for the second kind of interactions.

Although the above analysis is based on the solutions obtained by using Landau's assumption, the above conclusions can be applied to interpret the qualitative behavior of the meson clouds in question. Consequently, we are inclined to imagine the situation for the multiple production of particles in such a way that the produced particles will be clearly divided into two parts, one of which is the very high energy particles occupying the front part and the other of which is the fluid particles. These two parts will also be separated from each other in the experimental data, because there must be clear-cut differences between the two parts in the distributions of the particle number (for example, K-π ratio), of the momentum and of other quantities. Particularly, the existence of irreversible processes results in the slowing-down of the speed and the increment of the number of the fluid particles. In other words, the irreversible processes strengthen the tendency to separate the above two parts from each other.

It seems that such considerations are consistent with the recent experiments[6] of cosmic ray performed by the Japanese group and the Bristol group. In these experimental data one can find that the energy spectrum of the γ-ray number obtained from high energy jets ($\simeq 10^{14}$ e.v.) falls rapidly down and is cut off at the energies 10^{12} e. v.$\sim 10^{13}$ e.v.. The γ-rays in question are produced in the decay of neutral pions, so that the energy spectrum of neutral pions has perhaps the same form as that of the γ-rays. Thus, it may be plausible to regard these neutral pions as the fluid particles in our imagination mentioned above. It seems that similar evidence is found in the observation of muons in cosmic rays. The gradient of the number-energy curve changes critically from a large value to a somewhat small value at a definite energy. The muons with energies below this critical value may be the fluid particles, while the muons with energies higher than the critical value may be considered to be the front particles. Although the experimental evidence is not yet established, our considerations may play a role in suggesting how to analyse the extremely high energy phenomena. The group of particles with higher energies would inform us about the interactions at very short distances.

Acknowledgement

The authors would like to thank Professor Z. Koba for helpful discussions and Dr. S. Frautschi for careful reading of the manuscript.

Appendix A. Correlation length and mass flow

There are correlation lengths associated with the various quantities, such as the momentum density, the stress density and so on. As an example, we shall obtain the correlation length associated with the momentum density. This correlation length is defined by the width of the non-vanishing region of the function

$$\mathcal{G}(\boldsymbol{x}, \boldsymbol{x}') = \langle \{ g_1(\boldsymbol{x}, 0), \quad g_1(\boldsymbol{x}', 0) \} \rangle_0, \tag{A·1}$$

where $g_i(\boldsymbol{x}, 0)$ is a component of the momentum density operator (3·4). Here $\phi(\boldsymbol{x}, 0)$ is expressed by the following Fourier transform;

$$\phi(\boldsymbol{x}, 0) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\boldsymbol{k}}{(2\sqrt{k^2+m^2})^{1/2}} (a_k e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + a_k{}^* e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}), \tag{A·2}$$

where a_k and a_k^* are the well-known annihilation and creation operators. Substituting (A·2) into (A·1) and using

$$\left. \begin{aligned} \langle a_k^* a_{k'} \rangle &= n_k \delta(\boldsymbol{k}-\boldsymbol{k}'), \\ \langle a_k a_{k'}^* \rangle &= (n_k+1) \delta(\boldsymbol{k}-\boldsymbol{k}'), \end{aligned} \right\} \tag{A·3}$$

the function $\mathcal{G}(\boldsymbol{x}, \boldsymbol{x}')$ is written as a sum of products of the following functions or their derivatives:

$$\begin{aligned} A(\boldsymbol{x}-\boldsymbol{x}') &= \frac{1}{(2\pi)^3} \int \frac{d^3\boldsymbol{k}}{\sqrt{k^2+m^2}} (n_k + \tfrac{1}{2})\, e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}, \\ B(\boldsymbol{x}-\boldsymbol{x}') &= \frac{1}{(2\pi)^3} \int d^3\boldsymbol{k} \sqrt{k^2+m^2}\, (n_k + \tfrac{1}{2})\, e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}, \end{aligned} \tag{A·4}$$

where

$$n_k = \frac{1}{e^{\sqrt{k^2+m^2}/T}-1}\ . \qquad (\mathrm{A}\cdot 5)$$

Hence $\mathcal{G}$ consists of products of two integrals involving n_k, and products of an integral containing n_k in its integrand and an n_k-independent integral. The product of two integrals without n_k is cancelled out by the vacuum term. When $T \gg m$, the spatial variation of $\mathcal{G}$ is mainly determined by the integral containing n_k, which vanishes unless $|\boldsymbol{x}-\boldsymbol{x}'| \lesssim (1/T)$. This means that the correlation length is the order of $(1/T)$. Thus we shall write the correlation length ξ_0 in the form

$$\xi_0 = a'/T, \quad a' \text{ being of the order 1.} \qquad (\mathrm{A}\cdot 6)$$

It is easily found that the correlation lengths associated with the other quantities are of the same order ξ_0.

As is mentioned in § 1 and § 2, the hydrodynamical description is to be considered as the asymptotic form in which each region of the order ξ_0 is regarded as a point in the fluid. Thus, we can treat the local system, whose size is of the order ξ_0, as if it moves like a mass point. This fact permits us to define a meaningful mass flow. In fact, the criterion formulated in the footnote on page 599 in I is satisfied in our case, because the right-hand side of this equation is proportional to the delta function $\delta^{(4)}(x-x')$, when x and x' are space-like, as far as each interval of the order ξ_0 is regarded as a point. Hence the mass flow has a definite meaning, so that the local velocity can be constructed from the mass flow as in I.

Appendix B. Green's function of one meson in a medium

Here we shall discuss briefly the Green's function of one meson in a medium. The Green's function $G(x, x')$ of one meson is defined by

$$G(x, x') = \mathrm{T}_r\{\rho \mathrm{T}(\phi(x),\ \phi(x'))\} \qquad (\mathrm{A}\cdot 7)$$

in a medium represented by the density matrix ρ. The function G contains the one meson propagator in vacuum as a contribution of the vacuum term of ρ. If we introduce artificially an external source $J(x)$ of mesons into the Hamiltonian, $\phi(x)$ obeys the field equation

$$(\Box - m^2)\phi(x) = J(x) + ig_s\bar{\psi}(x)\gamma_5\psi(x) \qquad (\mathrm{A}\cdot 8)$$

for the *ps*-coupling system of meson and nucleon. With the help of $J(x)$, the function $G(x, x')$ can also be defined by

$$G(x, x') = \lim_{J\to 0}\frac{\delta\langle\phi(x)\rangle}{\delta J(x')}. \qquad (\mathrm{A}\cdot 9)$$

Thus the function $G(x, x')$ obeys the equation

$$(\Box - m^2)G(x, x') = \delta(x-x') + \lim_{J\to 0} ig_s \frac{\delta}{\delta J(x)} \operatorname{tr}(\gamma_5 K(x, x')),$$

where tr stands for the trace with respect to Dirac's indices and $K(x, x')$ is the one nucleon Green's function in a medium defined by

$$K(x, x') = \mathrm{T}_r\{\rho(\mathrm{T}\psi(x)\bar{\psi}(x))\}. \qquad (A\cdot 10)$$

K contains also the one nucleon propagator in vacuum. If we assume the existence of the inverse function $K^{-1}(x, x')$ of $K(x, x')$, the last term can be rewritten as

$$\lim_{J\to 0} ig_s \frac{\delta}{\delta J(x')} \operatorname{tr}(\gamma_5 K(x, x')) = \int \Pi(x, x'') G(x'', x') d^4x'',$$

where

$$\Pi(x, x'') = -ig_s \operatorname{tr}\left(\int \gamma_5 K(x, \xi) \Gamma_5(\xi, \eta; x'') K(\eta, x) d^4\xi d^4\eta\right). \qquad (A\cdot 11)$$

Here $\Gamma_5(\xi, \eta; x'')$ is the vertex part in a medium and is defined by

$$\Gamma_5(\xi, \eta; x'') = \lim_{J\to 0} \frac{\delta K^{-1}(\xi, \eta)}{\delta\langle\phi(x'')\rangle}. \qquad (A\cdot 12)$$

The c-number operator (A·11) is nothing but Π, which appeared in (2·4).

As has been seen in the above formulation, the calculation of Π can be conducted in a way quite similar to the vacuum field theory. Of course, Π contains the vacuum self-energy of one meson, to be ascribed to renormalization. However, Π has additional terms which correspond to the effective mass in a medium and to the imaginary part representing dissipation. The calculations in perturbation theory lead to the value (2·9) to the lowest order. In the case of pv-coupling, we can formulate the theory as mentioned here.

Appendix C. Relaxation times and transport coefficients

Here we shall interpret in detail the calculations of the relaxation times and the transport coefficients. For example, we calculate the heat conductivity and the relaxation times associated with it.

The heat conductivity κ is obtained from the formula

$$\kappa = \frac{1}{VT} \langle [\mathcal{P}_1(0)]^2 \rangle_0 \tau_0, \qquad (A\cdot 13)$$

where the relaxation time τ_0 (τ or σ) is defined by

$$\frac{1}{\sigma^2} = \frac{2}{\pi} \frac{-\langle [\mathcal{P}_1(0), \mathcal{H}_1(0)]^2 \rangle_0}{\langle [\mathcal{P}_1(0)]^2 \rangle_0} \quad \text{(Gaussian)}, \qquad (A\cdot 14)$$

or

$$\frac{1}{\tau}=\sqrt{\frac{\pi}{2}}\frac{\langle\{[\dot{P}_1(0),\ \overline{\mathcal{H}}_1(0)],[\mathcal{H}_1(0),\dot{P}_1(0)]\}\rangle_0}{\langle[\dot{P}_1(0)]^2\rangle_0}\quad\text{(Lorentzian)}. \tag{A·15}$$

Here $\overline{\mathcal{H}}_1(\omega)$ is the Fourier transform of $\mathcal{H}_1(t)$. From the definition (2·12) and one under (3·5), we get

$$\dot{P}_1(0)=\int\frac{d^3\boldsymbol{k}}{2}[a_k a_k{}^*+a_k{}^* a_k]k_1, \tag{A·16a}$$

$$\begin{aligned}\mathcal{H}_1(0)&=\frac{\lambda\langle\phi^2\rangle_0}{(2\pi)^{3/2}}\iiint\frac{d^3\boldsymbol{k}}{\sqrt{2\mathcal{E}_k}}\frac{d^3\boldsymbol{k}'}{\sqrt{2\mathcal{E}_k'}}d\omega\\&\quad\times[a_k a_{k'}\chi^*(\boldsymbol{k}+\boldsymbol{k}',\ \omega)+a_k{}^*a_{k'}{}^*\chi^*(-\boldsymbol{k}-\boldsymbol{k}',\ \omega)\\&\quad+a_k a_{k'}{}^*\chi^*(\boldsymbol{k}-\boldsymbol{k}',\ \omega)+a_k{}^*a_{k'}\chi^*(\boldsymbol{k}'-\boldsymbol{k},\ \omega)],\end{aligned} \tag{A·16b}$$

$$\begin{aligned}\overline{\mathcal{H}}_1(0)&=\frac{\lambda\langle\phi^2\rangle_0}{(2\pi)^2}\iint\frac{d^3\boldsymbol{k}}{\sqrt{2\mathcal{E}_k}}\frac{d^3\boldsymbol{k}'}{\sqrt{2\mathcal{E}_k'}}\\&\quad\times[a_k a_{k'}\chi(-\boldsymbol{k}-\boldsymbol{k}',\ -\mathcal{E}_k-\mathcal{E}_{k'})+a_k{}^*a_{k'}{}^*\chi(\boldsymbol{k}+\boldsymbol{k}',\ \mathcal{E}_k+\mathcal{E}_{k'})\\&\quad+a_k a_{k'}{}^*\chi(\boldsymbol{k}'-\boldsymbol{k},\ \mathcal{E}_{k'}-\mathcal{E}_k)+a_k{}^*a_{k'}\chi(\boldsymbol{k}-\boldsymbol{k}',\ \mathcal{E}_k-\mathcal{E}_{k'})],\end{aligned} \tag{A·16c}$$

where $\mathcal{E}_k=\sqrt{k^2+m^2}$. In what follows, we use the approximation $T\gg m$.

The mean square deviation $\langle[\dot{P}_1(0)]^2\rangle_0$ is easily obatined in the form

$$\langle[\dot{P}_1(0)]^2\rangle_0\simeq\frac{4}{\pi^2}T^5V, \tag{A·17}$$

where the relation $[\delta(\boldsymbol{k})]_{k=0}=(V/(2\pi)^3)$ has been used. After some calculations, we get

$$-\langle[\dot{P}_1(0),\ \mathcal{H}_1(0)]^2\rangle_0\simeq\frac{20}{\pi^3}(\lambda T^2)^2T^7V. \tag{A·18}$$

Thus we obtain the Gaussian relaxation time

$$\sigma\simeq\frac{\pi}{\sqrt{10}(\lambda T^2)}\cdot\frac{1}{T}. \tag{A·19}$$

Similar calculations lead to

$$\langle\{[\dot{P}_1(0),\ \overline{\mathcal{H}}_1(0)][\mathcal{H}_1(0),\ \dot{P}_1(0)]\}\rangle_0\simeq\sqrt{\frac{2}{\pi}}\frac{20(\lambda T^2)^2}{\pi^2}T^6V, \tag{A·20}$$

so that one gets

$$\tau=\frac{1}{5(\lambda T^2)^2}\cdot\frac{1}{T}. \tag{A·21}$$

The heat conductivity κ is easily obtained from (A·13) and (A·19) or (A·21).

The calculations of $\eta_{(s)}$ and $\eta_{(v)}$ are quite similar to the above. We write only the mean square deviations of the stress, that is,

$$\left.\begin{aligned}\langle[\mathcal{J}_{12}(0)]^2\rangle_0 &\simeq \frac{4}{5\pi^2}T^5 V,\\ \langle[\mathcal{J}_{11}(0)]^2\rangle_0 &\simeq \frac{12}{5\pi^2}T^5 V.\end{aligned}\right\} \tag{A·22}$$

Here it is noted that the quantity $\mathcal{J}_{ik}(0)$ is the fluctuating part of the stress.

References

1) M. Namiki and C. Iso, Prog. Theor. Phys. **18** (1957), 591.
This will be quoted as I in the present paper. The related papers are cited there.
2) D. I. Blokhintsev, Proc. CERN Symposium 1956, Vol. 2, 155; Zurn. Eksp. Teor. Fiz. **32** (1957), 350.
3) S. M. Rytov, Zurn. Eksp. Teor. Fiz. **33** (1957), 166.
M. Namiki, J. Inst. Elect. Com. Japan **41** (1558) 259; ibid. to be published.
4) L. van Hove, Physica **21** (1955), 517; ibid. **23** (1957) 441.
R. Kubo and K. Tomita, J. Phys. Soc. Japan **9** (1954), 888.
R. Kubo, ibid. **12** (1957), 570.
R. Kubo, M. Yokota and S. Nakajima, ibid. **12** (1957), 1203.
M. Toda, ibid. **13** (1958), 1266.
C. Bloch, Nuclear Physics **4** (1957), 503.
S. Hayakawa, T. Sasakawa, K. Tomita, M. Yasuno and M. Yokota, Prog. Theor. Phys. **21** (1959), 85.
5) S. Z. Belen'kij and L. D. Landau, Usp. Fiz. Nauk **56** (1955), 309.
S. Amai, H. Fukuda, C. Iso and M. Satō, Prog. Theor. Phys. **17** (1956), 241.
6) J. Nishimura, Private communication.